Noncollinear interlayer exchange coupling caused by interface spin-orbit interaction

Ke Xia, Weiyi Zhang, and Mu Lu
National Laboratory of Solid State Microstructures and Department of Physics, Nanjing University, Nanjing 210093, China

Hongru Zhai
Center of Material Analysis, Nanjing University, Nanjing 210093, China

(Received 8 October 1996)

The interlayer exchange couplings between neighboring ferromagnetic layers in ferromagnetic-nonmagnetic multilayer structures are derived analytically in the frame of the extended Anderson *s-d* mixing model with spin-orbit interaction. After transforming the extended Anderson mixing model into an *s-d* exchange model and taking into account the spin frustration at interfaces, the second-order perturbation calculation naturally gives rise to both the noncollinear Dzyaloshinski-Moriya (DM) exchange coupling as well as the usual isotropic Ruderman-Kittel-Kasuya-Yosida exchange coupling between the neighboring ferromagnetic layers. The isotropic and anisotropic exchange couplings have a decaying oscillatory behavior as a function of spacer layer thickness, but they differ by a phase factor of $\pi/2$. While the exact value of the DM coupling depends on the material parameters as well as the interface *s-d* mixing effect, a rough estimate suggests that it is significant. Thus our result offers an alternative explanation to the noncollinear exchange coupling as was observed in the experiments. [S0163-1829(97)02718-5]

I. INTRODUCTION

The indirect magnetic coupling between ferromagnetic (FM) layers across a nonmagnetic (NM) metal spacer has been extensively studied both experimentally and theoretically. An oscillatory dependence of the magnetic coupling strength with the thickness of the spacer layer has been observed in a large number of systems.[1] According to the theory of RKKY-like coupling,[2] the problem of magnetic interlayer exchange coupling includes two aspects: first, the interaction between ferromagnetic layer and conduction electrons of nonmagnetic metal layers; second, the way of the spin polarization propagating through the nonmagnetic metal spacer. It was pointed out by several authors[3] that mixing between localized *d* states of the magnetic layer and conducting states of the nonmagnetic layer at the interface is responsible for interlayer coupling. Shi *et al.*[4] studied the magnetic interlayer coupling using the two-impurity Anderson *s-d* mixing model, their approach being rather sophisticated and relying on numerical calculations. Bruno and Chappert[2] studied the oscillatory behavior of the interlayer coupling within the frame of RKKY theory. The calculation was performed almost in an analytical manner and the results are physically transparent. These two approaches all led to a Heisenberg-type interlayer coupling, and thus the magnetic moments of FM layers align collinearly. The spin-orbit interaction was neglected. Recently, noncollinear alignments among neighboring ferromagnetic layers were also observed in a number of ferromagnetic-nonmagnetic multilayer structures.[5–9] In order to explain the experimental results, an additional phenomenological term $J_2(\vec{M}_1\cdot\vec{M}_2)^2$ was proposed which is referred to as biquadratic interlayer coupling.

The origin of the biquadratic coupling has been studied by several groups.[10–16] Some of them attributed the biquadratic coupling to the extrinsic properties of structure such as (1) fluctuation of the spacer thickness,[10] (2) loose spin at interfaces and in the spacer layers,[11] and (3) a magnetic-dipole field resulting from rough interfaces.[12] In these studies, nonideal interfaces were considered and further experimental support of the connection between interface roughness and biquadratic coupling strength is required. There are also a number of calculations carried out for an ideal interface on the angular dependence of the intrinsic exchange coupling for trilayers,[13–16] these calculations predicting a biquadratic term and that the interlayer coupling oscillates as a function of spacer thickness with half of the period of the bilinear term. However, the value of the biquadratic coupling strength is too small to explain the experimental results.[11,12]

Moriya[17] has shown that in low-symmetry magnetic crystals, the spin-orbit interaction can lead to an anisotropic coupling of Dzyaloshinsky-Moriya (DM) type, $\vec{D}\cdot(\vec{S}_1\times\vec{S}_2)$. Here, $\vec{D}$ is a vector proportional to the spin-orbit interaction and depends on the symmetry of crystals. $\vec{S}_1$ and $\vec{S}_2$ are the localized magnetic moments. However, a more careful study by Shekhtman *et al.*[18,19] showed that a overlooked hidden symmetry makes the Moriya expression for the single-bond anisotropic superexchange coupling isomorphic to the symmetry of an isotropic one, and a weak ferromagnetic moment emerges from the superexchange coupling only when more than a single bond is considered and only as a result of frustration.

The effect of the spin-orbit interaction on the exchange coupling between magnetic impurities has been studied by several groups. In the 1980s, Fert and Levy[20] proposed a similar DM-type coupling in addition to the isotropic RKKY coupling after taking into account the spin-orbit scattering of conduction electrons by the localized states of magnetic impurities, such as Co and Pt, in CuMn spin glass alloy. Their study showed that a DM-type coupling exists when the inversion symmetry with respect to the midpoint between the two magnetic impurities is broken. They compared the DM term to the usual isotropic one in CuMn spin glass and found that the DM-type coupling was quite significant. Later,

 © 1997 The American Physical Society

Staunton *et al.*[21] studied a relativistic RKKY coupling between two magnetic impurities in the language of scattering theory; the spin polarization and spin-orbit interaction were treated on equal footing. Apart from the isotropic term, the exchange coupling between two magnetic impurities in a host metal also contains a DM square term and a pseudodipolar interaction term. The nonspherically symmetric nature of scattering potentials is the cause of the anisotropic interaction and dipolar interaction. In magnetic layered structures, the space inversion symmetry is also broken and frustration exists near the interfaces between ferromagnetic and nonmagnetic layers ,[18,19] and the spin-orbit interaction will also play a role in the interlayer exchange coupling.

In this paper, we study the influence of the spin-orbit interaction and the broken space inversion symmetry on the interlayer exchange coupling in magnetic layered structures. In Sec. II, an Anderson s-d mixing model is extended to include the spin-orbit scattering effect and the Schrieffer-Wolff transformation is used to obtain a generalized s-d exchange model. This exchange model contains both isotropic and anisotropic spin couplings between conduction electrons and localized states. Then in Sec. III, under second-order perturbation theory, a general form of the exchange coupling between two ferromagnetic layers via conduction electrons is derived. To demonstrate the significance of the noncollinear DM interlayer exchange coupling, we have derived an analytical result within the free electron approximation. Our results show that the strength of the DM-type interlayer coupling is proportional to the spin-orbit interaction and oscillates with spacer layer thickness; it has the same period as that of the usual RKKY interlayer coupling but with a phase shift of $\pi/2$. Thus, the noncollinear DM coupling becomes maximum when the usual RKKY exchange coupling vanishes. This offers an alternative explanation to the 90° alignment of the neighboring ferromagnetic layers as was observed in the experiments.

II. GENERALIZED *s-d* EXCHANGE MODEL

It has been shown that the hybridization of s-band electrons and d-band electrons at interfaces between magnetic layers and nonmagnetic layers is responsible for the interlayer exchange coupling in the ferromagnetic-nonmagnetic multilayer structures. Usually the single-electron mixing potential is assumed to be spin independent, but it holds only in the absence of a spin-orbit interaction. The mixing potential becomes spin dependent when the spin-orbit interaction is taken into account.[22] In this paper, we will consider the effect of the spin-orbit interaction and study how it affects the interlayer exchange coupling between the ferromagnetic layers.

The Anderson s-d mixing Hamiltonian in the presence of spin-orbit interaction can be expressed as

$$H=H_0+H_1,$$

$$H_0=\sum_{\vec{k},s}\epsilon_{\vec{k}}n_{\vec{k}s}+\sum_{s}\epsilon_d n_{ds}+Un_{d\uparrow}n_{d\downarrow},$$

$$H_1=\sum_{\vec{k},ss'}C^{\dagger}_{\vec{k}s}(V_0I+\vec{V}\cdot\vec{\sigma})^{ss'}_{\vec{k}d}C_{ds'}+\text{c.c.},\tag{1}$$

where $C^{\dagger}_{\vec{k}s}$ and $C^{\dagger}_{ds}$ are the creation operators for the conduction electron with momentum $\vec{k}$ and localized d-electrons, respectively. s is the spin index, and $\epsilon_{\vec{k}}$ and ϵ_d are the corresponding energies. U is the Coulomb repulsion between opposite-spin electrons located on the d orbital. I is a 2×2 unit matrix and σ_i's are the Pauli matrices. The $\vec{k}s$ and ds' states are mixed by the potential $(V_0I+\vec{V}\cdot\vec{\sigma})^{ss'}_{\vec{k}d}$, where V_0 is the usual spin-independent s-d mixing potential and $\vec{V}$ is the spin-dependent mixing potential arising from the spin-orbit interaction of the localized d states.

Schrieffer and Wolff[23] showed that the Anderson s-d mixing model is almost equivalent to the s-d exchange model. We will show below how the spin-dependent mixing potential influences the s-d exchange model. Following Ref. 23, we make a canonical transformation $\overline{H}=e^{S}He^{-S}$ under the condition that the first-order term $[H_0,S]-H_1$ vanish; then

$$\overline{H}=H_0+\tfrac{1}{2}[S,H_1]+\tfrac{1}{3}[S,[S,H_1]]+\tfrac{1}{8}[S,[S,[S,H_1]]]+\cdots.\tag{2}$$

$[A,B]$ denotes the commutator relation. The choice of S is similar to the spin-independent case

$$S=\sum_{\vec{k}\alpha}\sum_{ss'}\left[\frac{C^{\dagger}_{\vec{k}s}(V_0I+\vec{V}\cdot\vec{\sigma})^{ss'}_{\vec{k}d}C_{ds'}n^{\alpha}_{d-s'}}{\epsilon_{\vec{k}}-\epsilon_{\alpha}}-\frac{C^{\dagger}_{ds}n^{\alpha}_{d-s}(V_0I+\vec{V}\cdot\vec{\sigma})^{ss'}_{\vec{k}d}C_{\vec{k}s'}}{\epsilon_{\vec{k}}-\epsilon_{\alpha}}\right],\tag{3}$$

where $\alpha=\pm$, $\epsilon_+=\epsilon_d+U$, and $\epsilon_-=\epsilon_d$. n^{α}_{d-s} are the projection operators with $n^{+}_{d-s}=n_{d-s}$ and $n^{-}_{d-s}=1-n_{d-s}$, respectively.

It is easy to prove that S satisfies the condition $[H_0,S]=H_1$ and $\overline{H}$ can be approximately expressed as H_0+H_2 with H_2 given by

$$H_2=\tfrac{1}{2}[S,H_1].\tag{4}$$

After neglecting the terms which are irrelevant to s-d exchange coupling, one obtains

$$H_2=-\sum_{\vec{k}\vec{k}'}C_{\vec{k}'\vec{k}}\left[B_{\vec{k}'\vec{k}}\psi^{\dagger}_{\vec{k}'}\frac{\vec{\sigma}}{2}\psi_{\vec{k}}\cdot\vec{S}_d+i\vec{T}_{\vec{k}'\vec{k}}\cdot\left(\psi^{\dagger}_{\vec{k}'}\frac{\vec{\sigma}}{2}\psi_{\vec{k}}\times\vec{S}_d\right)+\left(\psi^{\dagger}_{\vec{k}'}\frac{\vec{\sigma}}{2}\psi^{\dagger}_{\vec{k}'}\overleftrightarrow{A}\vec{S}_d\right)\right],\tag{5}$$

with

$$C_{\vec{k}'\vec{k}}=\left[\frac{1}{\epsilon_{\vec{k}}-\epsilon_d-U}+\frac{1}{\epsilon_{\vec{k}'}-\epsilon_d-U}-\frac{1}{\epsilon_{\vec{k}}-\epsilon_d}-\frac{1}{\epsilon_{\vec{k}'}-\epsilon_d}\right],\tag{6}$$

$\psi_{\vec{k},(d)}=(C_{\vec{k},(d)\uparrow},C_{\vec{k},(d)\downarrow})^T$, and $\vec{S}_d=\psi^{\dagger}_d(\vec{\sigma}/2)\psi_d$. Here, $B_{\vec{k}'\vec{k}}=V_0^{\vec{k}'d}V_0^{d\vec{k}}-\frac{1}{3}(V_1^{\vec{k}'d}V_1^{d\vec{k}}+V_2^{\vec{k}'d}V_2^{d\vec{k}}+V_3^{\vec{k}'d}V_3^{d\vec{k}})$ is the usual isotropic s-d exchange coupling term, $\vec{T}_{\vec{k}'\vec{k}}=(V_0^{\vec{k}'d}\vec{V}^{d\vec{k}}-\vec{V}^{\vec{k}'d}V_0^{d\vec{k}})$ is the DM-type s-d exchange coupling term, and

$$\overleftrightarrow{A}_{\vec{k}'\vec{k}}=\begin{pmatrix} M_{xx} & V_1^{\vec{k}'d}V_2^{d\vec{k}}+V_2^{\vec{k}'d}V_1^{d\vec{k}} & V_1^{\vec{k}'d}V_3^{d\vec{k}}+V_3^{\vec{k}'d}V_1^{d\vec{k}} \\ V_2^{\vec{k}'d}V_1^{d\vec{k}}+V_1^{\vec{k}'d}V_2^{d\vec{k}} & M_{yy} & V_2^{\vec{k}'d}V_3^{d\vec{k}}+V_3^{\vec{k}'d}V_2^{d\vec{k}} \\ V_3^{\vec{k}'d}V_1^{d\vec{k}}+V_1^{\vec{k}'d}V_3^{d\vec{k}} & V_3^{\vec{k}'d}V_2^{d\vec{k}}+V_2^{\vec{k}'d}V_3^{d\vec{k}} & M_{zz} \end{pmatrix}, \tag{7}$$

with

$$M_{xx}=+\tfrac{4}{3}V_1^{\vec{k}'d}V_1^{d\vec{k}}-\tfrac{2}{3}V_2^{\vec{k}'d}V_2^{d\vec{k}}-\tfrac{2}{3}V_3^{\vec{k}'d}V_3^{d\vec{k}},$$
$$M_{yy}=-\tfrac{2}{3}V_1^{\vec{k}'d}V_1^{d\vec{k}}+\tfrac{4}{3}V_2^{\vec{k}'d}V_2^{d\vec{k}}-\tfrac{2}{3}V_3^{\vec{k}'d}V_3^{d\vec{k}},$$
$$M_{zz}=-\tfrac{2}{3}V_1^{\vec{k}'d}V_1^{d\vec{k}}-\tfrac{2}{3}V_2^{\vec{k}'d}V_2^{d\vec{k}}+\tfrac{4}{3}V_3^{\vec{k}'d}V_3^{d\vec{k}}, \tag{8}$$

is the magnetic pseudodipole interaction. Note that the exchange coupling between the localized spin and conduction electron in Eqs. (5)–(8) has exactly the same form as the one derived by Moriya for the exchange coupling between localized spins. Therefore, it also seems to be isomorphic to the symmetry of isotropic case.[18,19] However, the parameters $\hat{d}$ and θ (Refs. 18 and 19) in our case are $\vec{k}$ dependent; there exists frustration between different $\vec{k}$ electrons, and the degeneracy problem encountered in the Moriya's exchange coupling is generally absent in the multilayer system. This point can be seen even more clearly after the Fourier transformation of $\vec{T}_{\vec{k}'\vec{k}}$ and $\overleftrightarrow{A}_{\vec{k}'\vec{k}}$; then the relation between $\vec{T}_{\vec{k}'\vec{k}}$ and $\overleftrightarrow{A}_{\vec{k}'\vec{k}}$ is destroyed, and the Hamiltonian in real space does not have the feature needed to be transformed into the isotropic case. From the above expressions, one finds that $C_{\vec{k}'\vec{k}}=C_{\vec{k}\vec{k}'}$, $B_{\vec{k}'\vec{k}}=B_{\vec{k}\vec{k}'}$, $\vec{T}_{\vec{k}'\vec{k}}=-\vec{T}_{\vec{k}\vec{k}'}$, and $\overleftrightarrow{A}_{\vec{k}'\vec{k}}=\overleftrightarrow{A}_{\vec{k}\vec{k}'}$. In high-symmetry bulk crystals, space inversion symmetry requires $\vec{T}_{\vec{k}'\vec{k}}=\vec{T}_{\vec{k}\vec{k}'}$ and thus $\vec{T}_{\vec{k}'\vec{k}}=0$. This is not the case in ferromagnetic-nonmagnetic multilayer structures where space inversion symmetry is generally broken, especially near interfaces, and thus $\vec{T}_{\vec{k}'\vec{k}}$ can have a nonzero value. As we will see below, this term contributes to the noncollinear interlayer exchange coupling between the neighboring ferromagnetic layers. In general $\vec{V}$ is smaller than V_0; this is reasonable since spin-orbit interaction is weak in comparison with the usual s-d mixing potential. So to the first order of the spin-orbit interaction, the extended s-d exchange term has the form

$$H_2=-\sum_{\vec{k}\vec{k}'n}\tfrac{1}{2}C_{\vec{k}'\vec{k}}\psi^{\dagger}_{\vec{k}'}\vec{\sigma}\psi_{\vec{k}}\cdot(B_{\vec{k}'\vec{k}}\vec{S}_n-i\vec{T}_{\vec{k}'\vec{k}}\times\vec{S}_n)\times e^{i(\vec{k}'-\vec{k})\cdot\vec{R}_n}, \tag{9}$$

with $\vec{R}_n$ denoting the site of the localized magnetic moment, and $B_{\vec{k}'\vec{k}}\approx V_0^{\vec{k}'d}V_0^{d\vec{k}}$. H_2 describes the scattering potential of conduction electrons by localized magnetic impurities when the spin-orbit interaction is included.

III. INTERLAYER COUPLING

The interlayer exchange coupling between ferromagnetic layers has been studied previously using the Green's function method[4] and the fourth-order Schrieffer-Wolff transformation[24] for the case without a spin-orbit interaction; the results involve numerical computations. In order to get a physically transparent result, we follow the approach of Ref. 2. To the second-order perturbation in terms of H_2, the exchange coupling between two ferromagnetic layers separated by a nonmagnetic metal spacer can be expressed by

$$H_{\text{eff}}=\sum_{nm}\sum_{\vec{k}\vec{k}'}\sum_{ss'}\frac{\langle\vec{k}s|H_2|\vec{k}'s'\rangle\langle\vec{k}'s'|H_2|\vec{k}s\rangle}{\epsilon_{\vec{k}}-\epsilon_{\vec{k}'}}. \tag{10}$$

After a tedious calculation, one obtains

$$H_{\text{eff}}=\sum_{nm}[J_H(z)\vec{M}_n\cdot\vec{M}_m+\vec{J}_{\text{DM}}(z)\cdot(\vec{M}_n\times\vec{M}_m)]. \tag{11}$$

Here z is the spacer layer thickness and M_n is the magnetization of the nth ferromagnetic layer. (mn) implies a summation over pairs of nearest-neighbor ferromagnetic layers. $J_H(z)=(1/2\pi)\int_{-\infty}^{\infty}e^{iq_zz}dq_zJ_H(q_z)$ and $\vec{J}_{\text{DM}}(z)=(1/2\pi)\int_{-\infty}^{\infty}e^{iq_zz}dq_z\vec{J}_{\text{DM}}(q_z)$. $J_H(q_z)$ and $\vec{J}_{\text{DM}}(q_z)$ are given by

$$J_H(\vec{q})=\frac{1}{2}\sum_{\vec{k}}|C_{\vec{k},\vec{k}+\vec{q}}|^2\frac{|B_{\vec{k},\vec{k}+\vec{q}}|^2f_{\vec{k}}(1-f_{\vec{k}+\vec{q}})}{\epsilon_{\vec{k}}-\epsilon_{\vec{k}+\vec{q}}} \tag{12}$$

and

$$\vec{J}_{\text{DM}}(\vec{q})=i\sum_{\vec{k}}|C_{\vec{k},\vec{k}+\vec{q}}|^2\frac{B_{\vec{k},\vec{k}+\vec{q}}\vec{T}_{\vec{k},\vec{k}+\vec{q}}f_{\vec{k}}(1-f_{\vec{k}+\vec{q}})}{\epsilon_{\vec{k}}-\epsilon_{\vec{k}+\vec{q}}}, \tag{13}$$

respectively. $f_{\vec{k}}$ is the Fermi distribution function. In Eqs. (12) and (13), we have redefined $B_{\vec{k},\vec{q}}=B_{\vec{k},\vec{k}+\vec{q}}$ and $\vec{T}_{\vec{k},\vec{q}}=\vec{T}_{\vec{k},\vec{k}+\vec{q}}$; it is easy to verify $B_{\vec{k},\vec{q}}=B_{\vec{k},-\vec{q}}$ and $\vec{T}_{\vec{k},\vec{q}}=-\vec{T}_{\vec{k},-\vec{q}}$. So $B_{\vec{k},\vec{q}}$ is an even function of $\vec{q}$ and $\vec{T}_{\vec{k},\vec{q}}$ is a odd function of $\vec{q}$.

Further analysis requires the band structure of the conduction electron and full knowledge of the matrix elements $B_{\vec{k},\vec{q}}$, $\vec{T}_{\vec{k},\vec{q}}$. To simplify the calculation, we assume below that the conduction electron band can be approximated by the free electron band and matrix elements are slowly varying functions of wave vector $\vec{k}$. In this case, $J_H(\vec{q})$ and $\vec{J}_{\text{DM}}(\vec{q})$ can be calculated analytically and have the forms

$$J_H(\vec{q})=-\frac{mk_F|CB_{k_Fq}|^2}{16\pi^2\hbar^2}\left[1+\frac{4k_F^2-q^2}{4k_Fq}\ln\left|\frac{q+2k_F}{q-2k_F}\right|\right] \tag{14}$$

and

$$\vec{J}_{\mathrm{DM}}(\vec{q})=-\frac{imk_F|C|^2B_{k_Fq}\vec{T}_{k_F\vec{q}}}{8\pi^2\hbar^2}\left[1+\frac{4k_F^2-q^2}{4k_Fq}\ln\left|\frac{q+2k_F}{q-2k_F}\right|\right]. \tag{15}$$

$J_H(\vec{q})$ and $\vec{J}_{\mathrm{DM}}(\vec{q})$ are even and odd functions of $\vec{q}$, respectively.

To calculate the interlayer exchange coupling between two ferromagnetic layers, we need only the components of $J_H(\vec{q})$ and $\vec{J}_{\mathrm{DM}}(\vec{q})$ with $\vec{q}=(0,0,q_z)$ since the multilayer structures are translation invariant in the layer. After expanding $B_{\vec{k},\vec{q}}$ and $\vec{T}_{\vec{k},\vec{q}}$ in power series of q_z and keeping the lowest order, $B_{k_F,q_z}\simeq B$ and $\vec{T}_{k_F,q_z}\simeq\vec{\tau}q_z$. With these simplifications, we finally obtain for a large spacer layer thickness

$$J_H(z)=-\frac{m(CB)^2}{64\pi^2\hbar^2}\frac{\sin(2k_Fz)}{z^2} \tag{16}$$

and

$$\vec{J}_{DM}(z)=-\frac{mC^2B\vec{\tau}2k_F}{16\pi^2\hbar^2}\frac{\cos(2k_Fz)}{z^2}. \tag{17}$$

Here, m is the electron mass. As usual, the singularities of $J(q_z)$ and $\vec{J}_{\mathrm{DM}}(q_z)$ at $q=\pm 2k_F$ are responsible for the long-range oscillatory behavior of the interlayer coupling. The long period can be similarly obtained by replacing $2k_F$ with $2k_F-G$ and G is the translational vector in reciprocal space. $J_H(z)$ and $\vec{J}_{\mathrm{DM}}(z)$ have the same oscillation period at a large spacer layer thickness, but they differ by a phase factor of $\pi/2$. $|\vec{J}_{\mathrm{DM}}(z)|$ becomes dominant when $J_H(z)$ vanishes. The contribution of the spin-orbit interaction cannot be suppressed in an ideal interface even if the spin-dependent s-d mixing potential is weaker than spin-independent s-d mixing potential. The interplay between the isotropic and anisotropic exchange couplings can be easily seen from the interlayer coupling energy which can be written as

$$E(\theta,z)=-\frac{mC^2B}{32\pi^2\hbar^2}\frac{M^2}{z^2}[B\sin(2k_Fz)\cos(\theta)+8k_F\tau\cos(2k_Fz)\sin(\theta)]. \tag{18}$$

θ is the angle between the magnetic moments of two layers and τ is the projection of $\vec{\tau}$ in the direction perpendicular to magnetizations. When $|\sin(2k_Fz)|$ is near zero, $|\cos(2k_Fz)|$ is almost 1 and the minimum of $E(\theta,z)$ is at $\theta=90°$ or $-90°$. In principle, any θ is possible depending on the competition between the isotropic and anisotropic interlayer coupling strengths. Recent direct observation of the 50° coupled magnetization profile in a Cu/Cr(100) multilayer structures lends further support to our argument.[25]

IV. DISCUSSION AND CONCLUSION

Whether the anisotropic DM exchange coupling plays a dominant role or not in the noncollinear coupling depends on the ratio of $|\vec{V}|/V_0$. Although the exact value of the DM exchange coupling is determined by material parameters as well as the interface s-d mixing effect and is difficult to calculate, a rough estimate of its order of magnitude can be obtained from previous experimental and theoretical works.

Moriya[17] estimated that the DM term is roughly $(\Delta g/g)$ times the isotropic superexchange term in weak ferromagnetic crystals, where g is the gyromagnetic ratio and Δg is the deviation from the value for a free electron.[26] By comparing the DM exchange coupling with the usual RKKY coupling in the Co-diluted CuMn spin glass alloy, Fert and Levy[20] found that the ratio of the DM coupling to the RKKY coupling is of the order of 0.1. In magnetic multilayers, the value of $J_{\mathrm{DM}}(z)$ depends on the spin-orbit interaction constant as well as the asymmetry of the system. If we take the spin-orbit interaction constant to be the same as that in the Co-diluted CuMn spin glass alloy, the magnitude of the DM exchange coupling can reach 10% of the isotropic RKKY exchange coupling. Another way to estimate the ratio of the DM term to the usual RKKY term is to consider the anisotropic magnetoresistance in ferromagnetic alloys, which is due to spin-orbit scattering. The ratio $|\vec{V}|/V_0$ was found to be about 0.03 in the measurement of the anisotropic resistance in Ni-based ferromagnetic alloy.[27] As the ratio $J_{\mathrm{DM}}/J_H\simeq 4|\vec{V}|/V_0$, it is again of the order of 0.1. We assume that this ratio has the same order of magnitude in the $3d$-based ferromagnetic layered structures.

Note that the relative strength of 0.1 between 90° coupling and the linear coupling is in much better agreement with experiments than the relative magnitude of biquadratic coupling predicted by previous theories based on intrinsic origins, which is only of the order of 10^{-2}–10^{-3}.[13–16] Furthermore, the DM term obtained by us has the same decaying form as that of the RKKY term, whereas the biquadratic coupling decays faster with the spacer layer thickness than the quadratic coupling does. We would also like to mention that the phase difference of $\pi/2$ between the DM term and RKKY term makes the DM term especially important since there always exists a region where DM coupling becomes dominant. Thus, the fact that 90° coupling is observed when the spacer thickness is roughly between the ferromagnetic and antiferromagnetic coupling is not only because the linear coupling is then weak but also because the DM term is at its maximum.

To fully test the validity of the role played by DM exchange coupling, more quantitative analyses of the DM term and its temperature dependence are needed, which require detailed information of the spin-orbit scattering effect as well as s-d mixing and other effects near interfaces. Such a study can provide further checks for the mechanism of noncollinear coupling proposed in this paper.

In summary, we have studied the interlayer exchange coupling in ferromagnetic-nonmagnetic multilayer structures based on an extended Anderson *s*-*d* mixing model. Both the isotropic RKKY coupling as well as anisotropic DM coupling are derived analytically using the Schrieffer-Wolff transformation and second-order perturbation theory. While the DM term vanishes in the crystals with space inversion symmetry, it plays a crucial role in multilayer structures. The DM-type interlayer coupling oscillates with spacer layer thickness and has the same period as that of the usual RKKY term at a large spacer layer thickness, but with a phase shift of $\pi/2$. This phase difference makes the DM-type interlayer exchange coupling non-negligible. Our rough estimate suggests that DM exchange coupling plays an important role in some trilayers and magnetic multilayer structures.

ACKNOWLEDGMENTS

We would like to acknowledge partial financial support from the National Natural Science Foundation of China and the Education Commission of China. W.Z. wishes to thank ICTP at Trieste for its hospitality, where part of the work was carried out.

[1] S. S. P. Parkin, N. More, and K. P. Roche, Phys. Rev. Lett. **64**, 2304 (1990).
[2] P. Bruno and C. Chappert, Phys. Rev. B **46**, 261 (1992).
[3] D. M. Deaven, D. S. Rokhsar, and M. Johnson, Phys. Rev. B **44**, 5977 (1991).
[4] Z. P. Shi, P. M. Levy, and J. L. Fry, Phys. Rev. B **49**, 15 159 (1994).
[5] M. Ruhrig, R. Schaefer, A. Hubert, R. Mosler, J. A. Wolf, S. Demokritov, and P. Grunberg, Phys. Status. Solidi A **125**, 635 (1991).
[6] J. Ungaeris, R. J. Celotta, and D. T. Pierce, J. Magn. Magn. Mater **127**, 205 (1993).
[7] Z. Celinski, B. Heinrich, and J. F. Cochran, J. Magn. Magn. Mater **145**, L1 (1995).
[8] U. Rucker, S. Demokritov, E. Tsymbal, P. Grunberg, and W. Zinn, J. Appl. Phys. **75**, 387 (1995).
[9] S. Adenwalla, G. P. Felcher, E. E. Fullerton, and S. D. Bader, Phys. Rev. B **53**, 2474 (1996)
[10] J. C. Slonczewski, Phys. Rev. Lett. **67**, 3172 (1991).
[11] J. C. Slonczewski, J. Appl. Phys. **73**, 5957 (1993).
[12] S. Demokritov, E. Tsymbal, P. Grunberg, W. Zinn, and I. K. Schuller, Phys. Rev. B **49**, 720 (1994).
[13] R. P. Erickson, K. B. Hathaway, and J. R. Cullen, Phys. Rev. B **47**, 2626 (1993).
[14] D. M. Edwards, J. M. Ward, and J. Mathon, J. Magn. Magn. Mater. **126**, 380 (1993).
[15] J. C. Slonczewski, J. Magn. Magn. Mater. **126**, 374 (1993).
[16] J. Inoue, J. Magn. Magn. Mater. **136**, 233 (1994).
[17] T. Moriya, Phys. Rev. Lett. **4**, 228 (1960).
[18] L. Shekhtman, O. Entin-Wohlman, and Amnon Aharony, Phys. Rev. Lett. **69**, 836 (1992).
[19] L. Shekhtman, O. Entin-Wohlman, and Amnon Aharony, Phys. Rev. B **47**, 174 (1992).
[20] A. Fert and P. M. Levy, Phys. Rev. Lett. **44**, 1538 (1980).
[21] J. B. Staunton. B. L. Gyorffy, J. Poulter, and P. Strange, J. Phys. C **21**, 1595 (1988).
[22] D. Errandonea, J. Phys. C **7**, 9439 (1995).
[23] J. R. Schrieffer and P. A. Wolff, Phys. Rev. **149**, 491 (1966).
[24] Q. Zhang and P. M. Levy, Phys. Rev. B **34**, 1884 (1986).
[25] A. Schreyer, J. F. Ankner, M. Schäfer, Th. Zeidler, H. Zabel, C. F. Majkrzak, and P. Grünberg (unpublished).
[26] S. Chikazumi, *Physics of Magnetism* (Wiley, New York, 1964), p. 47.
[27] I. A. Campbell, A. Fert, and O. Jaoul, J. Phys. C Suppl. **1**, S95 (1970).