Coherent charge transport in metallic proximity structures

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We develop a detailed microscopic analysis of electron transport in normal diffusive conductors in the presence of proximity-induced superconducting correlations. We calculated the linear conductance of the system, the profile of the electric field, and the densities of states. In the case of transparent metallic boundaries the temperature-dependent conductance has a nonmonotonic ''reentrant'' structure. We argue that this behavior is due to nonequilibrium effects occurring in the normal metal in the presence of both superconducting correlations and the electric field. Low transparent tunnel barriers suppress nonequilibrium effects and destroy the reentrant behavior of the conductance. If the wire contains a loop, the conductance shows Aharonov-Bohm oscillations with a period $\Phi_0 = h/2e$ as a function of the magnetic flux Φ inside the loop. The amplitude of these oscillations also demonstrates the reentrant behavior. It vanishes at $T=0$ and decays as $1/T$ at relatively large temperatures. The latter behavior is due to low-energy correlated electrons which penetrate deep into the normal metal and ''feel'' the effect of the magnetic flux Φ. We point out that the density of states and thus the ''strength'' of the proximity effect can be tuned by the value of the flux inside the loop. Our results are fully consistent with recent experimental findings. [S0163-1829(97)02002-X]

I. INTRODUCTION

Recent progress in nanolithographic technology has revived interest in both experimental and theoretical investigations of electron transport in various mesoscopic proximity systems consisting of superconducting and normal metallic layers. In such systems the Cooper pair wave function of a superconductor penetrates into a normal metal at a distance which increases with decreasing temperature.[1] At sufficiently low temperatures this distance becomes large and the whole normal metal may acquire superconducting properties. Although this phenomenon was already understood more than 30 years ago and intensively investigated during the past decades, recently novel physical features of metallic proximity systems have been discovered[2–8] and studied theoretically (see Refs. 9–21 and further references therein).

In this paper we study the influence of the proximity effect on the transport properties of a diffusive conductor in the limit of relatively low temperatures and voltages. We will assume that this conductor is brought in direct contact with a superconducting reservoir which serves as an effective injector of Cooper pairs into a normal metal. We will show that if the system contains no tunnel barriers, there are two different physical regimes which determine the system conductance in different temperature intervals. It is well known that the proximity-induced superconducting correlation between electrons in a diffusive normal metal survives at a distance of order $\xi_N \sim \sqrt{\mathcal{D}/T}$, where $\mathcal{D} = v_F l_{\mathrm{imp}}/3$ is the diffusion coefficient. As T is lowered the proximity-induced

superconductivity expands into the normal metal and, consequently, the ''normally conducting'' part of the system effectively shrinks in size. This effect results in an increase of the conductance of a normal metal. At sufficiently low temperature the length ξ_N becomes of the order of the size of the normal layer and the system behavior becomes sensitive to the physical choice of the boundary condition at the edge of the normal wire opposite to that attached to a superconductor.

One possible choice of this boundary condition corresponds to the assumption that a nontransparent barrier is present at the edge of this wire. Then electrons cannot diffuse out of the wire, the proximity-induced superconducting correlation survives everywhere in the system, and a real gap in the quasiparticle spectrum develops in the N metal.[22] The value of this gap is of the order of $\varepsilon_g \sim \min(\Delta, \mathcal{D}/L^2)$, where Δ is the bulk superconducting gap and L is the length of the normal wire.

Another possible situation corresponds to the presence of a large normal reservoir N' directly attached to the N wire by means of a highly transparent contact. In this case even at very low T the proximity-induced Cooper pair amplitude is essentially nonhomogeneous in the N metal. Indeed, close to a superconductor this amplitude is large, whereas in the vicinity of a normal reservoir it is essentially suppressed. Thus, strictly speaking, the whole N wire cannot be characterized by the real gap in its quasiparticle spectrum. In the absence of a potential barrier between N and N' this gap is obviously equal to zero at the NN' interface and, as will be demon-

strated, everywhere in the normal metal. Nevertheless, it turns out that the density of states in the N metal shows a soft pseudogap which is again of the order of ε_d. In other words, the spatially averaged normalized density of states $N_N(\varepsilon)$ in the N wire at small $\varepsilon \lesssim \varepsilon_d$ is smaller than its normal state value $N_N < N(0)$ but always remains nonzero. It increases with increasing ε and reaches the value $N_N = N(0)$ at $\varepsilon \gtrsim \varepsilon_d$. This is the key point for understanding the low-temperature behavior of the conductance of our system. As the temperature increases from zero, higher and higher values of ε contribute to the current, and the system conductance, due to the increase of N_N with ε, increases with T. This regime takes place until the temperature reaches the value $T \sim \varepsilon_d$ where the crossover to a high-temperature behavior takes place. Note that similar behavior of the normal metal conductance in the presence of proximity-induced superconductivity has been recently found by Nazarov and Stoof.[23]

An interesting feature of the system without tunnel barriers is that at $T=0$ its conductance *exactly* coincides with that of a normal metal with no proximity effects. This result has been first obtained by Artemenko, Volkov, and Zaitsev[24] for the case of a normal-superconducting constriction. Although this result has already been around for many years, the physical meaning of this result, if any, still needs to be understood. At first sight the linear conductance of the system at $T=0$ should be smaller than in the normal state because of the presence of the (pseudo)gap in the normal density of states (DOS) N_N at low energies. Why is this not the case?

In order to answer this question we should recall the well-known fact that in the presence of nonequilibrium effects the current flowing in a superconductor depends not only on the normal DOS but is characterized by a set of generalized DOS.[25] Our problem is just a particular example of a nonequilibrium superconductor: On the one hand, superconducting correlations penetrate into the normal metal and the Cooper pair amplitude is nonzero there; on the other hand, in the absence of low transparent tunnel barriers the electric field also penetrates into the N metal and drives the quasiparticle distribution function out of equilibrium. We will argue that in this situation one of the generalized DOS [below we define it as $N_S(\varepsilon)$], which is nonzero in the N layer due to the presence of proximity-induced superconducting correlations at low energies, plays an important role and also contributes to the system conductance. In other words, in the presence of the electric field inside the system *both uncorrelated and correlated electrons contribute to a dissipative current*. This is the reason why in the presence of proximity-induced superconductivity the system conductance is never smaller than its normal state value although the normal DOS $N_N(\varepsilon) < N(0)$ at low energies.[26]

We would like to emphasize that the situation is entirely different in the presence of low transparent tunnel barriers. Provided their resistances are much larger than that of the N metal the whole voltage drop takes place at these barriers and the electric field does not penetrate into the N layer. In this case only uncorrelated electrons contribute to the dissipative current and therefore only the normal DOS N_N matters. As a result the temperature dependence of the system conductance changes. We will demonstrate that with de-

creasing barrier transparency a crossover takes place in which the effective conductance decreases monotonically with T, characteristic of two serial NIS' tunnel junctions (S' is now the diffusive normal conductor with the proximity-induced gap).

Note that both types of behavior, namely, reentrant and monotonically decreasing with T conductance, have been observed in the experiments.[3,7,8] Furthermore, we would like to point out that both densities of states $N_N(\varepsilon)$ and $N_S(\varepsilon)$ can be investigated in one experiment. We will come back to this point further below.

When the system contains a mesoscopic loop of a normal metal, the conductance as a function of the magnetic flux through the loop shows oscillations with period $\Phi_0 = h/2e$ (superconducting flux quantum). Although the Cooper pair amplitude (and thus the supercurrent) in the ring is exponentially small at $T \gg \epsilon_d$, the amplitude of these oscillations decays only as $\propto 1/T$. This again illustrates an important difference in the behavior of kinetic (conductance) and thermodynamic (supercurrent) quantities. Below we will argue that in the systems considered here even at large $T \gg \epsilon_d$ the behavior of the first quantity is dominated by correlated low-energy electrons with $\epsilon \lesssim \epsilon_d$ penetrating far into the normal metal whereas the only important contribution of electrons with $\epsilon \sim T$ is to the second one. Again the presence of the electric field inside the N metal is crucially important for this effect. At low T the oscillation amplitude again shows the reentrant behavior and vanishes in the limit $T \to 0$ as T^2. Clear experimental evidence for a $1/T$ decay of the conductance oscillations has been recently reported in Ref. 6.

Finally we point out that by making use of the geometry with a metallic loop one can easily tune the densities of states of the system by applying a magnetic flux Φ inside this loop. We will show that, e.g., for $\Phi = \Phi_0/2$ the proximity effect in the normal region "after" the loop is completely suppressed and the normal DOS $N_N(\epsilon) = N(0)$ there. This effect can be investigated experimentally and used for further studies of proximity-induced superconductivity in normal metallic structures.

The structure of our paper is as follows. In Sec. II we briefly describe the general kinetic approach based on quasi-classical Green functions in the Keldysh technique and define the physical quantities of interest. Then a detailed analysis of these quantities [conductance (Sec. III), DOS (Sec. IV), and electric field (Sec. V)] will be presented. Sections VI and VII are devoted to the extension of our analysis to the proximity systems containing mesoscopic normal metal loops with a magnetic flux. The main results of the present paper are summarized in Sec. VIII. Further details related to different geometric realizations of the proximity systems with loops are presented in the Appendix.

II. KINETIC ANALYSIS

A. General formalism

Let us consider a quasi-one-dimensional normal conductor of length $2L$ with a superconducting strip of thickness $2d_s$ attached to a normal metal on top of it and two normal reservoirs attached to its ends (see Fig. 1). The length L is
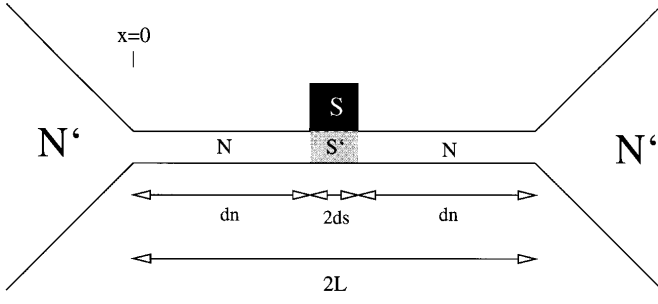
FIG. 1. The experimental system under consideration.

assumed to be much larger than the elastic mean free path l_{imp} but much shorter than the inelastic one. This geometrical realization has a direct relation to that investigated in the experiments.[3,5,8] Two large normal reservoirs N' are assumed to be in thermodynamic equilibrium at the potentials V and 0, respectively. In contrast to the case of a ballistic constriction[27,28] the potential drop within the system is distributed between the interfaces and the conductor itself. The general approach to calculate the conductance of these structures was developed in Refs. 9, 10, and 12. In what follows we shall apply this method to analyze the temperature dependence of the NS proximity structure of Fig. 1.

Such an experimental realization allows one to prepare a structure without effective tunnel barriers in the direction of the current flow. Even with ''perfect'' samples in a usual sandwich geometry, a natural barrier shows up due to the inevitable mismatch of Fermi velocities between different materials. This could well be one of the reasons why in previous experiments with sandwichlike structures the reentrant behavior of the conductance was not detected.[29]

The electron transport through the metallic system can be described by the equations for a matrix of quasiclassical Green functions $\check{G}$ in the contact:[30,31]

$$\check{G} = \begin{pmatrix} \hat{G}^R & \hat{G}^K \\ 0 & \hat{G} \end{pmatrix}, \tag{1}$$

where $\hat{G}^A$, $\hat{G}^R$, and $\hat{G}^K$ are, respectively, the impurity-averaged advanced, retarded, and Keldysh Green functions. These functions are in turn matrices in the Nambu space:

$$\hat{G}^R = \hat{\sigma}_z g^R + i \hat{\sigma}_y f^R, \qquad \hat{G}^A = -(\hat{G}^R)^*$$

and

$$\hat{G}^K = \hat{G}^R \hat{f} - \hat{f} \hat{G}^A.$$

Here the distribution function $\hat{f} = f_l + \hat{\sigma}_z f_t$, where $f_l = \tanh(\varepsilon/2T)$ and f_t describes deviation from equilibrium. Taking advantage of the normalization condition for the normal and the anomalous Green functions $(g^R)^2 - (f^R)^2 = 1$ it is convenient to parametrize $g^R = \cosh\theta$, $f^R = \sinh\theta$, where $\theta \equiv \theta_1 + i\theta_2$ is a complex function. Deep in the bulk superconductor it is equal to $\theta_s = 1/2\ln[(\Delta+\varepsilon)/(\Delta-\varepsilon)] - i\pi/2$ for $\varepsilon < \Delta$ and $\theta_s = (1/2)\ln[(\varepsilon+\Delta)/(\varepsilon-\Delta)]/2$ for $\varepsilon > \Delta$ [here and below we omit the indices $R(A)$].

The current I and the electrostatic potential ϕ are expressed through $\check{G}$ as

$$I = \frac{\nu DS}{2} \int_{-\infty}^{\infty} d\varepsilon \, \mathrm{Sp}[\hat{\sigma}_z \check{G} \partial_x \check{G}]^K, \tag{2}$$

$$\phi(x) = \int_0^{\infty} d\varepsilon \, \mathrm{Tr}\hat{g}^K(x,\varepsilon) = \int_0^{\infty} d\varepsilon f_t(x,\varepsilon) \nu_\varepsilon(x), \tag{3}$$

where ν is the the density of states, $\nu_\varepsilon(x) = \mathrm{Re}[g_\varepsilon^R(x)]$, and S is the cross-sectional area of the N conductor.

Being expressed in terms of the function $\theta(\varepsilon,x)$ the equations[30,31] for the Green functions and the distribution function for the N metal take a particularly simple form

$$D\partial_x^2 \theta + 2i\varepsilon \sinh\theta = 0, \tag{4}$$

$$\partial_x[D(\cosh^2\theta_1)\partial_x f_t] = 0, \tag{5}$$

x is the coordinate along the N conductor. Here we neglected the processes of inelastic relaxation and put the pair potential in the normal metal equal to zero $\Delta_N = 0$ assuming the absence of electron-electron interactions there.

Before we come to a detailed solution of the problem let us point out that the conclusion about the anomalous behavior of the system conductance can be reached already from the form of Eq. (5). Indeed it is quite clear from Eq. (5) that the effective diffusion coefficient $D_{eff} = D\cosh^2\theta_1$ increases in the N regions with proximity-induced superconductivity and, therefore, the electric field is partially expelled from these regions. This energy-dependent field modulation is controlled by the solution for $\theta(\varepsilon,x)$ and is directly related to the physical origin of the anomalous temperature dependence of the system conductance discussed below.

Equations (4) and (5) should be supplemented by the boundary conditions at the interfaces of the normal metal N. Assuming that the anomalous Green function of the large normal reservoirs N' is equal to zero from[32,10] we obtain

$$\xi_N^* \gamma_B \partial_x \theta = \pm \sinh\theta,$$

$$\xi_N^* \gamma_B \cosh\theta_1 \partial_x f_t = \pm \cosh\theta_2 (f_t - f_t(x=0,2L)), \tag{6}$$

where $\gamma_B = R_b/\rho_N \xi_N^*$ is the interface resistance parameter, R_b is the specific resistance of the interface between the N conductor and the N' reservoirs, ρ_N is the resistivity of the N metal, and $\xi_N^* = \sqrt{D_N/2\pi T_c}$ is the temperature-independent characteristic length scale in N [note that the coherence length in N, $\xi_N(T) = \sqrt{D_N/2\pi T}$, is T dependent].

In general we should also fix the boundary condition at the interface between the N metal and the superconductor. For the case of perfect transparency of this interface and for a typical thickness of the normal layer $w_N \sim \sqrt{S}$, Cooper pairs easily penetrate into it due to the proximity effect and the Green functions of the N metal at relatively low energies for $d \leq x \leq d + 2d_s$ are equal to those of a bulk superconductor $\theta = \theta_s$ (the influence of finite transparency of the NS contact will be discussed below). In this sense the region of a normal metal situated directly under the superconductor can be also treated as a piece of a superconductor S' and the solution of

Eqs. (4) and (5) needs to be found only for $0 < x < d$ (without loss of generality we will stick to a symmetric configuration).

Cooper pairs penetrate into the normal conductor also in the case of an imperfectly transparent NS interface. As demonstrated below, the energy gap is induced in the S' region in this case. As a result, for a sufficiently long N wire, which is only considered here, the presence of the barrier at the NS interface will not influence the results derived for the system conductance.

B. Physical quantities of interest

Proceeding along the same lines as in Ref. 12 we arrive at the final expression for the current,

$$I = \frac{1}{2R} \int_0^\infty d\varepsilon \left[\tanh\left(\frac{\varepsilon + eV}{2T} \right) - \tanh\left(\frac{\varepsilon - eV}{2T} \right) \right] D(\varepsilon), \quad (7)$$

where $D(\varepsilon)$ defines the effective transparency of the system,[12]

$$D(\varepsilon) = \frac{1 + r}{r/[\cosh\theta_1(x=0,\varepsilon)\cos\theta_2(x=0,\varepsilon)] + (1/L)\int_0^L dx\, \mathrm{sech}^2\theta_1(x,\varepsilon)}, \quad (8)$$

where $R = R_b + R_N$, $r = R_b/R_N \equiv \gamma_B \xi_N^*/L$, and R_N is the resistance of the N metal.

Let us consider the case of a sufficiently long normal conductor $d^2 \gg \mathcal{D}/\Delta$. Then at low temperatures $T \ll \Delta$ the interesting energy interval is restricted to $\varepsilon \ll \Delta$. For such values of ε the contribution of the S' part of the normal conductor shows no structure and can be easily taken into account with the aid of the obvious relations

$$\int_0^L dx\, \mathrm{sech}^2\theta_1(x,\varepsilon) = \int_0^d dx\, \mathrm{sech}^2\theta_1(x,\varepsilon) + d_s \mathrm{sech}^2\theta_{s,1} \quad (9)$$

and $\mathrm{sech}^2\theta_{s,1} = (1 - \varepsilon^2/\Delta^2)$ (no barrier at the NS interface) or $\mathrm{sech}^2\theta_{s,1} = (1 - \varepsilon^2/\Delta_{gN}^2)$ (the barrier is present at the NS interface). Due to this, we will discuss only the properties of the N part ($0 < x < d$). For the sake of completeness we will also demonstrate the effect of finite d_s at the end of our calculation.

For the differential conductance of the N part $0 \leq x \leq d$ normalized to its normal ("nonproximity") value in the zero bias limit Eq. (7) yields

$$\bar{G}_N = \left(\frac{R\, dI}{dV} \right)_{V=0} = \frac{1}{2T} \int_0^\infty d\varepsilon\, D(\varepsilon) \mathrm{sech}^2(\varepsilon/2T). \quad (10)$$

Analogously the normalized zero-bias electrostatic potential distribution reads

$$\phi_0(x) = \lim_{V \to 0} \frac{\phi(x)}{V}$$

$$= \frac{1}{2Td} \int_0^\infty d\varepsilon\, D(\varepsilon) \nu_\varepsilon(x) \mathrm{sech}^2(\varepsilon/2T)$$

$$\times \int_x^d dx'\, \mathrm{sech}^2[\theta_1(x')]. \quad (11)$$

The normal density of states is given by the normal Green's function via the standard relation $\nu_\epsilon(x) = N(0)\mathrm{Re}[g_\epsilon(x)]$ which enters into the conductance in the form $\cosh^2\theta_1 = (\mathrm{Re}\,g)^2 + (\mathrm{Im}\,f)^2$ together with a "correlation DOS" $\eta_\epsilon(x) = N(0)\mathrm{Im}[f_\epsilon(x)]$. The importance of the latter

quantity for understanding the effects discussed here has been already pointed out in the Introduction. We will discuss the features of these local densities as well as the averaged ones:

$$N_N(\epsilon) = \int d\bar{x}\,\nu_\epsilon(\bar{x}), \quad N_S(\epsilon) = \int d\bar{x}\,\eta_\epsilon(\bar{x}).$$

As already mentioned the "correlation DOS" η belongs to the set of generalized densities of states familiar from the standard theory of nonequilibrium superconductivity.[25,34] It reflects the presence of superconducting correlations at low energies. E.g., in a BCS superconductor this function reads

$$\eta = \frac{\Delta\,\Theta(\Delta - \epsilon)}{\sqrt{\Delta^2 - \epsilon^2}}$$

.

In our case this function is not only energy, but also space dependent due to the fact that the proximity-induced superconducting correlation decays inside the normal metal. But its physical meaning remains the same as in standard nonequilibrium superconductivity theory:[25] η plays a role whenever the quasiparticle distribution function of a superconductor is driven out of equilibrium. This happens, e.g., in the well-known problems of charge relaxation[34] and charge imbalance.[35] It happens also here due to a simultaneous presence of the electric field and the proximity-induced superconducting correlation in the normal metal.

C. Influence of finite barrier transparency at the top NS interface

Let us consider the effect of a tunnel barrier at the NS interface in more detail.

Under the assumption that the N wire thickness is small $w_N \ll \xi_N^*$ the equation for θ in the region $0 \leq x \leq 2d_s$ underneath the superconducting terminal can be derived by the method of Ref. 36:

$$\mathcal{D}\partial_x^2\theta + 2i\tilde{\varepsilon}\sinh\theta + \tilde{\Delta}\cos\theta = 0, \quad (12)$$

where the effective order parameter $\tilde{\Delta} = \sin\theta_s/\gamma_B^{NS}$ the effective energy $\tilde{\varepsilon} = \varepsilon + \cos\theta_s/\gamma_B^{NS}$ and $\gamma_B^{NS} = (R_b/\rho_N)(w_N/\xi_N^{*2})$

is the interface transparency parameter. Here θ_s is the solution in S which is set equal to the bulk value $\tan^{-1}(i\Delta/\varepsilon)$, a good approximation for thin N film, $w_N \ll \xi_N^*$. With these substitutions Eq. (12) in the N film has a form similar to that in a superconductor. This equation is valid for $\gamma_B^{NS} > (w_N/\xi_N^*)^2$, i.e., for sufficiently small transparency of the NS interface: $\langle D \rangle < l_N/w_N$.

As follows from Eq. (12), superconducting properties of the N layer are described in terms of the energy-dependent coherence length

$$\xi_N(\epsilon) = \{\hbar D_N/2[\gamma_B^{-2} - \epsilon^2 - 2i\gamma_B^{-1}\epsilon\cos\theta_S]^{1/2}\}^{1/2}, \quad (13)$$

which determines an exponential decay of $N_N(x,\epsilon)$ with x .

The expression for $\xi_N(\epsilon)$ has a pole at the gap energy $\epsilon = \Delta_{gN}$, which signals the decay of quasiparticles entering N at $\epsilon < \Delta_{gN}$. At high energy $\epsilon \gg \pi T_c/\gamma_B^{NS}$ the well-known result $\xi_N(\epsilon) = (\hbar D_N/2\epsilon)^{1/2}$ is reproduced, whereas at low energies $\epsilon \ll \pi T_c/\gamma_B^{NS}$ one obtains $\xi_N = (\hbar D_N\gamma_B^{NS}/2\pi T_c)^{1/2}$. Thus the effective length scale in N increases with the decrease of the SN interface transparency.

It is straightforward to calculate the gap energy Δ_{gN} assuming the ''rigid'' boundary conditions $\theta_s = \tan^{-1}(i\Delta/\varepsilon)$ either from the pole of $\xi_N(\epsilon)$ or, making use of the solution $\theta = \tanh^{-1}[\sinh\theta_s/(\cosh\theta_s - i\varepsilon\gamma_B^{NS})]$ and calculating the quasiparticle density of states $N_N(\varepsilon) = \mathrm{Re}(\cosh\theta)$. Subsituting the expression for θ_s into this solution one arrives at the equation for the energy gap

$$t^3 + 2Ct^2 + (C^2 - 1)t - 2C = 0, \quad (14)$$

where $t = \sqrt{1 - (\Delta_{gN}/\Delta)^2}$ and $C = \pi T_c/\gamma_B^{NS}\Delta$. The general solution is rather cumbersome; therefore here we present only its asymptotic forms.

The gap is given by $\Delta_{gN}/\Delta = 1 - 2(\gamma_B^{NS}\Delta/\pi T_c)^2$ for large transparency of the NS interface, $\gamma_B^{NS}\Delta/\pi T_c \ll 1$, and by $\Delta_{gN} = \Delta/(1 + \gamma_B^{NS}\Delta/\pi T_c)$ for small transparency, $\gamma_B^{NS}\Delta/\pi T_c \gg 1$. In the latter case (the McMillan limit), the expression for the gap may be written as $\Delta_{gN} = \pi T_c/\gamma_B^{NS} \equiv \hbar v_{FN}\langle D \rangle/4w_N$. The gap originates from the finite average lifetime $\tau_N = 2w_N/\langle D \rangle v_{FN}$ for quasiparticles in the N layer with respect to Andreev scattering from S, since a contribution of gapless quasiparticle trajectories parallel to the NS interface is eliminated in the diffusive regime. The dependence of Δ_{gN} on γ_B^{NS} in the whole range of transparencies is presented in Fig. 2. Note that under a substitution $w_N = \pi/k_F$ the above expressions reproduce the result of Ref. 33 for a gap induced in a two-dimensional electron gas in contact with a superconductor. We also note that in the case of two superconducting terminals attached to N the gap aquires a phase factor $\Delta_{gN} \to \cos(\varphi/2)\Delta_{gN}$, φ being the phase difference between the terminals.

At the subgap energies $\varepsilon \leq \Delta_{gN}$ the solution in N is $\theta \simeq 1/2\ln[(\Delta_{gN}+\varepsilon)/(\Delta_{gN}-\varepsilon)] - i\pi/2$ whereas above the gap, at $\varepsilon > \Delta_{gN}$, $\theta \simeq 1/2\ln[(\varepsilon+\Delta_{gN})/(\varepsilon-\Delta_{gN})]$, in complete analogy with the solutions in a bulk superconductor but with reduced gap. Therefore the results for conductance of N depend on the relation between the gap Δ_g and the Thouless energy $\varepsilon_d = D/d^2$. Since for the case of a sufficiently long N wire, which is the only one considered in this paper, the condition
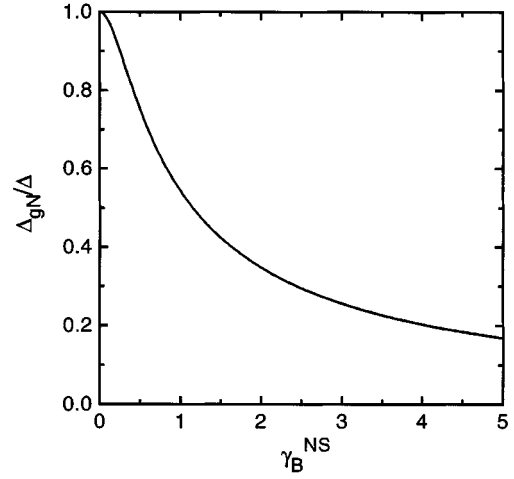

FIG. 2. Reduced gap on the normal side of the NS boundary in the presence of barriers.

$\Delta_{gN} \gg \varepsilon_d$ is satisfied, the presence of the barrier at the NS interface does not influence our results for the system conductance.

III. CONDUCTANCE

A. Perfectly transparent boundaries

The analysis of the problem can be significantly simplified in the case of perfectly transparent interfaces ($\gamma_B = 0$). In this case the boundary conditions are

$$\theta(0) = 0, \quad (15)$$

$$\theta(d) = \theta_S, \quad (16)$$

for the contact to the normal and the superconducting reservoirs, respectively. The effective transparency of the N part then reads

$$D(\varepsilon) = \left(\frac{1}{d}\int_0^d dx\,\mathrm{sech}^2[\theta_1(x)]\right)^{-1}. \quad (17)$$

As was already pointed out for relatively long normal conductors and at low T only the energies $\varepsilon \ll \Delta$ give an important contribution to the conductance. In this case the typical energy scale is defined by the Thouless energy $\epsilon_d = D/d^2 \ll \Delta, \Delta_{gN}$. For these energies we can set $\theta_S = -i\pi/2$. Let us first put $T = 0$. Then the thermal distribution factor $\mathrm{sech}^2(\varepsilon/2T)/(2T)$ reduces to a δ function and we have

$$\bar{G}_N(T=0) = D(0); \quad (18)$$

i.e., we only need the solution of Eq. (4) with boundary conditions (15) at $\varepsilon = 0$, which is $\theta = -i(\pi/2)\bar{x}$. This does not depend on $\mathcal{D}$, and so the correlations are destroyed by the influence of the boundary conditions but not by thermal excitation or by impurity scattering. From here, we can calculate the conductance

$$\bar{G}_N(T=0) = 1; \quad (19)$$

i.e., at $T=0$ the system conductance exactly coincides with its normal state value (cf. Refs. 24 and 23). This result, however, by no means implies the destruction of the proximity-induced superconductivity in the N layer. Later on, we will demonstrate that the DOS and the electrical field are completely different from their values in the normal state and in fact only due to the additional contribution of correlated electrons the system conductance does not fall below its normal state value.

In the case $T \ll \varepsilon_d$ we can calculate θ perturbatively. From $\mathcal{D}\partial_x^2\theta = -2i\varepsilon\sinh\theta_0(x)$ and Eq. (15) we get

$$\theta = -\frac{8}{\pi^2}\frac{\varepsilon}{\varepsilon_d}[\bar{x} - \sin(\bar{x}\pi/2)] - i\frac{\pi}{2}\bar{x}.$$

Keeping only leading order terms in ϵ/ϵ_d, we get

$$\bar{\sigma}_N = 1 + A\frac{T^2}{\epsilon_d^2}, \qquad (20)$$

where

$$A = \frac{64}{3\pi^4}\left(\frac{5}{6} - \frac{8}{\pi^2}\right) \approx 0.049$$

is a universal constant. This means that for low temperatures $\bar{G}_N(T)$ grows quadratically on the scale of ε_d and approaches the crossover towards the high-temperature regime discussed below.

In the limit $T \gg \epsilon_d$ (where we still have $T \ll \Delta$), the contribution of the low-energy components to the thermally weighted integral for $\bar{G}_N(T)$ is $\propto 1/T$ as we will see below and can therefore be neglected. We only have to take into account the solutions of Eq. (4) for energies $\epsilon \gg \epsilon_d$. It is well known (see, e.g., Ref. 12), that for this energy range the solution of Eq. (4) together with Eq. (15) reads

$$\tanh[\theta(\bar{x})/4] = \tanh\left(\frac{i\pi}{8}\right)e^{k(\bar{x}-1)}, \qquad (21)$$

where $k = d\sqrt{-2i\varepsilon/\mathcal{D}}$. By using obvious substitutions and multiple-argument relations for hyperbolic functions, we arrive at the following identity:

$$\int_{\bar{x}}^1 d\bar{x}\,\overline{\text{sech}}^2[\theta_1(\bar{x})] = (1-\bar{x}) - 4\sqrt{\frac{\varepsilon_d}{\varepsilon}}$$

$$\times \int_0^{\text{Re}(k)(1-\bar{x})} \frac{q(y)dy}{[1+q(y)]^2}, \qquad (22)$$

where $q(y) = 4(3+2\sqrt{2})e^{-2y}\sin^2 y/(e^{-2y}+3+2\sqrt{2})^2$.

For calculating $D(\varepsilon)$ we can, as the integrand becomes exponentially small for $y \gtrsim \text{Re}(k) \gg 1$, take the upper bound to infinity, such that it becomes a universal constant. From there we can calculate the conductance in this limit

$$\bar{G}_N(T) = 1 + B\sqrt{\frac{\varepsilon_d}{T}}, \qquad (23)$$

where again $B = 0.42$ is a universal constant.

These results have a simple physical interpretation. Superconductivity penetrates into the normal part up to $\xi_N = \sqrt{D/2\pi T}$, whereas the rest stays normal, and so the total
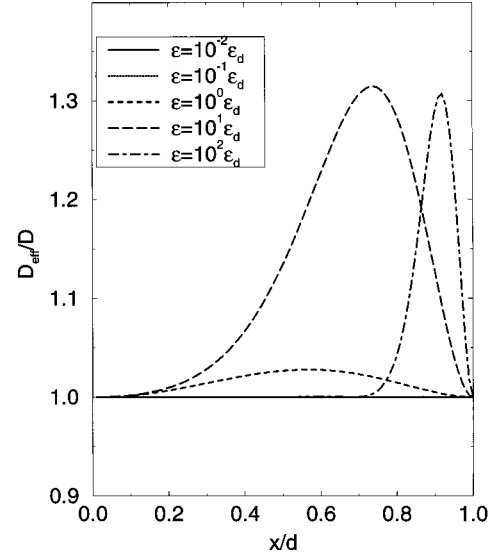

FIG. 3. Local effective diffusion constant.

voltage drops over a reduced distance $d - \xi_N$. Thus the resistance of the structure is reduced according to Ohm's law. In terms of the conductance, this means

$$\bar{G}_N = 1 + B'\frac{\xi_N}{d}, \qquad (24)$$

which is equivalent to Eq. (23).

Let us point out that at both ends of the N metal the local effective diffusion constant $\mathcal{D}_{\text{eff}} = \cosh^2\theta_1\mathcal{D}$ is not enhanced (see Fig. 3) in comparison to its normal state value, because either the Cooper pair amplitude (at the NN' boundary) or the electric field (at the NS boundary) is equal to zero due to the imposed boundary conditions. Inside the N metal the value $\mathcal{D}_{\text{eff}}$ becomes higher due to nonequilibrium effects in the presence of superconducting correlations ($\eta \neq 0$). This effect is small at very low energies and becomes more pronounced at $\varepsilon \sim \varepsilon_d$.

For temperatures comparable to ε_d the problem was treated numerically. The results show excellent agreement with our analytical expressions obtained in the corresponding limits.

The numerical results (see Fig. 4) confirm that for $\varepsilon_d \ll \Delta$ the universal scaling with T/ε_d is excellently fulfilled, the conductance peak with the height of about 9% taking place at $T \approx 5\varepsilon_d$ (cf. Ref. 23). This peak becomes smaller if we take into account the influence of finite d_S keeping d fixed (Fig. 5). The qualitative features, however, remain the same.

B. Tunnel barriers

Let us now assume that a tunnel barrier, is present at the N'-N interface. If one lowers the transparency of this barrier a crossover takes place to the behavior, demonstrating monotonically decreasing conductance with T (Fig. 6), which is typical for two serial NIS tunnel junctions. Figure 6 demonstrates the crossover with increasing $r = \gamma_B\xi_N^*/d$. The inset shows the Arrhenius plot for the case of $\gamma_B\xi_N^*/d \gg 1$ which illustrates the activated tunnel-like behavior.
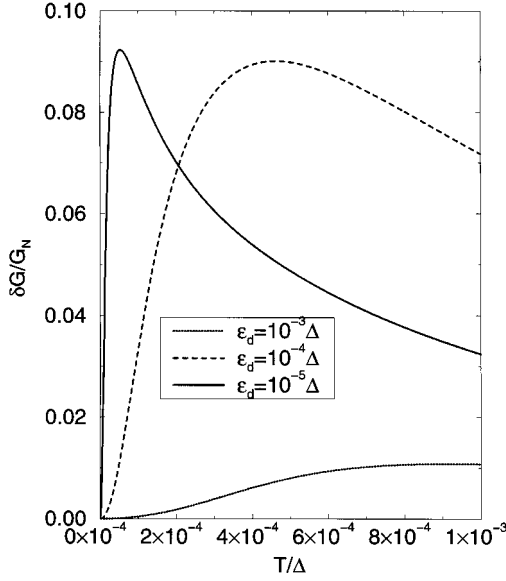
FIG. 4. Conductance in the case of transparent barriers.

Formally this is due to the term $r/\cosh\theta_1(x=0,\varepsilon)\cos\theta_2(x=0,\varepsilon)$ in the denominator of Eq. (8), i.e., the contribution of the barrier at the N'-N interface. In the small transparency limit $r\gg1$, expression (8) reduces to the standard tunnel formula. The physical reason for this behavior is evident. For $r\ll1$ the presence of a tunnel barrier is not important, the electric field penetrates inside the normal metal, and we come back to the picture discussed above for perfectly transparent boundaries in which both normal and correlation DOS play a significant role. If, however, the resistance of a tunnel barrier dominates over the Drude resistance of the normal metal, $r\gg1$, the whole voltage drop is concentrated at the barrier, nonequilibrium effects in the N metal are absent, and therefore only the normal density of states enters into the system conductance.
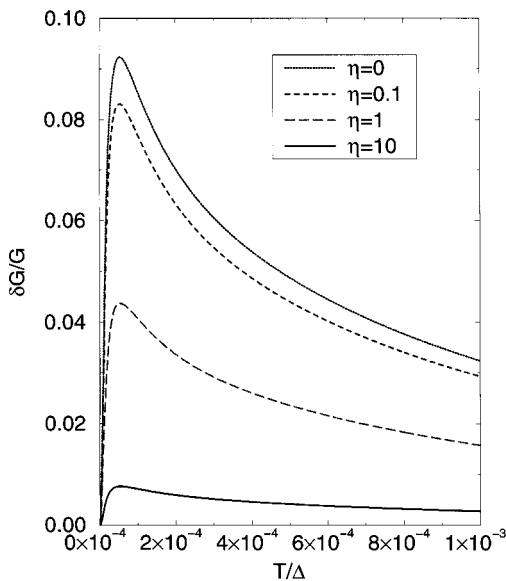
FIG. 5. Conductance normalized to the total length of the normal wire. $\eta:=d_S/d$, $\epsilon_d=10^{-5}\Delta$.
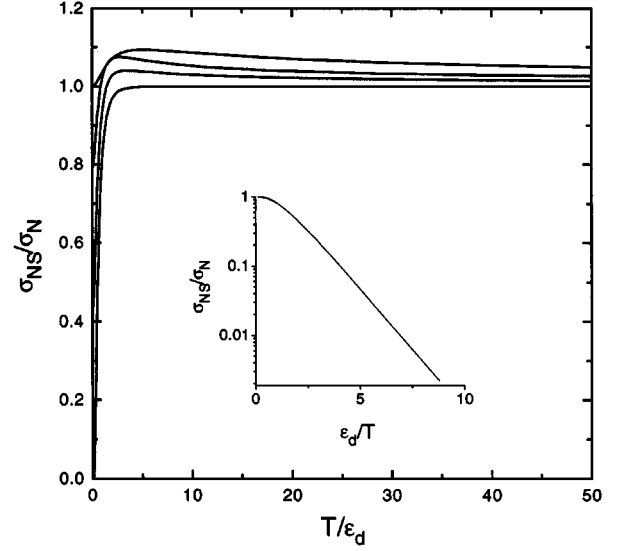
FIG. 6. Conductance in the presence of tunneling barriers. $\gamma_b\xi_N^*/d=0,1,2,5,\infty$ (top to bottom).

An additional effect is that a real gap instead of a soft pseudogap develops in the case of low transparent tunnel barriers. The crossover between these two regimes is discussed in more detail below.

Note that both types of behavior, namely, nonmonotonic and monotonically decreasing with T conductance, have been observed in the experiments.[3]

IV. DENSITY OF STATES

A. Averaged density

From our approximate solutions of the preceding sections, the densities of states can be easily calculated. For $\epsilon=0$ we have $N_N=N_S=N(0)(2/\pi)$. At low energies $\epsilon\ll\epsilon_d$ there are quadratic corrections

$$N_{N/S}=N(0)\left[\frac{2}{\pi}\pm A_{1/2}\left(\frac{\epsilon}{\epsilon_d}\right)^2\right],$$

with

$$A_1=\frac{64}{\pi^5}\left(1-\frac{8}{\pi^2}\right)\approx0.0396$$

and

$$A_2=\frac{16}{\pi^4}\left(1+\frac{2}{\pi^5}\right)\approx0.198.$$

For high energies, the densities approach their normal values, again with square-root corrections $N_N=N(0)(1-B_1\sqrt{\epsilon_d/\epsilon})$ and $N_S=B_2\sqrt{\epsilon_d/\epsilon}$ with $B_1\approx0.321$ and $B_2\approx0.75$.

Together with our numerical data (see Fig. 7), this demonstrates the presence of a soft pseudogap in the density of states below the energy ϵ_d. Similar results have also been discussed in Ref. 22.
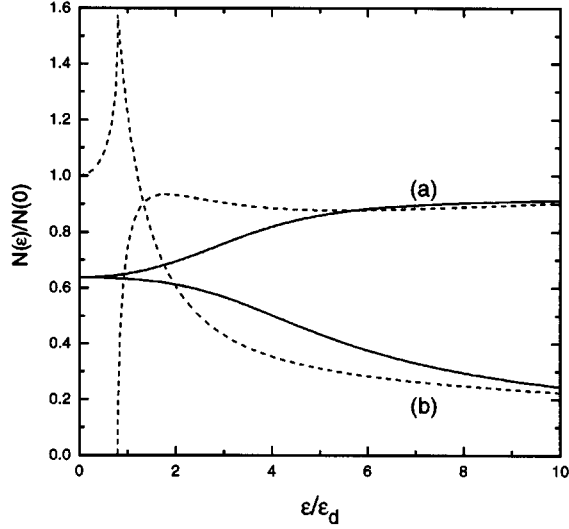
FIG. 7. Averaged normal and correlation DOS. Solid line, transparent interfaces; dashed line, strong tunneling barrier at the N'-N interface.

B. Spatial dependence and tunneling experiments

It is also interesting to investigate the spatial dependence of the DOS in the normal layer. Figure 8 show local normal DOS N_N calculated for perfectly transparent ($\gamma_B=0$) and nontransparent ($\gamma_B=\infty$) NN' interfaces, respectively, at different distances from the NS boundary: $x/d=0.1, 0.2, \ldots, 1$. The difference between these two cases is quite obvious: Whereas for $\gamma_B=0$ the normal DOS at low energies is always finite, becoming larger at larger values of x, for $\gamma_B=\infty$ a real gap in the density of states clearly shows up at all energies. Similar results have been recently discussed in Refs. 38–40. The overall behavior of the local correlation DOS at each value of x is similar to its average value.

It is important to emphasize that both $\nu(x)$ and $\eta(x)$ are measurable quantities and can be directly probed in experiments. Recently the spatial and energy dependence of the normal DOS has been studied in tunneling experiments.[41] The data[41] show qualitative agreement with theoretical predictions. The results obtained here suggest that much better agreement can be achieved if one takes into account smearing of the proximity-induced gap in the normal metal due to the diffusion of normal electrons from the external circuit (which plays the role of the N' reservoir) through the NN' boundary. For nontransparent NN' boundaries ($\gamma_B=\infty$) this process can be neglected and a real gap develops in the N metal [Fig. 8(b)]. As no such gap was found in Ref. 41, we believe that diffusion of normal excitations into the N layer from the external circuit should plays an important role in these experiments. In other words, the experimental situation appears to be closer to that described by the boundary condition $\gamma_B=0$ with a soft pseudogap [Fig. 8(a)] than to the case $\gamma_B=\infty$ (see, e.g., Ref. 40). The dependence of this effect on the size of the N layer is depicted in Fig. 9.

Making use of the Usadel equation one can easily recover simple analytic expressions for the density of states at a distance x_0 away from the NS boundary. For a N wire of the total length d at $\epsilon \ll \epsilon_d$ we obtain
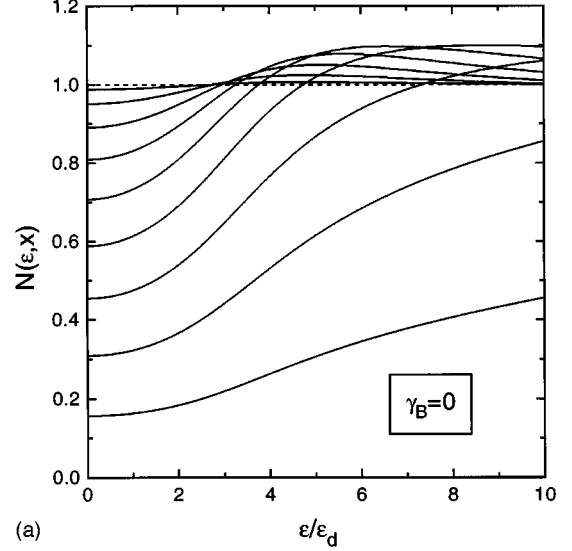
(a)
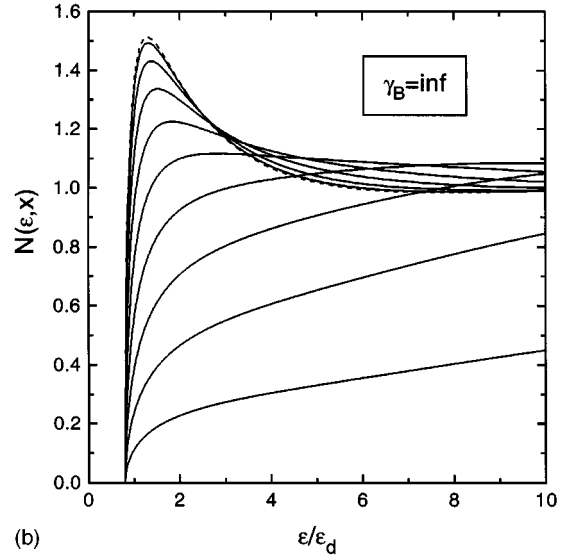
(b)

FIG. 8. Local DOS for different N-N' boundaries. (a) Transparent and (b) non-transparent.

$$\frac{N_N(\epsilon,x_0)}{N(0)} = \alpha + \frac{\epsilon^2}{\epsilon_d^2}\beta, \qquad (25)$$

where α and β describe the size effect,

$$\alpha = \sin\left(\frac{\pi}{2}\frac{x_0}{d}\right),$$

$$\beta = \frac{32}{\pi^4}\alpha\left[1 - \frac{x_0}{d} - \cos\left(\frac{\pi}{2}\frac{x_0}{d}\right)\right]^2.$$

Thus for $x_0 \ll d$ the normal DOS at zero energy and $x=x_0$ is proportional to $1/d$. Neglecting the charging effects (which in principle can also be important[41]) for the differential conductance of the tunneling probe we find

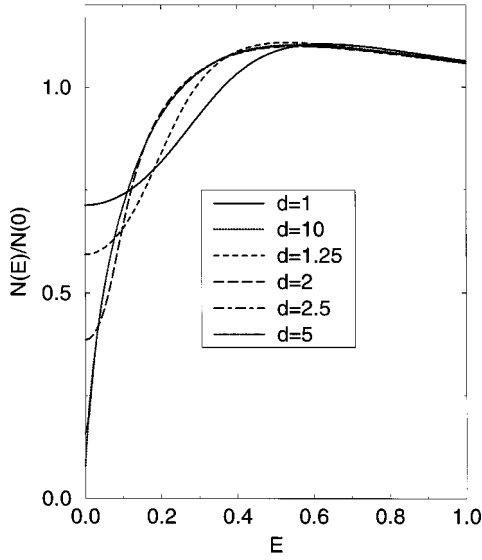$$R_T\frac{dI}{dV}\bigg|_{V=0} = \alpha + \frac{2\pi^2}{3}\frac{T^2}{\epsilon_d^2}\beta. \qquad (26)$$

FIG. 9. Size effect on the local normal DOS. Here, the density of states at a fixed distance $x = 0.5$ from the NS boundary is plotted for different values of the total length L of the N part.

These results demonstrate that the depairing effect of the N' reservoir needs to be taken into account on an equal footing with the pair breaking due to inelastic scattering.[40,41]

Let us also point out that one can also extract information about the correlation DOS by making two kinds of measurements with the same sample. Indeed, by measuring the conductance of the system (or a part of it) with no tunnel barriers one obtains information about the combination of N_N and N_S entering the expression for the system conductance G, whereas performing the tunnel experiments[41] one probes only the normal DOS N_N. Then the correlation DOS can be easily recovered.

V. ELECTRIC FIELD AND CHARGE

In this section we shall discuss only the case of perfectly transparent interfaces.

From our solutions we can calculate the electric field and the charge by using Eq. (11) and Poisson's equation. The field shows essentially nonmonotonic behavior. At $T = 0$ we have $E(\bar{x}) = \cos(\bar{x}\pi/2) - (\pi/2)(\bar{x} - 1)\sin(\bar{x}\pi/2)$. At high temperatures $T \gg \epsilon_d$, the field is constant $E = 1$ far from the superconductor where no correlation remains $(1 - \bar{x} \gg \xi_\epsilon)$ and it changes linearly near the superconductor: $E(\bar{x}) = B_4(1 - \bar{x})\sqrt{T/\epsilon_d}$ with $B_4 \approx 2.59$; however, it still overshoots in between these regimes (see Fig. 10).

We see that close to the superconductor the electric field monotonicly decreases with temperature as superconductivity becomes stronger there. Farther from the NS boundary the field shows a complicated behavior overshooting the normal state value (the total voltage drop is fixed) in the region where superconducting correlation starts decaying either due to thermal effects (high T) or due to the presence of a normal reservoir (low T). The local resistivity is maximally lowered there and the layer of polarization charges is formed (see Fig. 10). These results emphasize again the importance of nonequilibrium effects for understanding the behavior of the system conductance.
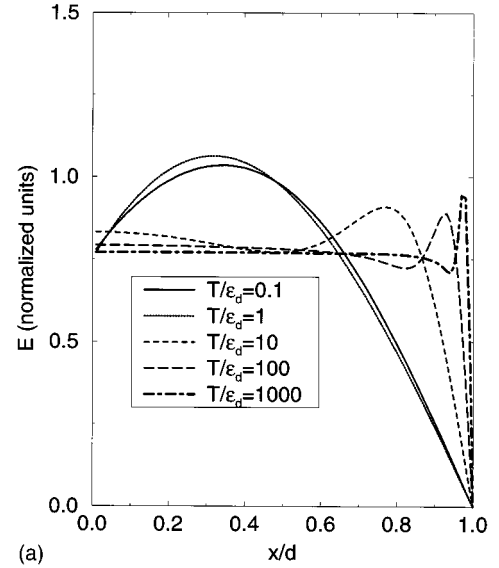
(a)
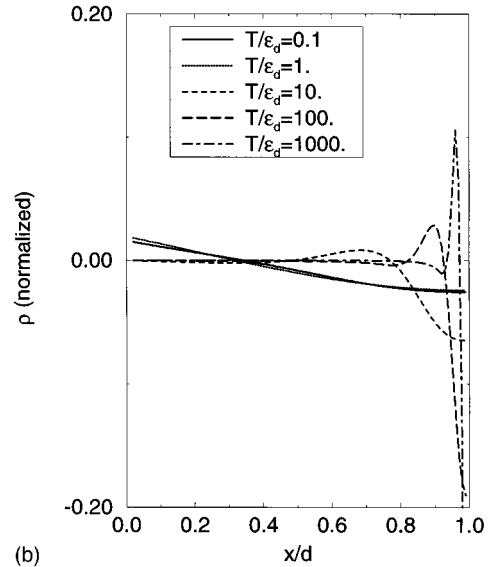
(b)

FIG. 10. Electrostatics within the wire. (a) Distribution of the electrical field. (b) Distribution of the electrical charge.

VI. EXTENSION TO SYSTEMS CONTAINING A LOOP

Recently, the properties of proximity wires containing a loop (see, e.g., Fig. 11) have attracted much experimental[6–8] and theoretical[43,46] interest.

If the wire was a real superconductor, the magnetic flux would induce a supercurrent into the ring. As a function of Φ, this current has the period of the *superconducting* flux quantum $\Phi_0 = h/2e$.

To describe these type of systems, our kinetic scheme has to be extended in several places.

We define the Green's functions in the loop as

$$G = \cosh u_\epsilon,$$

$$F = \sinh u_\epsilon e^{i\varphi_\epsilon},$$

$$\varphi_\epsilon = \chi_\epsilon - 2e \int_0^{\vec{x}} d\vec{l}\, \vec{A}(\vec{l}),$$

where the integration goes around the loop. In the presence of a vector potential, we have to introduce gauge-independent derivatives

$$\nabla \rightarrow \nabla - 2ie\vec{A}.$$

This means that instead of solving the Usadel equation with a vector potential, we can perform a gauge transformation and map onto a system without a magnetic field having phase χ instead of ϕ. As the definition of the Green's functions has to be unique everywhere in the loop, we have $\lim_{x\rightarrow 0+}\varphi(x)=\lim_{x\rightarrow 0-}\varphi(x)(\mathrm{mod}2\pi)$ or

$$\lim_{x\rightarrow 0+}\chi(x)-\lim_{x\rightarrow 0-}\chi(x)=\frac{2e\Phi}{\hbar}(\mathrm{mod}2\pi)$$

after gauge transformation. Here, Φ is the magnetic flux in the ring.

This mapping shows that the magnetic field induces a supercurrent j_ϵ^S (screening current) into the system. We want to neglect any conversion between this supercurrent and the dissipative current, and so both are conserved seperately. This allows the application of the kinetic scheme which has been developed for systems without a phase gradient[12] but can be generalized to any system where the dissipative current is conserved.

The Usadel equation then reads[42,21]

$$\mathcal{D}\frac{d^2}{dx^2}u_\epsilon = -2i\epsilon\sinh u_\epsilon + \frac{\mathcal{D}}{2}\left(\frac{d\chi_\epsilon}{dx}\right)^2\sinh 2u_\epsilon \qquad (27)$$

and has to be solved together with the equation for the conservation of the supercurrent:

$$\frac{d}{dx}j_\epsilon^S = 0, \quad j_\epsilon^S = |\sinh u_\epsilon|^2\frac{d\chi_\epsilon}{dx}. \qquad (28)$$

In order to match the Green's functions at branching points we use the standard continuity condition as no tunnel barriers are assumed to be there. From the Usadel equation in matrix form,

$$\mathcal{D}\nabla(\check{g}_\epsilon\nabla\check{g}_\epsilon)+i\epsilon[\tau_z,\check{g}_\epsilon]=0,$$

follows, for any branching point (see also Ref. 43),

$$\sum_{i=1}^N A_i\check{g}_\epsilon\frac{\partial}{\partial x_i}\check{g}_\epsilon=0,$$

where the sum runs over matching branches, $\partial/\partial x_i$ denotes the derivative in the direction of branch i, and A_i is the cross-section area of branch i. Using our definitions, we get

$$\sum_i A_i\frac{\partial u_\epsilon}{\partial x_i}=0, \quad \sum_i A_i\frac{\partial\chi_\epsilon}{\partial x_i}=0. \qquad (29)$$

These conditions are equivalent to current conservation, and so this is a ''Green's functions Kirchhoff law.'' For $N=1$ Zaitsev's boundary condition[45] for a normal-vacuum boundary is reproduced; $N=2$ is equivalent to the trivial statement that the Green's function derivatives are continuous within a branch.
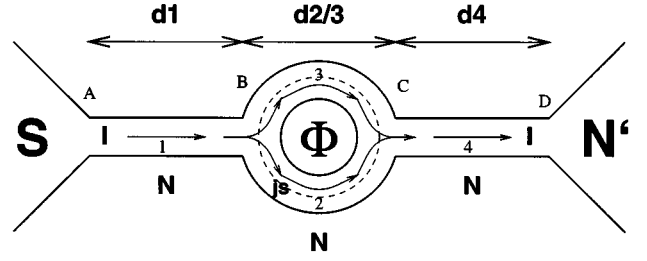

FIG. 11. The system under consideration. S and N are, respectively, superconducting and normal reservoirs. The wire is made of normal conducting material.

For the calculation of the total transparency $D=1/m$, we can use the fact that the m_i fulfill Ohm's law just by their definition:

$$m=\frac{d_1m_1+d_4m_4+(1/d_2m_2+1/d_3m_3)^{-1}}{d_1+(1/d_2+1/d_3)^{-1}+d_4}.$$

VII. MAGNETORESISTANCE OSCILLATIONS

Equations (27) and (28) together with boundary conditions (6) and branching conditions (29) have been solved numerically and also analytically in some limiting cases. For the numerical solution, the problem was mapped onto a simpler boundary value problem without any fitting point. As the system of equations is unstable, we used the relaxation method[45] instead of the shooting method.

For convenience, we have chosen $d_1=d_2=d_3=d_4$ and $A_1=2A_2=2A_3=A_4$, which simplifies the conditions (29). The effect of geometry on the conductance oscillations will be discussed in the Appendix (see, also, Fig. 11). The Thouless energy of just one branch will be labeled as $\epsilon_d=D/d_i^2$.

A. T-dependent amplitude of $h/2e$ oscillations

For $T=0$, only quasiparticles with the energy $\epsilon=0$ contribute to the conductance. From Eqs. (27) and (6) we can conclude that $u_{\epsilon=0}$ is a purely imaginary function, and so the total conductance of the system is equal to its normal state value, being independent of Φ. In other words, there exist no conductance oscillations at $T=0$ (cf. Ref. 37 and 46).

At nonzero temperatures the system conductance depends on the magnetic flux inside the loop with the period equal to the flux quantum Φ_0 (see Fig. 12). With the aid of simple analytic arguments (see the Appendix) one can conclude that at low temperatures the amplitude of the conductance oscillations increases as T^2 (see Fig. 13).

In order to establish the temperature dependence of this amplitude at higher $T\gg\epsilon_d$ it is convenient to make use of the fact that for electrons with sufficiently large energies, $\epsilon\gtrsim\epsilon_d$, the superconducting correlation is destroyed already before they reach the loop. Thus at such energies the transparency of the whole structure $D(\epsilon)$ should be insensitive to the particular value of the flux inside the loop. In other words, calculating the flux-dependent part of the system conductance we can take into account only the contribution of
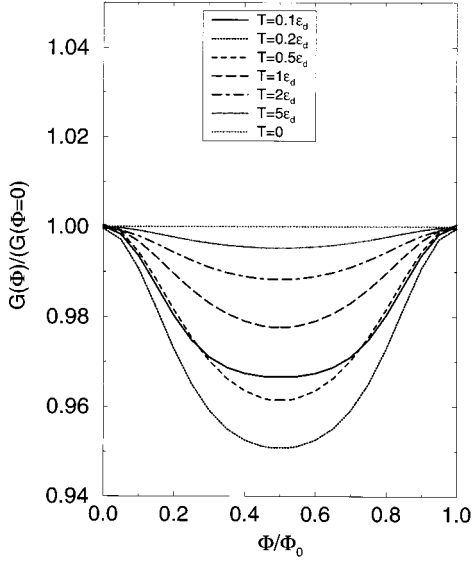
FIG. 12. $h/2e$-periodic structure of the conductance, normalized to the zero-field conductance.

low-energy quasiparticles which remain correlated in the loop area. E.g., for the amplitude of the conductance oscillations we get

$$\Delta G(T) = G_{h/4e}(T) - G_0(T)$$

$$= \frac{1}{2T}\int_0^\infty d\epsilon [D_{h/4e}(\epsilon) - D_0(\epsilon)]\,\mathrm{sech}^2(\epsilon/2T)$$

$$\approx \frac{1}{2T}\int_0^{\epsilon_c} d\epsilon [D_{h/4e}(\epsilon) - D_0(\epsilon)]\,\mathrm{sech}^2(\epsilon/2T)$$

$$\approx \frac{\epsilon_c}{2T}\Delta D_{\mathrm{av}}, \qquad (30)$$

where ϵ_c is the cutoff parameter of order ϵ_d, and ΔD_{av} is a constant.

The results of our numerical analysis fully support the simple analytic arguments presented above. The system transparency $D(\epsilon)$ is depicted in Fig. 14(a) for different values of the flux Φ. The value $D(\epsilon)$ depends on Φ only at low
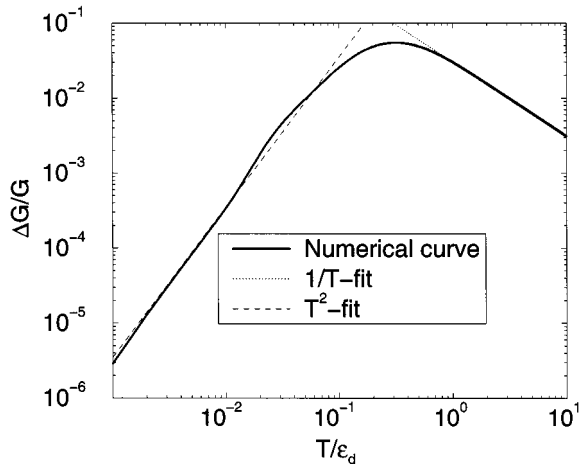

FIG. 13. Oscillation amplitude.
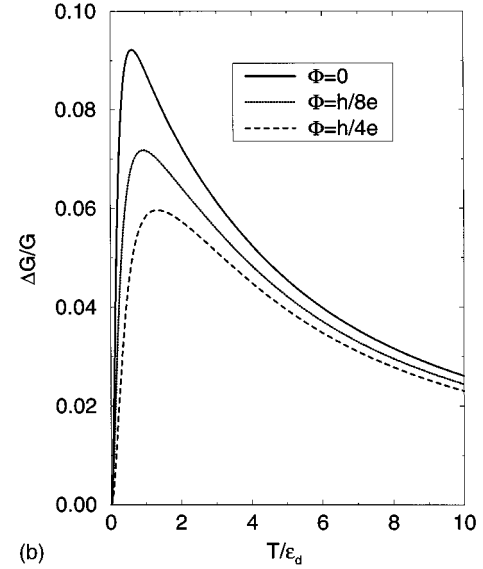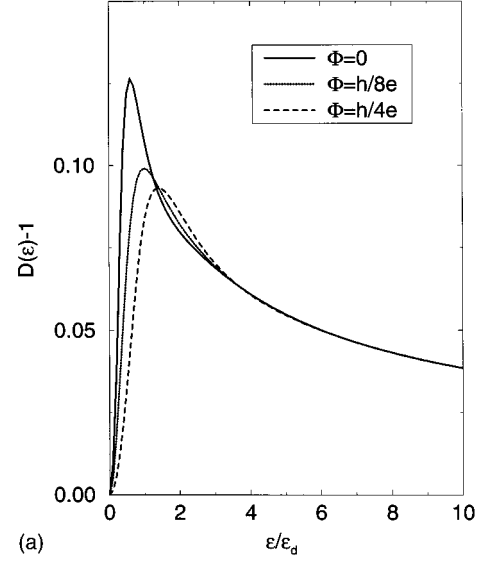

FIG. 14. Temperature dependence of tranport properties at different fluxes. (a) Transparency and (b) conductance.

energies, whereas for $\epsilon \gtrsim \epsilon_d$ all curves merge. In accordance to our simple estimate (30) this leads to a $1/T$ decay of the oscillation amplitude ΔG at large T [see Figs. 14(b) and 12]. Also the T^2 behavior of ΔG in the low-temperature limit is recovered (Fig. 12).

The $1/T$ behavior of ΔG has also been found in recent experiments.[6,7] We would like to point out that a slow power-law decay of the conductance due to a dominating contribution of low-energy quasiparticles just emphasizes the physical difference between kinetic and thermodynamic quantities, like the supercurrent, which decays exponentially with increasing T.

B. Flux-dependent DOS

As was already discussed the simultaneous presence of correlated electrons and the electric field in the normal metal causes nontrivial nonequilibrium effects, the description of which involves two densities of states $\nu(x)$ and $\eta(x)$. In the
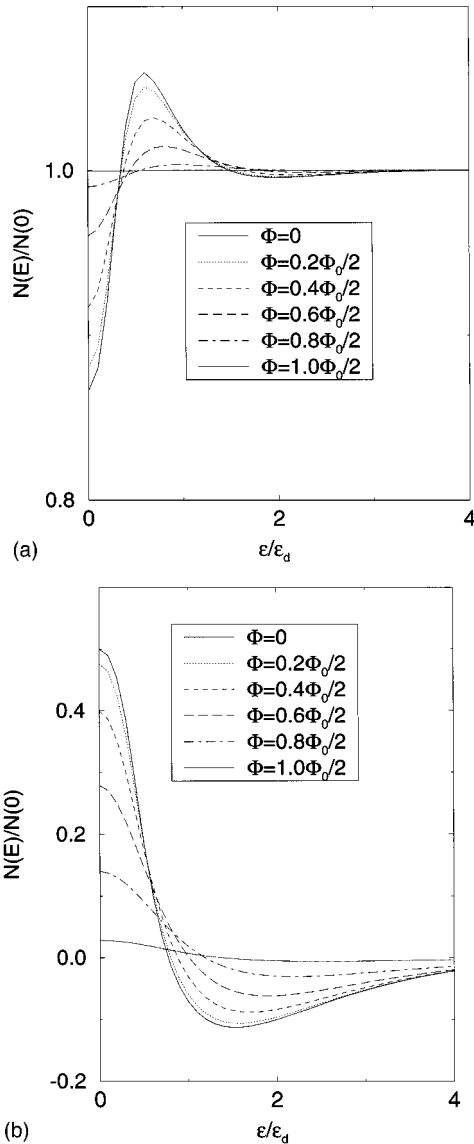
FIG. 15. Flux dependence of the two densities of states in point C of the system. (a) Normal and (b) correlations.

presence of the normal metal loop with the magnetic flux Φ in our system there appears a possibility to tune both normal and correlation DOS by changing the value of Φ. For the system depicted in Fig. 11 these densities of states can be easily calculated. As one might expect for the region between the superconductor and the loop (between points A and B) this dependence is quite weak and both DOS practically coincide with those calculated above for a wire without the loop. On the other hand, in the region between the loop and the normal reservoir N' (between points C and D) the quantities $\nu(x)$ and $\eta(x)$ are very sensitive to the flux Φ.

The normal and correlation DOS for point C (Fig. 11) are presented in Fig. 15. We see that with increasing value of the magnetic flux the proximity-induced pseudogap decreases and vanishes completely as the flux approaches the value $\Phi_0/2$. For such a value of Φ the proximity effect in the region ''after'' the loop is completely destroyed, the pseudogap is fully suppressed, and the normal DOS coincides with $N(0)$ at all energies. Accordingly the correlation

DOS vanishes at $\Phi = \Phi_0/2$. Thus in this case the resistance of the region between points C and D is equal to its normal state value at all T.

These results demonstrate that ''the strength'' of the proximity effect in our system can be regulated by the external magnetic flux. This might serve as an additional experimental tool for investigation of proximity-induced superconductivity in normal metalic structures. In particular we believe that it would be interesting to repeat the tunneling experiments[41] in the presence of the loop with the magnetic flux. Such experiments would provide a direct probe of the dependence of the densities of states on Φ [Fig. 15(a)].

VIII. SUMMARY AND OUTLOOK

We have used a microscopic kinetic analysis to describe the transport properties of superconductor-normal metal proximity structures. In the case of transparent intermetallic boundaries we demonstrated a reentrant behavior of the system conductance with temperature. This behavior was attributed to nonequilibrium effects occurring in the normal metal in the presence of proximity-induced superconductivity and the electric field. We argued that under these conditions both uncorrelated (''normal'') and correlated (''superconducting'') electrons contribute to the system conductance which is henceforth defined by a combination of two densities of states: the normal DOS N_N and the correlation DOS N_S. The latter is known to play an important role whenever the quasiparticle distribution function in a superconductor is driven out of equilibrium.[25]

We studied the energy dependence of both these quantities and demonstrated that if the normal metal is brought in direct contact to a superconductor on one side and a large normal reservoir N' on the other side, the normal DOS N_N shows a soft pseudogap at energies below the Thouless energy ϵ_d. This effect is due to an interplay between correlated and uncorrelated electrons penetrating into the N layer, respectively, from a superconductor and a normal reservoir. If a low transparency tunnel barrier is present at the NN' interface, the diffusion of normal excitations into the N metal is suppressed, the influence of a superconductor prevails, and a real gap in the density of states develops.

Low transparent tunnel barriers also prevent the electric field from penetration into the N layer, thus suppressing nonequilibrium effects there. We demonstrated that with the aid of a proper combination of the systems with and without tunnel barriers one can directly probe both energy and spatial dependences of both densities of states N_N and N_S in one experiment.

We extended our analysis to proximity systems containing the normal metal loop with the magnetic flux Φ. We demonstrated that the conductance of such systems as a function of Φ oscillates with the period equal to the flux quantum $\Phi_0 = h/2e$. The amplitude of these oscillations ΔG also shows the reentrant behavior being equal to zero at $T = 0$, increasing as T^2 at $T \lesssim \epsilon_d$ and decaying as $1/T$ at $T \gtrsim \epsilon_d$. We argued that even at high temperatures, $T \gg \epsilon_d$, low-energy electrons with $\epsilon \lesssim \epsilon_d$ are only responsible for the conductance oscillations leading to the power-law decay of ΔG at large T. We pointed out that the densities of states N_N and N_S can be tuned (decreased) by applying the magnetic flux
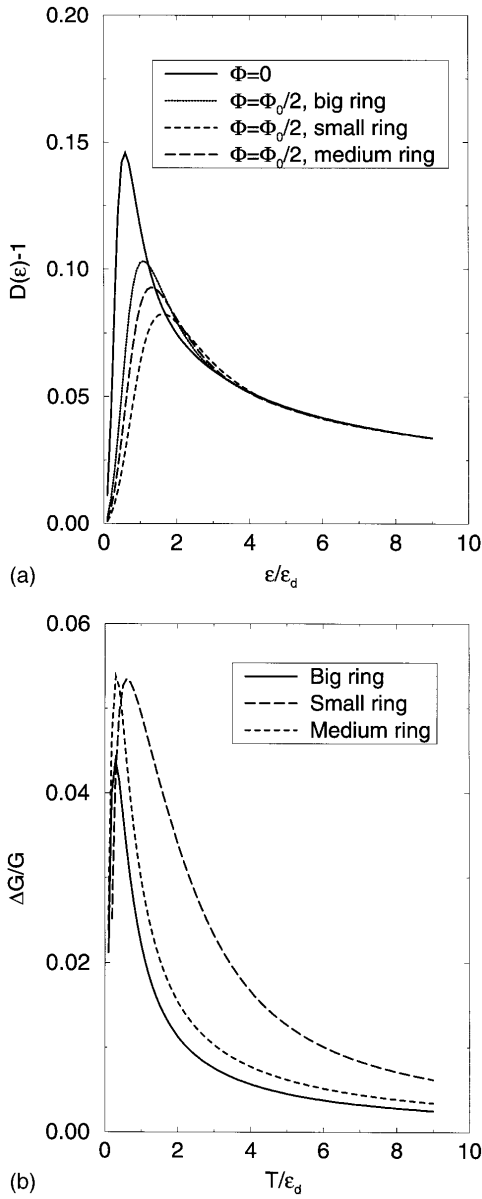
FIG. 16. Size effects onto the conductance oscillations. Small: $d_1=d_4=1.2$, $d_2=d_3=0.6$. Medium: $d_1=d_2=d_3=d_4=1$. Large: $d_1=d_4=0.8$, $d_2=d_3=1.4$. (a) Transparency and (b) amplitude of the conductance oscillations.
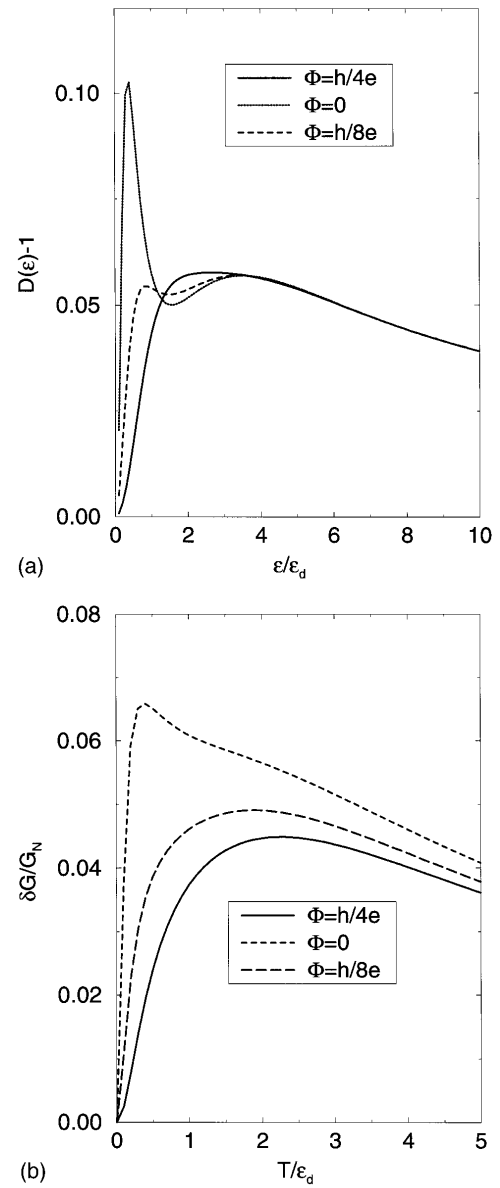
FIG. 17. Transport quantities for the system with $A_1=A_2=A_3=A_4$ and $d_1=d_2=d_3=d_4$ displaying the cross-section effect. (a) Transparency and (b) conductance.

Φ. In particular, if the flux in the loop is equal to the half of the flux quantum $\Phi=\Phi_0$, the proximity effect in the region "after" the loop is completely suppressed, the normal DOS $N_N=N(0)$ at all energies, and $N_S=0$. This effect can be also directly probed in tunneling experiments and used for further studies of proximity-induced superconductivity in normal metallic systems.

An interesting subject is the sensitivity of the above-discussed conductance anomalies to the symmetry of the order parameter in the superconducting terminals. Recently, the proximity effect in normal-metal–insulator–d-wave superconductor (N-I-S) junction was studied in Ref. 47 for the case of a clean N metal. The formation of subgap bound states near interface was shown. Although the influence of disorder in N metal has not yet been studied, it is clear, qualitatively, that the behavior of the N-metal conductance will be sensitive to the angle-averaged densities of states in a superconductor near the NS interface. Then the bound states would certainly influence the conductance anomalies discussed in the present paper, although our qualitative conclusion concerning the reentrant conductance behavior remains unchanged. Quantitative study of these effects will be the subject of future work.

APPENDIX: A CLOSER LOOK AT THE $h/2e$ OSCILLATIONS

Here we present further details related to the effect of geometry on the behavior of the proximity NS systems containing a mesoscopic normal metal loop with the magnetic flux (see Fig. 11). In the three subsections we will keep $A_1=2A_2=2A_3=A_4$ for simplicity allowing different values for the d_i but restricting ourself to symmetric loops $d_2=d_3$.

1. Low-temperature behavior

For examining the low-energy range, which is dominant for the conductance oscillations at any temperature as stated above, we proceed perturbatively from the case $\epsilon=0$, $\phi=0$ first to finite flux, then to finite ϵ. As the Usadel equation is quadratic in $d\chi/dx$, the value u does not distinguish between the upper and the lower branches of the ring (2 or 3 in Fig. 11), and so we will not make a difference in the notation.

We start from $\epsilon=0$, $\Phi=0$, where u is purely imaginary and therefore yields $D=1$. For a finite value of the flux but $\epsilon=0$, we get a purely imaginary correction and therefore $D_{\phi,0}=1$. This correction is quadratic in the flux as the right-hand side of the equation is quadratic in the phase. The finite energy correction at zero flux is a real function and is quadratic in ϵ.

Thus proceeding perturbatively we find $\text{Re}\,u\propto\epsilon\Phi^2$, and from the expansion

$$u=-i\frac{\pi x}{2d_\Sigma}+i\phi^2 g_{d_1,d_2,d_3,d_4}(x)+\epsilon h_{d_1,d_2,d_3,d_4}(x)$$
$$+\epsilon\phi^2 k_{d_1,d_2,d_3,d_4}(x)+i\epsilon\phi^2 l_{d_1,d_2,d_3,d_4}(x),$$

with $d_\Sigma=d_1+d_2+d_4$, in the leading order we get

$$D(\epsilon)=\left(\frac{1}{d_\Sigma}\int_0^{d_\Sigma}\frac{dx}{\cosh^2[u_1(x)]}\right)^{-1}$$

$$=\left[\frac{1}{d_\Sigma}\int_0^{d_\Sigma}dx\left(1-\frac{\epsilon^2}{2}(h+\Phi^2 k)^2\right)+O(\epsilon^4,\phi^4)\right]^{-1}$$

$$=\frac{1}{d_\Sigma}\left(1+\frac{\epsilon^2}{2}(\lambda_{d_1,\ldots,d_4}-\phi^2\mu_{d_1,\ldots,d_4})+O(\epsilon^4,\phi^4),$$

where the coefficients are defined as

$$\lambda=\int_0^{d_\Sigma}dx\,h^2(x),\quad\mu=4\int_0^{d_\Sigma}dx\,h(x)k(x).$$

Thus at low T both the transparency and the conductance depend quadratically on energy and flux. Further analytic expressions are presented in Ref. 48.

It is remarkable that for $d_3=0$ due to Eq. (6) we have $\sinh u=0$ at point C. Therefore, the current conservation condition (28) can be fulfilled for $j=0$, $\chi=\pm\phi/2$ (different signs refer to different branches), and so the phase gradient is zero almost everywhere and the Usadel equation does not contain the phase any more. Thus no magnetoresistance oscillations occur in this case.

2. Cutoff energy

Let us estimate the cutoff energy ϵ_c.

Consider the case $\epsilon\gg\epsilon_{d_1+d_2}=D/(d_1+d_2)^2$. For $\Phi=0$ we again have, Eq. (21), $u=4\arctan[(i\pi/8)e^{-k^R x}]$, and so for small ϕ we can proceed perturbutavely. As the supercurrent is exponentially small, we can approximate the phase profile as

$$\frac{d\chi}{dx}=\frac{1}{2\sinh k_1 d_2}e^{k_1(x-d_1-d_2/2)},$$

and so the influence of the magnetic flux is concentrated within a distance $\max\{\xi_{N,\epsilon},d_2\}$ from point C (see Fig. 11). However, as u is exponentially small there, the oscillations of the transparency are exponentially suppressed, so we can estimate $\epsilon_c\lesssim\epsilon_{d_1+d_2}$, which depends only on the sum of these length, but not on d_1 alone (see also Fig. 16).

The key feature of Fig. 16(b) is the strong flux dependence for systems with small rings. This fact can be also recovered from the Usadel equation: For $d_2\ll d$ one estimates $d\chi/dx\approx\Phi/2\Phi_0 d_2$. As this enters quadratically and in the end we have to integrate over the ring only once, the contribution of the ring is roughly $\propto 1/d_2$.

3. Cross-section effect

For the sake of simplicity, we have restricted ourselves up to now to the case $2A_2=A_1=A_3$. As this condition might not be fulfilled in real experiments it is worthwhile to check whether the main features of our analysis survive for other values of $A_{1,2,3}$. In order to do that we performed calculations also for the case $A_1=A_2=A_3$. The results are similar to those obtained before, showing an additional dip structure in the transparency at intermediate energies [Fig. 17(a)] and a slightly deformed G in the same energy interval [Fig. 17(b)].

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