

## Delocalization in continuous disordered systems

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Continuous one-dimensional models supporting extended states are studied. These delocalized states occur at well-defined values of the energy and are consequences of simple statistical correlation rules. We explicitly study alloys of  $\delta$ -barrier potentials as well as alloys and liquids of quantum wells. The divergence of the localization length is studied and a critical exponent  $\frac{2}{3}$  is found for the  $\delta$ -barrier case, whereas for the quantum wells we find an exponent of 2 or  $\frac{2}{3}$  depending on the well's parameters. These results support the idea that correlations between random scattering sequences break Anderson localization. We further calculate the conductance of disordered superlattices. At the peak transmission the relative fluctuations of the transmission coefficient are vanishing. [S0163-1829(97)11116-X]

### I. INTRODUCTION

Recently, the interest has increased substantially in order to find theoretical evidence for the breaking of Anderson localization by internal correlations in disordered systems.<sup>1-17</sup> For instance, in Ref. 1 a tight-binding Hamiltonian with site correlations between the diagonal and the off-diagonal potentials was studied. A divergence of the localization length was obtained with a critical exponent  $\nu=2$  outside the band edges. In Refs. 2-6 a simple discrete model with correlations expressed by pairing (dimer) was studied. Their main result was the existence of a divergent localization length at some critical energies. In fact, this divergence is related to the existence of delocalized states found explicitly in Ref. 7, which can be related to random phases and thus to extended states. Similar dimer models for classical systems were studied in Ref. 8. Using the interrelation between a disordered Kronnig-Penney system and the dimer model (Poincaré map),<sup>9-12</sup> an infinite set of such delocalized states exist. The generalization of the dimer model to the  $N$ -mer case was considered, for instance, in Refs. 13-16. In this context perturbative methods were used in Ref. 17.

In this way, delocalized states have been found “contradicting” the usual belief that all eigenstates are localized in one dimension (1D). In fact, this is only apparent because the theorems related to localization in 1D consider strictly uncorrelated random potentials (see, for instance, Refs. 18-22). Interesting numerical simulations were carried out for binary disordered systems in Ref. 23.

Usually these correlated disordered systems support delocalized states for well-defined energies. In this respect, a “band of delocalization” can be defined when the length  $L$  of the sample is smaller than the localization length  $L_c(E)$ , which diverges near the critical energy  $E_c$ . Quantitatively, the region of delocalization can be defined approximately as

$$L < L_c(E) \sim \frac{1}{|E - E_c|^\nu}, \tag{1}$$

where  $\nu$  has been determined explicitly in some models. Evidently this definition of a band of delocalization is contro-

versial because it depends on the existence of states in the interval  $|E - E_c|$  and, moreover, is very different from the usual one related to true delocalized (unnormalizable) states. For instance, in the dimer model<sup>2-4</sup> a band of delocalized states is found to follow as  $\Delta E \sim \sqrt{L}$ , where  $L$  is proportional to the number of impurities  $N$ , nevertheless, the total number of states grows as  $L$  and therefore the relative number of delocalized states tends to zero as  $1/\sqrt{L}$ . Furthermore, it has recently been shown that additional perturbations on the dimer potentials lead to a suppression of these extended states.<sup>24</sup> Therefore these delocalization properties become more relevant in finite systems, as presented in the last part of this paper. In quasi-one-dimensional systems, such as models with dimerized interchain couplings,<sup>25</sup> leading to a cantor-set-like conducting band, the finite temperature transmission can be greatly enhanced by the resonance energies.

As noticed, internal correlations break strong localization. In this paper we consider some simple models showing delocalization properties. For a general point of view we consider the 1D disordered Hamiltonian of a particle in a random potential,

$$H = \frac{p^2}{2m} + \sum_l V_l(x - x_l), \tag{2}$$

$$V_l(x) \begin{cases} \neq 0, & -a_l < x < a_l \\ = 0 & \text{otherwise,} \end{cases} \tag{3}$$

thus, a sequence of scattering barriers centered at the random position  $x_l$  without overlap. Every barrier is symmetric with random support  $2a_l$  and the distance between two barriers is given by

$$x_{l+1} - x_l = d_l, \tag{4}$$

which is a random quantity.

Between consecutive barriers the particle propagates freely and its wave function can be expressed as

$$\psi_l(x) = A_l e^{ikx} + B_l e^{-ikx} \quad \text{for } x_l + a_l < x < x_{l+1} - a_{l+1}, \tag{5}$$

where  $k$  is the wave number related to the energy by  $E = \hbar^2 k^2 / 2m$ . Theoretical group arguments<sup>26-28</sup> relate the amplitudes  $(A_{l+1}, B_{l+1})$  to  $(A_l, B_l)$  by means of

$$\begin{pmatrix} A_{l+1} \\ B_{l+1} \end{pmatrix} = \begin{pmatrix} e^{-ikx_l} & 0 \\ 0 & e^{ikx_l} \end{pmatrix} \begin{pmatrix} \alpha_l & \beta_l \\ \beta_l^* & \alpha_l^* \end{pmatrix} \begin{pmatrix} e^{ikx_l} & 0 \\ 0 & e^{-ikx_l} \end{pmatrix} \begin{pmatrix} A_l \\ B_l \end{pmatrix}, \quad (6)$$

where the elements  $\alpha, \beta$  of the collision matrix are inter-related like  $|\alpha|^2 - |\beta|^2 = 1$  and we further assume  $\beta^* = -\beta$  (symmetric barrier). This form of the collision matrix is related to the fact that the time-independent Schrödinger equation is real.

Defining the wave function just after the collision with the  $l$ th barrier as

$$\psi_l^+ = A_l e^{ik(x_l + a_l)} + B_l e^{-ik(x_l + a_l)}, \quad (7)$$

a condition of delocalization is

$$\psi_{l+1}^+ = \pm \psi_l^+, \quad (8)$$

which relates the elements of the collision matrix as

$$\alpha_l e^{ik(d_l + a_{l+1} - a_l)} + \beta_l^* e^{-ik(d_l + a_{l+1} + a_l)} = \pm 1. \quad (9)$$

The above delocalization condition is a sufficient one and, as expected, is not verified for totally uncorrelated systems. Condition (9) yields using  $|\alpha|^2 - |\beta|^2 = 1$  the following simple delocalization condition:

$$|\alpha_l| = 1 \quad \text{and} \quad \beta_l = 0, \quad (10)$$

which leads to  $|A_{l+1}| = |A_l|$  and  $|B_{l+1}| = |B_l|$ .

Condition (9) inter-relates the random parameters like  $a_l$  and  $d_l$  in the disordered system and is therefore a source of correlation between these random parameters. In fact, in some simple systems, relations (9) and (10) can be satisfied and we present two examples: The  $\delta$ -barriers alloy and the quantum well liquid alloy with inter-related random parameters.

To end this section, we remark that any relation similar to Eq. (6) can be formally written as a tight-binding Schrödinger equation (Poincaré map) by considering

$$\hat{\alpha}_l = \alpha_l e^{ik(d_l + a_{l+1} - a_l)}, \quad \hat{\beta}_l = \beta_l e^{ik(d_l + a_{l+1} + a_l)} \quad (11)$$

and

$$D_{l+1} \psi_{l+1}^+ + D_l \psi_{l-1}^+ = V_l \psi_l^+, \quad (12)$$

where

$$D_l = \frac{D(k)}{2i(\text{Im} \hat{\alpha}_l - \text{Im} \hat{\beta}_l)}$$

and

$$V_l = \{D_{l+1}(\hat{\alpha}_l + \hat{\beta}_l^*) + D_l(\hat{\alpha}_{l-1}^* - \hat{\beta}_{l-1}^*)\}. \quad (13)$$

We notice that the above expression is only a formal one. For models such as the  $\delta$ -barriers sequence,  $D_l$  is well defined for any  $k$  with the arbitrary choice  $D(k) = 2i \sin k$ . Nevertheless, in other systems singularities for some values of the wave number  $k$  can appear.

## II. DELOCALIZATION IN A $\delta$ -BARRIER SEQUENCE

Consider a sequence of  $\delta$  potentials that are statistically distributed over lattice sites (alloy). Thus consider the sequence

$$V_l(x) = V_l \delta(x - x_l) \quad \text{and} \quad d_l = d, \quad (14)$$

where  $V_l$  are random uncorrelated parameters and  $d$  is a constant lattice parameter. In this case, the elements of the collision matrix are given by

$$\alpha_l = 1 + i \frac{V_l}{2k} \quad \text{and} \quad \beta_l = i \frac{V_l}{2k}. \quad (15)$$

The delocalization condition (9) becomes explicitly

$$e^{ikd} - \frac{V_l}{k} \sin(kd) = \pm 1. \quad (16)$$

In general, for any arbitrary momentum  $k$ , this condition does not hold because of the random quantity  $V_l$ . Nevertheless, if  $k = n\pi/d$  ( $n \in Z^*$ ), as first observed by Ishii,<sup>19</sup> we have a set of delocalized states where  $|\psi_{l+1}| = |\psi_l|$ . At first sight this is very surprising because of the simplicity of the disordered model. Evidently, correlations exist and are related to the choice  $d_l = d$  for any  $l$ . In fact, assuming a sample with  $N$  barriers, the usual  $2N$  random parameters characterizing the uncorrelated system are reduced to  $N$  because of the constraint  $d_l = d$ .

In this model the physical interpretation is simple, in fact as long as  $k = n\pi/d$  the electron does not ‘feel’ the random potential because the distance  $d$ , between consecutive barriers, is a multiple of its wavelength.

At this point, it is interesting to study the divergence of the localization length  $L_c(E)$  near the critical energy  $E_c = (1/2m)(\hbar n\pi/d)^2$ . Using the Poincaré map (12) for this model we have

$$\psi_{l+1} + \psi_{l-1} = 2 \left\{ \cos kd \frac{V_l}{k} \sin kd \right\} \psi_l. \quad (17)$$

For  $\epsilon = E_c - E \ll 1$  and  $\hbar/2m$  taken as unity, Eq. (17) can be rewritten

$$\psi_{l+1} + \psi_{l-1} = \pm \left( 2 - V_l \epsilon \frac{d^3}{n^2 \pi^2} \right) \psi_l. \quad (18)$$

This last model was extensively studied in the limit  $\epsilon \ll 1$  by Derrida and Gardner.<sup>29</sup> They calculated the complex Lyapunov exponent  $\gamma$ , where the real part corresponds to the inverse localization length and the imaginary part to  $\pi$  times the integrated density of states. Their results can be expressed as follows:

$$\begin{aligned} \text{Re}(\gamma) &\simeq K_1 \epsilon^{2/3} \langle V_n^2 \rangle^{1/3} \frac{d}{(n\pi)^{2/3}}, \\ \text{Im}(\gamma) &\simeq K_2 \epsilon^{2/3} \langle V_n^2 \rangle^{1/3} \frac{d}{(n\pi)^{2/3}}, \end{aligned} \quad (19)$$

where  $K_1 = 0.29 \dots$  and  $K_2 = 0.16 \dots$  and  $\langle \rangle$  is the aver-



$$L_c(E) \sim \frac{1}{|E - E_c|^\nu} \text{ where } \nu = \begin{cases} \frac{2}{3} & \delta\text{-barrier and } k_c(d-2a)/\pi \\ \text{an integer for the quantum well} & \\ 2 & \text{otherwise.} \end{cases} \quad (26)$$

#### IV. FLUCTUATIONS AND CONDUCTANCE

In one-dimensional disordered systems, the relative fluctuations of the transmission coefficients diverge with the size of the system.<sup>28</sup> Therefore the average transmission is a bad statistical quantity. However, at the critical energies discussed above, the states are deterministic and therefore one would expect no fluctuations. In the following we evaluate the relative fluctuations explicitly and calculate the conductance for superlattices around these critical energies.

Around the critical energy  $E_c$  we evaluated the localization length dependence on energies, therefore  $T_N \sim e^{-L/L_c}$ , where  $T_N$  is the transmission coefficient after  $N$  barriers and  $N$  is proportional to  $L$ . The relative fluctuations are then given by

$$\frac{\Delta T}{T} \sim N \frac{\Delta L_c}{L_c^2}. \quad (27)$$

Using the central limit theorem, valid for the logarithm of the product of random determinant one matrices, the relative fluctuations of the localization length can be expressed as

$$\frac{\Delta L_c}{L_c} \sim N^{-1/2}, \quad (28)$$

hence using Eqs. (26)–(28)

$$\frac{\Delta T}{T} \sim N^{1/2} |E - E_c|^\nu. \quad (29)$$

In usual one-dimensional disordered systems the average distance between eigenvalues is of the order  $N^{-1}$ ; therefore, using  $|E - E_c| \sim N^{-1}$ , we obtain

$$\frac{\Delta T}{T} \sim |E - E_c|^{\nu-1/2}. \quad (30)$$

This demonstrates that for the case of interest here, both cases  $\nu=2$  and  $\nu=\frac{2}{3}$  have vanishing relative fluctuations near the critical energies. This fluctuation analysis gives also a bound on the possible divergence exponent  $\nu$ . In fact as long as  $\nu > \frac{1}{2}$ , the average transmission around the critical energies is a good statistical quantity.

Many physical systems can be very well approximated with either rectangular well potentials or  $\delta$  potentials. This is the case for superlattices of heterostructures. One can grow very precise, up to the atomic precision, layers of GaAs. Then one dopes differently one layer with Al and then again the same layer and one obtains in this way a single so-called  $\delta$ -doped layer. This can then be repeated as often as one wishes. This last structure is very well described in the direction of the layers by  $\delta$  impurity models. Instead of doping differently only one layer we can also dope a finite number

of layers differently. In this case the potential looks more like a rectangular well potential, where the amplitude depends on the concentration of the doping used. In this way it is possible to grow one-dimensional quantum wires where both directions of the layers are etched, two-dimensional systems where only one of the direction of the layers is etched, and finally three-dimensional systems.

The two probe conductance is simply given by the sum of the transmission coefficients,<sup>31</sup> thus

$$G(\mu) = \sum_{k|E=\mu} T(k) \frac{e^2}{h}, \quad (31)$$

where  $\mu$  is the chemical potential.

For a one-dimensional quantum wire the conductance is just given by the transmission coefficient, as there is only one conducting channel. Around the critical energies we evaluated the localization length dependence on energies; therefore using  $T_N \sim e^{-N/L_c}$ , where  $T_N$  is the transmission coefficient after  $N$  barriers, we obtain conductance peaks at the critical energies of the form

$$G(\mu) \sim e^{-\gamma|\mu - E_n|^\nu}, \quad (32)$$

where  $\mu$  is the chemical potential,  $\gamma$  depends on the size and disorder of the system, and  $E_n$  are the critical energies. For the  $\delta$  doping  $E_n = (1/2m)(\hbar n \pi/d)^2$  and  $\nu = \frac{2}{3}$ . For the rectangular case we have one critical energy  $E_c$  and the random Al doping potentials are given by

$$V_l = E_c - \frac{n_l^2 \pi^2}{a^2} \quad \text{with } n_l \text{ a random integer} \quad (33)$$

and  $a$  is the width of the doped layers. The width  $d_l$  of the undoped layers can be random and we obtain  $\nu=2$ . For the special case where  $d_l=d$  is constant the value of the exponent  $\nu$  depends on the ratio  $E_c(d-2a)^2/\pi^2$ . If this ratio is an integer the exponent is  $\nu=\frac{2}{3}$  but remains  $\nu=2$  otherwise. The interesting feature of this system is that it would behave as a perfect filter. The bandwidth could be controlled by the size of  $\gamma$  and the desired critical energy by the choice of the dopants.

In order to evaluate the conductance of the three-dimensional system, as defined above, we have to sum over all possible channels. In the directions  $(x, y)$  perpendicular to the grown layers the density of states is simply the usual box filling density. In the infinite limit we have to integrate over the Brillouin zone which yields

$$G \sim \int_{k_x^2 + k_y^2 \leq \mu} e^{-\gamma|\mu - k_x^2 - k_y^2 - E_c|^\nu} dk_x dk_y. \quad (34)$$

The plots of the conductance from Eq. (34) as a function of the chemical potential are given in Fig. 2 for different system sizes characterized by  $\gamma$  and for the two cases  $\nu=2$  and  $\nu=\frac{2}{3}$ .

The two-dimensional case is presented in Fig. 3. In this case the behavior is consequently different for  $\nu=2$  and  $\nu=\frac{2}{3}$ . Indeed for  $\nu=2/3$  we have a singular derivative, which could be observed experimentally, in order to distinguish between the two cases.

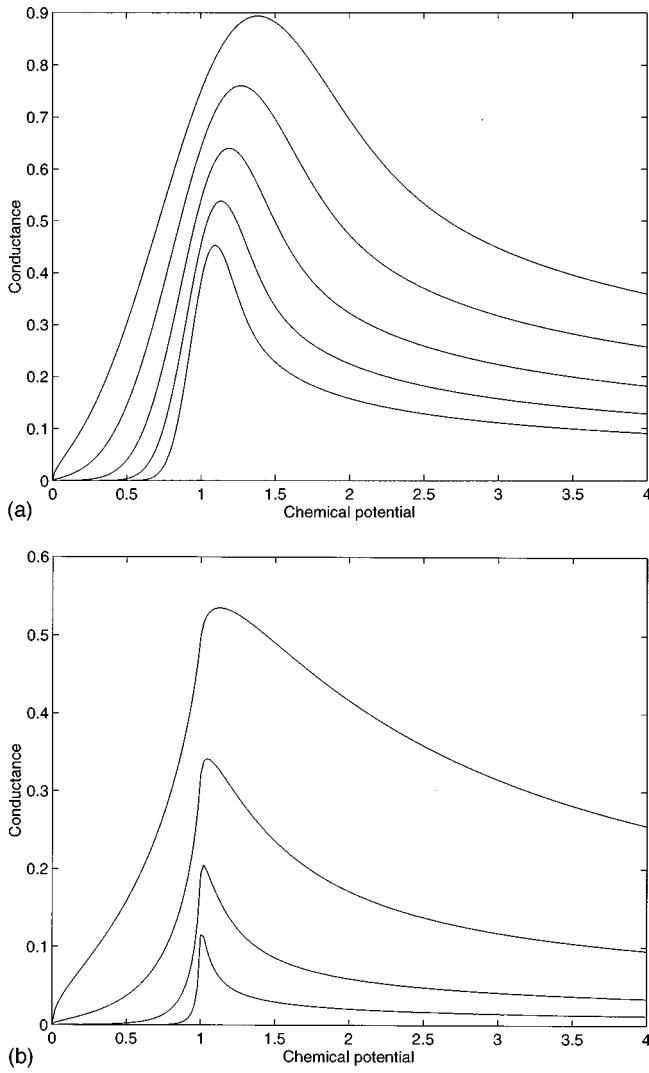


FIG. 2. Conductance for  $\nu=2$  and  $\nu=\frac{2}{3}$  (right graph) as a function of the chemical potential, for the case where the resonance energy is  $E_c=1$ . The different curves represent different values of  $\gamma=2^n$ , with  $n=1,2,3,4,5$ . The uppermost curve represents the case  $\gamma=2$ .

The quantitative values can also be estimated. In the  $\delta$ -doping case the typical Fermi wavelength is of the order of 200 Å and the typical layer thickness is of the order of 5 Å. This means that we have to construct  $\delta$ -doped layers with around 40 layers in between. In order to obtain the disorder we just have to choose randomly the dopant or the concentration of the doping for the  $\delta$  layers. The same reasoning can be held for rectangular potentials, where one just has to choose a particular potential configuration.

## V. CONCLUSIONS

In the first section we demonstrated the existence of extended states in continuous disordered systems with  $\delta$  impurities placed on a lattice and obtained for the exponent describing the divergence of the localization length  $\nu=\frac{2}{3}$ . Hence this model demonstrates the existence of extended states in systems with only shape disorder. In the second section, using rectangular potentials we demonstrated that

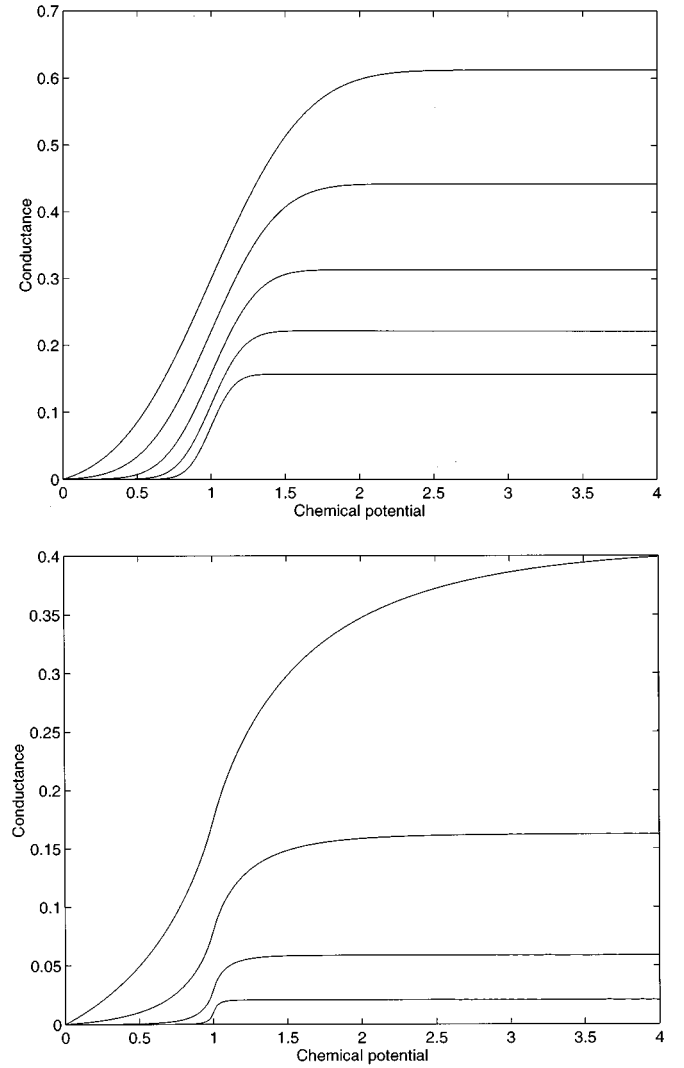


FIG. 3. Conductance for the two-dimensional case with the same parameters as in Fig. 2.

models with shape *and* space disorder can also exhibit extended states. The only constraint on the disorder distribution is that it must be discrete for the shape disorder but can be continuous for the space disorder. In this case the correlation in disorder leading to the existence of delocalized states is simply the fact that the distribution mentioned above is discrete and obeys relation (24). Therefore one has to be very careful when one discusses disordered systems using discrete disorder distributions as there can exist singularities in the spectrum. The exponents of the localization length divergence are very interesting. In fact, depending on the parameters of the quantum wells, we obtain different exponents, either  $\nu=\frac{2}{3}$  or  $\nu=2$ .

The relative fluctuations of the conductance around these critical energies are vanishing. Therefore the conductance is a well-defined statistical average and represents the transport properties of these disordered systems.

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