dc transport in quantum wires

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The influence of the electron-electron interaction on the two-terminal dc conductance of one-dimensional quantum wires is studied. A cancelation between the effect of the electron-electron interaction on the current and on the external electric field is the reason for the universal value, $e^2/2\pi\hbar$ per mode, of the dc conductance of a clean wire. The effect of the renormalization of the electric field on the dc conductance in the presence of an interplay between the electron-electron interaction and backward scattering due to an impurity is considered. [S0163-1829(96)52544-0]

It is well understood now that for noninteracting electrons a two-terminal dc conductance G of a clean quantum wire is $e^2/2\pi\hbar$ per mode.[1] Since the electron-electron (e-e) interaction renormalizes the current, it was generally accepted after the calculation of Ref. 2 that the conductance should be renormalized to the value $K_\rho e^2/2\pi\hbar$, where the parameter K_ρ is related to the density-density electron interaction ($K_\rho=1$ in the absence of the interaction). In this paper we explain why despite the fact that the current is renormalized, the conductance is not. In a clean wire with an e-e interaction $G=e^2/2\pi\hbar$. This occurs because the electric field is also renormalized by the e-e interaction. The dc conductance of a clean one-dimensional (1D) electron liquid is a property in which the effect of the e-e interaction on the electric field cancels out its effect on the current.[3–5] The question of the renormalization of the external electric field to the total one demands special care in one dimension because the only possible electric field is longitudinal. The influence of the renormalization of the electric field on the conductance when there is an interplay between an impurity backward scattering and the e-e interaction is also discussed.

Recently Tarucha *et al.*[6] measured the conductance of a quantum wire formed from $Al_{0.35}Ga_{0.65}As/AlGa$ modulation-doped heterostructures. Based on the temperature dependence of the conductance, it was found following the analysis of Refs. 7 that $K_\rho\sim 0.7$. However, in contrast to the earlier predictions, the conductance per mode was very close to the universal value $e^2/2\pi\hbar$. To explain the results of Ref. 6 the experimental device was modeled[8] by a system with two line segments attached to the central part of the quantum wire. The temperature dependence of the conductance was controlled by an impurity located in the central part where the e-e interaction parameter was K_ρ^W, while the interaction parameter in the attached segments was $K_\rho^L \neq K_\rho^W$. For the length of the segments much larger than the length of the central part, the quantization of the conductance was determined by K_ρ^L only, and was given by $K_\rho^L e^2/2\pi\hbar$ in this theory. Eventually, it was assumed that the line segments represent the leads where the electrons are free, i.e., $K_\rho^L=1$, and in this way the discrepancy between the theory and the experiment was settled. On the other hand, it has been argued by Kawabata[4] (see also Ref. 5) and independently by us[3] (in the context of edge states in the quantum Hall regime), that when the conductance is defined as the response to the total field, rather than to the external one, the dc conductance of a clean 1D system is not influenced by the e-e interaction. The renormalization of the electric field by the e-e interaction was ignored in Refs. 8, as well as in earlier publications. In this paper we clarify the role of this effect.

Let us consider the case of a clean wire when the density-density electron interaction exists and may be not homogeneous. First we will show the universality of the dc conductance in a procedure similar to the one elaborated in the context of the edge states in quantum Hall devices.[3] Then, we extend the consideration of Ref. 9 to the case of interacting electrons and show how to calculate the two-terminal conductance in the Kubo formalism.

The dc conductance of a quantum wire connecting two reservoirs (leads) is given by

$$G=eI/(\mu_R^1-\mu_L^2)=eI/(\mu_R^2-\mu_L^1), \tag{1}$$

where I is the current (I does not depend on x in the dc limit), $\mu_{R(L)}^1$ is the chemical potential of the right (left) moving electrons near the left reservoir and $\mu_{R(L)}^2$ is the chemical potential of the right (left) moving electrons near the right reservoir. The second equality in Eq. (1) follows from the fact that in equilibrium the conductance calculated for electrons or for holes should give the same result. To find the current I the continuity equation will be used. In the absence of backward scattering of electrons we can apply the continuity equation for the left- and right-moving electrons separately,

$$J_{R,L}(p)=\frac{i}{p}\frac{d}{dt}e\rho_{R,L}(p)=\frac{e}{\hbar p}[H,\rho_{R,L}(p)], \tag{2}$$

where H is the Hamiltonian of the system. The density operators $\rho_{R,L}(p)=\Sigma_{k\approx\pm k_F}a_{k+p}^\dagger a_k$ have the standard 1D commutation relations:[10]

$$[\rho_R(-p),\rho_R(p')]=[\rho_L(p),\rho_L(-p')]=\frac{pL}{2\pi}\delta_{p,p'},$$

$$[\rho_L(p),\rho_R(p')]=0, \tag{3}$$

where L is the length of the wire. For operators commuting like that, performing commutation is equivalent to differentiation, i.e., $[F\{\rho_{R,L}\},\rho_{R,L}(p)]=\pm(pL/2\pi)\partial F\{\rho_{R,L}\}/\partial\rho_{R,L}(-p)$. Since in one dimension the Hamiltonian is a functional of the $\rho_{R,L}$ operators, we can rewrite $J_{R,L}$ in Eq. (2) as

$$\frac{1}{L}J_{R,L}(p)=\pm\frac{e}{2\pi\hbar}\frac{\partial H}{\partial\rho_{R,L}(-p)}. \tag{4}$$

On the other hand, by definition, the chemical potentials of the right- and left-moving species of electrons are given by

$$\frac{1}{L}\mu_{R,L}(p)=\frac{\partial H}{\partial\rho_{R,L}(-p)}. \tag{5}$$

Thus, for the total current $J=J_R+J_L$ we obtain that $J(p)=(e/2\pi\hbar)[\mu_R(p)-\mu_L(p)]$. This result holds for any momentum p and therefore it can be represented also in space i.e., at any point x the current $J(x)=(e/2\pi\hbar)[\mu_R(x)-\mu_L(x)]$. Since in the dc limit the current $I=\langle J\rangle$ does not depend on x,

$$\mu_R^1-\mu_L^1=\mu_R^2-\mu_L^2=(2\pi\hbar/e)I. \tag{6}$$

Finally, the combination of Eqs. (1) and (6) leads to $\mu_L^1=\mu_L^2=\mu_L$, $\mu_R^1=\mu_R^2=\mu_R$ in the dc limit, and, correspondingly,

$$G=e^2/2\pi\hbar. \tag{7}$$

Note that separately each of J_R, J_L, μ_R and μ_L is influenced by the e-e interaction, while in the particular ratio defining the conductance the renormalization of the chemical potential difference cancels out the renormalization of the current.

The above treatment is in the spirit of Landauer's approach.[1] Now we consider the conductance of the 1D electron gas using the Kubo formalism. In a two-terminal measurement the electrons accelerated by the total electric field inside the wire dissipate their energy in the reservoirs. The total electric field $E^{\rm tot}(x)$ is built from the external field and the induced one. Since the electric field vanishes inside the reservoirs, the dc conductance of the two-terminal device is given by

$$G=\int_0^L I(x)E^{\rm tot}(x)dx\Bigg/\left(\int_0^L E^{\rm tot}(x)dx\right)^2. \tag{8}$$

Let us define a tensor $\sigma(x,x')$, such that

$$I(x)=\int_0^L\sigma(x,x')E^{\rm tot}(x')dx'. \tag{9}$$

It follows from the Kubo formula that $\sigma(x,x')$ is a divergenceless tensor in the dc limit, i.e., $d\sigma(x,x')/dx=0$; see Appendix A of Ref. 9. This property of $\sigma(x,x')$ together with Eqs. (8) and (9) yields

$$G=\sigma(x_0,x_0'), \tag{10}$$

where x_0,x_0' are arbitrary points inside the wire. The location of these points can be chosen so as to simplify the calculation of $\sigma(x_0,x_0')$.

Let us consider again the case of a clean wire with an inhomogeneous e-e interaction $V(x,y)$. For a 1D electron liquid the Hamiltonian of the problem can be expressed in terms of conjugated bosonic operators $\phi(x)$ and $\tilde{\phi}(x)$. The operator $\phi(x)$ is related to the electron density operator $\rho(x)$ as $-(1/\sqrt{\pi})\,d\phi(x)/dx=\rho(x)$; the operator $\tilde{\phi}(x)$ has a similar relation with the current operator. The fluctuations of the charge density are described by the Tomonaga–Luttinger Hamiltonian,

$$H_0=\frac{v_F}{2L}\sum_p p^2\tilde{\phi}_p\tilde{\phi}_{-p}+\frac{v_F}{2L}\sum_{p,q}[\delta_{p,q}p^2+pqV(p,-q)/(\pi v_F L)]\phi_p\phi_{-q}, \tag{11a}$$

where ϕ_p and $\tilde{\phi}_p$ are the Fourier transforms of the operators $\phi(x)$ and $\tilde{\phi}(x)$, and $V(p,q)$ is the Fourier transform of $V(x,y)$; here we set $e=\hbar=1$. When an external electric field $E^{\rm ext}(x,t)$ is applied, the term

$$H_1=-\frac{1}{\sqrt{\pi}L}\sum_p\phi_p E_p^{\rm ext}(t) \tag{11b}$$

should be added to the Hamiltonian. This term describes the interaction of the local dipole moment with the external electric field.

The current operator in a 1D system is $J(p)=(i/\sqrt{\pi})[H_0,\tilde{\phi}_p]=i(v_Fp/\sqrt{\pi})\tilde{\phi}_p$, where the commutation relations $[\tilde{\phi}_{-q},\phi_p]=(L/p)\,\delta_{p,q}$ have been used. Then, the current $I=\langle J\rangle$ induced by the external electric field is

$$I_{-\omega}(-q)=-i\frac{v_Fq}{\sqrt{\pi}}\langle\tilde{\phi}_{-\omega,-q}\rangle, \tag{12}$$

where $\tilde{\phi}_{\omega,q}=\int dt e^{i\omega t}\langle e^{i(H_0+H_1)t}\tilde{\phi}(q)e^{-i(H_0+H_1)t}\rangle$. As a result (see, e.g., Chap. 3 of Ref. 11)

$$I_{-\omega}(-q)=-i\frac{v_Fq}{\pi}\sum_p C_\omega(q,p)E_{\omega,p}^{\rm ext}, \tag{13}$$

where C is the retarded correlation function of $\tilde{\phi}$ and ϕ. The function C obeys the Dyson equation,

$$C_\omega(q,p)=C_\omega^0(p)\,\delta_{p,q}+C_\omega^0(q)\frac{1}{\pi L}\sum_k qkV(q,-k)D_\omega(k,p), \tag{14}$$

where D is the full propagator of ϕ and $C_\omega^0(p)=-(1/2p)[(\omega+v_Fp+i\gamma)^{-1}+(\omega-v_Fp+i\gamma)^{-1}]$. The total electric field is the sum of the external and the induced fields, $E^{\rm tot}=E^{\rm ext}+E^{\rm ind}$. The induced field $E^{\rm ind}$ arises as a result of the redistribution of the density of the electrons,

$$E_{\omega,p}^{\rm ind}=-\frac{1}{L}\sqrt{\pi}\sum_q pqV(p,-q)\langle\phi_{-\omega,-q}\rangle. \tag{15}$$

Since $\langle\phi_{-\omega,-q}\rangle=-(v_F/\sqrt{\pi})\,\Sigma_pD_\omega(q,p)E_{\omega,p}^{\rm ext}$, the induced field is related to $E_{\omega,p}^{\rm ext}$ as

$$E^{\rm ind}_{\omega,p}=\frac{v_F}{L}\sum_{qk}\ pqV(p,-q)D_\omega(q,k)E^{\rm ext}_{\omega,k}. \qquad (16)$$

With the help of the Dyson equation (14) and Eq. (16) the relation between $E^{\rm tot}_{\omega,p}$ and $E^{\rm ext}_{\omega,q}$ can be obtained:

$$C^0_\omega(p)E^{\rm tot}_{\omega,p}=\sum_q\ C_\omega(p,q)E^{\rm ext}_{\omega,q}. \qquad (17)$$

This result corresponds to a well-known fact in the diagrammatic technique, that when the conductance is calculated with the help of the density correlation function only the irreducible part of the correlation function is involved. The response to the external electric field is given by a series of diagrams containing polarization bubbles and starting with an external field. The total field is given by diagrams of the same type. Therefore the response to the total electric field is given by the irreducible part of the correlation function. The importance of this fact to the calculation of the conductance of quantum wires was emphasized recently by Kawabata.[4] Substitution of Eq. (17) in the expression for the current, Eq. (13), yields

$$I_{-\omega}(-q)=-i\frac{qv_F}{\pi}C^0_\omega(q)E^{\rm tot}_{\omega,q}. \qquad (18)$$

From this relation one can obtain the conductance G for the two-terminal dc transport:

$$G=\sigma(0,0)=(-i)\frac{1}{\pi L}\sum_q\ qv_FC^0_{\omega=0}(q)=1/2\pi. \qquad (19)$$

Since the wire is attached to the reservoirs the electron states in the wire have a finite width γ, which we assume to be larger than the level spacing. Under this assumption the sum over momenta in Eq. (19) was transformed to an integral. After restoring the constants e and $\hbar$ the dc conductance of a clean wire becomes $G=e^2/2\pi\hbar$, i.e., it is not influenced by the e-e interaction.

The above consideration was performed for an arbitrary e-e interaction including the case when it is spatially inhomogeneous. Let us discuss now a system of the type considered in Refs. 8 in which the e-e interaction exists only in the central part (of length $L_{\rm int}$) and is absent in the segments attached to the central part of the wire. One can check that if the dc conductance is calculated ignoring the renormalization of the electric field, i.e., using $C_\omega(p,q)$ rather than $C^0_\omega(p)$ in Eq. (19), then there appear corrections $\sim(\gamma L_{\rm int}/v_F)(V/v_F)$. However, these corrections are not noticeable when the region of the interaction, $L_{\rm int}$, is short. In the treatment of Refs. 8, v_F/γ corresponds to the length of the wire L, and the limit $L_{\rm int}/L\to 0$ was considered.

Let us discuss now a system with a backward scattering defect inside the wire. Our goal now is to determine the effect of the renormalization of the external electric field on the conductance of this system. We will follow the same line of consideration as above. The conductance is given by

$$G=(-i)\frac{v_F}{\pi L}\sum_{q,p}\ q\tilde{\mathcal{C}}^0_{\omega=0}(q,p), \qquad (20)$$

where $\tilde{\mathcal{C}}^0_{\omega=0}(q,p)$ is the irreducible (with respect to the e-e interaction) part of the retarded correlation function of the operators $\tilde{\phi}$ and ϕ in the presence of the impurity backward scattering and the interaction. The full correlation function $\tilde{\mathcal{C}}$ is related to its irreducible part via the Dyson equation,

$$\tilde{\mathcal{C}}=\tilde{\mathcal{C}}^0+\tilde{\mathcal{C}}^0W\tilde{\mathcal{D}}. \qquad (21)$$

Here $W(k,-q)=(1/\pi L)\,qkV(q,-k)$ and the matrix $\tilde{\mathcal{D}}$ is the correlation function of ϕ operators. (Henceforth we use matrix notation.) The matrices $\tilde{\mathcal{D}}$ and $\tilde{\mathcal{C}}$ carry information about the backward scattering in the presence of interaction:

$$\tilde{\mathcal{D}}=D+D\mathcal{T}D,\tilde{\mathcal{C}}=C+C\mathcal{T}D, \qquad (22)$$

where $\mathcal{T}$ is the effective scattering matrix of the ϕ operators due to the impurity term, and the matrices D and C are the correlators in the absence of the impurity. After some transformations we obtain

$$\tilde{\mathcal{C}}^0=C^0(D^0)^{-1}(\tilde{\mathcal{D}}^{-1}+W)^{-1}, \qquad (23)$$

where D^0 and C^0 are the irreducible parts of the correlators D and C, see Eq. (14). Thus, the calculation of the conductance is reduced to the inversion of operators. To perform the inversion we will assume that the impurity backward scattering is local, while the e-e interaction inside the wire is homogeneous and short range, i.e., the elements of the matrix $\mathcal{T}$ do not depend on the momenta and $W(q,-k)=(1/\pi)\,V_0q^2\delta_{q,k}$. Now the inversion can be done straightforwardly and one obtains

$$\tilde{\mathcal{C}}^0=C^0+\frac{1}{1+{\rm tr}\mathcal{T}(D-D^0)}C^0\mathcal{T}D^0. \qquad (24)$$

With the use of Eq. (20) the conductance $G(\mathcal{T})$ is determined if the scattering matrix $\mathcal{T}$ is known.

When one ignores the effect of the renormalization of the electric field the full correlator $\tilde{\mathcal{C}}$ is used instead of $\tilde{\mathcal{C}}^0$ in Eq. (20). The quantity obtained will be denoted by G'. In contrast to to the conductance G, which is the response to the total electric field, G' describes the response to the external field. Using Eq. (22) $G'(\mathcal{T})$ can be found:

$$G'(\mathcal{T})=(-i)\frac{v_F}{\pi L}\sum_{q,p}\ qC_{\omega=0}(q)[\,\delta_{q,p}+\mathcal{T}(q,p)D_{\omega=0}(p)]. \qquad (25)$$

The relations (24) and (25) enable us to eliminate $\mathcal{T}$ and to express G as a function of G':

$$G=\frac{K_\rho G'}{2\pi(K_\rho-1)G'+K_\rho}, \qquad (26)$$

where $K_\rho=1/\sqrt{1+V_0/v_F\pi}$. The structure of Eq. (26) reflects the fact that in the presence of a backward scattering center the effect of the electric field renormalization depends not only on the e-e interaction, but also on the interplay between the backscattering and the e-e interaction. The quantity G', rather than G, has been extensively studied in the recent years by diverse techniques[7,12,13] in the perturbative and nonperturbative regimes. Equation (26) allows one to use these results to recalculate the conductance in order to

include the effect of the renormalization of the electric field. The results of the experiment[6] can be interpreted now without using the model with two line segments attached to the wire.[8] At high temperatures $G' \approx K_\rho/2\pi$ and Eq. (26) reproduces the universal value of the conductance G, while when the temperature decreases the deviation of the conductance from the ideal value is given by a power law correction. Both features have been observed in the experiment.[6]

Until now the simplified case of a single mode wire has been considered. In a real quantum wire a few modes exist due to spin and subbands corresponding to quantization of transversal motion. In the absence of backward scattering Eq. (19) can be easily generalized to the case when $N>1$ modes are occupied. Namely, $v_F C^0_{\omega=0}(q)$ should be substituted by $\Sigma_{n=1}^{N} v_F^n C^{0,n}_{\omega=0}(q)$ where n is the mode index. This yields the conductance of a multimode wire $G_N = Ne^2/2\pi\hbar$. To generalize Eq. (26) we consider the most symmetric case when the Fermi velocities in all channels are identical, $v_F^n = v_F$, and only an interaction of the form $\rho(x)V_0\delta(x-y)\rho(y)$ is present, where ρ is the total electron density. Then

$$G_N = \frac{K_\rho^N G'_N}{2\pi\left(\frac{K_\rho^N - 1}{N}\right)G'_N + K_\rho^N}, \tag{27}$$

where $K_\rho^N = 1/\sqrt{1+NV_0/v_F\pi}$ and the conductancelike quantity G'_N is the response to the external electric field. In the absence of backward scattering $G'_N = NK_\rho^N/2\pi$, resulting in $G_N = Ne^2/2\pi\hbar$.

To summarize, we have studied the influence of the e-e interaction on the two-terminal conductance of quantum wires. It was shown, by two different approaches, that the universal value of the conductance in a clean wire is a result of a cancelation of the effects of the e-e interaction on the current and on the external electric field. In addition, for a system with a backward scattering center we have found the relation of the dc conductance to the response to the external electric field.

The last remark concerns the relation of the edge state electrons, under the condition of the quantum Hall effect (QHE), to the interacting 1D electron gas.[14] It is a rather common belief that the physics of the edge states in the fractional QHE with $\nu = 1/(2p+1)$ and the physics of the interacting 1D electron gas are equivalent when the filling factor $\nu = K_\rho$. The fact that $G' = K_\rho e^2/2\pi\hbar$ and the Hall conductance $\sigma_{xy} = \nu e^2/2\pi\hbar$ is one of the reasons for that point of view. In this connection we would like to emphasize that ν is not completely equivalent to K_ρ. In the fractional QHE the filling factor ν appears through the commutation relations of the operators of the electron density, but not as a result of the density-density interaction of the edge state electrons. For that reason the effect of the electric field renormalization has no connection with the factor ν, and therefore σ_{xy} does contain it. On the other hand, the Hall conductance of the edge states is not modified by an interedge e-e interaction,[3] precisely in the same way as in the case of a clean wire.

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