Impurity scattering effect on the specific-heat jump in anisotropic superconductors

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The specific-heat jump at a normal-superconducting phase transition in an anisotropic superconductor with nonmagnetic impurities is calculated within a weak-coupling mean-field approximation. It is shown that its dependence on the impurity concentration is remarkably different for $d_{x^2-y^2}$-wave and $(d_{x^2-y^2}+s)$-wave states. This effect may be used as a test for the presence of an s-wave admixture in the cuprates. [S0163-1829(97)04417-2]

There now exists a considerable experimental evidence supporting d-wave superconductivity in the cuprates, but the most direct probes of the superconducting state such as the electromagnetic penetration depth, photoemission, and quantum phase interference measurements neither confirm nor exclude a possible small s-wave admixture in a predominantly $d_{x^2-y^2}$ superconductor.[1] The linear temperature dependence of the penetration depth at low temperatures[2] observed in $YBa_2Cu_3O_{7-\delta}$ (YBCO) agrees with the theoretical predictions for a d-wave state.[3] However, the measurements only went down to about 1 K and an exponential behavior below this temperature, indicating a small nonzero gap minimum cannot be eliminated. Even by taking data at much lower temperatures, the presence of a small s-wave component in the order parameter cannot be entirely excluded in the penetration depth experiments. Similar constraints limit the angle resolved photoemission spectroscopy (ARPES) method. Although ARPES data[1,4,5] are consistent with a $d_{x^2-y^2}$ scenario in $Bi_2Sr_2CaCu_2O_8$ (BSCCO) as well as in YBCO, the experiments cannot decide with an accuracy greater than an instrumental resolution if the order parameter completely vanishes at the $d_{x^2-y^2}$ nodal lines. This leaves the possibility of a small (<2 meV) s-wave admixture. Therefore the above experimental methods do not rule out the presence of a small isotropic component, but place an upper bound on the minimum of the gap function. As analyzed in Ref. 1, the emerging picture from the Josephson experiments supports a scenario of a real mixture of s and $d_{x^2-y^2}$ states in YBCO, but also does not definitely confirm the presence of the s-wave component. The existence of even a small s-wave admixture in a d-wave superconductor may be tested by thermodynamic measurements in the presence of nonmagnetic impurities. It is well known that the d-wave state is strongly suppressed by the defects,[6,7] and the s-wave state is not affected by the nonmagnetic scatterers.[8] In the case of a $(d+s)$-wave superconductor a power-law T_c suppression[9] should be observed above a certain impurity-doping level and the thermodynamic properties at large impurity concentration should resemble those of the s-wave state. In fact the critical temperature of YBCO is decreased below 12 K by the electron-irradiation,[1] and the Pr-doping or ion-beam damage lead to a long tail T_c suppression[10] characteristic for a small nonzero value of the gap function integrated over the Fermi surface.[9] However, despite the electron-irradiation removing the planar oxygens produces the nonmagnetic defects,[11] it has not been determined whether the scattering centers created by Pr doping and ion-beam damage in YBCO are purely nonmagnetic.

In the present paper we suggest that more significant features attributed to the s-wave part of the order parameter may be seen in the specific-heat measurements. We study a nonmagnetic impurity effect on the specific-heat jump at a superconducting-normal phase transition in anisotropic superconductors and show that the result depends on the Fermi surface (FS) averages of the first four powers of the superconducting order parameter. A particularly large difference in the specific-heat jump between the states with a nonzero and a zero value of the order parameter FS average is observed. We suggest that this measurement may be used as a test for the presence of an s-wave admixture in a $d_{x^2-y^2}$ state. $\hbar=k_B=1$ is taken throughout the paper.

We consider the effect of potential scattering by nonmagnetic, noninteracting impurities on the order parameter with its orbital part defined as

$$\Delta(\mathbf{k})=\Delta e(\mathbf{k}), \tag{1}$$

where $e(\mathbf{k})$ is a momentum-dependent function normalized by taking its average value over the Fermi surface $\langle e^2\rangle=\int_{FS}dS_k n(\mathbf{k})e^2(\mathbf{k})=1$, where $\int_{FS}dS_k$ represents the integration over the Fermi surface and $n(\mathbf{k})$ is the angle-resolved FS density of states, which obeys $\int_{FS}dS_k n(\mathbf{k})=1$. This normalization gives Δ the meaning of the absolute magnitude of the order parameter. The function $e(\mathbf{k})$ may belong to a one-dimensional (1D) irreducible representation of the crystal point group or may be given by a linear combination of the basis functions of different 1D representations. The impurity effect is studied in the t-matrix approximation.[12,13] This approach introduces two parameters describing the scattering process: $c=1/(\pi N_0 V_i)$ and $\Gamma=n_i/(\pi N_0)$, where N_0, V_i, and n_i are, respectively, the overall density of states at the Fermi level, the impurity (defect) potential, and the impurity concentration. We assume an s-wave scattering by the impurities, that is V_i does not have an internal momentum dependence. It is particularly convenient to think of c as a measure of the scattering strength, with $c\rightarrow 0$ in the unitary limit and $c\gg 1$ for weak scattering, i.e., the Born limit.

The amplitude of the order parameter is determined by the mean-field self-consistent equation

$$\Delta(\mathbf{k}) = -T\sum_{\omega}\sum_{\mathbf{k}'} V(\mathbf{k},\mathbf{k}') \frac{\tilde{\Delta}(\mathbf{k}')}{\tilde{\omega}^2+\xi_{k'}^2+|\tilde{\Delta}(\mathbf{k}')|^2}, \quad (2)$$

where T is the temperature, ξ_k is the quasiparticle energy, $\omega=\pi T(2n+1)$ (n is an integer), and $V(\mathbf{k},\mathbf{k}')$ is the phenomenological pair potential taken as

$$V(\mathbf{k},\mathbf{k}') = -V_0 e(\mathbf{k}) e(\mathbf{k}'). \quad (3)$$

We have assumed a particle-hole symmetry of a quasiparticle spectrum. The renormalized Matsubara frequency $\tilde{\omega}(\mathbf{k})$ and the renormalized order parameter $\tilde{\Delta}(\mathbf{k})$ are then given by

$$\tilde{\omega}=\omega-\Sigma_0, \quad \tilde{\Delta}(\mathbf{k})=\Delta(\mathbf{k})+\Sigma_1 \quad (4)$$

with the self-energies defined as

$$\Sigma_0=-\Gamma\frac{g_0}{c^2+g_0^2+g_1^2}, \quad \Sigma_1=\Gamma\frac{g_1}{c^2+g_0^2+g_1^2} \quad (5)$$

and the g_0, g_1 functions determined by the self-consistent equations

$$g_0=\frac{1}{N_0\pi}\sum_{\mathbf{k}}\frac{\tilde{\omega}}{\tilde{\omega}^2+\xi_k^2+|\tilde{\Delta}(\mathbf{k})|^2} \quad (6)$$

$$g_1=\frac{1}{N_0\pi}\sum_{\mathbf{k}}\frac{\tilde{\Delta}(\mathbf{k})}{\tilde{\omega}^2+\xi_k^2+|\tilde{\Delta}(\mathbf{k})|^2}. \quad (7)$$

To proceed further, we restrict the wave vectors of the electron self-energy and pairing potential to the Fermi surface and replace $\Sigma_{\mathbf{k}}$ by $N_0\int_{\mathrm{FS}} dS_k n(\mathbf{k})\int d\xi_k$. Integrated over ξ_k the gap equation (2) can be transformed after a standard procedure[14] into

$$\ln\left(\frac{T}{T_{c_0}}\right)=2\pi T\sum_{\omega>0}\left(f_\omega-\frac{1}{\omega}\right), \quad (8)$$

where the f_ω function is defined as

$$f_\omega=\int_{\mathrm{FS}} dS_k n(\mathbf{k})\frac{\tilde{\Delta}(\mathbf{k})e(\mathbf{k})}{\Delta[\tilde{\omega}^2+|\tilde{\Delta}(\mathbf{k})|^2]^{1/2}}. \quad (9)$$

We expand Eq. (8) in powers of Δ^2 around $\Delta=0$ using the relations (4)–(7). Keeping up to the fourth power terms in Δ we get the gap equation in the Ginzburg-Landau regime

$$\ln\left(\frac{T}{T_{c_0}}\right)=-f_0-\frac{1}{2}f_1\left(\frac{\Delta}{2\pi T}\right)^2+\frac{1}{4}f_2\left(\frac{\Delta}{2\pi T}\right)^4 \quad (10)$$

where the coefficients are given by

$$f_0=-2\pi T\sum_{\omega>0}\left((f_\omega)_{\Delta=0}-\frac{1}{\omega}\right), \quad (11)$$

$$f_1=-(2\pi T)^3\sum_{\omega}\left(\frac{df_\omega}{d\Delta^2}\right)_{\Delta=0}, \quad (12)$$

$$f_2=2(2\pi T)^5\sum_{\omega}\left(\frac{d^2f_\omega}{d(\Delta^2)^2}\right)_{\Delta=0}. \quad (13)$$

Taking the derivatives with respect to Δ^2

$$\frac{d}{d\Delta^2}=\frac{\partial}{\partial\Delta^2}+\sum_{\omega}\left\{\frac{d\tilde{\omega}}{d\Delta^2}\frac{\partial}{\partial\tilde{\omega}}+\frac{d\tilde{\Delta}(\mathbf{k})}{d\Delta^2}\frac{\partial}{\partial\tilde{\Delta}(\mathbf{k})}\right\} \quad (14)$$

and with a use of the relations given in Eqs. (4)–(7) we calculate f_0 and f_1 coefficients

$$f_0(\varrho)=(1-\langle e\rangle^2)\left[\psi\left(\frac{1}{2}+\varrho\right)-\psi\left(\frac{1}{2}\right)\right], \quad (15)$$

$$\begin{aligned} f_1(\varrho)=&2\langle e\rangle[2\langle e^3\rangle+5\langle e\rangle^3-7\langle e\rangle]\varrho^{-2}\left[\psi\left(\frac{1}{2}+\varrho\right)\right.\\ &\left.-\psi\left(\frac{1}{2}\right)\right]+2\langle e\rangle[-2\langle e^3\rangle-3\langle e\rangle^3+5\langle e\rangle]\varrho^{-1}\\ &\times\psi^{(1)}\left(\frac{1}{2}+\varrho\right)+4\langle e\rangle^2[1-\langle e\rangle^2]\varrho^{-1}\psi^{(1)}\left(\frac{1}{2}\right)\\ &+\frac{1}{2}[-\langle e^4\rangle+3\langle e\rangle^4+4\langle e\rangle\langle e^3\rangle-6\langle e\rangle^2]\\ &\times\psi^{(2)}\left(\frac{1}{2}+\varrho\right)-\frac{1}{2}\langle e\rangle^4\psi^{(2)}\left(\frac{1}{2}\right)+\frac{1}{6}[2(\langle e\rangle^2-1)^2\\ &\times(c^2+1)^1-\langle e\rangle^4+2\langle e\rangle^2-1]\varrho\,\psi^{(3)}\left(\frac{1}{2}+\varrho\right), \end{aligned} \quad (16)$$

where $\varrho=[\Gamma/(c^2+1)]/(2\pi T)$ and ψ, $\psi^{(n)}$ ($n=1,2,3$) are the polygamma functions.[15] In the unitary limit $c\to 0$ and $\varrho=\Gamma/(2\pi T)$. Alternatively for weak scattering ($c\gg 1$) we keep only the terms linear in $1/c^2$ in a Taylor's expansion which leads to the Born approximation scattering rate $\varrho=\pi N_0 n_i V_i^2/(2\pi T)$ and $\varrho/(c^2+1)=0$. Coefficients f_0 and f_1 involve three different types of the Fermi surface averages of the order parameter namely, $\langle e\rangle$, $\langle e^3\rangle$, and $\langle e^4\rangle$. These averages enter the free energy and determine the thermodynamic properties at the phase transition. In this paper we discuss a specific-heat jump at T_c, $\Delta C(T_c)=C_S(T_c)-C_N(T_c)$, where $C_S(T_c)$ and $C_N(T_c)$, respectively are the specific heat of the superconducting and normal state, $C_N(T_c)=(2\pi^2/3)N_0T_c$. We obtain[14] from Eq. (10)

$$\frac{\Delta C(T_c)}{C_N(T_c)}=\frac{12}{(f_1)_{T=T_c}}\left[1+T_c\left(\frac{df_0}{dT}\right)_{T=T_c}\right]^2 \quad (17)$$

and finally, f_0 from Eq. (15) yields

$$\frac{\Delta C(T_c)}{C_N(T_c)}=\frac{12}{f_1(\varrho_c)}\left[1+(\langle e\rangle^2-1)\varrho_c\psi^{(1)}\left(\frac{1}{2}+\varrho_c\right)\right]^2, \quad (18)$$

where ϱ_c is ϱ at $T=T_c$. This rather cumbersome formula, when considered along with Eq. (16), reduces significantly for the $\langle e\rangle=0$ case:

$$\frac{\Delta C(T_c)}{C_N(T_c)}=\frac{12[1-\varrho_c\psi^{(1)}[(1/2)+\varrho_c]]^2}{\frac{\mu}{6}\varrho\,\psi^{(3)}[(1/2)+\varrho_c]-(1/2)\langle e^4\rangle\psi^{(2)}[(1/2)+\varrho_c]}, \quad (19)$$

where $\mu=(1-c^2)/(1+c^2)$. For an appropriate choice of $\langle e^4\rangle$ value, $\Delta C(T_c)/C_N(T_c)$ from Eq. (19) agrees with the result obtained by Hirschfeld *et al.*[13] as well as that by Suzumura and Schulz[16] in the Born limit.

It is informative to discuss the limiting cases of Eq. (18), that is a pure system where $\varrho_c=0$ and a highly impure one with $\varrho_c\to\infty$ in which $T_c\to 0$ suppressed by the impurities. Using a series representation of the f_1 function[17] we get in the $\varrho_c=0$ limit

$$\left(\frac{\Delta C(T_c)}{C_N(T_c)}\right)_{\varrho_c=0}=-\frac{24}{\psi^{(2)}(1/2)\langle e^4\rangle}\approx\frac{1.426}{\langle e^4\rangle}. \tag{20}$$

The $\varrho_c\to\infty$ limit is obtained with a use of Eq. (16) and asymptotic forms of polygamma functions.[15] There are two cases to distinguish here. First, when the Fermi surface average of the order parameter $\langle e\rangle\neq 0$ then

$$\left(\frac{\Delta C(T_c)}{C_N(T_c)}\right)_{\varrho_c\to\infty}=-\frac{24}{\psi^{(2)}(1/2)}\approx 1.426 \tag{21}$$

and the second,[18] with $\langle e\rangle=0$, which leads to

$$\left(\frac{\Delta C(T_c)}{C_N(T_c)}\right)_{\varrho_c\to\infty}=0. \tag{22}$$

We note, that a specific-heat jump value in the $\varrho_c\to\infty$ limit for a nonzero value of $\langle e\rangle$ given by Eq. (21) agrees with that of an isotropic s-wave superconductor. This fact has a simple intuitive interpretation. A nonzero Fermi surface average of the order parameter leads to an asymptotic power-law critical temperature suppression for large impurity concentration[9] $T_c\sim(T_{c_0})^{1/\langle e\rangle^2}[\Gamma/(c^2+1)]^{(1-1/\langle e\rangle^2)}$, therefore T_c is almost constant for large Γ values. The impurity effect, then, in the large impurity concentration range is the same as in the case of s-wave superconductivity, where T_c is not changed by the nonmagnetic impurities. Indeed, as it has been shown for the representative order parameters,[19,20] the gap anisotropy is smeared out by the isotropic scattering when $\langle e\rangle\neq 0$ and the density of states approaches that of an isotropic s-wave superconductor. Alternatively, for $\langle e\rangle=0$ we observe a strong impurity-induced suppression of the critical temperature[6,7] leading to a zero value at finite impurity concentration, which is reflected by a zero specific-heat jump limit value in Eq. (22). As a nonzero value of $\langle e\rangle$ can be achieved only when $e(\mathbf{k})$ contains a component belonging to an identity representation of the crystal point group, the measurement of the specific-heat jump at the phase transition in the limit of $T_c\to 0$ (and large impurity concentration) may be used as a stringent test for the occurrence of even a small A_{1g} admixture to the order parameter. It should be noted that the effect at large impurity concentration for $\langle e\rangle\neq 0$ [Eq. (21)] is independent of the amount of the s-wave content in the order parameter, however, as we discuss below, it may be hard to detect for a very small s-wave component as it would require an experiment at low temperatures.

We discuss our results in a context of high-T_c superconductivity, considering a $d_{x^2-y^2}$ state,[1] that is the order parameter given by Eq. (1) with $e(\mathbf{k})=(k_x^2-k_y^2)\langle(k_x^2-k_y^2)^2\rangle^{-1/2}$. As we have mentioned above, our main result that is the value of the specific-heat jump at $T_c\to 0$ is independent of the amplitude of the s-wave component and its origin. However, in order to establish a quantitative behavior of $\Delta C(T_c)$ in a whole range of impurity doping we must work with a particular level of s-wave admixture. We do this by assuming that the s-wave component is an artifact of an orthorhombic anisotropy of the system and relate the amount of the s-wave admixture to the degree of this anisotropy.[9,21] This approach gives a semimicroscopic justification for the $(d+s)$-wave state. The orthorhombicity in the case of YBCO means that the a- and b-crystal axes in the CuO_2 planes become inequivalent, which leads, with a simple approximation of an elliptical Fermi surface, to the following form of an energy band:[9]

$$\xi_{\mathbf{k}}=c_xk_x^2+c_yk_y^2-\epsilon_F, \tag{23}$$

where a ratio of the effective masses c_x/c_y is a dimensionless parameter describing the orthorhombic anisotropy of the Fermi surface and ϵ_F is the Fermi energy. It is easy to see within this model, that a $(d_{x^2-y^2}+s)$ state emerges from $d_{x^2-y^2}$ in a natural way due to the orthorhombic distortion of the crystal lattice. A straightforward calculation based on a transformation from an elliptical FS to a circular one shows that the normalized $d_{x^2-y^2}$ order parameter defined on the FS given by Eq. (23) can be represented on a circular FS as

$$\Delta(\mathbf{k})=\Delta\,\frac{1+(c_x/c_y)}{[(3/2)-(c_x/c_y)+(3/2)(c_x/c_y)^2]^{1/2}}\times\left[\cos 2\varphi+\frac{1-(c_x/c_y)}{1+(c_x/c_y)}\right], \tag{24}$$

where φ is the polar angle. In order to clarify the terminology, we will refer to the circular Fermi surface when classifying the superconducting states. Therefore, as a $d_{x^2-y^2}$ we define a state with $e(\mathbf{k})$ proportional to $\cos 2\varphi$ and the states with a nonzero s-wave contribution are called $(d_{x^2-y^2}+s)$. We note, that the order parameter from Eq. (24) is $d_{x^2-y^2}$ when $c_x/c_y=1$ only, that is for a tetragonal symmetry, otherwise it contains a nonzero s-wave component proportional to $(1-c_x/c_y)$. In Table I we present as the functions of the orthorhombic anisotropy parameter c_x/c_y the Fermi surface averages which enter the Ginzburg-Landau coefficients f_0 and f_1 given in Eqs. (15) and (16). We emphasize, that the assumption of the orthorhombic asymmetry as the mechanism producing the s-wave admixture in the order parameter does not affect the results since only the amplitude of this component matters in the calculation. Thus one can obtain the same results in a more phenomenological way assuming the presence of the s-wave phase and taking its level as given by $\langle e\rangle$ in Table I for the c_x/c_y values considered in this paper.[22]

Based on the discussion of the specific-heat jump for a large impurity concentration in Eqs. (21) and (22) we can discuss this limit for d- and $(d+s)$-wave superconductors. For a pure $d_{x^2-y^2}$ state $(c_x/c_y=1)$ we have $\langle e\rangle=0$. Therefore the specific-heat jump decreases to zero with a critical temperature driven to zero by impurities as in Eq. (22). On the other hand, even a slight s-wave component yields $\langle e\rangle\neq 0$ and the specific-heat jump increases and reaches a

TABLE I. The elliptical Fermi surface averages of the powers of the normalized order parameter $e(\mathbf{k})=(k_x^2-k_y^2)\langle(k_x^2-k_y^2)^2\rangle^{-1/2}$.

$\nu\left(\frac{c_x}{c_y}\right)$	$\left[\frac{3}{2}-\frac{c_x}{c_y}+\frac{3}{2}\left[\frac{c_x}{c_y}\right]^2\right]^{-1/2}$
$\langle e\rangle$	$\nu\left(\frac{c_x}{c_y}\right)\left[1-\frac{c_x}{c_y}\right]$
$\langle e^2\rangle$	1
$\langle e^3\rangle$	$\nu^3\left(\frac{c_x}{c_y}\right)\left[\frac{5}{2}\left[1-\left[\frac{c_x}{c_y}\right]^3\right]-\frac{3}{2}\frac{c_x}{c_y}\left[1-\frac{c_x}{c_y}\right]\right]$
$\langle e^4\rangle$	$16\nu^4\left(\frac{c_x}{c_y}\right)\left[\frac{35}{128}\left[1+\frac{c_x}{c_y}\right]^4-\frac{5}{4}\left[1+\frac{c_x}{c_y}\right]^3+\frac{9}{4}\left[1+\frac{c_x}{c_y}\right]^2 -2\left[1+\frac{c_x}{c_y}\right]+1\right]$

finite nonzero value at $T_c\to 0$ given by Eq. (21). Below, we present the specific-heat jump at the phase transition normalized by the specific heat in a normal state as a function of the normalized impurity scattering rate $\varrho_c T_c/T_{c_0}$ in the Born limit [Fig. 1(a)], where $\varrho_c T_c/T_{c_0}=\pi N_0 n_i V_i^2/(2\pi T_{c_0})$ and in the unitary limit [Fig. 1(b)] with $\varrho_c T_c/T_{c_0}=\Gamma/(2\pi T_{c_0})$. Note that $N_0=(c_x c_y)^{-1/2}S/(2\pi\hbar^2)$, where S is a sample surface area, hence T_{c_0} is different for different values of the $c_x c_y$ product. In the Figs. 2(a) and 2(b) we show the same $\Delta C(T_c)/C_N(T_c)$ data versus the normalized critical temperature T_c/T_{c_0}. The considered states contain a small s-wave admixture varying from about 8 to 16 %, therefore we observe a strong T_c suppression by the nonmagnetic impurities and a fast decrease in the specific-heat jump as long as a significant d-wave component is present. Once it is almost destroyed and the s-wave part, which is insensitive to the nonmagnetic defects, prevails, the BCS normalized specific-heat jump value of about 1.426 is restored in a sudden increase of $\Delta C(T_c)/C_N(T_c)$. The general tendency of the T_c suppression, given by Eq. (10) at $\Delta=0$, changes at that doping level too and the critical temperature asymptotically goes to zero (Fig. 3). For the sake of comparison we show in Fig. 4 the specific-heat jump $\Delta C(T_c)$ normalized by $C_N(T_c)$ as a function of the impurity scattering rate $\varrho_c T_c/T_{c_0}$ in the Born and unitary limits for the $(s+d_{x^2-y^2})$ state, where the s-wave component is large (~60%) and the $d_{x^2-y^2}$ part is considered as minor.

One can notice from the above figures that the unitary and Born scattering limits differ for small values of the pair-breaking parameter and fall on the same curve in the range where practically the s-wave superconductivity is only left. The pair-breaking parameter $\varrho_c T_c/T_{c_0}$, however, has a different meaning in either case.

We have mentioned before that a detection of a small s-wave component would need a measurement at low temperatures. For instance, in a superconductor of the critical temperature in a clean limit $T_{c_0}=90$ K an s-wave content of about 7.4% ($\langle e\rangle\simeq 0.074$) can be observed at a temperature

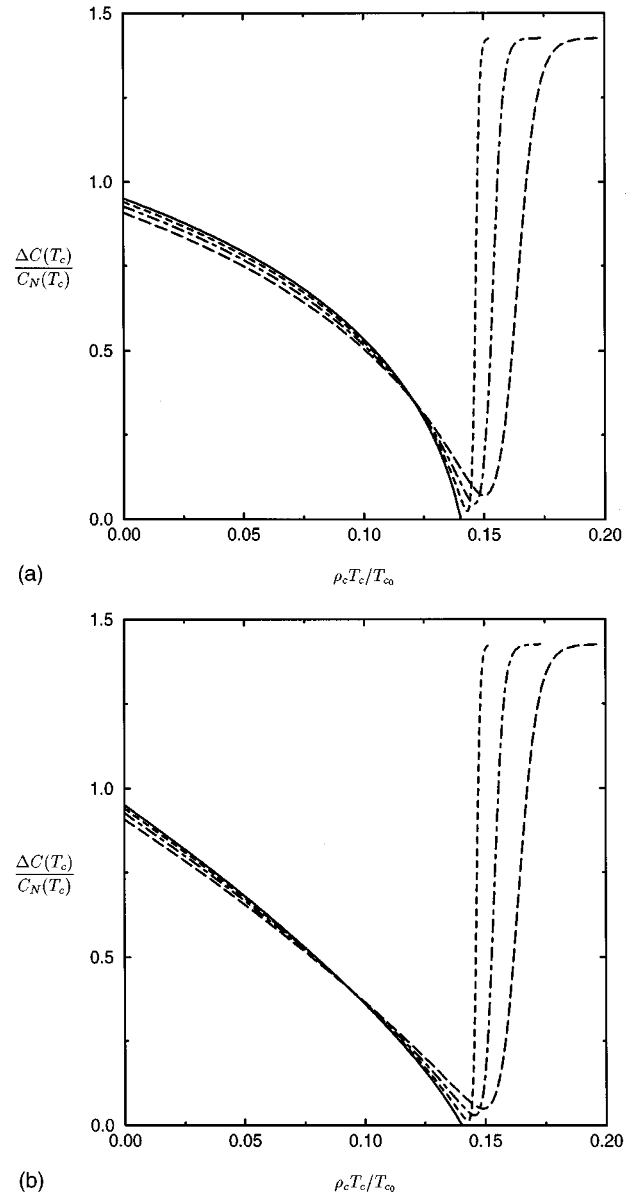

FIG. 1. Jump in specific heat at T_c normalized by the normal-state specific heat at T_c as a function of the normalized impurity scattering rate for $c_x/c_y=1$, i.e., $\langle e\rangle=0$ (solid), $c_x/c_y=0.9$, i.e., $\langle e\rangle\simeq 0.0742$ (short dashed), $c_x/c_y=0.85$, i.e., $\langle e\rangle\simeq 0.1139$ (dot dashed), $c_x/c_y=0.8$, i.e., $\langle e\rangle\simeq 0.1552$ (long dashed): (a) Born limit, (b) unitary limit.

of ~2.5 K, which is the estimated position of the $\Delta C(T_c)/C_N(T_c)$ minimum in Figs. 2(a) and 2(b). This minimum is a place where a distinct signal from the s-wave component appears, therefore its position is of special interest for possible experiments. We have found the minimum coordinates $(\varrho_c T_c/T_{c_0})^*$ (Fig. 5) and $(T_c/T_{c_0})^*$ (Fig. 6) as the functions of the order parameter FS average value $\langle e\rangle$, which multiplied by 100% gives the s-wave fraction in per cent in the normalized to unity order parameter $\langle e^2\rangle=1$. A plot of $(\varrho_c T_c/T_{c_0})^*$ vs $\langle e\rangle$ in Fig. 5 may also be of experimental use, since the scattering rate $\varrho_c T_c/T_{c_0}$ is proportional to the impurity concentration, which can be estimated in the measurements. As one can see from Figs. 5 and 6 the measurements at low temperatures are required for small s-wave ad-

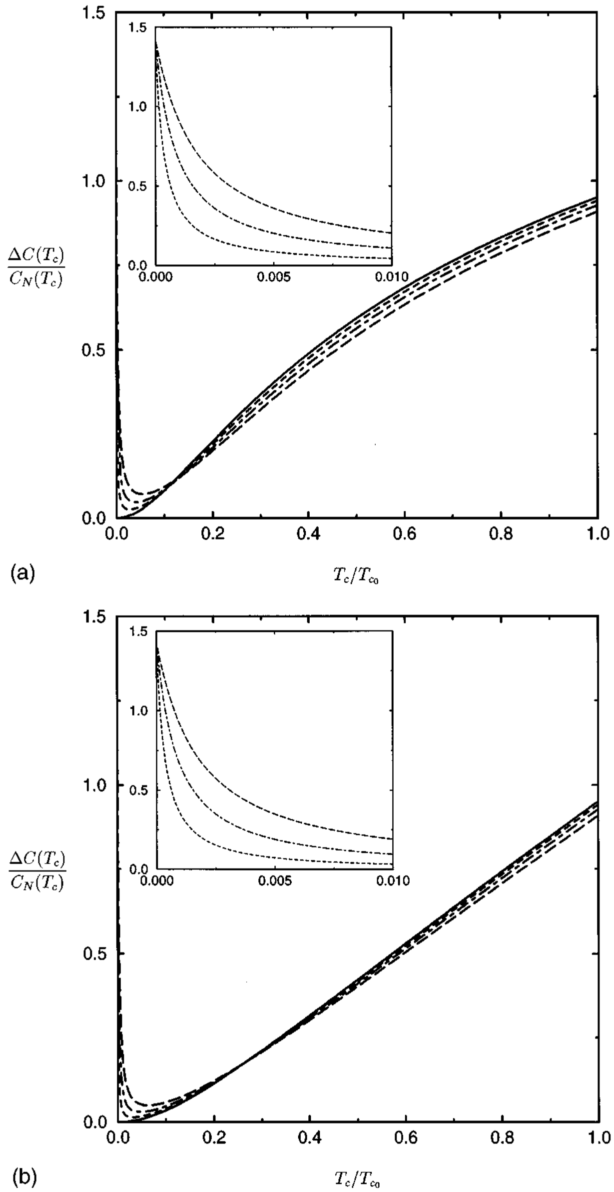

FIG. 2. Jump in specific heat at T_c normalized by the normal-state specific heat at T_c as a function of the normalized critical temperature T_c/T_{c_0} for $\langle e\rangle=0$ (solid), $\langle e\rangle\simeq 0.0742$ (short dashed), $\langle e\rangle\simeq 0.1139$ (dot dashed), $\langle e\rangle\simeq 0.1552$ (long dashed): (a) Born limit, (b) unitary limit. The insets show $\Delta C(T_c)/C_N(T_c)$ in the range of small T_c.

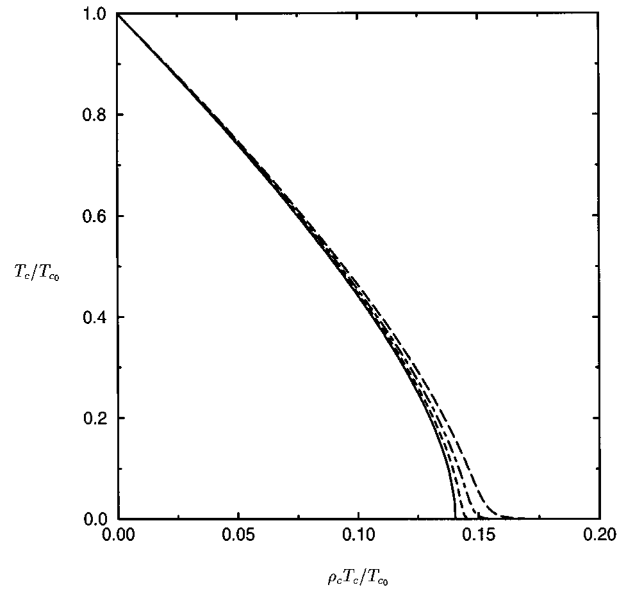

FIG. 3. Normalized critical temperature T_c/T_{c_0} as a function of the normalized impurity scattering rate for $\langle e\rangle=0$ (solid), $\langle e\rangle\simeq 0.0742$ (short dashed), $\langle e\rangle\simeq 0.1139$ (dot dashed), $\langle e\rangle\simeq 0.1552$ (long dashed).

mixtures, however, they are to be performed at the phase transition which should be accessible as long as T_c is measurable. Assuming that a possible s-wave admixture is of the order of magnitude of the experimental resolution error ($\sim$2.5 meV) in the ARPES measurements[5] of the smallest energy gap values, we can estimate its fraction as a ratio 2.5 meV/34 meV $\simeq$0.07, where 34 meV is a measured maximum $|\Delta|$ value. Therefore from Fig. 6 we find that the abrupt rise in the normalized specific-heat jump should be observed at $T_c\sim 2.5$ K in a superconductor of critical temperature in the absence of impurities $T_{c_0}=90$ K. Experiments[23] investigating the disorder effect on the specific-heat jump at T_c in YBCO show $\Delta C(T_c)/T_c$ suppression to zero with the increasing impurity concentration. However, the magnetic defects, which act as the pair breakers on both the d-wave and the s-wave states, were probably present in these studies.

It is noteworthy that the effect of an abrupt rise in the specific-heat jump at T_c may be observed even in the purely d-wave superconductors in the presence of a perpendicular magnetic field ($\mathbf{H}\|c$ axis). The s-wave component in this case may be induced by the vortices.[24]

We have derived the specific-heat jump from a mean-field weak-coupling theory, neglecting the fluctuations and the strong-coupling effects. As the observation of thermody-

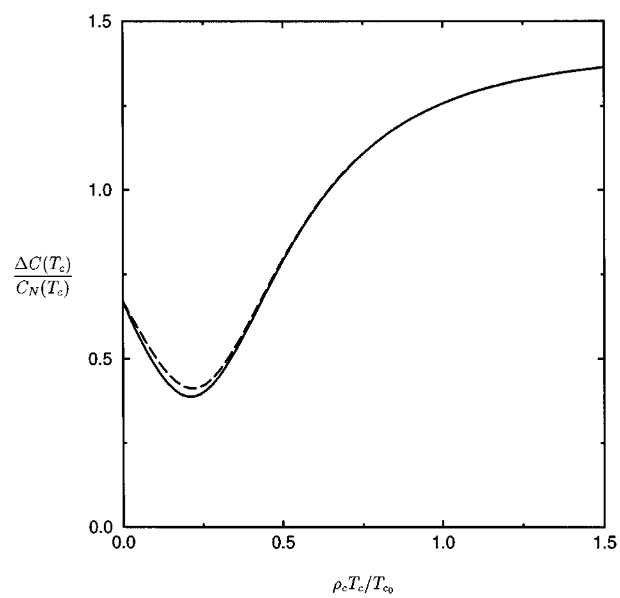

FIG. 4. Jump in specific heat at T_c normalized by the normal-state specific heat at T_c as a function of impurity scattering rate for $\langle e\rangle\simeq 0.6058$ ($c_x/c_y=0.3$) in the Born (dashed) and unitary (solid) limits.

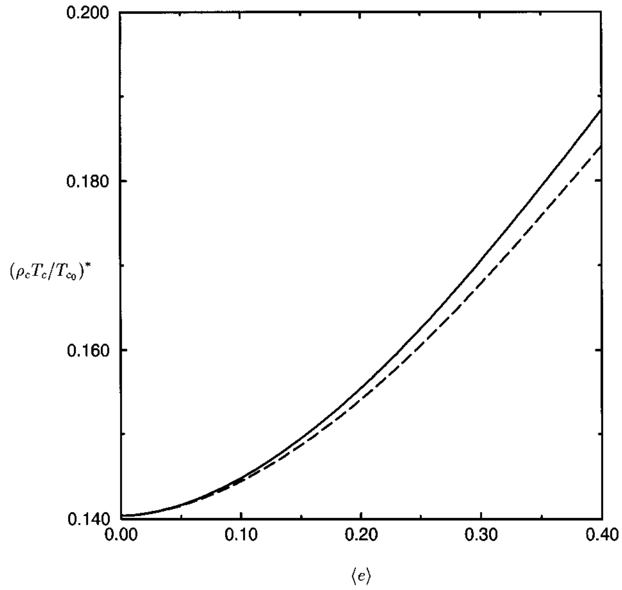

FIG. 5. Position of the minimum in the normalized specific-heat jump $\Delta C(T_c)/C_N(T_c)$ on $\varrho_c T_c/T_{c_0}$ axis [Figs. 1(a) and 1(b)] as a function of the s-wave component content $\langle e \rangle$ in the Born (dashed) and unitary (solid) limits.

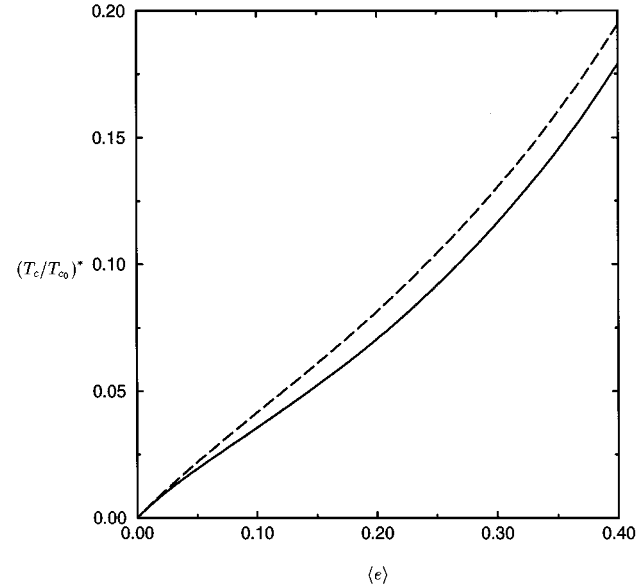

FIG. 6. Position of the minimum in the normalized specific-heat jump $\Delta C(T_c)/C_N(T_c)$ on T_c/T_{c_0} axis [Figs. 2(a) and 2(b)] as a function of the s-wave component content $\langle e \rangle$ in the Born (dashed) and unitary (solid) limits.

namic fluctuations in the specific heat of crystals of YBCO has been reported,[25] we expect our BCS result to be modified by the deviations from the mean-field approximation. We hope, however, that the feature of a sharp upturn in the specific heat will be still present. The strong-coupling corrections will rescale the scattering rates[7] and may change the magnitude of the specific-heat jump.[26,27]

In conclusion, we have calculated the electronic specific-heat difference between the superconducting and normal state at the phase transition as a function of the nonmagnetic impurity scattering rate in the general case of an anisotropic superconductor. We have found that the result depends on the symmetry of the order parameter, given by a function $e(\mathbf{k})$, and that of the Fermi surface through the FS averages $\langle e \rangle$, $\langle e^3 \rangle$, and $\langle e^4 \rangle$. A remarkably different dependence of the specific-heat jump on the impurity concentration for the systems with $\langle e \rangle = 0$ and $\langle e \rangle \neq 0$ is observed. We suggest that this effect may be used as a test for the s-wave component in the order parameter of the cuprates.

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