PHYSICAL REVIEW B VOLUME 55, NUMBER 16 15 APRIL 1997-II

Absolute band gaps and electromagnetic transmission in quasi-one-dimensional comb structures

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(Received 16 September 1996; revised manuscript received 17 December 1996)

We demonstrate the existence of absolute gaps in the band structure of a quasi-one-dimensional electromagnetic comb composed of a one-dimensional wave guide along which an infinity of side branches are grafted periodically. We show that the width of the gaps is very sensitive to the length of the side branches, to the periodicity, as well as to the contrast in dielectric properties of the constituent materials. Nevertheless, relatively wide gaps still remain when the constituent materials are identical. We also present results of the transmission coefficient of an electromagnetic wave propagating along the wave guide for a finite number of side branches. For an increasing number of side branches the behavior of the transmission coefficient parallels the calculated band structure of the infinite comblike structure. The convergence, as concerns the band-gap limits, can be achieved for most of the gaps for a small number of side branches ($N \cong 10-20$). [S0163-1829(97)10916-X]

I. INTRODUCTION

Ten years ago, Yablonovitch,[1] inspired by experiments on Rydberg atoms and Penning-trapped electrons,[2,3] suggested that a periodic dielectric structure possessing an electromagnetic band gap could lead to inhibited spontaneous emission. The spontaneous emission inhibition in such gaps could be utilized to enhance the performance of semiconductor lasers and other quantum devices. Moreover, John[4] highlighted the relationship between forbidden electromagnetic band gaps and the localization of photons. In addition to these proposals, the problem of propagation of electromagnetic waves in composite media has received a great deal of attention. Of particular interest is the existence of photonic gaps in the electromagnetic band structure of artificial materials called ''photonic crystals.''

At the outset, theoretical and experimental works focused on three-dimensional photonic crystals. By employing the full vector Maxwell's equations, the existence of a pseudogap in the photonic band structure of a face-centered-cubic lattice of dielectric spheres has been established.[5,6] It was proved that spheres arranged in the diamond structure[7] as well as nonspherical dielectric inclusions placed on a fcc lattice[8] possess absolute band gaps. Complete band gaps were further obtained in two-dimensional photonic crystals constituted of periodic arrays of dielectric rods embedded in a dielectric background. Square,[9,10] triangular,[11–14] and hexagonal[15,16] lattices were investigated. The propagation of electromagnetic waves in one-dimensional systems such as superlattices[17–20] has also been studied extensively during the last two decades.

In all these composite systems the contrast in dielectric properties between the constituent materials and the composition of the inhomogeneous material are emerging as critical parameters in determining the existence of gaps.[21] These numerous studies open a question regarding the occurrence of band gaps for electromagnetic waves in homogeneous systems by tailoring their geometry.

In this paper, we pursue the appealing possibility of devising a comblike structure of one-dimensional wave guides exhibiting stop bands. This structure is composed of a backbone (or substrate) wave guide along which finite side branches are grafted periodically. The analogy between the electromagnetic waves and vibrations in one dimension and recent results[22] showing vibrational gaps in comblike structures suggest the possibility of opening gaps in the electromagnetic band structure of wave guide networks with similar geometry.

This study is conducted within the frame of the interface response theory of continuous media which we recall briefly in Sec. II. This theory allows the calculation of the Green's functions of a network structure in terms of the Green's functions of its elementary constituents. Three network structures are then considered, namely a single side branch on an infinite one-dimensional wave guide, an infinite periodic comblike structure, and a finite comb with two semi-infinite leads. The first one is shown in Sec. III to give rise to well-defined zeros of transmission due to resonances between the side branch and the backbone. These resonances are enlarged to absolute gaps in the limit of an infinite periodic comb. Because of the periodicity it is also shown in Sec. IV that additional gaps form. Finally, in Sec. V, we calculate the transmission coefficient for electromagnetic waves of a finite comb. Despite its finite size this device retains most of the features of the infinite periodic one. This work demonstrates the possibility of designing simple homogeneous networks of one-dimensional wave guides with absolute band gaps. Further conclusions on the extension of this work are drawn in Sec. VI.

II. INTERFACE RESPONSE THEORY OF CONTINUOUS MEDIA

A. Overview

In this paper, we study the propagation of electromagnetic waves in composite systems composed of one-dimensional continuous segments (or branches) grafted on different substrates. This study is performed with the help of the interface response theory[23] of continuous media which permits us to calculate the Green's function of any composite material. In what follows, we present the basic concepts and the fundamental equations of this theory.

Let us consider any composite material contained in its space of definition D and formed out of N different homogeneous pieces situated in their domains D_i. Each piece is bounded by an interface M_i, adjacent in general to j ($1\leqslant j \leqslant J$) other pieces through subinterface domains M_{ij}. The ensemble of all these interface spaces M_i will be called the interface space M of the composite material.

The elements of the Green's function $g(DD)$ of any composite material can be obtained from[23]

$$g(DD)=G(DD)-G(DM)G^{-1}(MM)G(MD)+G(DM)G^{-1}(MM)g(MM)G^{-1}(MM)G(MD), \tag{1}$$

where $G(DD)$ is the Green's function of a reference continuous medium and $g(MM)$, the interface elements of the Green's function of the composite system. The inverse $g^{-1}(MM)$ of $g(MM)$ is obtained for any points in the space of the interfaces $M=\{\cup M_i\}$ as a superposition of the different $g_i^{-1}(M_i,M_i)$,[23,24] inverse of the $g_i(M_i,M_i)$ for each constituent i of the composite system. The latter quantities are given by the equation

$$g_i^{-1}(M_i,M_i)=\Delta_i(M_i,M_i)G_i^{-1}(M_i,M_i), \tag{2}$$

where

$$\Delta_i(M_i,M_i)=I(M_i,M_i)+A_i(M_i,M_i) \quad (I \text{ is the unit matrix}), \tag{3}$$

and

$$A_i(X,X')=V_{c_i}(X'')G_i(X'',X')|_{X''=X}, \tag{4}$$

where $\{X,X''\}\in M_i$ and $X'\in D_i$.

In Eq. (4), the cleavage operator V_{c_i} acts only in the surface domain M_i of D_i and cuts the finite or semi-infinite size block out of the infinite homogeneous medium.[23] A_i is called the surface response operator of block i.

The new interface states can be calculated from[23]

$$\det[g^{-1}(MM)]=0 \tag{5}$$

showing that, if one is interested in calculating the interface states of a composite, one only needs to know the inverse of the Green's function of each individual block in the space of their respective surfaces and/or interfaces.

Moreover, if $U(D)$ (Ref. 25) represents an eigenvector of the reference system, Eq. (1) enables one to calculate the eigenvectors $u(D)$ of the composite material

$$u(D)=U(D)-U(M)G^{-1}(MM)G(MD)+U(M)G^{-1}(MM)g(MM)G^{-1}(MM)G(MD). \tag{6}$$

In Eq. (6), $U(D)$, $U(M)$, and $u(D)$ are row vectors. Equation (6) enables one also to calculate all the waves reflected and transmitted by the interfaces as well as the reflection and the transmission coefficients of the composite system. In this case, $U(D)$ must be replaced by a bulk wave launched in one homogeneous piece of the composite material.[25]

B. Inverse surface Green's functions of the elementary constituents

We report here the expression of the Green's function of a homogeneous isotropic infinite dielectric medium. For the sake of simplicity, we restrict ourselves to nonmagnetic media. We give also the inverse of the surface Green's function for the semi-infinite medium with a free surface and for the slab of thickness d.

1. Green's function of an infinite medium

We consider an infinite medium "i" associated to the Cartesians coordinates system (O,x_1,x_2,x_3). It has been established[26,27] that the Fourier transformed Green's function between two points $\mathbf{X}(x_1,x_2,x_3)$ and $\mathbf{X}'(x_1',x_2',x_3')$ of this medium is given as

$$G_i(\vec{k}_\parallel x_3,x_3')=-\frac{e^{-\alpha_i|x_3-x_3'|}}{2F_i}, \tag{7}$$

where $\vec{k}_\parallel$ is a two-dimensional wave vector in the plane (x_1Ox_2).

For electromagnetic waves with which we are dealing in this paper, α_i and F_i are given as

$$\alpha_i=i\left[\frac{\omega^2}{c^2}\,\varepsilon_i(\omega)-k_\parallel^2\right]^{1/2}=i\alpha_i', \tag{8a}$$

$$F_i=\alpha_i \quad \text{for the } s \text{ polarization}, \tag{8b}$$

$$F_i=-\frac{\omega^2}{c^2}\,\frac{\varepsilon_i(\omega)}{\alpha_i} \quad \text{for the } p \text{ polarization}, \tag{8c}$$

where ω is the angular frequency of the wave, c the speed of light in vacuum, and $\varepsilon_i(\omega)$ the relative permittivity for the homogeneous isotropic dielectric medium i.

Equation (7) may be generalized to other excitations as elastic waves in solids or liquids[25] and electrons.[28] The Green's function for a one-dimensional infinite wave guide is obtained by setting $\vec{k}_\parallel=0$ in Eqs. (8). In this one-dimensional case, the parameter F_i has the same value for the s and p polarizations.

2. Inverse surface Green's functions of the semi-infinite medium

One considers a semi-infinite medium "i" with a "free surface" located at the position $x_3=0$ in the direction Ox_3 of the Cartesians coordinates system (O,x_1,x_2,x_3) and infinite in the two other directions. In this case,[29]

$$g_i^{-1}(MM)=g_i^{-1}(00)=-F_i. \tag{9}$$

3. Inverse surface Green's functions of the slab

One considers a slab of width d_i bounded by two free surfaces located on $x_3=0$ and $x_3=d_i$ in the direction Ox_3 of

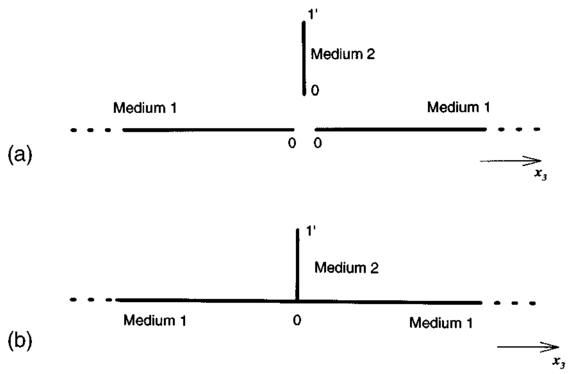

FIG. 1. (a) Elementary constituents of the wave guide with a single grafted segment of length $d_2=d$. (b) Wave guide with a single grafted segment of length $d_2=d$.

the Cartesians coordinates system (O,x_1,x_2,x_3) and infinite in the two other directions. In this case[29]

$$g_i^{-1}(MM)=\begin{pmatrix} -\frac{F_iC_i}{S_i} & \frac{F_i}{S_i} \\ \frac{F_i}{S_i} & -\frac{F_iC_i}{S_i}\end{pmatrix} =\begin{pmatrix} g_i^{-1}(0,0) & g_i^{-1}(0,d_i) \\ g_i^{-1}(d_i,0) & g_i^{-1}(d_i,d_i)\end{pmatrix}, \tag{10}$$

where F_i has the same meaning as above and

$$C_i=ch(\alpha_i d_i), \tag{11a}$$

$$S_i=sh(\alpha_i d_i). \tag{11b}$$

One can see that in the interface domain M corresponding to interfaces $x_3=0$ and $x_3=d_i$, the surface Green's function is a 2×2 square matrix. To obtain the Green's function for one-dimensional segments of wave guides, one needs only to take the limit of $\vec{k}_\parallel\to 0$ in Eq. (10). In order to study elementary electromagnetic excitations, we calculate the surface Green's function for different composite systems composed of finite segments grafted on a one-dimensional wave guide.

III. PROPAGATION OF ELECTROMAGNETIC WAVES IN AN INFINITE LINE WITH ONE GRAFTED FINITE SEGMENT

One considers a quasi-one-dimensional composite system formed out of a finite segment of length d grafted on an infinite wave guide line [see Fig. 1(a)]. In order to calculate the surface Green's function in this case, we construct this system with two semi-infinite lines constituted of the same dielectric material 1 and a segment of dielectric material 2 of finite length d. These three blocks are coupled at their ends [see Fig. 1(b)]. For the two semi-infinite lines and for the finite segment, the interface domains correspond to site 0 and sites 0 and $1'$, respectively.

The inverse surface Green's functions $g_1^{-1}(MM)$ for the two semi-infinite lines and $g_2^{-1}(MM)$ for the finite segment are given by Eqs. (9) and (10) with $i=1$ and $i=2$, respectively ($d_2=d$). Media 1 and 2 are one-dimensional and in the expressions of parameters α_i and F_i ($i=1,2$) [Eqs. (8a) and (8b)], $\vec{k}_\parallel=0$.

In this case, the interface domain of the composite system reduces to site 0 and the finite segment contribution to the surface Green's function of the composite system takes the form

$$\tilde{g}_2(0,0)=g_2(0,0)=-\frac{C_2}{F_2S_2} \quad\text{and}\quad \tilde{g}_2^{-1}(0,0)=-\frac{F_2S_2}{C_2}. \tag{12}$$

Superposing these different contributions, one deduces[24] that the inverse surface Green's function of the composite system is

$$g^{-1}(0,0)=2g_1^{-1}(0,0)+\tilde{g}_2^{-1}(0,0)=-2F_1-\frac{F_2S_2}{C_2} \tag{13}$$

and

$$g(0,0)=-\frac{C_2}{F_2S_2+2F_1C_2} \tag{14}$$

with $C_2=ch(\alpha_2 d)=\cos(\alpha_2' d)$ and $S_2=sh(\alpha_2 d)=i\ \sin(\alpha_2' d)$, where $\alpha_2'=(\omega/c)[\varepsilon_2(\omega)]^{1/2}$.

Equation (6) allows us to calculate the transmission coefficient of this composite system. Consider $U(x_3)=e^{-\alpha_1 x_3}$, a bulk propagating wave coming from $x_3=-\infty$. Using this incident wave in Eq. (6), one obtains the transmitted wave $u(x_3')$ with $x_3'\geqslant 0$ as

$$u(x_3')=\frac{2F_1C_2}{F_2S_2+2F_1C_2}\,e^{-\alpha_1 x_3'}. \tag{15}$$

We deduce from Eq. (15) that the transmission coefficient is

$$T=\left|\frac{2F_1C_2}{F_2S_2+2F_1C_2}\right|^2. \tag{16}$$

We observe that this coefficient equals zero when $C_2=0$, i.e.,

$$\alpha_2'=\left(m+\frac{1}{2}\right)\left(\frac{\pi}{d}\right), \tag{17a}$$

where m is a positive integer. The variations of T versus the dimensionless quantity $\alpha_2' d$ are reported in Fig. 2 in the case of identical media 1 and 2 for the backbone and for the branch. T is equal to zero for $\alpha_2' d$ odd multiple of $\pi/2$ and reaches its maximum value of 1 for $\alpha_2' d$ multiple of π. For this composite system, there exists an infinite set of forbidden frequencies ω_g such as

$$\omega_g=\frac{c}{[\varepsilon_2(\omega)]^{1/2}}\left(m+\frac{1}{2}\right)\left(\frac{\pi}{d}\right) \tag{17b}$$

corresponding to eigenmodes of the grafted finite segment. This grafted segment behaves as a resonator and this simple composite system filters out the frequencies ω_g. One can notice that the existence of transmission zeros has been already demonstrated in wave guides with a resonantly coupled stub for electrons[30] and phonons.[31,22] This phenomenon is related to the resonances associated with the finite

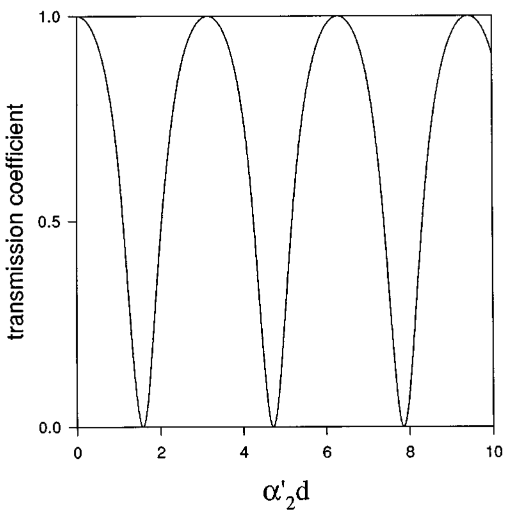

FIG. 2. Transmission coefficient versus the dimensionless quantity $\alpha_2' d$ for the wave guide with a single grafted segment of length d in the case of identical media 1 and 2. ($\alpha_2' = (\omega/c)[\varepsilon_2]^{1/2}$, where ε_2 is the relative permittivity of medium 2.)

additional path offered to the wave propagation. We can now consider more complex structures of the composite system containing a larger number of side branches.

IV. ONE-DIMENSIONAL INFINITE BACKBONE WITH A PERIODIC ARRAY OF FINITE SEGMENTS: INFINITE COMB

We treat the case of a comblike structure composed of finite segments (medium 2) of length d_2 grafted periodically with lattice spacing d_1 on an infinite substrate (medium 1) (see Fig. 3). Let us first write the surface Green's function of this composite system. The infinite line can be modeled as an infinite number of finite segments (one-dimensional slab) of length d_1 in the direction x_3, each one being glued to two neighbors. The interface domain is constituted of all the connection points between finite segments. In what follows, these connection points will be called ''sites'' and each site on the infinite chain will be defined by the integer n such as $-\infty < n < +\infty$. On each site n, a finite segment of length d_2 is connected. The respective contributions of media 1 and 2 to the inverse surface Green's function of the composite system are given by Eqs. (10) and (12), respectively. The inverse surface Green's function of the composite system is then obtained as an infinite banded matrix $g_\infty^{-1}(MM)$ defined

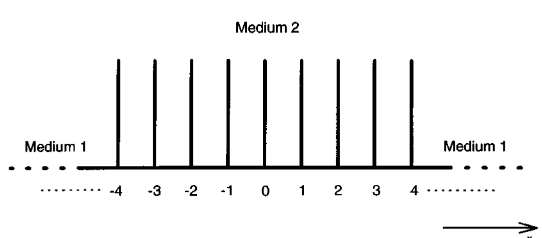

FIG. 3. Wave guide with a periodic array of grafted segments of length d_2 distant from each other by a length d_1. Each medium i ($i=1$ for the backbone and $i=2$ for the side branches) is characterized by the relative permittivity ε_i.

in the interface domain constituted of all the sites n. The diagonal and off-diagonal elements of this matrix are given respectively, by $-(2F_1C_1/S_1 + F_2S_2/C_2)$ and F_1/S_1.

Taking advantage of the translational periodicity of this system in the direction x_3, this matrix can be Fourier transformed as

$$[g_\infty(\mathbf{k},MM)]^{-1} = \frac{2F_1}{S_1}[-\xi + \cos(kd_1)], \tag{18}$$

where k is the modulus of the one-dimensional reciprocal vector $\mathbf{k}$ and $\xi = C_1 + (F_2/2F_1)(S_1S_2/C_2)$.

The dispersion relation of the infinite periodic comblike wave guide is given by Eq. (5), i.e., $[g_\infty(\mathbf{k},MM)]^{-1} = 0$ and is expressed in the simple form

$$\cos(kd_1) = \xi. \tag{19}$$

There exists forbidden frequencies for $C_2 = \cos(\alpha_2' d_2) = 0$, which correspond to the zeros of transmission [see Eqs. (17)] of an infinite substrate with a single grafted segment. On the other hand, in the $\mathbf{k}$ space, the surface Green's function is

$$g_\infty(\mathbf{k},MM) = \frac{S_1}{F_1}\frac{1}{\{-2[\xi - \cos(kd_1)]\}}. \tag{20}$$

After inverse Fourier transformation, Eq. (20) gives[32]

$$g_\infty(n,n') = \frac{S_1}{F_1}\frac{t^{|n-n'|+1}}{t^2-1}, \tag{21}$$

where the integers n and n' refer to the sites ($-\infty < n,n' < +\infty$) on the infinite line. The parameter t is defined as follows:

$$t = \begin{cases} \xi - \sqrt{\xi^2 - 1}, & \xi > 1 \\ \xi + \sqrt{\xi^2 - 1}, & \xi < -1 \\ \xi \pm i\sqrt{1-\xi^2}, & -1 < \xi < +1 \end{cases}$$

$$\text{with } t + \frac{1}{t} = 2\xi \quad \text{and} \quad |t| < 1. \tag{22}$$

We now focus on the dispersion relation of this composite system. Equation (19) can be written explicitly as

$$\cos X - \frac{\sqrt{\beta}}{2}\frac{\sin X \, \sin(\gamma\sqrt{\beta}X)}{\cos(\gamma\sqrt{\beta}X)} = \cos(kd_1), \tag{23}$$

where $X = \alpha_1' d_1$, $\beta = \varepsilon_2/\varepsilon_1$, and $\gamma = d_2/d_1$.

Let us first consider the particular case where media 1 and 2 are identical ($\alpha_2' = \alpha_1'$) with $d_1 = d_2$, i.e., $\beta = \gamma = 1$. In this very simple case, the resolution of Eq. (23) is strictly analytical. The width $\Delta\omega$ of the gaps is given by values of X such that

$$\frac{3\cos^2 X - 1}{2\cos X} = \pm 1. \tag{24}$$

We then deduce the gap width

$$\Delta\omega = \frac{c}{[\varepsilon_1(\omega)]^{1/2}}\frac{\Delta X}{d_1}. \tag{25}$$

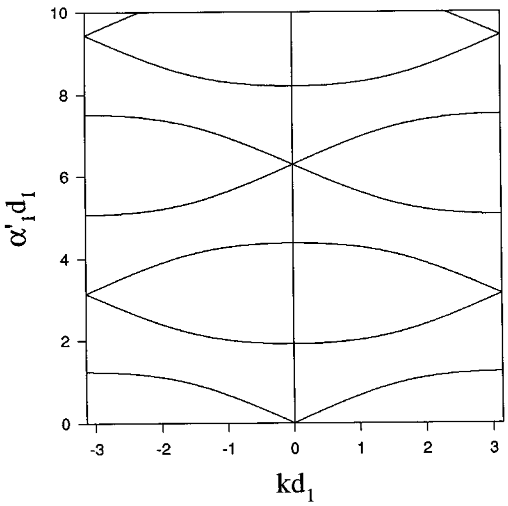

FIG. 4. Electromagnetic band structure of the infinite periodic comb with $\beta=\gamma=1$ ($\beta=\varepsilon_2/\varepsilon_1$ and $\gamma=d_2/d_1$). The plot is given in terms of $\alpha_1' d_1$ ($\alpha_1'=(\omega/c)[\varepsilon_1]^{1/2}$) versus the dimensionless quantity kd_1 ($-\pi\leqslant kd_1\leqslant+\pi$), where k is the modulus of the propagation vector. One observes two absolute gaps of identical width between the first and the second band and between the third and the fourth band.

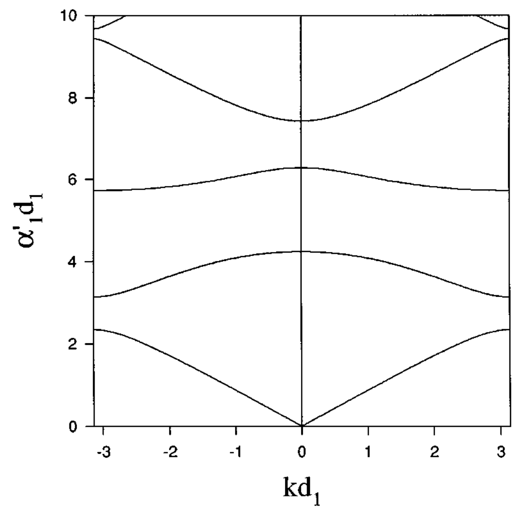

FIG. 5. The same as in Fig. 4 with $\beta=\varepsilon_2/\varepsilon_1=1$ and $\gamma=d_2/d_1=0.3$. In this case one observes three absolute gaps between the first and the second band, the second and the third band, and the third and the fourth band.

$X_1=0$, $X_2=\pi$, $X_3=1.2309$, and $X_4=1.9106$ are solutions of Eq. (24) in the interval $[0,\pi]$. In Eq. (25), ΔX stands for the difference between X_4 and X_3. From Eq. (23), one observes that the width of the gaps is governed by the quantity $\gamma\sqrt{\beta}$. Figure 4 represents the band structure of the infinite comb composite in terms of $\alpha_1' d_1$ versus kd_1 in the case $\beta=\gamma=1$ for $-\pi\leqslant kd_1\leqslant+\pi$. There exist two gaps of identical width ΔX between the first and the second band and between the third and the fourth band. These gaps appear around values of X corresponding to odd multiple of $\pi/2$ associated with the zeros of transmission of the grafted resonators. The second and the third bands meet at $kd_1=\pm\pi$ and there is no gap between these two bands. Along the $\alpha_1' d_1$ axis, the band structure repeats periodically with a 2π period.

Equation (25) gives the width of the gaps in the electromagnetic band structure of this particular one-dimensional composite system. These gaps appear around the frequencies

$$\omega_g=\frac{c}{[\varepsilon_1(\omega)]^{1/2}}\left(m+\frac{1}{2}\right)\left(\frac{\pi}{d_1}\right), \tag{26}$$

where m is a positive integer. This equation shows that according to the value of d_1, the first forbidden band ($m=0$) exists in different frequency domains of the electromagnetic spectrum. More precisely, if one considers $\varepsilon_1=9$, which corresponds to a material (alumina composite) often used in dielectric composite studies,[33] the first forbidden band appears in the microwaves domain for $0.05<d_1<25$ mm, in the infrared domain for $0.05<d_1<50$ μm and in the range of visible electromagnetic radiations for $333<d_1<500$ Å. Therefore, the one-dimensional nature of our model retains its validity to the microwave and infrared domains. Indeed the diameter of the finite grafted segments must be small compared to their length. Recent improvements[34,35] in the manufacturing of materials has permitted the fabrication of long Co or Cu wires of small diameter (from 50 to 300 Å), suggesting the possibility of designing such one-dimensional composite materials. For $\varepsilon_1=9$ and $d_1=10$ mm, the width of the first gap is 1.08 GHz, which is of the same order of magnitude as the gap width obtained in Ref. 33 for a two-dimensional dielectric composite. Moreover by decreasing d_1, one increases $\Delta\omega$.

In order to study the influence of the geometry of the comblike system on its electromagnetic band structure, we compute the band structure for $\beta=1$ and $\gamma\neq1$. For instance, Fig. 5 shows the band structure for $\beta=1$ and $\gamma=0.3$. In the low frequency domain, one observes gaps between the first and the second band, the second and the third band, and the third and the fourth band. Contrary to the second gap, which is associated with a zero of transmission of a single resonator, the first and the third gaps appear at $\alpha_1' d_1$ values different from an odd multiple of $\pi/2$. Therefore, these gaps must result from the ''superlattice'' nature of our quasi-one-dimensional wave guide with periodic side branches.

We have investigated the variation of the width of the first three absolute gaps in the $[\beta,\gamma]$ plane. We report in Figs. 6(a)–6(c) three-dimensional maps of these widths in the intervals $0.1<\beta<2.1$ and $0.1<\gamma<2.1$. The locus of the maxima of the first gap width [see Fig. 6(a)] is given by the condition $\gamma\sqrt{\beta}\sim0.5$. This condition appears as a ridge in the 3D map. The maximum value of 1.375 of the first gap width is located at the point $\beta=2.1$ and $\gamma=0.3578$. The width of the second gap attains its minimum value of zero, for the same condition $\gamma\sqrt{\beta}\sim0.5$. These minima correspond to a valley in the 3D map of Fig. 6(b). There exist two valleys where the third gap width is zero [see Fig. 6(c)], one corresponding again to the condition $\gamma\sqrt{\beta}\sim0.5$ and the other one for $\gamma\sqrt{\beta}\sim2$. The first condition ($\gamma\sqrt{\beta}\sim0.5$) is realized along the deepest valley in the 3D map.

As a general rule, the widest gaps are obtained for small values of β and γ. However, it is important to keep in mind

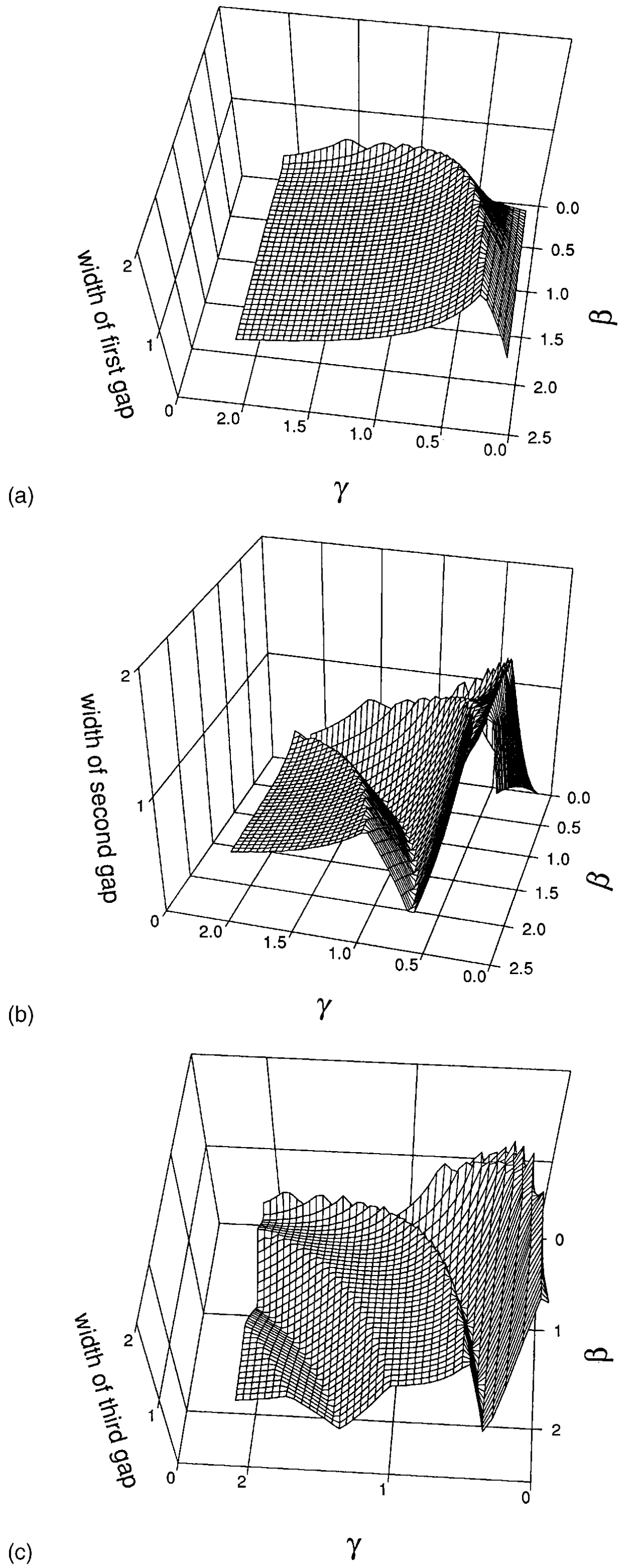

FIG. 6. (a) Variation of the width of the first absolute gap [in units of $\alpha_1' d_1 = (\omega/c)\sqrt{\varepsilon_1} d_1$] in the $[\beta,\gamma]$ plane ($\beta = \varepsilon_2/\varepsilon_1$, $\gamma = d_2/d_1$). This gap appears in the band structure between the first and the second band (see, for instance, Fig. 5). (b) The same as in (a) for the second absolute gap. This gap appears in the band structure between the second and the third band (see, for instance, Fig. 5). (c) The same as in (a) for the third absolute gap. This gap appears in the band structure between the third and the fourth band (see, for instance, Fig. 5).

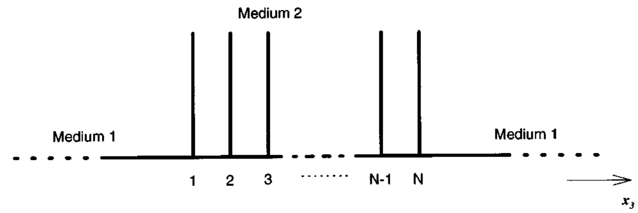

FIG. 7. Wave guide with a finite number N of grafted branches arranged periodically.

that even with the two identical constituent materials ($\beta = 1$) this composite system exhibits relatively wide gaps.

We have also performed a study of the electromagnetic band structure of an infinite two-dimensional wave guide. This ''brushlike'' structure is composed of finite segments of identical length (medium 2) grafted on each site of a periodic square array of one-dimensional wave guides (medium 1). We obtained wide absolute band gaps extending throughout the two-dimensional Brillouin zone. In the particular case of two identical constituent materials with a network periodicity equal to the length of the grafted segments (i.e., $\beta = \gamma = 1$), these gaps are narrower than in the corresponding comblike structure.

V. TRANSMISSION COEFFICIENT OF THE FINITE COMB

Infinite electromagnetic combs are not physically realizable. We investigate in this section a finite comb with a behavior similar to the infinite one.

We consider in this section the quasi-one-dimensional electromagnetic comb represented in Fig. 7. This composite system is constructed out of a finite comb cut out of the infinite periodic system of Fig. 3, which is subsequently connected at its extremities to two semi-infinite leading lines. The finite comb is therefore composed of N segments (medium 2) of length d_2 grafted periodically with a lattice spacing d_1 on a finite line (medium 1). For the sake of simplicity, the semi-infinite leads are assumed to be constituted of the same material as medium 1. We calculate analytically the transmission coefficient of a bulk electromagnetic wave coming from $x_3 = -\infty$.

The system of Fig. 7 is constructed from the infinite comb of Fig. 3. In a first step, one suppresses the segments linking sites 0 and 1, and sites N and $N+1$. For this new system composed of a finite comb and two semi-infinite combs, the inverse surface Green's function, $g_t^{-1}(MM)$, is an infinite banded matrix defined in the interface domain of all the sites n, $-\infty < n < +\infty$. The matrix is similar to the one associated with the infinite comb. Only a few matrix elements differ, namely, those associated with the sites $n = 0$, $n = 1$, $n = N$, and $n = N+1$.

The cleavage operator $V_{\rm cl}(MM) = g_t^{-1}(MM) - g_\infty^{-1}(MM)$ (Ref. 23) is the following 4×4 square matrix defined in the interface domain constituted of sites 0, 1, N, $N+1$:

$$V_{\rm cl}(MM) = \begin{pmatrix} w & -v & 0 & 0 \\ -v & w & 0 & 0 \\ 0 & 0 & w & -v \\ 0 & 0 & -v & w \end{pmatrix}, \qquad (27a)$$

where

$$w=\frac{F_1C_1}{S_1} \quad \text{and} \quad v=\frac{F_1}{S_1}. \tag{27b}$$

In a second step, two semi-infinite leads constituted of the same material as medium 1 are connected to the extremities $n=1$ and $n=N$ of the finite comb. With the help of the interface response theory, one deduces that the perturbing operator $V_p(MM)$ allowing the construction of the system of Fig. 7 from the infinite comb is then defined as the 4×4 square matrix [see Eq. (9)]:

$$V_p(MM)=\begin{pmatrix} w & -v & 0 & 0\\ -v & w-F_1 & 0 & 0\\ 0 & 0 & w-F_1 & -v\\ 0 & 0 & -v & w\end{pmatrix}. \tag{28}$$

On the other hand, using Eq. (21), one can write the elements of the surface Green's function of the infinite comb for $n,n'=0,1,N,N+1$ in the form of a 4×4 square matrix $g_r(MM)$:

$$g_r(MM)=\frac{S_1}{F_1}\frac{t}{t^2-1}\begin{pmatrix} 1 & t & t^N & t^{N+1}\\ t & 1 & t^{N-1} & t^N\\ t^N & t^{N-1} & 1 & t\\ t^{N+1} & t^N & t & 1\end{pmatrix}. \tag{29}$$

Using Eqs. (28) and (29), one obtains the matrix operator $\Delta(MM)=I(MM)+V_p(MM)g_r(MM)$ in the space M of sites 0, 1, N, and $N+1$. For the calculation of the transmission coefficient, we only need the matrix elements $\Delta(1,1)$, $\Delta(1,N)$, $\Delta(N,1)$, and $\Delta(N,N)$, which can be set in the form of a 2×2 matrix $\Delta_s(MM)$,

$$\Delta_s(MM)=\begin{pmatrix}\Delta(1,1) & \Delta(1,N)\\ \Delta(N,1) & \Delta(N,N)\end{pmatrix}=\begin{pmatrix}1+At & At^N\\ At^N & 1+At\end{pmatrix} \tag{30a}$$

with

$$A=-\frac{[t-(C_1-S_1)]}{(t^2-1)}. \tag{30b}$$

The surface Green's function $d_s(MM)$ of the finite comb with two connected semi-infinite leads in the space of sites 1 and N is

$$\begin{aligned} d_s(MM)&=g_s(MM)\Delta_s^{-1}(MM)\\ &=\frac{S_1}{F_1}\frac{t}{t^2-1}\frac{1}{\det\Delta_s(MM)}\\ &\quad\times\begin{pmatrix}1+At(1-t^{2N-2}) & t^{N-1}\\ t^{N-1} & 1+At(1-t^{2N-2})\end{pmatrix}\end{aligned} \tag{31}$$

with

$$g_s(MM)=\frac{S_1}{F_1}\frac{t}{t^2-1}\begin{pmatrix}1 & t^{N-1}\\ t^{N-1} & 1\end{pmatrix} \tag{32}$$

and

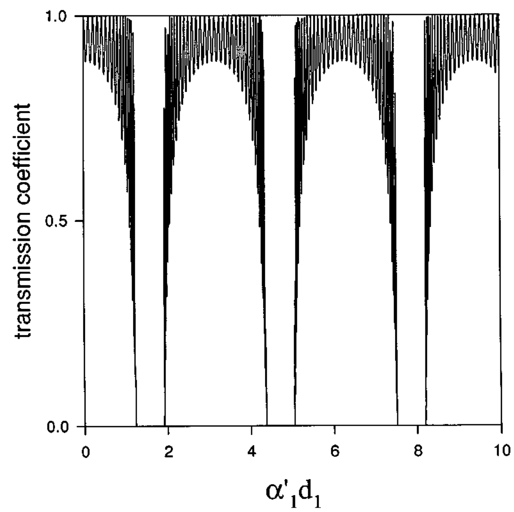

FIG. 8. Transmission coefficient for the finite comb with $\beta=\varepsilon_2/\varepsilon_1=1$ and $\gamma=d_2/d_1=1$ and $N=20$ grafted branches.

$$\det\Delta_s(MM)=1+2At+A^2t^2(1-t^{2N-2}). \tag{33}$$

In Eq. (32), $g_s(MM)$ is the matrix constituted of elements of $g_r(MM)$ associated with sites 1 and N. We now calculate the transmission coefficient with a bulk electromagnetic wave coming from $x_3=-\infty$, $U(x_3)=e^{-\alpha_1x_3}$. Substituting this incident wave in Eq. (6) and considering Eqs. (7) and (31), we obtain the transmitted wave $u(x_3')$ with $x_3'\geqslant Nd_1$ as

$$u(x_3')=-2S_1\frac{t^N}{t^2-1}\frac{e^{-\alpha_1[x_3'-(N-1)d_1]}}{\det\Delta_s(MM)}. \tag{34}$$

One deduces that the transmission coefficient is

$$T=\left|\frac{2S_1(t^2-1)t^N}{[1-t(C_1-S_1)]^2-t^{2N}[t-(C_1-S_1)]^2}\right|^2. \tag{35}$$

One can easily check that for $N=1$, which corresponds to the single grafted segment on an infinite line [see Fig. 1(b)], Eq. (35) leads to Eq. (16). In what follows, we study the variation of T versus $\alpha_1'd_1$ for different values of N in various finite combs.

Figure 8 represents the variation of the transmission coefficient T versus $\alpha_1'd_1$ for $\beta=1$, $\gamma=1$, and $N=20$. Despite the finite number of grafted segments, T approaches zero in regions corresponding to the observed gaps in the electromagnetic band structure of Fig. 4. Next we analyze the evolution of T as the number of grafted segments N increases in the case $\{\beta=1,\ \gamma=0.3\}$. The first zero of transmission is situated at $\alpha_1'd_1=(\pi/2)(1/0.3)$ as shown in Fig. 9(a). As the number of grafted segments increases to 4, a gap forms around the zero of transmission and the coefficient of transmission is strongly reduced around $\alpha_1'd_1=\pi$, 2π, 3π,... [see Fig. 9(b)]. This depression forms gaps for larger finite numbers of segments. In Fig. 9(c), one can observe for $N=20$ that nearly absolute gaps have appeared at π and 2π. At $\alpha_1'd_1=3\pi$, T has not yet converged to zero for $N=20$. One would need to increase the number of grafted segments to open a nearly absolute gap in this vicinity. These results parallel the calculated electromagnetic band structure of the infinite superlattice shown in Fig. 5. Therefore, the conver-

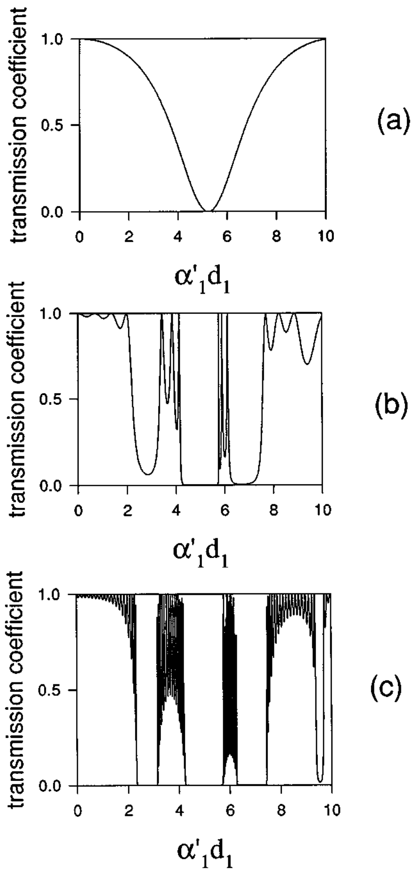

FIG. 9. Transmission coefficient for the finite comb with $\beta=\varepsilon_2/\varepsilon_1=1$ and $\gamma=d_2/d_1=0.3$. (a) $N=1$ grafted branch. (b) $N=4$ grafted branches. (c) $N=20$ grafted branches.

gence, as concerns the limits of the band gaps, can be achieved in general for a reasonably small number of side branches, usually N less than 10 to 20. However, higher values of N may be needed at those frequencies where the two conditions for the existence of gaps (namely $\alpha_1'd_1$ integer multiple of π or $\alpha_2'd_2$ half-integer multiple of π) become coincident.

The evolution of T for a finite quasi-one-dimensional comblike structure suggests the possibility of designing electromagnetic devices mimicking the behavior of the periodic infinite comblike wave guide. These devices may serve as frequency filters with wide stop bands.

VI. CONCLUSIONS

We have investigated the propagation of electromagnetic waves in simple networked wave guides with a comblike structure. There exist zeros in the transmission spectrum of a quasi-one-dimensional wave guide with a single grafted side branch. These zeros of transmission give rise to absolute band gaps in the electromagnetic band structure of an infinite periodic comb. Additional gaps form due to the periodic nature of the structure. Moreover, these features are still present in the transmission spectrum of a simple device constructed from a finite comb. In these systems, the gap width is controlled by geometrical parameters including the length of the side branches, the periodicity of the comb, as well as the contrast in dielectric properties of side branch material and the backbone. Nevertheless, the electromagnetic band structure exhibits relatively wide gaps for homogeneous systems where the branches and the substrate are constituted of the same material. We have also shown that devices composed of finite numbers of grafted side branches exhibit a behavior similar to that of an infinite periodic comb.

At this stage it is worth pointing out again the conditions of validity of the model. In all our calculations we have assumed that the cross section of the wave guide is small compared to its linear dimension, that is, the wave guide may be considered as a one-dimensional medium. We have seen in Sec. IV that this condition can be approached in the infrared and microwave regions of the electromagnetic spectrum as recent manufacturing techniques permit the fabrication of extremely thin wires.[34,35] However it would be interesting to verify the extension of the band gaps in the two-dimensional Brillouin zone of thicker wires. The calculation of the band structure in this case ($\vec{k}_\parallel\neq 0$) will be the subject of future work as well as the study of electromagnetic properties of more complex structures.

Finally, it is worthwhile mentioning that the Green's function calculation presented in this work assumes the vanishing of the electric field derivative at the free end of the grafted segments. The dispersion relation and transmission coefficients will be different when using another boundary condition, namely, the vanishing of the electric field at the grafted segments free ends [then, in Eq. (12), C_2 and S_2 have to be interchanged]. The physical consequence of this boundary condition will be the possibility of a cutoff frequency (i.e., a forbidden band which commences at zero frequency) in the spectrum of the periodic comb structure, which will be discussed in a forthcoming paper.

ACKNOWLEDGMENTS

One of us (P.A.D.) would like acknowledge financial support from the ''Laboratoire de Dynamique et Structure des Matériaux Moléculaires,'' U.F.R. de Physique, Université de Lille I, where this work was done. This work is partially supported by ''le Conseil Regional Nord-Pas de Calais.''

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