Infrared conductivity of cuprate metals: Detailed fit using Luttinger-liquid theory

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(Received 18 September 1995; revised manuscript received 27 November 1995)

Measurements of infrared conductivity in the normal state of the cuprate layer metals show a characteristic behavior in the plane of the layers which is in essential agreement among many experiments. A simple parametrization of this behavior, proposed originally by Collins and Schlesinger, and exploited by Bontemps and her group, which gives an adequate fit over frequencies from a few hundred cm^{-1} to $>5000\ cm^{-1}$, is that the phase angle of the complex conductivity is independent of frequency. This fit is shown to be a natural consequence of Luttinger-liquid theory with charge-spin separation, and determines the exponent of the singularity at the Fermi surface to be $\sim 0.15 \pm 0.05$. [S0163-1829(97)00917-X]

The infrared conductivity of the high-T_c cuprates in the normal state has a characteristic deviation from the normal ''Drude'' behavior of metals, which has sometimes been described as an additional, distinct ''midinfrared absorption'' and sometimes as an extended tail of the low-frequency peak. Schlesinger,[1] some years ago, analyzed his data on the reflectvity of single crystals of $YBCO_7$ in terms of the conventional expression

$$\sigma = \frac{ne^2}{m(i\omega + 1/\tau)} \tag{1}$$

with frequency-dependent parameters $m(\omega)$ and $1/\tau(\omega)$, which showed remarkably simple behavior (see Fig. 1): $1/\tau$ is proportional to ω, and m has a slow, approximately logarithmic variation. There is, in fact, little difference in the data among actual experiments, as opposed to interpretations, on good materials, so we may take Fig. 1 as typical of optimally doped cuprates, since it is in essence a heuristic description of the data.

Bontemps and collaborators[2] have used a similar plot to describe data over a wide range of frequencies, up to around 8000 cm^{-1}, using transmission and reflection data on films of a number of cuprates, most but not all closely related to YBCO (see Figs. 2 and 3). With this wide frequency range the family resemblance of all of the data becomes striking, particularly plotted using Schlesinger's parameters. I believe that there would be little disagreement as to the general characteristics of the actual data among these and other experimentalists, except that less highly doped YBCO samples show ''spin-gap'' deviations at the lower end of the range ($\leqslant 500\ cm^{-1}$),[3] and that quite impure samples may have a small residual resistivity.

The data of Bontemps *et al.* are, as pointed out by them, not very reliable at low frequencies because of substrate effects on the film data. We emphasize its smoothness over a broad frequency range, and its good agreement as to general trend with the single-crystal data of Schlesinger and Collins, which is accurate up to 1000 cm^{-1}. Note that while the dependence on T should be similar to that on ω according to our theoretical ideas, the T dependence will be considerably modified by thermal expansion and oxygen redistribution and/or loss, and can be studied only over at most 20 % of the range. Thus the fact that the T dependence appears linear seems fortuitous and is somewhat sample dependent.

We describe a detailed fit to the data of Figs. 2 and 3 using the ''Luttinger-liquid'' hypothesis for the electronic state of the two-dimensional (2D) normal metal.[4] This result depends only on rather general properties of the theory but is totally dependent on its non-Fermi-liquid nature.

The basis of the fit is the remark that the cuprates are in the ''holon nondrag regime'' of Luttinger-liquid transport theory.[4,5,8] This is the regime where charge excitations (''holons'') are scattered sufficiently rapidly that they do not recohere with the spinons after the accelerated electron decays into charge and spin excitations.

We restate the argument briefly. There are three regimes in the theory, depending on the relative rates of scattering $(1/\tau_h)$ and of the decay of the electron into the charge and spin eigenexcitations (holons and spinons). This latter rate is about $\hbar\omega$ and also determines the rate of the inverse process (by detailed balance) of holon-spinon scattering (equivalent to recombination and decay) to be $\sim(\hbar\omega)^2/t_\parallel$. If

$$\frac{\hbar}{\tau_h} < \frac{(\hbar\omega)^2}{t_\parallel},$$

we are in the ''holon drag'' regime of weak scattering, which has not yet been analyzed but may look Fermi-liquid-like. If

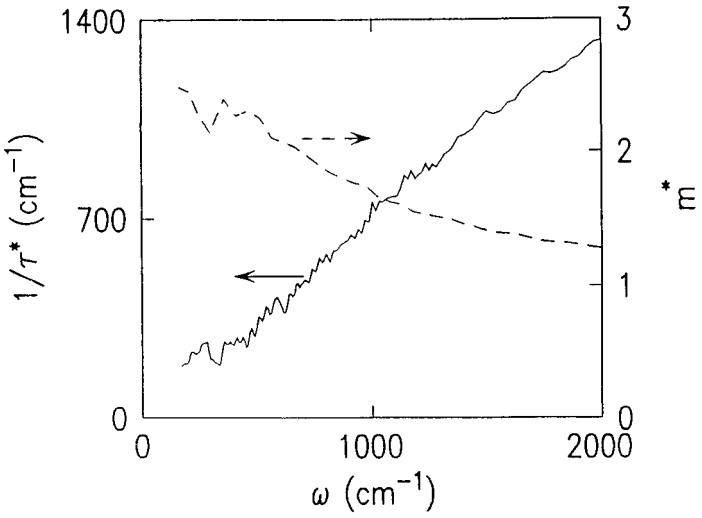

FIG. 1. Schlesinger's original data on the IR spectrum of YBCO. This is repeated in Fig. 2 as the crossed square points (Ref. 1).

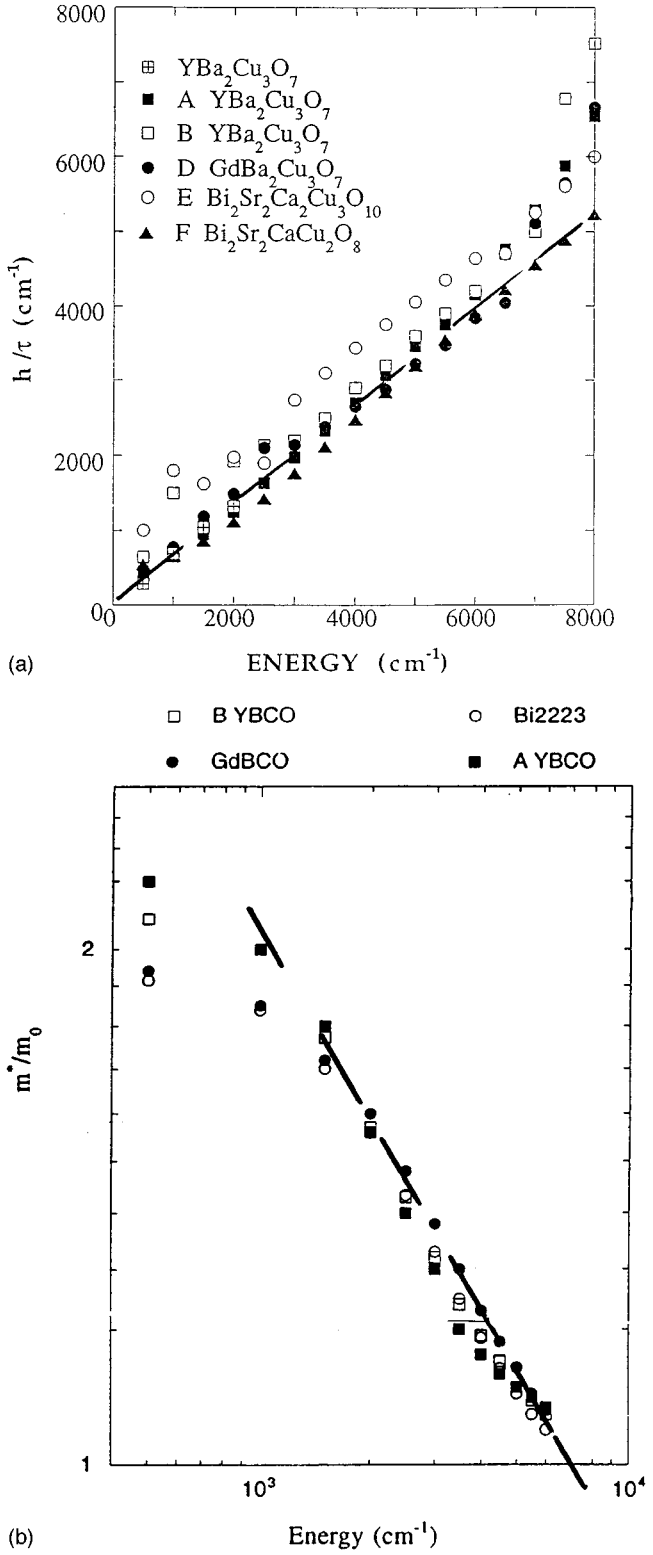

FIG. 2. (a) The Bontemps group's data on a group of cuprates from Ref. 2: $1/\tau$ vs ω. Further, more recent data are given in (b) and reported in Ref. 11. (b) (a)+(b) Same set of data, m^* vs ω.

$\hbar\omega > \hbar/\tau_h > (\hbar\omega)^2/t_\parallel$, we are in the ''holon nondrag'' regime where the major dissipative process is the decay and vertex corrections are damped out by holon scattering.[5] The conductivity is given by the simple one-loop diagram (Fig. 3)

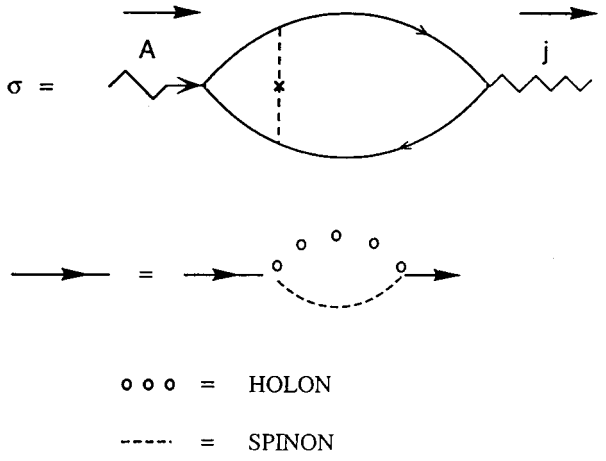

FIG. 3. Primitive diagram for the conductivity. Vertex corrections are omitted for reasons given in the text.

$$\sigma(\omega) \propto \frac{1}{\omega} \int dx \int dt G^e(x,t) G^h(x,t) e^{i\omega t}. \tag{2}$$

G^e and G^h are the exact (interacting) one-electron Green's functions for electrons and holes, respectively. The physical process which controls the rate of entropy production is the decay of the electrons and holes into spin and charge excitations, but this is enabled to act as a resistivity mechanism by the fact that the momentum decays because the charge is then scattered by the lattice. The process is analogous to phonon scattering in the phonon nondrag regime, where the momentum decay occurs by the scattering of the phonons by the lattice, which prevents phonon drag, while the entropy production is caused by phonon emission which is momentum conserving and controls the observed resistivity. The phonon gas is not dragged along by the electrons, and analogously, in the holon-drag regime the gas of charge excitations is stationary, the current being carried by spinons because of the backflow of the charge gas. Ogata in Ref. 5 has shown that the vertex corrections which would invalidate Eq. (2) and restore the Ward identities in a pure sample are cut off by the mean free path for charge scattering in this regime. Note that in this regime neither the conventional residual resistivity nor phonon resistivity appear, and they are replaced by the ''linear T'' resistivity when the sample is ''impure enough.'' In the third regime, $\hbar/\tau_h > \hbar\omega$, charge-spin separation is irrelevant and a kind of localizing behavior is observed, which will be discussed in a future publication.

The above arguments have been questioned by many people, so I will attempt to restate them. When an electron is accelerated by the electric field, it immediately begins to decay into the eigenexcitations which we describe by holon and spinon degrees of freedom. This decay rate is clearly given by the width of the relevant single-particle Green's function G_1. However, this is not an entropy-producing process if the inverse process proceeds at the rate given by detailed balance, $\sim\omega^2 N(E)$, i.e., if the electron recoheres at an equally rapid rate. This fact is signaled by the backscattering terms in ordinary Boltzmann transport theory including ''phonon drag'' terms, or by the Ward identities between self-energy and vertex corrections in diagrammatic perturba-

tion theory, which express the underlying translation invariance. These would, and probably do in fact, remove the effect of electron decay on resistivity in very pure crystals, as in many of the organic metals. In the holon nondrag regime, where the mean free time of charge excitations is less than $\omega^2 N(E)$, the electron does not recohere, and the rate of entropy production is controlled by the electron decay. The charge excitations are approximately pinned by the impurities and the current may be thought of as carried by the spinons. [Recent analyses of thermopower measurements confirm the ''two-fluid'' nature of the flow in these materials (Coleman and Tsvelik).[12]] While spinon excitations are nominally neutral, they can carry a backflow current, as shown by Nozieres and collaborators (private communication). Incidentally, these arguments invalidate conventional renormalization-group calculations of transport responses which do not allow for distinct scattering rates for the separate charge and spin excitations. Since these calculations have never been correlated with experiment, this is no reason for uneasiness.

As the paper by Ogata (Ref. 5) shows, the above verbal explanation is far simpler than a diagrammatic calculation, and in fact the physical description of the process in Ref. 5 is flawed. In fact, in the Luttinger liquid such direct calculations are not to be trusted very firmly, since it is the nature of the Luttinger liquid that vertex corrections, if they must be included, will be singular; conventional transport theory is not applicable, and special methods such as the above are necessary.

We can evaluate Eq. (2) very simply using the fact that $G_1(x,t)$ is a homogeneous function of (x,t) considered as a single variable. This is the consequence of the fact that all excitations have a finite Fermi velocity. For the Fermi liquid,

$$G_{\mathrm{FL}} \propto \frac{e^{ik_F x}}{x - v_F t}, \tag{3}$$

homogeneous of order (-1), while for the 1D Luttinger liquid,

$$G_{\mathrm{LL}} \propto \frac{e^{ik_L x}}{\sqrt{(x-v_s t)(x-v_c t)}\,(x^2 - v_c^2 t^2)^{\alpha/2}}, \tag{4}$$

which is homogeneous of order $(-1-\alpha)$. For the 2D liquid, G is an average of an expression like Eq. (3) or Eq. (4) over the Fermi surface. For the Fermi liquid, the relevant G in momentum and frequency space may be approximated by

$$G(p,\omega) \simeq \frac{1}{\hbar\omega - (p - \mathbf{p}_f)\cdot \mathbf{v_F}},$$

where p_f and v_F are at the projection of $\mathbf{p}$ on the Fermi surface along $v_{\mathbf{F}}$, p assumed close to the Fermi surface. A similar construction for the Luttinger liquid will give a pair of variables $\Delta p = p - p_F$, ω, in which the Green's function will again be homogeneous of order $-(1-\alpha)$, but this function has no simple formal expression. Nonetheless, we may in general write, as the appropriate law for scaling of the low-frequency excitation spectrum,

$$G(x,t) = \frac{1}{t^{1+\alpha}} F\left(\frac{x}{v_F t}\right)(\mathrm{LL}), \tag{5}$$

$$G(p,\omega) = \frac{1}{\omega^{1-\alpha}} F\left(\frac{(p-p_F)v_F}{\omega}\right), \tag{6}$$

where F will depend on the parameters v_c/v_F, v_s/v_F as functions of position on the Fermi surface. Equation (5) reduces to the Fermi-liquid expression (3) if $\alpha \to 0$. By a simple scaling argument, we find

$$\sigma(\omega) = \frac{\mathrm{const}}{(i\omega)^{1-2\alpha}}. \tag{7}$$

Equation (7) holds up to an upper frequency cutoff $\Omega = \wedge/\hbar$ of the order of the electron bandwidth $\wedge$. The sum rule on conductivity will be satisfied if the coefficient in Eq. (7) is set so that

$$\sigma(\omega) = \frac{ne^2}{i\omega m_0}\left(\frac{i\omega}{\Omega}\right)^{2\alpha} \frac{2\alpha}{\sin \pi\alpha}. \tag{8}$$

Here m_0 is the sum-rule mass,

$$\int \sigma(\omega)\, d\omega = \frac{ne^2}{m_0},$$

which should not be far from the band mass: Eq. (8) contains all intraband mass renormalization effects.

I would remind the reader that for the Fermi liquid the integral (2) is not convergent without a finite lifetime giving an imaginary part $\hbar/\tau$ to the energy denominator in G. This gives the characteristic ''Drude'' behavior of ordinary metals, with σ falling off as $1/\omega^2$ at high frequencies. The Luttinger liquid is qualitatively different from a Fermi liquid with small Z.

Equation (8) contains only two free parameters, n/m_0 and α (the upper cutoff Ω merely scales m_0 and is not independent). Neither can vary much: m_0 must not be much bigger than the band mass, and c-axis Hall data[6] among others tell us that n is the conventional band filling $\propto 1-\delta$. α for the 1D Hubbard model is $\leqslant 1/8$, but models with $\alpha > 1/8$ exist. There is no fundamental theory of α in 2D. Vague indications from gauge theory[7] suggest $1/6 (2\alpha = 1/3)$, while the tomographic picture[8] might suggest rough agreement with 1D.

The data give two measures of 2α, one from the slope of $1/\tau$ vs ω and one from the dependence of m on ω. These two numbers agree, which is evidence for the quality of the Kramers-Kronig transform carried out in Refs. 1 and 2. For low values of α, it would be difficult to distinguish marginal Fermi-liquid theory[9] ($m^* \propto \ln\omega$) from our power-law result; but the relatively large value of α, twice confirmed, argues against this. In addition, the MFLT has no explicit mechanism of resistivity, i.e., no explicit sink for momentum, such as we have, and seems simply to use the same ''bubble''

diagram without justification. The slope of $1/\tau$ [dashed line in Fig. 2(a)] is $\sim 0.7 \pm 0.1$ which gives $\alpha \simeq 0.15 \pm 0.05$. The median slope is used for the dashed line in Fig. 2(b), which as you can see is an adequate fit, although the power-law form is not constrained much by the data. On the other hand, the analytic properties of σ require that if it really has a constant phase angle [as Fig. 2(a)] it must be a power of $(i\omega)$ (or logarithmic in the limit $\alpha = 0+$).

Let us summarize the achievements of the Luttinger-liquid hypothesis, coupled with the concept of the holon-nondrag regime. The original motivation, which was satisfied by this idea, was to explain the absence of phonon-scattering effects or, in most cases, of residual impurity scattering, both of which should be large in most of these materials. Let it be explicit that the separation of charge and spin, though it fails to appear in the formal expression (5) or (6), which depends only on the ''Fermi-surface'' exponent, α, is essential to the entire theory because of the concept of ''holon drag.''

Now we see that the theory leads to a unique scaling form for the conductivity which holds over almost 2 decades of frequency and for a number of cuprates. Particularly important, in my view, is the fact that the expression scales from >5000 cm^{-1} to <500 cm^{-1}, a property which no alternative theory motivates in any natural way.

It is interesting that other groups (especially Bozovic[10]) see indications of similar behavior in the ''midinfrared conductivity'' of a number of other materials, mostly those with other symptons of strong correlation phenomena. With considerable caution because of the existence of other transport regimes, we would consider a Luttinger-liquid explanation for some of these cases.

I would like to acknowledge especially extensive discussions with Nicole Bontemps, as well as the use of her data. I also was stimulated by discussions with R. Laughlin and E. Abrahams. This work was supported by the NSF, Grant No. DMR-9104873.

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