Cooper-pair charge solitons: The electrodynamics of localized charge in a superconductor

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One-dimensional arrays of small-capacitance Josephson junctions exhibit a current-voltage curve that is characterized by a zero-current state for bias voltage below a threshold voltage V_t. The threshold voltages can be modulated with an external magnetic field B, which tunes effective Josephson coupling between adjacent electrodes of the array. The dependance of V_t on B is well explained by a model where V_t is the injection voltage for a Cooper-pair charge soliton. [S0163-1829(96)52634-2]

The Coulomb blockade of Cooper-pair tunneling (CBCPT), which exists in small-capacitance superconducting tunnel junctions, can be regarded as the quantum mechanical complement of the Josephson effect.[1–3] In this article we demonstrate the CBCPT in one-dimensional arrays of Josephson junctions. We give an explanation which requires one to go beyond the zero-dimensional, lumped element approximations embodied in the ''orthodox theory'' of the single junction.[3] Electrodynamic considerations of a series network of junctions, result in a description of current flow in terms of the Cooper-pair charge soliton, which is a relativistically invariant model of an extended charge quantum $(2e)$ in one dimension.[4–6]

A small-capacitance Josephson junction can have a Coulomb energy, $E_Q=Q^2/(2C)$, which is comparable to the Josephson coupling energy, $U_J=E_J\cos(\phi)$. The charge $Q=CV$ is given by the potential difference, V, across the junction, and ϕ is the difference across the junction of the quantum mechanical phase describing the coherent state of the charge density. The junction Hamiltonian is that of a particle in a periodic potential, so that the Schrödinger equation will have Bloch wave solutions.[3] Thus, we can define a quasicharge, q, as the wave number of the plane wave part of the wave function (in analogy to the crystal momentum of electrons in a periodic potential).

Restricting ourselves to the lowest energy band, $E^0(q)$, the junction current and voltage are then related to the quasicharge by a set of relations which are complementary to the Josephson relations

$$\frac{dE^0}{dq}=V=V_c\,\text{saw}(q), \tag{1a}$$

$$I=\frac{dq}{dt}. \tag{1b}$$

The junction voltage is a $2e$ periodic function of the quasicharge, q, which we note by an amplitude, V_c, multiplied by $2e$ periodic function, saw(q), having an amplitude of 1. Both V_c and saw(q) are derived from properties of the Mathieu functions, and both depend on the ratio E_J/E_C, where $E_C=e^2/(2C)$.[3,7] For $E_J/E_C>1$, the shape is sinusoidal, saw$(q)\simeq\sin(\pi q/e)$, and the amplitude, V_c, decreases to zero exponentially in the ratio $\sqrt{E_J/E_C}$. For $E_J/E_C<1$, the shape is saw-tooth-like, with $V_c\simeq e/C$, [see Fig. 2(b) of Ref. 7].

Measurement of the quasicharge requires that the impedance seen by the single junction be greater than the resistance quantum $R_Q=h/(4e^2)=6.45$ kΩ. Experiments on single junctions biased with special high resistance leads[8] have demonstrated the CBCPT, thus confirming this general theoretical picture based on the idea of a definite quasicharge. In this article, we present experiments on 1D series arrays of small-capacitance Josephson junctions, where the CBCPT can be observed *without* a special high impedance source of charge. Several experiments have demonstrated the CBCPT in 2D arrays,[9–11] and the Coulomb blockade for single electrons has been studied in 1D arrays.[12] However, Cooper-pair tunneling has only been examined in very short 1D arrays.[13] In a long one-dimensional array, an interesting analysis based on sine-Gordon solitons is possible, and quantitative comparison with theory can be made.

Consider first a uniform one-dimensional series array of junctions. The current along the array, $I(x,t)$, and the potential of the electrodes, $V(x,t)$, are functions of the spatial coordinate x along the array. In the continuum limit, the current and voltage are related by a set of differential equations

$$\partial_x V=-l_0\partial_t I-v_c\text{saw}(q), \tag{2a}$$

$$\partial_x I=-c_0\partial_t V, \tag{2b}$$

where $l_0=L_0/\Delta x$ and $c_0=C_0/\Delta x$ are the distributed inductance and capacitance to the ground conductor, and $v_c=V_c/\Delta x$ is a critical electric field. With the exception of the $v_c$saw(q) term, these equations are the TEM transmission line equations describing electromagnetic waves. Combining Eq. (2) with Eq. (1b) and introducing a dimensionless quasicharge $\chi=\pi q/e$ (so that the saw function is periodic on the interval $\chi\in\{0,2\pi\}$), we arrive at,

$$(1/c^2)\,\partial_{tt}\chi-\partial_{xx}\chi+(1/\lambda_S^2)\,\text{saw}(\chi)=0, \tag{3}$$

where the length $\lambda_S=\sqrt{2e/(2\pi c_0 v_c)}$, and $c=1/\sqrt{c_0 l_0}$ is the electromagnetic wave velocity. This equation admits soliton solutions which are Lorentz invariant. In the limit $E_J/E_C>1$, saw$(\chi)\simeq\sin(\chi)$ and Eq. (3) is the sine-Gordon equation, with well-known soliton solutions.[14] In the oppo-

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site limit, $E_J/E_C<1$, a soliton solution[4] describes a potential distribution which decays exponentially from a point, with characteristic length λ_S. In either limit, the potential distribution with spatial extent $2\lambda_S$ is the result of a localized charge quantum $(2e)$ in the array. As $E_J/E_C\rightarrow 0$, $V_c\rightarrow e/C$, and $\lambda_S\sim\Delta x\sqrt{C/C_0}$, which is the electrostatic screening length of the charge quantum localized to one electrode. As $E_J/E_C\rightarrow\infty$, $\lambda_S\rightarrow\infty$ due to the delocalization of charge. An equation similar to Eq. (3) has been derived within the context of single electron tunneling,[4] where its validity has been questioned.[15] The existence of Eq. (3) for Cooper pairs was mentioned in Refs. 4, 5, and 15, and examined in Ref. 6.

There is a direct analogy with the system considered here and the one-dimensional parallel array of Josephson junctions (dual system), where one can derive a sine-Gordon equation for the Josephson phase variable $\phi(x,t)$.[16] In that case the ''kink'' solution for $\phi(x)$ describes the distributed supercurrent which gives rise to a magnetic flux soliton associated with a vortex. In our case of the series one-dimensional array, the ''kink'' solution for $\chi(x)$ describes the x component of the electric field (in the tunnel barriers of a discrete array), and the electrostatic potential (of the electrodes in a discrete array) associated with one excess Cooper pair sitting at the center of the kink. The way in which charge and flux quantization are treated on equal footing in these two classical (but complementary) models is intuitively appealing.

Several series arrays of small-capacitance Josephson junctions have been fabricated and measured. The junctions were made of Al, with AlO_x tunnel barriers, and were formed by the usual shadow evaporation technique.[17,18] The array discussed here had $N=255$ junctions in series. The connection between nearest-neighbor electrodes was actually two junctions in parallel, forming a dc-SQUID (superconducting quantum interference device). This geometry allowed tuning of the Josephson coupling between nearest neighbors with an external magnetic field.

A scanning electron microscopy (SEM) micrograph of part of the array is shown in Fig. 1(a), and an equivalent circuit is shown in Fig. 1(b). Due to a misalignment of the angle used for evaporation of the top electrode, alternating loops in the array have equal area. The effect of this lattice with a basis of two loop sizes, is clearly seen in the magnetic field dependence of the measured current-voltage $(I\text{-}V)$ curve, where two distinct periods of oscillation with magnetic field are observed.

The array was symmetrically biased through two 1 MΩ current measurement resistors, so that the potentials at the edges of the array were above and below the ground potential by equal amounts. The ground plane was located 250 μm below the array. Figure 2 shows the $I\text{-}V$ characteristic of the array measured at $T<50$ mK, for several magnetic fields between zero and the first minimum in E_J. We see a distinct threshold voltage for the onset of current through the array, which increases as E_J is suppressed with the magnetic field. This Coulomb blockade feature was smeared at higher temperature, and fully disappeared for $T>700\ \text{mK}\approx E_C/k_B$.

The threshold voltage is plotted versus the magnetic field in Fig. 3. Peaks in V_t are seen at magnetic fields correspond-

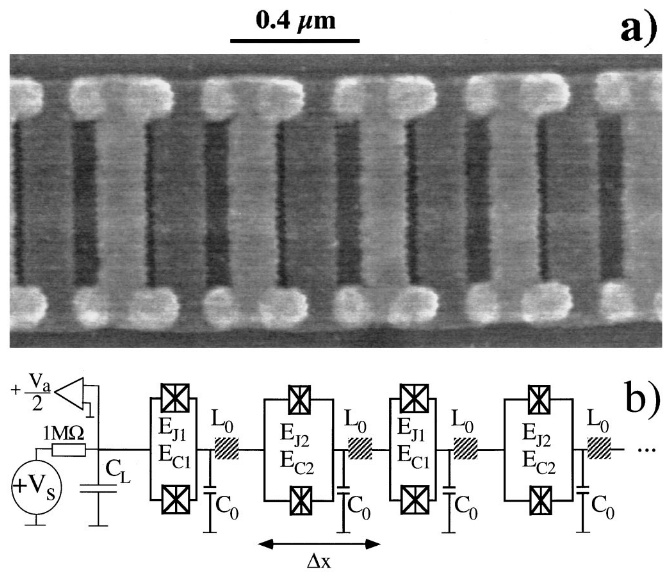

FIG. 1. (a) A SEM micrograph of a section of the array of SQUID's showing the alternating loop areas; and (b) the equivalent circuit showing one side of the symmetric bias arrangement.

ing to $(n+1)/2$ flux quanta, $\Phi_0=h/2e$, in one of the two loop areas. From the measured periodicity of V_t with magnetic field, we could accurately determine the loop areas $A_1=0.18\ \mu m^2$ and $A_2=0.13\ \mu m^2$. The areas correspond to loops defined by current paths through the center of the superconducting electrodes, as would be the case when the magnetic field is either penetrating into, or expelled from the bulk electrodes.

By design, the ratio A_1/A_2 is equal to the ratio of the tunnel junction areas of the two dissimilar SQUID's. This ratio, together with the measured normal state resistance of the entire array $(2.17\ \text{M}\Omega)$ allows one to determine the two normal state resistances between nearest-neighbor electrodes $R_{N1}=9.8\ \text{k}\Omega$, and $R_{N2}=7.2\ \text{k}\Omega$. Here all numbers refer to the two parallel junctions of a loop, as one effective junction connecting nearest neighbors. We calculate the Josephson coupling energies $E_{J1}^0=66\ \mu\text{eV}$ and $E_{J2}^0=89\ \mu\text{eV}$, where $E_J=(R_Q/R_N)(\Delta_0/2)$, and $\Delta_0=200\ \mu\text{eV}$ is the superconducting energy gap of our Al electrodes. From previous experience with these junctions we know the specific capacitance is roughly 45 fF/μm^2 which, together with a rough measurement of the junction area from the SEM micrograph gives $E_{C1}=59\ \mu\text{eV}$ and $E_{C2}=44\ \mu\text{eV}$ for the two nearest-neighbor charging energies.

For voltages $V>V_t$ the current must be associated with Cooper-pair tunneling because it can be completely suppressed as E_J is suppressed to zero (see Fig. 2). Nonetheless, this current is not completely coherent as there is some finite voltage drop across the array, and thus dissipation in the array. For the moment, we will ignore dissipation and explain the threshold voltage as the voltage required to inject a Cooper-pair charge soliton into the array.

The dynamic equation (3), with the addition of some damping terms, would allow one to calculate the $I\text{-}V$ curve of the array. The boundary conditions are given by the voltages measured at either end of the array, $V(x=0)=V_a/2$, $V(x=L)=-V_a/2$, which are independent of time due to the

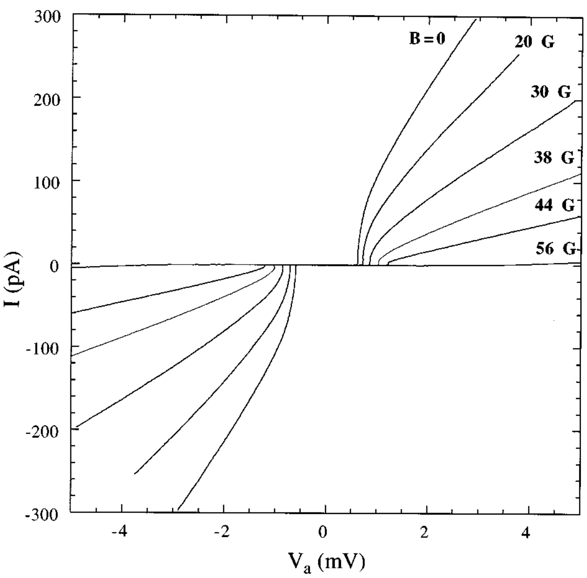

FIG. 2. The I-V characteristic of the array for several magnetic fields. A distinct threshold voltage is seen, which depends on magnetic field.

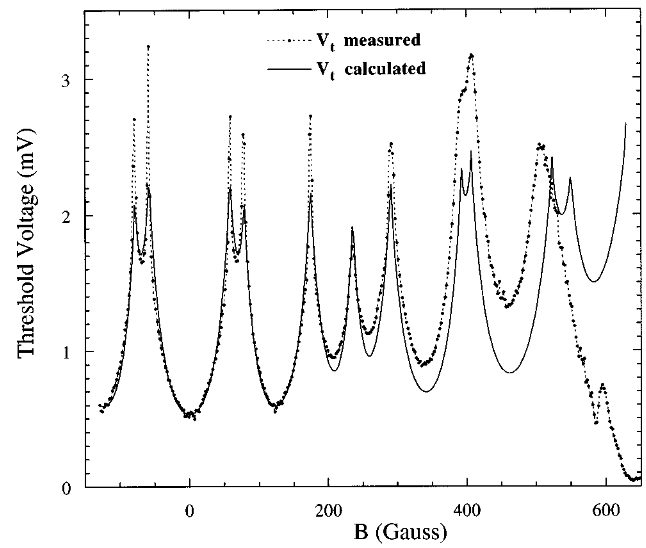

FIG. 3. The threshold voltage is shown versus magnetic field. The calculated curve explains the measured threshold voltage as being the injection voltage for a Cooper-pair charge soliton.

large capacitance of the leads, $C_L \gg C_0$. However, in this paper we wish to limit our discussion to the static case, or the stationary equation where the time derivative is zero. Here we consider an array of length $L \gg \lambda_S$, and approximate $\mathrm{saw}(\chi) \simeq \sin(\chi)$ (valid for $E_J/E_C > 1$). Analysis of the stationary sine-Gordon equation shows that solutions exist for V_a less than a threshold voltage,

$$V_t = 2\lambda_S v_c = (4/\sqrt{\pi})\sqrt{(e/C_0)\,V_c}. \quad (4)$$

The solutions for $V_a < V_t$ describe the tail of a Cooper-pair charge soliton penetrating into the array. For $V_a > V_t$, solitons and antisolitons are injected into opposite ends of the array. The current flow is described by a moving train of uniformly spaced solitons and antisolitons.

Extrapolating to the limit $E_J/E_C \to 0$ where $V_c = e/C$, we can compare Eq. (4) with the result of an electrostatic analysis of a discrete capacitor array.[12,15] We find that the threshold voltage given by Eq. (4) underestimates V_t by a factor of $2/\sqrt{\pi} = 1.13$. This 13% discrepancy is due to the approximation $V_c \mathrm{saw}(\chi) \simeq V_c \sin(\chi)$ where the E_J/E_C dependence of only the amplitude was properly accounted for. A full solution to Eq. (3) would account for this discrepancy. Thus, when there is no tunneling, the model recovers electrostatics.

In order to compare the experiment with theory, we need to account for the basis of two dissimilar junctions in series. This is easily done by coarse graining the system such that $2\Delta x \to \Delta x'$. In this case the effective critical voltage is simply $V_c' = V_{c1} + V_{c2}$, which is valid because the capacitance $C_0 \ll C_1 C_2/(C_1 + C_2)$. The critical voltages V_{c1} and V_{c2}, which depend on E_{J1}/E_{C1} and E_{J2}/E_{C2}, respectively, can be tuned with magnetic field, where

$$E_{J1}(B) = E_{J1}^0 |\cos(2\pi B A_1/\Phi_0)| \sqrt{1 - (B/B_C)^2}, \quad (5)$$

and similarly for $E_{J2}(B)$. The cosine term accounts for the SQUID modulation. We find that we do not need to account for any dissimilarity in the two parallel junctions forming the SQUID loop. The square-root term accounts for the suppression of Δ_0 by the penetrating magnetic field.

Figure 3 shows the measured threshold voltage versus the magnetic field, and a calculated curve. The theoretical curve required numerical analysis of the Mathieu equation so that one can calculate V_{c1} and V_{c2} for given values of $E_{J1}(B)/E_{C1}$ and $E_{J2}(B)/E_{C2}$.[7] To achieve the calculated curve shown in Fig. 3 we found it necessary to adjust the zero field Josephson coupling energies E_{J1}^0 and E_{J2}^0 each by a factor of 4.4 larger than the amount calculated from R_N. The charging energies E_{C1} and E_{C2} have each been decreased by 15% to account for the effect of the quasiparticle tunneling, which leads to an enhancement of the capacitance by an amount $\delta C = 3\pi\hbar/(32\Delta_0 R_N)$.[19,20] The factor e/C_0 was fixed to 9.9 mV, where the capacitance C_0 was calculated by dividing the capacitance of a strip, $C_a = \epsilon_{eff} L/[2\ln(8L/w)]$, having the array dimensions ($L = 51$ μm, $w = 1$ μm) by the effective number of junctions ($N/2 = 127.5$). The effective dielectric constant $\epsilon_{eff} = 4.4$ was determined from other measurements on this type of substrate.[21]

The discrepancy between the experimental and calculated curves near the peaks is in part due to the approximation, $\mathrm{saw}(\chi) \simeq \sin(\chi)$, which underestimates V_t by 13% when $E_J/E_C < 1$, near the peaks. There is also experimental error in the measured V_t, which was determined by taking the voltage at $I = 0.5pA$, the noise level for the measurement. This method was ambiguous only for one or two points at the peaks in V_t, where the current was suppressed to a minimum and no sharp threshold could be identified.

Apart from the peaks, the theoretical curve fits the data extremely well at low magnetic fields. This fit required adjusting only one parameter; E_{J1}^0 and E_{J2}^0 were multiplied by a factor of 4.4, their ratio being fixed. Without this factor we find that the calculation does not give small enough V_t near the minimum. Part of this factor might be accounted for by decreasing E_{C1} and E_{C2}, and at the same time increasing

e/C_0. Indeed, there is greater experimental uncertainty in E_C, which was estimated from the junction areas, than E_J^0 which was calculated from R_N. Another factor responsible for reducing V_t is the effect of next-nearest-neighbor and higher capacitances, which cannot be entirely neglected in our case because the distance to the ground plane was larger than the array length. The effects of dissipation by quasiparticle tunneling and movement of background charge could also further reduce the observed threshold voltage. Nonuniformity of the background potential, which is not included in our model, could also lead to modified soliton dynamics and threshold voltage. Further experiments with arrays of various lengths could sort these effects out.

At higher magnetic fields, the calculated curve misses the experimental data by increasing degree. The critical field used in the calculation was $B_c = 630$ G, where the measured V_t goes to zero. The simple form of the magnetic field suppression of the superconducting energy gap is valid only in the limit that the London penetration depth λ_L, is much larger than the width of the superconducting electrodes, which may not be valid for these samples. The rapid decrease of the measured threshold voltage above 500 G is due to onset of strong single electron tunneling. The junctions have $R_N \approx R_Q$, and thus there is only a very weak Coulomb blockade in the normal state.

Finally, we should discuss how our observations relate to experiments on two small-capacitance Josephson junctions in series, which have demonstrated how the *critical current* can be modulated with and external electric field.[13,22] Theoretical treatment of this effect[23,24] assumes the Josephson phase across the system to be a classical quantity (i.e., having a definite value). The measurement of a critical current is consistent with this assumption, which is explained as resulting from the low impedance of the electrodynamic environment, $Z_e \ll R_Q$. In contrast with these experiments, we have found that when many junctions are connected in series to form a one-dimensional array, a *critical voltage* can be observed, even when $E_J \gg E_C$. Furthermore, this critical voltage can be modulated with an external magnetic field. Such an observation implies quantum behavior of the Josephson phase, or a definite value of the quasicharge, which is the assumption used in developing the classical electrodynamic model applied here.

In conclusion, our experiments demonstrate the Coulomb blockade of Cooper-pair tunneling in 1D series arrays of Josephson junctions. Our analysis of the data is based on a continuum picture, where the electrodynamics of the excess localized Cooper pair is governed by a nonlinear wave equation which has soliton solutions. A stationary solution to this equation allows us to compare the measured and calculated threshold voltage as the Josephson coupling energy is tuned with a magnetic field. This simple theoretical model agrees well with the data if the ratio of the Josephson coupling energy to the charging energy is enhanced by a factor of 4.4.

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[1] P. W. Anderson, in *Lectures on the Many Body Problem*, edited by E. R. Caianiello (Academic, New York, 1964), Vol. 2, pp. 113–135.

[2] A. Widom, G. Megaloudis, T. D. Clark, and R. J. Prance, J. Phys. A **15**, 1561 (1982).

[3] K. K. Likharev and A. B. Zorin, J. Low Temp. Phys. **59**, 347 (1985).

[4] E. Ben-Jacob, K. Mullen, and M. Amman, Phys. Lett. A **135**, 390 (1989).

[5] D. V. Averin and K. K. Likharev, in *Mesoscopic Phenomena in Solids*, Vol. 30 of *Modern Problems in Condensed Matter Sciences*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (North-Holland, Amsterdam, 1991), Chap. 6, pp. 173–272.

[6] Z. Hermon, E. Ben-Jacob, and G. Schön, Phys. Rev. B **54**, 1234 (1996).

[7] A. B. Zorin, Phys. Rev. Lett. **76**, 4408 (1996).

[8] D. B. Haviland *et al.*, Z. Phys. B **85**, 339 (1991).

[9] L. J. Geerligs *et al.*, Phys. Rev. Lett. **63**, 326 (1989).

[10] C. D. Chen, P. Delsing, D. B. Haviland, and T. Claeson, *Low Dimensional Properties of Solids*, proceedings of the Nobel Jubilee Symposium, Gothenburg, Sweden (Physica Scripta T42, Swedish Royal Academy of Sciences, World Scientific, Singapore, 1992).

[11] P. Delsing *et al.*, Phys. Rev. B **50**, 3959 (1994).

[12] P. Delsing, in *Single Charge Tunneling, Coulomb Blockade Phenomena in Nanostructures*, Vol. 294 of *NATO ASI Series*, edited by H. Grabert and M. H. Devoret (Plenum, New York, 1992), Chap. 7, pp. 249–274.

[13] L. J. Geerligs, V. F. Anderegg, J. Romijn, and J. E. Mooij, Phys. Rev. Lett. **65**, 377 (1990).

[14] J Rubenstein, J. Math. Phys. **11**, 258 (1970).

[15] N. S. Bakhvalov, G. S. Kazacha, K. K. Likharev, and S. I. Serdyukova, Sov. Phys. JETP **68**, 581 (1989).

[16] K. K. Likharev, *Dynamics of Josephson Junctions and Circuits*, 1st ed. (Gordon and Breach, Philadelphia, 1986).

[17] J. Niemeyer, PTB-Mitt. **84**, 251 (1974).

[18] G. J. Dolan, Appl. Phys. Lett. **31**, 337 (1977).

[19] V. Ambegaokar, U. Eckern, and G. Schön, Phys. Rev. Lett. **48**, 1745 (1982).

[20] R. A. Ferrell, Physica C **152**, 10 (1988).

[21] P. Wahlgren, P. Delsing, and D. Haviland, Phys. Rev. B **52**, 2293 (1995)

[22] P. Joyez *et al.*, Phys. Rev. Lett. **72**, 2458 (1994).

[23] K. A. Matveev *et al.*, Phys. Rev. Lett. **70**, 2940 (1993).

[24] P. Joyez, Ph.D. thesis, Université de Paris 6, 1995, text in English.