Viscosity measurements in normal and superfluid ^{3}He

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Using a thick disk cavity of a torsional oscillator, we have measured the viscosity η and slip length ζ independently both in normal and superfluid ^{3}He. In the normal-liquid phase, superfluid fluctuation effects were seen clearly near the superfluid transition temperature. The ratio of the slip length to the viscous mean free path, ζ/λ_η was constant as predicted by theory, but its magnitude was about 2/3 of the theoretical prediction. In the superfluid phase, the temperature dependence of the observed value of ζ/λ_η supports the existence of Andreev reflections in superfluid ^{3}He. [S0163-1829(96)50834-9]

The Landau theory of Fermi liquids is known to describe well the properties of liquid ^{3}He at very low temperatures. The theory established many important concepts to understand interacting Fermi systems. Most properties of liquid ^{3}He predicted by the theory have been confirmed experimentally. However, the temperature dependence of the viscosity η has not been demonstrated experimentally. The experimental difficulty in the viscosity measurements is attributed mostly to long mean-free-path effects of ^{3}He quasiparticles.

Since the mean free path λ of a ^{3}He quasiparticle increases like T^{-2} in the degenerate temperature region, it could be quite large and become comparable to the typical size of the experimental cell or the viscous penetration depth δ. In this case, the hydrodynamic description of the liquid breaks down and it is difficult to derive the viscosity from experimental data.

In this temperature region, one existing theory that takes the long mean-free-path effect into account is the so-called ''slip theory.'' The theory introduces a new parameter, the slip length ζ, in the liquid velocity profile near the wall. The theory predicts that the ratio of this slip length ζ to the viscous mean free path λ_η is 0.58 and is independent of temperature for the normal Fermi liquid.[1,2] Most viscosity measurements are analyzed on the basis of this theory. So far, experiments for pure liquid ^{3}He have found that the magnitude of ηT^2 was not constant,[3] but it has been suggested that it might become so with a larger value of ζ/λ_η, at least in a certain temperature range.[4] Therefore, the magnitude of ζ/λ_η plays an important role in determining the temperature dependence of the viscosity. In this paper, we present independently derived experimental results of the viscosity and slip length from torsional-oscillator experiments.

We also show the results of viscosity and slip-length measurements for superfluid ^{3}He. In this case, the liquid properties are characterized by the properties of the Bogoliubov quasiparticles. The mean free path of the quasiparticle also can become very long, and we need to use slip theory. In early experiments, the viscosity decreased rapidly to zero as the temperature was decreased well below T/T_c, although the theory predicts finite viscosity even at $T=0$. The discrepancy between theory and experiments is considered because of the effects of a long mean free path and Andreev reflection of the quasiparticle. Therefore, what is measured was the effective viscosity.

If Andreev reflection exists in superfluid ^{3}He, an incident particlelike quasiparticle will be reflected as though it were a holelike quasiparticle at the wall, where the order parameter is distorted. In this reflection, the quasiparticle group velocity is reversed, but its momentum is almost unchanged. Therefore, the momentum transfer between the quasiparticle and the wall is negligibly small. The process acts to make the slip length large. The probability of the Andreev reflection increases with decreasing temperature as the relative number of low-energy quasiparticles increases at lower temperatures. As a result, the slip length becomes progressively large at low temperatures.

In the torsional-oscillator experiment, what we measure are the resonance frequency shift and the change of quality factor from those of the empty cell. The liquid properties then are related to these quantities. The form of the relations depend on the relative magnitude of the thickness d of the liquid disk and the viscous penetration depth δ. Using the slip boundary condition, the transverse acoustic impedance Z_1 can be expressed as

$$Z_1=\frac{d^3\omega^2}{4}\frac{\rho^2}{\eta}\left(\frac{1}{6}+\frac{\zeta}{d}\right)-\rho\frac{d\omega}{2}i,$$

for the case of $d\ll\delta$ (thin disk). Here, ω and ρ are the resonance angular frequency and the liquid density, respectively, and Z is defined as stress tensor divided by the wall velocity. The real part of Z_1 is related to the change of the inverse of quality factor $\Delta(Q^{-1})$, and the imaginary part of Z_1 is related to the shift of the resonance frequency Δf. In this case, the liquid density ρ (or normal component ρ_n for the superfluid) can be obtained from the frequency shift. This is why thin disks are adopted in many experiments. However, to derive the viscosity, one has to assume the magnitude and its temperature dependence of the slip length ζ.

For the case of thick disk, where $d\gg\delta$ and $\delta\ll R$ (R is the radius of the liquid disk), the impedance Z can be expressed as

$$Z_2=\sqrt{\frac{\omega\rho\eta}{2}}\left\{1-\left(1-\sqrt{\frac{2\omega\rho}{\eta}}\zeta\right)i\right\}$$

In this case, if ρ is known, one can determine η and ζ independently from the experiments. In the superfluid we used ρ_n data obtained by Parpia *et al.*[5] Note that the slip effect is

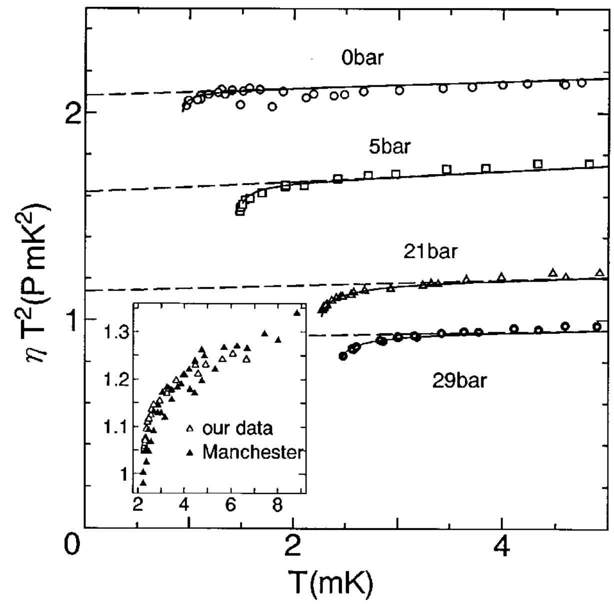

FIG. 1. The viscosity of η of the normal ^{3}He times T^2 as a function of temperature at various pressures. The dashed lines indicate the T-linear corrections for the ηT^2. The solid curves are the Emery's theoretical results showing the fluctuation effects above the superfluid transition (Ref. 9). The inset shows the similarity of our present data at 21 bar to that of Manchester group.

taken already into account in the expression of η. Therefore no correction is necessary. We only have to derive η and ζ from the measured data of the Δf and the $\Delta(Q^{-1})$.

The diameter and the height of our torsional oscillator cell are 11 mm and 6 mm, respectively, which is similar to those used by Ritchie *et al.*[6] For the temperature range of the present measurements, the viscous penetration depth can be as large as 1 mm. In this case, the analytical solution of the velocity field becomes inaccurate because the liquid near the corner is affected by both the side and upper or lower wall. Therefore, the velocity field of the liquid in the cavity was calculated numerically.

The cell body of the oscillator was machined from the Stycast 1266 and BeCu is used for the torsion rod. The resonance frequency is about 1490 Hz. The block diagram of the measuring circuit is similar to those of Archie's.[7]

In the present measurement, accurate adjustment of the phase position is very important. A small deviation of the phase determination leads to a large error in the value of slip length. The method we adopted for the determination of the resonance frequency and the Q value are as follows; first, we adjust roughly the phase position for the resonance. Then, we shift the phase $\pm 45°$, and read the drive voltage and the frequencies for each position. The position of the phase for the resonance is determined so that the drive voltage of the oscillator becomes equal to a $\pm 45°$ phase shift. The difference of these two frequencies gives the half width and the central value gives the resonance frequency. The Q value is calculated from these quantities.

The torsional oscillator is mounted on a copper nuclear demagnetization cryostat. The temperature of liquid ^{3}He is measured by LaCMN and the ^{3}He melting curve thermometer, which was calibrated using the Greywall temperature scale.[8]

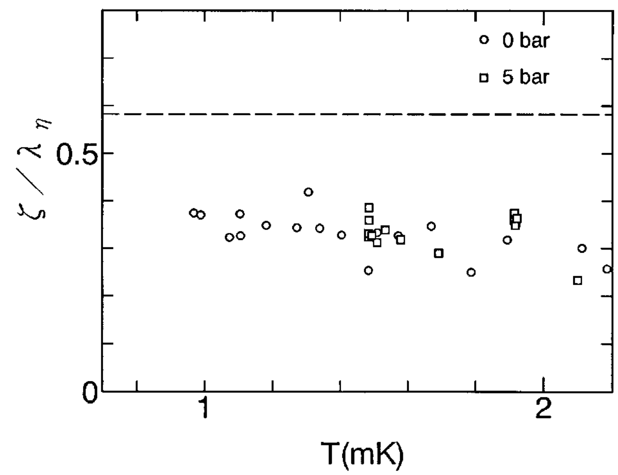

FIG. 2. The slip length ζ normalized by the viscous mean free path λ_η as a function of temperature at 0 and 5 bar in normal ^{3}He. The dashed line is the theoretical prediction by Jensen *et al.* (Ref. 1) and Einzel *et al.* (Ref. 2).

Figure 1 shows the experimental results of ηT^2 as a function of temperature at various pressures. The lowest temperature for each pressure corresponds to the superfluid transition temperature T_c. The broken lines show the best fit of the following equation for each pressure data.

$$\eta T^2 = \frac{1}{A - BT}.$$

The T-linear dependence of the ηT^2 is consistent with the previous measurements, but its magnitude is larger than those obtained by other groups.

The solid curves, which show the deviation from the broken line at the lowest temperatures, are the theoretical predictions by Emery[9] based on the fluctuation effect above the superfluid transition. To obtain the best fit to the experimental data, the value of the parameters $\Gamma_1 = 30$ and $\alpha = 0.5$ are adopted for all pressures. These values are within the range of values suggested by Emery. The agreement between theory and experiments is excellent. The existence of this form of deviations was suggested already by Hook *et al.*[4] (see the insert of Fig. 1). However, this behavior was, so far, not related to the fluctuation effect above the superfluid transition. Note that the viscosity in our results is independent of the value of the mean free path of ^{3}He quasiparticle or slip length: within the frame work of the slip theory, the effect is taken already into account. Therefore, we conclude that the observed deviation of the viscosity from the finite temperature correction is because of the fluctuation effects above the superfluid transition.

Figure 2 shows the results of ζ/λ_η derived from the same data with the viscosity measurements in the normal liquid ^{3}He. The viscous mean free path λ_η is determined from

$$\lambda_\eta = \frac{5\eta}{n p_F},$$

where n is the number density of ^{3}He atoms, p_F is the Fermi momentum, and η is the viscosity obtained in these experiments.

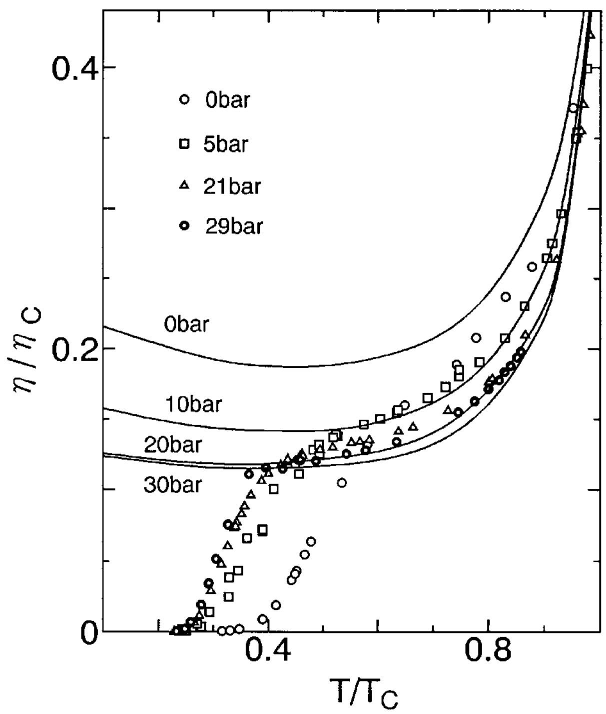

FIG. 3. The viscosity η of the superfluid ^{3}He normalized at T_c. The solid curves are the theoretical results by Einzel (Ref. 11).

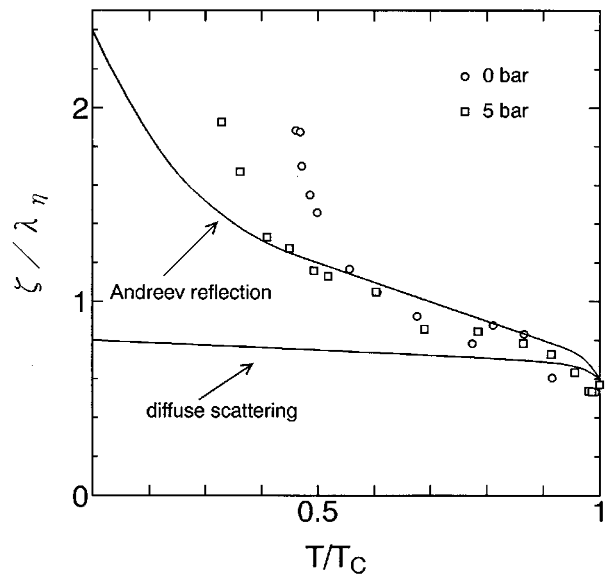

FIG. 4. The slip length ζ normalized by the viscous mean free path λ_η in the superfluid ^{3}He. The solid curves are the theoretical results at 0 bar by Einzel *et al.* (Ref. 12).

Within the experimental scatter, the ratio ζ/λ_η is independent of temperature as predicted by the theory, although its magnitude is considerably smaller than the theoretical value of 0.58. This discrepancy in the magnitude between theory and experiments also is seen in the earlier measurements and our results are consistent with those of the Cornell group.[10] Our results are obtained from the direct measurements of the slip length, the first experiments to our knowledge. The fact that the value ζ/λ_η is not larger than the theoretical prediction ensures that we apply the slip theory for the analysis of the experimental data.

Figure 3 shows the results of the viscosity measurements in the superfluid phase. The solid lines are the theoretical predictions.[11] Above $T/T_c=0.6$, the results are in fairly good agreement with theory, except for the zero bar data. The viscosity shows a rapid decrease just below T_c, and then tends to a constant value. In zero bar, the coincidence is not good. The reason is not clear at the moment, but one possibility is that the scattering parameter used in the theory might be larger than 0.68 (Ref. 11).

As long as the slip theory could apply, the results should not depend on the magnitude of the mean free path of the quasiparticle. The rapid decrease of the viscosity at the lowest temperatures, showing the deviation from the theoretical curve, is probably because of the extremely large mean free path of the quasiparticle, where the slip approximation can no longer be applicable.

Figure 4 shows the slip length normalized by the viscous mean free path in the superfluid phase of ^{3}He. The viscous mean free path λ_η is calculated as

$$\lambda_\eta=\frac{5\eta}{np_F}(Y_2Y_0)^{-1/2},$$

where Y_n is the generalized Yoshida function. The solid curves are the theoretical estimations by Einzel *et al.*[12] at 0 bar. The lower curve was obtained by assuming that the quasiparticle is scattered diffusively at the wall, and shows very weak temperature dependence. The upper curve was calculated including the Andreev reflection of the quasiparticle at the wall. In this case, the slip length increases progressively with decreasing temperature as the relative number of low energy Bogoliubov quasiparticle increases at lower temperatures.

The present data show an apparent increase of slip length with decreasing temperature in qualitative agreement with the theory including the Andreev reflection. The results support the existence of the Andreev reflection.[13–15]

In Fig. 4, in the temperature range of $0.6<T/T_c<1$, the data fall below the upper theoretical curve. On the other hand, they fall above the curve near the lowest temperatures. These discrepancies between theory and experiment may be explained as follows: in the theory, the order parameter profile is assumed to change as a step function. So, the calculation gives the upper bound of the slip length. We consider this as the possible reason for the deviations at higher temperatures. Below $T/T_c=0.5$, the quasiparticle mean free path increases rapidly with decreasing temperature. Therefore, as mentioned above, the slip approximation becomes inaccurate in this region.

In conclusion, we have measured both the viscosity and the slip length using a thick disk cavity of torsional oscillator. In the normal liquid, we first have derived the fluctuation effect in the viscosity above the superfluid transition. In the superfluid phase, the observed temperature dependence of the slip length supports the existence of the Andreev reflection in ^{3}He.

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