Spin precession and time-reversal symmetry breaking in quantum transport of electrons through mesoscopic rings

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We consider the motion of electrons through a mesoscopic ring in the presence of a spin-orbit interaction, Zeeman coupling, and magnetic flux. The coupling between the spin and the orbital degrees of freedom results in the geometric and the dynamical phases associated with a cyclic evolution of a spin state. Using a nonadiabatic Aharonov-Anandan phase approach, we obtain the exact solution of the system and identify the geometric and the dynamical phases for the energy eigenstates. Spin precession of electrons encircling the ring can lead to various interference phenomena such as oscillating persistent current and conductance. We investigate the transport properties of the ring connected to current leads to explore the roles of the time-reversal symmetry and its breaking therein with the spin degree of freedom being fully taken into account. We derive an exact expression for the transmission probability through the ring. We point out that the time-reversal symmetry breaking due to Zeeman coupling can totally invalidate the picture that spin precession results in an effective, spin-dependent Aharonov-Bohm flux for interfering electrons. We carry out numerical computation to illustrate the joint effects of the spin-orbit interaction, Zeeman coupling, and magnetic flux. By examining the resonant tunneling of electrons in the weak-coupling limit, we establish a connection between the observable time-reversal symmetry-breaking effects manifested by the persistent current and by the transmission probability. For a ring formed by a two-dimensional electron gas, we propose an experiment in which the direction of the persistent current can be determined by the flux dependence of the transmission probability. That experiment also serves to detect if the electron-electron interaction can qualitatively alter the electronic states. [S0163-1829(97)05816-5]

I. INTRODUCTION

The Aharonov-Bohm (AB) effect leads to a number of remarkable interference phenomena in mesoscopic systems, especially in rings.[1] Based on the discovery of the geometric phases,[2] including the adiabatic Berry phase[3] and the nonadiabatic Aharonov-Anandan (AA) phase,[4] it has been predicted that analogous interference phenomena can be induced by the geometric phases that originate from the interplay between electrons' orbital and spin degrees of freedom. Such interplay can be produced by external electric and magnetic fields, which lead to Zeeman coupling and the spin-orbit (SO) interaction, respectively.

Loss *et al.* first studied the textured ring embedded in inhomogeneous magnetic field.[5] They found the inhomogeneity of the field results in a Berry phase, which can produce the persistent currents. The effects of this Berry phase on conductivity were then discussed.[6] It was further pointed out that the adiabatic condition is not necessary for the geometric phase to exist, and the AA phase in textured rings can produce the persistent currents as well.[7]

On the other hand, the Aharonov-Casher (AC) effect[8] in mesoscopic systems has attracted much attention. Meir *et al.* showed that the SO interaction in one-dimensional (1D) rings results in an effective magnetic flux.[9] Mathur and Stone then pointed out that observable phenomena induced by the SO interaction are the manifestations of the AC effect in electronic systems.[10] These authors investigated the effects of the SO interaction on the persistent-current paramagnetism and the quantum transport in disordered systems and obtained specific reduction factors for harmonics in AB oscillations.[9–11] When the AC flux is not random, it can lead to interference phenomena as AB flux. Mathur and Stone proposed an observation of the AC oscillation of the conductance on semiconductor samples.[10] Balatsky and Altshuler[12] and Choi[13] studied the persistent currents produced by the AC effect.

Inspired by the study on textured rings, the AC effect has also been analyzed in connection with the spin geometric phase. Aronov and Lyanda-Geller considered the spin evolution in conducting rings and found that the SO interaction results in a spin-orbit Berry phase, which plays an interesting role in the transmission probability of the rings.[14] In their models, there is a Zeeman coupling from a uniform magnetic field, but the SO Berry phase can be caused by the SO interaction alone. So they have indeed shown the existence of the Berry phase in the AC effect. Since the SO interaction is usually not strong enough to guarantee the validity of the adiabatic approximation, a nonadiabatic treatment of the problem is necessary. In Ref. 15 we demonstrated the existence of a nonadiabatic AA phase in the AC effect in 1D rings. We found that the AC flux and local spin orientations of the electronic eigenstates are determined by a spin cyclic evolution. In particular, we showed that the AC phase comprises both the AA and the dynamical phases that are acquired in the cyclic evolution and the adiabatic limit of the AA phase is just the SO Berry phase. Based on this geometric phase approach for the AC effect, Oh and Ryu studied the persistent currents produced by the cylindrically symmetric SO interaction in 1D rings.[16]

As is well known, the SO interaction is time-reversal in-

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variant, while Zeeman coupling breaks the time-reversal symmetry (TRS). Many prior works have shown the significance of the TRS and its breaking with regard to various interference phenomena caused by the AB flux and SO interaction. It is therefore worthwhile to investigate if the coexistence of the SO interaction and Zeeman coupling can produce any new observable effect with the spin degree of freedom being fully taken into account. However, most of the previous studies have focused on the rings in the presence of Zeeman coupling or the SO interaction only. In Ref. 17 we have demonstrated that the competition between Zeeman coupling and the SO interaction can produce persistent currents through the TRS breaking in a many-electron ring with a complete set of current-carrying single-particle states. For the transport properties, Aronov and Lyanda-Geller[14] have derived a transmission probability for a conducting ring in the presence of both the SO interaction and the Zeeman coupling by making use of the concept of the Berry phase. Unfortunately, they failed to take into account correctly the different properties of the SO interaction and Zeeman coupling under the time-reversal transformation. As a result, they did not realize that their picture of the effective flux for the interference of spin-polarized electrons is actually invalidated by the TRS-breaking Zeeman coupling. Furthermore, even if the Zeeman coupling is absent, their expression for the effective flux induced by the SO interaction is still not complete. So the transport properties of a ring in the presence of both the SO interaction and Zeeman coupling have not been solved yet and the roles of TRS and its breaking therein need to be clarified.

In this paper we will discuss the transport properties of a ring in the presence of both the SO interaction and the Zeeman coupling. We will explore the roles of the TRS and its breaking in the transport phenomena when the spin degree of freedom is taken into account explicitly. We will also show the connection between the observable TRS-breaking effects manifested by the persistent current and by the transmission probability. Throughout the discussion, we will emphasize the TRS by investigating how the TRS-breaking Zeeman coupling affects the thermodynamic and transport properties of the system. The paper is organized as follows. First we identify the electronic states for the ring connected to current leads by making use of the exact solution of the closed ring. Then we derive an exact expression for the transfer matrices of the two ring branches (arms) by introducing four auxiliary spin states, which exhibit the orbital quantum number dependence of the spin orientations in electronic eigenstates. From the transfer matrices, we obtain the transmission probability of the ring by adopting the standard formulation developed in Ref. 18. Finally, we carry out some numerical calculations to illustrate the effects of the SO interaction and Zeeman coupling. We find that there is an interesting and observable correspondence between the TRS-breaking effects manifested by the transmission probability and by the persistent current. That correspondence, if experimentally verified or excluded in some specific ring, may serve to detect if the electron-electron interaction is of qualitative importance in determining electronic states. We conclude this paper with a summary of our results.

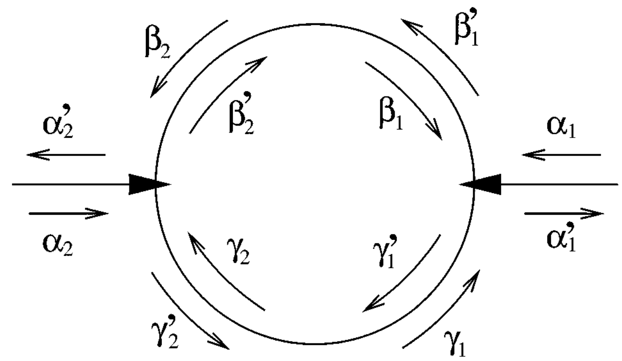

FIG. 1. Schematic representation of the electronic waves propagating through the ring connected to current leads. The right junction is located at $\theta=0$ and the left junction at $\theta=\pi$, with the upper branch lying within $(0,\ \pi)$ and the lower branch within $(\pi,\ 2\pi)$.

II. TRANSMISSION PROBABILITY

The Hamiltonian for an electron in the electric field $\mathbf{E}=-\nabla V$ and the magnetic field $\mathbf{B}=\nabla\times\mathbf{A}$ is

$$H=\frac{1}{2m_e}\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)^2+eV-\frac{e\hbar}{4m_e^2c^2}\boldsymbol{\sigma}\cdot\mathbf{E}\times\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)-\frac{ge\hbar}{4m_ec}\boldsymbol{\sigma}\cdot\mathbf{B}. \tag{1}$$

We consider a ring that is effectively one dimensional and the fields that are cylindrically symmetric, i.e., $\mathbf{E}=E(\cos\chi_1\mathbf{e}_r-\sin\chi_1\mathbf{e}_z)$, $\mathbf{B}=B(\sin\chi_2\mathbf{e}_r+\cos\chi_2\mathbf{e}_z)$ in the cylindrical coordinate system. For the ring lying in the xy plane with its center at the origin, the Hamiltonian is given by

$$H=\frac{\hbar^2}{2m_ea^2}\left[-i\frac{\partial}{\partial\theta}+\phi+\alpha(\sin\chi_1\sigma_r+\cos\chi_1\sigma_z)\right]^2+\frac{\hbar\omega_B}{2}(\sin\chi_2\sigma_r+\cos\chi_2\sigma_z), \tag{2}$$

with $\sigma_r=\sigma_x\cos\theta+\sigma_y\sin\theta$, $\alpha=-eaE/4m_ec^2$, and $\omega_B=-geB/2m_ec$, where a is the ring radius, θ is the angular coordinate, and ϕ is the enclosed magnetic flux in units of flux quantum. The exact solution for the closed ring is given in the Appendix.

To investigate the transport properties, we discuss the ring that is connected to external current leads, schematically illustrated in Fig. 1. We adopt the standard formulation developed in the study of quantum oscillations in 1D rings threaded by the AB flux.[18] In the upper and the lower branches, the wave amplitudes at one end are related to the wave amplitudes at the other end by the transfer matrices as

$$\begin{bmatrix}\beta_2\\ \beta_2'\end{bmatrix}=\underline{t}_{\mathrm{I}}\begin{bmatrix}\beta_1'\\ \beta_1\end{bmatrix},\qquad \begin{bmatrix}\gamma_1\\ \gamma_1'\end{bmatrix}=\underline{t}'_{\mathrm{II}}\begin{bmatrix}\gamma_2'\\ \gamma_2\end{bmatrix},$$

where t_{I} and t'_{II} denote the transfer matrices of the upper and lower branches, respectively, and they depend on the energy E of the incident wave. At the two junctions, the amplitudes of the three outgoing waves (α',β',γ') are related to the amplitudes of the incoming waves (α,β,γ) by

$$\begin{bmatrix}\alpha'\\ \beta'\\ \gamma'\end{bmatrix}=\begin{bmatrix}-(a+b) & \sqrt{\epsilon} & \sqrt{\epsilon}\\ \sqrt{\epsilon} & a & b\\ \sqrt{\epsilon} & b & a\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix},$$

where $a=\pm(\sqrt{1-2\epsilon}-1)/2$ and $b=\pm(\sqrt{1-2\epsilon}+1)/2$ with $0\leqslant\epsilon\leqslant 1/2$. When considering a wave incident from the right junction, we have $\alpha_1^\dagger\alpha_1=1$ and $\alpha_2=0$. The amplitude of the transmitted wave is

$$\alpha_2'=-\frac{\epsilon}{b^2}([b-a\quad,\quad 1]\otimes\sigma_0)\underline{t}_{\mathrm{I}}\Pi^{-1}\left(\begin{bmatrix}b-a\\ -1\end{bmatrix}\otimes\sigma_0\right)\alpha_1, \quad (3)$$

with Π given by

$$\Pi=\frac{1}{b^2}\left(\begin{bmatrix}b^2-a^2 & a\\ -a & 1\end{bmatrix}\otimes\sigma_0\right)\underline{t}'_{\mathrm{II}}\left(\begin{bmatrix}b^2-a^2 & a\\ -a & 1\end{bmatrix}\otimes\sigma_0\right)\underline{t}_{\mathrm{I}}-\mathbb{1}, \quad (4)$$

where σ_0 is the 2×2 unit matrix in spin space. This formulation is in general applicable to the derivation of the transmission probability through any ring, provided the corresponding transfer matrices are known. Note that in the study of the ring only threaded by the AB flux, electrons can be treated as spinless particles, so that all amplitudes are simply represented by complex numbers and the matrix σ_0 can be dropped. In this paper, α_1, α_1', . . . have to be represented by two-component spinors and $\underline{t}_{\mathrm{I}},\underline{t}'_{\mathrm{II}}$ are 4×4 matrices.

To derive an explicit expression for the two transfer matrices, we first identify the electronic states in the ring by making use of its cylindrical symmetry. If we write $\underline{t}_{\mathrm{I}},\underline{t}'_{\mathrm{II}}$ in a 2×2 matrix form, then each matrix element is a 2×2 matrix in spin space. We can easily conclude that the off-diagonal elements of $\underline{t}_{\mathrm{I}},\underline{t}'_{\mathrm{II}}$ are zero because of the conservation of $-i(\partial/\partial\theta)+\frac{1}{2}\sigma_z$, which indicates that in each branch any propagating wave with fixed energy can possess a well-defined momentum and pass each branch without reflection, as a result of the cylindrical symmetry of the external fields and the absence of scattering potential. So our task reduces to finding the four 2×2 matrices that, respectively, relate β_1' with β_2 and β_1 with β_2' for the upper branch and γ_2' with γ_1 and γ_2 with γ_1' for the lower branch. These four 2×2 matrices are the four nonzero diagonal elements of $\underline{t}_{\mathrm{I}},\underline{t}'_{\mathrm{II}}$ directly.

The electrons' tunneling through the ring is carried out by the energy eigenstate of the ring connected to two ideal conductors. Consider an incident wave with wave vector k_F. The corresponding eigenenergy of the steady transport state is $E_F=\hbar^2k_F^2/2m$. In the right conductor the electronic state is a superposition of the incident plane wave α_1 and the reflected plane wave α_1', while in the left conductor the propagating wave is just the transmitted plane wave α_2'. The state inside the ring is a superposition of four wave functions of energy E_F. They actually determine the four nonzero matrix elements defined above for the two diagonal transfer matrices.

To find the four components of the electronic wave inside the ring, we first use the energy expression

$$\frac{\hbar^2k_F^2}{2m}=E_{n,\mu}=\frac{\hbar\omega_0}{2}(n+\phi)^2+\frac{\hbar\omega_0}{2}(\alpha^2-\alpha\cos\chi_1)$$
$$+\frac{\hbar\omega_n}{2}(1-\mu\cos\beta_n)+\mu\alpha\hbar\omega_n\cos(\beta_n-\chi_1)$$
$$+\frac{\mu\hbar\omega_B}{2}\cos(\beta_n-\chi_2) \quad (5)$$

to find four solutions of n, which are positive $n_{+,+}$ and negative $n_{-,+}$, with $\mu=+$, and positive $n_{+,-}$ and negative $n_{-,-}$, with $\mu=-$. For arbitrary k_F, these quantum numbers are not integers in general. For each $n_{\lambda,\mu}$ we can obtain a wave function $\Psi_{n_{\lambda,\mu},\mu}$ that bears the same form as $\Psi_{n,\mu}$ of the closed ring, but with n being substituted by $n_{\lambda,\mu}$ and accordingly the spin tilt angle β_n being substituted by $\beta_{n_{\lambda,\mu}}$ from Eq. (A2). These four $\Psi_{n_{\lambda,\mu},\mu}$ are actually eigenstates of the Hamiltonian (2) at energy E_F, but the periodic boundary condition $\Psi_{n,\mu}(\theta)=\Psi_{n,\mu}(\theta+2\pi)$ is resolved due to the connection with external conductors. The electronic wave inside the ring is a superposition of the four $\Psi_{n_{\lambda,\mu},\mu}$ by which the eight amplitudes β_1,β_1', . . . can be represented. This is a natural conclusion from the steadiness of the electronic state that transports electrons at fixed energy E_F through the ring. With this understanding, we can derive the transfer matrices in terms of $\Psi_{n_{\lambda,\mu},\mu}$.

As shown in the Appendix, the Zeeman coupling brings the dependence on orbital quantum number to spin orientations. As a result, $\Psi_{n_{\lambda,+},+}$ and $\Psi_{n_{\lambda,-},-}$, which carry the clockwise ($\lambda=-$) or the counterclockwise ($\lambda=+$) wave, are of nonorthogonal spin states $\widetilde{\psi}_{n_{\lambda,-},-}(\theta)$ and $\widetilde{\psi}_{n_{\lambda,+},+}(\theta)$ unless in the absence of Zeeman coupling. To derive the transfer matrix associated with spin-polarized transport, it is crucial to distinguish the $\mu=+$ from the $\mu=-$ contribution for any wave propagating in fixed direction. For this purpose, we define four auxiliary spin states

$$\widetilde{\eta}_{\lambda,\mu}(\theta)=\frac{1}{R_\lambda}[\widetilde{\psi}_{n_{\lambda,\mu},\mu}(\theta)$$
$$-\widetilde{\psi}^\dagger_{n_{\lambda,-\mu},-\mu}(\theta)\widetilde{\psi}_{n_{\lambda,\mu},\mu}(\theta)\widetilde{\psi}_{n_{\lambda,-\mu},-\mu}(\theta)], \quad (6)$$

where $R_\lambda=1-|\widetilde{\psi}^\dagger_{n_{\lambda,\mu},\mu}(\theta)\widetilde{\psi}_{n_\lambda,-\mu}(\theta)|^2$. It is easy to verify the relations of redefined orthogonality and completeness,

$$\widetilde{\eta}_{\lambda,\mu}(\theta)^\dagger\widetilde{\psi}_{n_{\lambda,\nu},\nu}(\theta)=\delta_{\mu\nu} \quad (7)$$

and

$$\sum_\mu \widetilde{\psi}_{n_{\lambda,\mu},\mu}(\theta)\widetilde{\eta}_{\lambda,\mu}(\theta)^\dagger=\sigma_0. \quad (8)$$

In the upper branch, the wave propagating counterclockwise consists of the two components $\Psi_{n_{+,+},+}$ and $\Psi_{n_{+,-},-}$. β_1' and β_2 can thereby be expressed as

$$\beta_1' = c_1\Psi_{n_{+,+},+}(0) + c_2\Psi_{n_{+,-},-}(0),$$
$$\beta_2 = c_1\Psi_{n_{+,+},+}(\pi) + c_2\Psi_{n_{+,-},-}(\pi), \tag{9}$$

where c_1 and c_2 are two specific constants. Using Eqs. (A1) and (7), we obtain

$$\beta_2 = [e^{in_{+,+}\pi}\tilde{\psi}_{n_{+,+},+}(\pi)\tilde{\eta}^{\dagger}_{+,+}(0) + e^{in_{+,-}\pi}\tilde{\psi}_{n_{+,-},-}(\pi)\tilde{\eta}^{\dagger}_{+,-}(0)]\beta_1', \tag{10}$$

and therefore find the 2×2 matrix that is the first diagonal element of $\underline{t}_{\mathrm{I}}$. The other three matrix elements in diagonal $\underline{t}_{\mathrm{I}}$ and $\underline{t}_{\mathrm{II}}$ can be derived in the same way. We finally obtain the two transfer matrices in the form of

$$\underline{t}_{\mathrm{I}} = \begin{bmatrix} \sum_\mu e^{in_{+,\mu}\pi}\tilde{\psi}_{n_{+,\mu},\mu}(\pi)\tilde{\eta}^{\dagger}_{+,\mu}(0) & 0 \\ 0 & \sum_\mu e^{in_{-,\mu}\pi}\tilde{\psi}_{n_{-,\mu},\mu}(\pi)\tilde{\eta}^{\dagger}_{-,\mu}(0) \end{bmatrix}, \tag{11}$$

$$\underline{t}'_{\mathrm{II}} = \begin{bmatrix} \sum_\mu e^{in_{+,\mu}\pi}\tilde{\psi}_{n_{+,\mu},\mu}(0)\tilde{\eta}^{\dagger}_{+,\mu}(\pi) & 0 \\ 0 & \sum_\mu e^{in_{-,\mu}\pi}\tilde{\psi}_{n_{-,\mu},\mu}(0)\tilde{\eta}^{\dagger}_{-,\mu}(\pi) \end{bmatrix}. \tag{12}$$

From Eq. (3), the transmission probability for unpolarized incident electrons is $\langle \alpha_2'^{\dagger}\alpha_2'\rangle$ in which $\langle\ \rangle$ denotes an averaging over α_1 with fixed $\alpha_1^{\dagger}\alpha_1 = 1$. Explicitly, it is given by

$$T = \frac{1}{2}\sum_{i,j}\left|\left\{-\frac{\epsilon}{b^2}([b-a \quad , \quad 1]\otimes\sigma_0)\underline{t}_{\mathrm{I}}\Pi^{-1} \times\left(\begin{bmatrix} b-a \\ -1\end{bmatrix}\otimes\sigma_0\right)\right\}_{ij}\right|^2, \tag{13}$$

in which $\underline{t}_{\mathrm{I}}$, $\underline{t}'_{\mathrm{II}}$, and Π are all known.

In the absence of Zeeman coupling, the expression of the transmission probability can be greatly simplified and explicitly related to the spin-independent transmission probability through the ring threaded by the AB flux only. From Eq. (A2), it is obvious that if $\omega_B = 0$, $\tilde{\psi}_{n_{\lambda,\mu},\mu}$ are independent of $n_{\lambda,\mu}$ defined in Eq. (5) and can be denoted by $\tilde{\psi}_\mu$ with $\tilde{\eta}_{\lambda,\mu} = \tilde{\psi}_\mu$. Combining this fact with Eqs. (7)–(9), we see that in the absence of Zeeman coupling, the electronic wave in the ring actually consists of two orthogonal amplitudes, which propagate coherently and independently, with their local spin states being given by $\tilde{\psi}_\mu$. We then turn to the phase shift for spin-polarized electrons. When $k_F a$ is very large and the quasiclassical approximation is therefore applicable, it is worthwhile to write $n_{\lambda,\mu}$ in Eq. (5) as

$$n_{\lambda,\mu} = \lambda k_F a - \phi - \frac{1}{2}(1-\mu\cos\chi_{n_{\lambda,\mu}}) - \mu\alpha\cos\chi_{n_{\lambda,\mu}}, \tag{14}$$

where the last three terms on the right-hand side are $1/2\pi$ of the AB phase, the spin AA phase, and the dynamical phase contributed by the SO interaction, respectively. When the Zeeman coupling is absent, the last two terms give the $1/2\pi$ of the AC phase $\Phi^{\mu}_{\mathrm{AC}}/2\pi$ (Ref. 15) and

$$n_{\lambda,\mu} = \lambda k_F a - \phi + \frac{1}{2\pi}\Phi^{\mu}_{\mathrm{AC}} \tag{15}$$

becomes an exact relation without the quasiclassical approximation. Since Φ^{μ}_{AC} is n independent, Eq. (15) indicates that the effect of the SO interaction can be regarded as an AB effect of the effective flux $-\Phi^{\mu}_{\mathrm{AC}}/2\pi$ in units of Φ_0 for the locally polarized electron gases with local spin states $\tilde{\psi}_\mu$.

We can derive for $\omega_B = 0$ the transmitted amplitude

$$\alpha_2'(\phi,\alpha_1) = \sum_\mu [\tilde{\psi}^{\dagger}_\mu(0)\alpha_1]t\left(\phi - \frac{\Phi^{\mu}_{AC}}{2\pi}\right)\tilde{\psi}_\mu(\pi), \tag{16}$$

where $t(\phi)$ is the transmitted amplitude for the ring threaded by magnetic flux ϕ (Ref. 18) with vanishing the SO interaction. Equation (16) indicates clearly that the real electronic wave in the ring is a superposition of the two locally polarized waves, which enclose different effective fluxes and propagate independently. The transmission probability $T_{\mathrm{AB,AC}}$ is given by $\alpha_2'^{\dagger}\alpha_2'$:

$$T_{\mathrm{AB,AC}}(\phi,\alpha_1) = \sum_\mu |\tilde{\psi}^{\dagger}_\mu(0)\alpha_1|^2 T_{\mathrm{AB}}\left(\phi - \frac{\Phi^{\mu}_{\mathrm{AC}}}{2\pi}\right), \tag{17}$$

where $T_{\mathrm{AB}}(\phi) = t^{\dagger}t$ is the transmission probability of the ring threaded by the magnetic flux ϕ with the vanishing SO interaction. To see what happens for an unpolarized incident wave, we average $T_{\mathrm{AB,AC}}$ over α_1 and obtain $\overline{T}_{\mathrm{AB,AC}} = \Sigma_\mu T_{\mathrm{AB}}(\phi - \Phi^{\mu}_{\mathrm{AC}}/2\pi)/2$, which agrees with the relation predicted in Ref. 9 for general spin-independent ther-

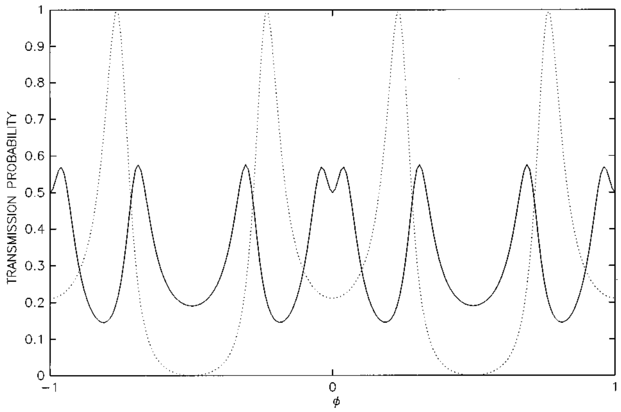

FIG. 2. Transmission probability as a function of the AB flux for $\epsilon=0.25$, $ka=60.239$, $a=1$ μm, and $\chi_2=\pi/6$. The dotted and the solid lines are associated with the absence of and the presence of the SO interaction of $\alpha=1.8$, respectively.

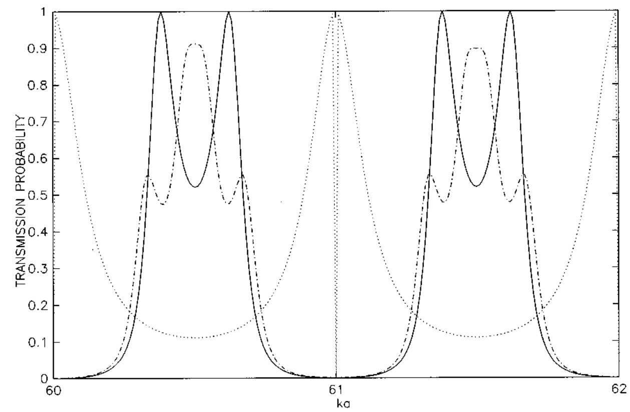

FIG. 3. Transmission probability as a function of the energy of incident electrons ($E_F=\hbar^2k^2/2m$) for $\epsilon=0.25$, $a=1$ μm, $\alpha=1.8$, $\chi_2=\pi/6$, and $\phi=0$. The dotted line corresponds to the absence of the SO interaction and Zeeman coupling, the solid line corresponds to the presence of the SO interaction only, and the dash-dotted line corresponds to the presence of both the SO interaction and the Zeeman coupling of $B=30$ G.

modynamic and transport quantities. However, in the competition with the SO interaction, the Zeeman coupling brings the n dependence to the spin orientations of energy eigenstates. The n-dependent spin precession then results in the n-dependent spin phases. It is seen that in the presence of the Zeeman coupling, the last two terms in Eq. (14) are n dependent and the effect of the spin phases can no longer be regarded as that from the effective flux, which must be independent of the specific orbital quantum numbers of the states.

A numerical calculation has been carried out to illustrate some essential characteristics of the transmission probability derived here. We find that the respective effects of the Zeeman coupling and SO interaction can be reflected by the resonance of the transmission probability in the weak-coupling limit at small ϵ. In particular, we can see an interesting correspondence between the TRS-breaking effects manifested by the transmission probability and by the persistent current.

We adopt the model of an InAs ring.[14] The Hamiltonian is of the form

$$H_{\mathrm{InAs}}=\frac{1}{2m}\left(\mathbf{p}-\frac{e\mathbf{A}}{c}\right)^2+\hbar\kappa[\boldsymbol{\sigma}\times\mathbf{p}]_z-\frac{ge\hbar}{4mc}\boldsymbol{\sigma}\cdot\mathbf{B}, \quad (18)$$

where $m=0.023m_e$ is the effective mass, $\hbar^2\kappa=6.0\times10^{-10}$ eV cm is the SO coefficient, and $g=15$. Here the effective electric field is in the z direction, hence $\chi_1=\pi/2$. For the ring of radius $a=1$ μm, the dimensionless coefficient α in Eq. (2) is found to be $ma\kappa=1.8$, which is large enough to result in an AC phase of order unity.[15] The Fermi velocity v_F is approximately 3×10^7 cm s^{-1}, corresponding to $|n_F|\approx60$.

The effective flux induced by the SO interaction and its effect on the transmission probability can be clearly seen in Fig. 2, where T_{AB} and $T_{\mathrm{AB,AC}}$ are plotted as functions of ϕ. The magnitude of the AC phase can actually be approximately measured by a comparison between the ϕ coordinates of the transmission probabilities' peaks in the absence and in the presence of the SO interaction. The energy splitting due to Zeeman coupling is illustrated in Fig. 3. For $\phi=0$ and $\omega_B=0$, since the Kramers degeneracy makes each two eigenstates of the closed ring have the same energy, at certain E_F the transmission probabilities in the two spin branches can reach their highest value 1 simultaneously, thereby making $T_{\mathrm{AB,AC}}=1$. After the Zeeman coupling is turned on, the resulted energy splitting destroys the simultaneous happenings of the resonances in the two spin branches and we see the maximum values of T decrease appreciably with the strength of Zeeman coupling.

With ϵ being even smaller, the energy dependence of the transmission probability manifests as an interesting TRS-breaking effect, which also has its corresponding observability in the persistent current. In Ref. 17 it has been demonstrated that in the presence of the SO interaction, the TRS-breaking mechanism due to Zeeman coupling is intrinsically different from that due to the AB flux. As the corresponding observable effect, it has been found that the direction of the persistent current induced by Zeeman coupling changes periodically with the particle number N with the periodicity $\Delta N=2$, while the direction of the persistent current induced by the AB flux never changes with the particle number. The dependence of the current direction on the particle number is actually the dependence on the Fermi energy. Such an energy dependence of the current direction, an equilibrium phenomenon as it is, can actually be manifested in the resonant tunneling of electrons, a transport phenomenon as it is, in the weak-coupling limit. For $\epsilon\to0$, the peaks of $T(E_F)$ locate at the eigenenergies $E_{n,\mu}$ of the closed ring.[18] In the presence of the SO interaction and a weak Zeeman coupling, the transmission probability is plotted as a function of the incident energy in Fig. 4. Every two peaks, which are closest to each other, locate at a pair of splitted energy levels, which come from the Kramers doublet ($\Psi_{n,\mu}$, $\Psi_{-n-1,-\mu}$) in the absence of Zeeman coupling. With the AB flux being zero, the energy splittings in all the splitted energy levels are the same. Here we use the first-order perturbation, which gives the energy correction but does not change the eigenfunction. As shown in Ref. 17, those eigenstates of the closed ring,

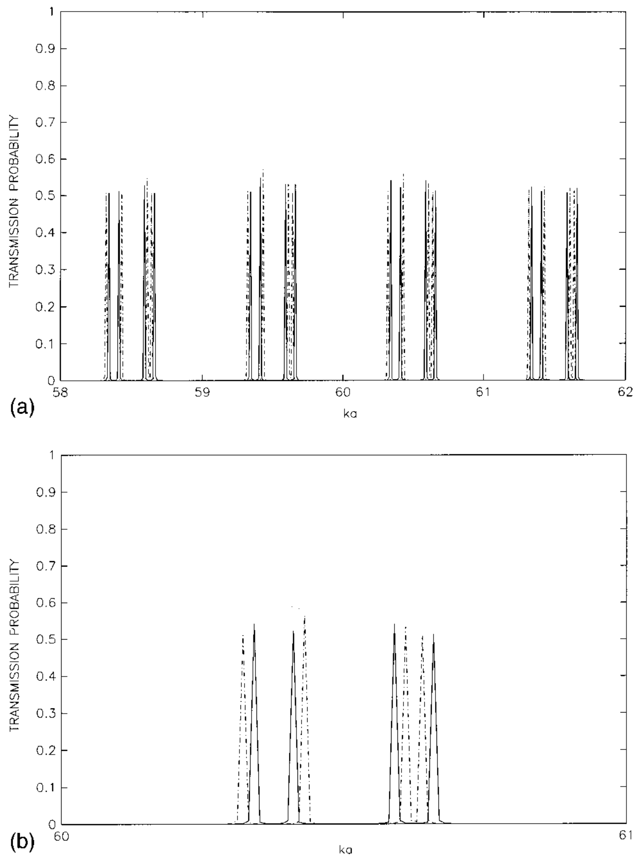

FIG. 4. Transmission probability as a function of the energy of incident electrons ($E_F = \hbar^2 k^2/2m$) for $\epsilon = 0.005$, $a = 1$ μm, $\alpha = 1.8$, and $\chi_2 = \pi/6$. The solid line corresponds to the presence of the Zeeman coupling of $B = 15$ G and the dash-dotted line corresponds to the presence of the same Zeeman coupling and a magnetic flux of $\phi = 0.02$. (a) Constant and alternating distances between paired peaks vs energy, represented by the solid and the dash-dotted lines, respectively. (b) Taken from (a) for a clear illustration of the effect caused by $\phi = 0.02$.

with increasing energy, have the spin orientations and current directions in a sequence of

$$\ldots, \quad [(+,d),(-,u)], \quad [(-,d),(+,u)],$$
$$[(+,d),(-,u)], \quad [(-,d),(+,u)], \quad \ldots, \tag{19}$$

where (s_1, s_2) refers to a single quantum state, with $s_1 = +$ (counterclockwise) or $-$ (clockwise) denoting the current direction and $s_2 = u$ (up) or d (down) denoting the spin orientation, and $[(s_1, s_2), (-s_1, -s_2)]$ refers to a pair of energy levels from the Kramers doublet. The eigenstate correspondence so identified for T leads to interesting resonance behavior, as depicted in Fig. 4. It is seen that when a small AB flux is added to distinguish the current directions, each two paired peaks are separated by a distance, which takes the larger or the smaller value alternatingly. The reason is already clear in the sequence (19). In essence, since the current direction determines the sign of the energy shift caused by a small AB flux, for $[(-,d),(+,u)]$ the energy splitting due to the small AB flux enhances that first caused by the Zeeman coupling, while for $[(+,d),(-,u)]$ the energy splitting due to the small AB flux cancels part of that first caused by the Zeeman coupling. We want to point out that the essential character of the above correspondence between the equilibrium and the transport properties can be quantitatively, but not be qualitatively, affected by the disorder or scattering potential in the ring as long as the single-particle picture holds for electronic states. In particular, such a correspondence, if experimentally verified or excluded in some specific ring, may serve to detect if the electron-electron interaction qualitatively alters the electronic states.

III. CONCLUSION AND DISCUSSION

In summary, we have studied the motion of electrons confined in the perfect ring in the presence of the cylindrically symmetric spin-orbit interaction and Zeeman coupling, and the magnetic flux. Starting from the exact solution for the closed ring, we have investigated the transport properties of the ring connected to current leads, with emphasis on the roles of the TRS and its breaking therein. We have provided the numerical results for illustrating the joint effects of the spin-orbit interaction, Zeeman coupling, and magnetic flux. From the resonance behavior of the transmission probability in the weak-coupling limit, we have found the observable correspondence between the TRS-breaking effects manifested by the persistent current and by the transmission probability as long as the single-particle picture of electronic states holds. As the relation between transmission probability and persistent current is discussed for 1D noninteracting electrons with spin, the effect of the electron-electron interaction is yet to be investigated. It is interesting to note that, for 2D interacting electrons, the effects of the electron-electron interaction on the persistent current and conductance has been discussed, but only in the spinless case.[19] On the other hand, in the presence of disorder, though the exact calculation can no longer be carried out, our observation that the TRS breaking due to Zeeman coupling invalidates the effective flux picture is still correct. In addition, the shift of transmission peaks depicted in Fig. 4(b) does not change qualitatively.

APPENDIX A: GEOMETRIC PHASE AND EXACT SOLUTION

The eigenvalue equation of the closed ring can be solved through a straightforward diagonalization, as presented in Ref. 17. Here we adopt the geometric phase approach[15] in order to identify the geometric and the dynamical phases in current-carrying eigenstates, which are responsible for transporting electrons when the ring is connected to current leads. How the phases and the spin orientations jointly affect the transmission probability will be elaborated on in Sec. II.

The cylindrical symmetry of the system leads to the conservation of total angular momentum $-i\partial/\partial\theta + \frac{1}{2}\sigma_z$, which means that the eigenstates of the Hamiltonian (2) are of the form $\Psi_{n,\mu}(\theta) = \exp(in\theta)\tilde{\psi}_{n,\mu}(\theta)/\sqrt{2\pi}$, in which $\mu = \pm$, n are arbitrary integers, and the spin states are given by

$$\tilde{\psi}_{n,+}(\theta) = \begin{bmatrix} \cos\dfrac{\beta_n}{2} \\ e^{i\theta}\sin\dfrac{\beta_n}{2} \end{bmatrix}; \qquad \tilde{\psi}_{n,-}(\theta) = \begin{bmatrix} \sin\dfrac{\beta_n}{2} \\ -e^{i\theta}\cos\dfrac{\beta_n}{2} \end{bmatrix}, \tag{A1}$$

where β_n is θ independent. From $\Psi^{\dagger}_{n,\mu}\sigma_i\Psi_{n,\mu}$ as a function of θ, it is readily seen that the local spin orientation at θ is in the direction of $\mu(\cos\beta_n\mathbf{e}_z+\sin\beta_n\mathbf{e}_r)$. The explicit expression for the spin tilt angle β_n can be obtained by introducing a cyclic evolution of the spin state for electrons encircling the ring, as presented in Ref. 15. The geometric and the dynamical phases associated with the spin precession can thereby be identified for all of the energy eigenstates to determine the whole energy spectrum. For $\Psi_{n,\mu}$ we obtain the spin tilt angle

$$\tan\beta_n=\frac{2\alpha\omega_n\sin\chi_1+\omega_B\sin\chi_2}{2\alpha\omega_n\cos\chi_1+\omega_B\cos\chi_2-\omega_n} \tag{A2}$$

and the geometric and the dynamical phases $\delta_{n,\mu}$ and $\gamma_{n,\mu}$,

$$\delta_{n,\mu}=-\pi(1-\mu\cos\beta_n), \tag{A3}$$

$$\gamma_{n,\mu}=-\mu\pi\left[2\alpha\cos(\beta_n-\chi_1)+\frac{\omega_B}{\omega_n}\cos(\beta_n-\chi_2)\right], \tag{A4}$$

where ω_n is given by $\omega_0(n+\frac{1}{2}+\phi)$ with $\omega_0=\hbar/ma^2$. Here the geometric AA phase $\delta_{n,\mu}$ is the $-1/2$ of the solid angle subtended by a circuit traced on a sphere by the local spin orientation of $\Psi_{n,\mu}$. It is readily seen that the Zeeman coupling makes the spin orientations of electronic eigenstates depend on the orbital quantum number. The consequence of such an interplay between the spin and the orbital degrees of freedom will be explored when we discuss the transport properties of the ring. With use of β_n, $\delta_{n,\mu}$, and $\gamma_{n,\mu}$, the eigenvalue $E_{n,\mu}$ of $\Psi_{n,\mu}$ is found to be

$$E_{n,\mu}=\frac{\hbar\omega_0}{2}(n+\phi)^2+\frac{\hbar\omega_0}{2}(\alpha^2-\alpha\cos\chi_1)-\frac{\hbar\omega_n}{2\pi}(\delta_{n,\mu}+\gamma_{n,\mu}). \tag{A5}$$

The first term in the right-hand side represents the energy from orbital motion, the second term the zero-point energy, while the third term comes from the spin precession originating from the interplay between the spin and the orbital degrees of freedom.

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