

# Quantum theory of the coercive force and the capping effect for magnetic multilayers

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A quantum theory of coercivity is established for magnetic systems. The concept of a metastable state is adopted and an order parameter that is actually the gap of the magnon excitations is used to tell the stability of the metastable state and to determine the coercive force of the system. Quantum theory is applied to study the capping effects in a magnetic double-film structure. A model Hamiltonian is presented for such a system, in which there are several magnetic layers with perpendicular easy-axis anisotropy capped by a few layers with in-plane easy-axis anisotropy. The Hamiltonian is diagonalized under the harmonic approximation and the magnon excitation gap is determined. Numerical calculations are carried out and the experimental results about the coercive force in the double-film structure are explained qualitatively. Finally, we show that there are two types of phase transition in the capping effect. The results show that the capping effect can be significantly enhanced if one selects the anisotropy and the number of capping layers appropriately. [S0163-1829(96)07337-7]

## I. INTRODUCTION

Magnetic hysteresis has been known for more than a century. Many phenomenological models have been developed to describe such a behavior.<sup>1-4,7</sup> Although they are different when seen at a first glance, the main point is the same. A pinning energy should be introduced into the total energy function to describe the impedance for the changes of magnetizations, then the magnetizations are calculated by some standard techniques (see, molecular field theory) setting different conditions for the cases that the field decreases from a positive saturation value  $H_{\max}^+$  and that it increases from a negative saturation value  $H_{\max}^-$ .<sup>1</sup> Although the standard phenomenological approaches had provided a qualitative description for the hysteresis behaviors, they suffered some disadvantages. For example, a clear definition for the coercivity of the system is not stated explicitly; and since the theories are phenomenological ones, some assumptions and experience parameters are needed.

On the other hand, magnetic multilayer systems are attracting attention from both theoretical and experimental sides recently because they have good potential to be used as recording media and magnetic devices. Many studies have been carried out for a double-film structure which is a recording thin film with perpendicular easy-axis capped by another one whose magnetic easy-axis lies in the plane.<sup>5-12</sup> The hysteresis behaviors of such a double-film structure are rather interesting, the coercive force of the entire structure is much smaller than that of a single recording thin film (so-called "capping effect").<sup>5,6</sup>

The present work is devoted to establishing a quantum micromagnetic theory of the coercive force for a magnetic system. As an illustration of our method, we will apply it to study the double-film system. Although the model in the present paper is an oversimplified one, we wish to get some qualitative understanding of the capping effect. A concept of the metastable state is proposed to understand the hysteresis loop of a magnetic system, and the energy gap of magnon excitations in the applied field is defined as the order parameter to monitor the stability of the metastable state. A general procedure for determining the coercive force of a magnetic multilayer at the zero temperature is presented. The extension to the finite temperature case is also discussed.

This paper is organized as follows. In Sec. II, a model Hamiltonian and its local coordinates (LC) representation is presented for the double-film structure, then a Bose transformation is applied to the Hamiltonian. Diagonalization of the harmonic Hamiltonian part is given in Sec. III. Section IV is devoted to illuminating the concept of the metastable state and describing a general method for calculating the coercive force of a magnetic coupled multilayer. The capping effects are discussed in Sec. V, and the conclusions are summarized in the last section.

## II. HAMILTONIAN AND LC REPRESENTATION

We study a superlattice which is constructed by  $N$  anisotropic magnetic layers coupled each other. The model Hamiltonian is

$$\begin{aligned}
H = & -\frac{1}{2} \sum_{m,m'} \sum_{\mathbf{R},\mathbf{R}'} I_{m,m'}(\mathbf{R},\mathbf{R}') \mathbf{S}_m(\mathbf{R}) \cdot \mathbf{S}_{m'}(\mathbf{R}') \\
& -h \sum_{m,\mathbf{R}} S_m^z(\mathbf{R}) + \sum_{m \leq L} \sum_{\mathbf{R}} D_m (S_m^x(\mathbf{R}))^2 \\
& + \sum_{m > L} \sum_{\mathbf{R}} D_m (S_m^z(\mathbf{R}))^2, \quad (1)
\end{aligned}$$

where  $m, m'$  are the number of the layers, and  $\mathbf{R}, \mathbf{R}'$  are the vectors of lattices on the  $(x, y)$  plane. The first  $L$  layers of the structure are the capping material with an in-plane magnetic easy axis along the  $x$  direction, and the remaining  $(N-L)$  layers are the recording materials whose easy axis is perpendicular to the film along the  $z$  direction.  $I_{m,m'} (> 0)$  are the exchange constants and only the nearest-neighbor interaction is considered in this model.  $\{D_m (< 0)\}$  are the single-ion anisotropy constants. An external magnetic field  $h$  is applied along the  $z$  direction.

Following Ref. 8, the local coordinates (LC)  $\{x_m, y_m, z_m\}$  are introduced for each layer. The  $y_m$  directions are always kept in the original direction  $y$ , and the  $x_m, z_m$  axes of the  $m$ th layer are obtained by rotating an angle  $\theta_m$  around  $y$  axis. After the LC transformation, the Hamiltonian  $H$  can be expressed by the components of the spin operators in the local coordinates as  $H(S_m^x, S_m^y, S_m^z)$ . Then, a Bose transformation, such as the Holstein-Primakoff<sup>13</sup> (H-P) or the complete Bose transformation<sup>14</sup> (CBT), is used to express the Hamiltonian as a Bose system. In the present paper, a harmonic approximation is accepted. In such case, all Bose transformations are the same. After the LC and the Bose transformations, the Hamiltonian becomes<sup>8</sup>

$$H = U_0 + H_1 + H_2 + \dots, \quad (2)$$

where

$$\begin{aligned}
U_0 = & \text{const} - \frac{N_s S^2}{2} \sum_{m,m' (\neq m)} I_{m,m'} \cos(\theta_m - \theta_{m'}) \\
& - h S N_s \sum_m \cos \theta_m + S N_s \left( \frac{1}{2} - S \right) \sum_{m \leq L} D_m \cos^2 \theta_m \\
& + S N_s \left( \frac{1}{2} - S \right) \sum_{m > L} D_m \sin^2 \theta_m, \quad (3)
\end{aligned}$$

$$\begin{aligned}
H_1 = & N_s \sqrt{2S} \sum_{m \leq L, \mathbf{R}} \left[ \frac{S}{2} \sum_{m'} I_{m,m'} \sin(\theta_m - \theta_{m'}) + \frac{h}{2} \sin \theta_m \right. \\
& \left. - \frac{1}{4} \left( 1 - \frac{S}{2} \right) D_m \sin 2\theta_m \right] [a_m^\dagger(\mathbf{R}) + a_m(\mathbf{R})] \\
& + N_s \sqrt{2S} \sum_{m > L, \mathbf{R}} \left[ \frac{S}{2} \sum_{m'} I_{m,m'} \sin(\theta_m - \theta_{m'}) + \frac{h}{2} \sin \theta_m \right. \\
& \left. + \frac{1}{4} \left( 1 - \frac{S}{2} \right) D_m \sin 2\theta_m \right] [a_m^\dagger(\mathbf{R}) + a_m(\mathbf{R})], \quad (4)
\end{aligned}$$

$N_s$  in Eqs. (3) and (4) is the number of lattice sites in each layer.

In  $\mathbf{k}$  space of  $(x, y)$  plane,  $H_2$  can be expressed as

$$\begin{aligned}
H_2 = & \sum_{m,m'} \sum_{\mathbf{k}} F_{m,m'}(\mathbf{k}, \theta) a_m^\dagger(\mathbf{k}) a_{m'}(\mathbf{k}) \\
& + \sum_{m,m'} \sum_{\mathbf{k}} G_{m,m'}(\mathbf{k}, \theta) [a_m^\dagger(\mathbf{k}) a_{m'}^\dagger(-\mathbf{k}) \\
& + a_m(\mathbf{k}) a_{m'}(-\mathbf{k})], \quad (5)
\end{aligned}$$

where

$$\begin{aligned}
F_{m,m}(\mathbf{k}, \theta) = & I_{m,m} Z S (1 - \gamma_k) + D_m \left( S - \frac{1}{2} \right) \\
& \times (\cos^2 \theta_m - 2 \sin^2 \theta_m) + \sum_{m'} S I_{m,m'} \\
& \times \cos(\theta_m - \theta_{m'}) + h \cos \theta_m, \quad m \leq L, \quad (6)
\end{aligned}$$

$$\begin{aligned}
F_{m,m}(\mathbf{k}, \theta) = & I_{m,m} Z S (1 - \gamma_k) + D_m \left( S - \frac{1}{2} \right) \\
& \times (\sin^2 \theta_m - 2 \cos^2 \theta_m) + \sum_{m'} S I_{m,m'} \\
& \times \cos(\theta_m - \theta_{m'}) + h \cos \theta_m, \quad m > L, \quad (7)
\end{aligned}$$

$$F_{m,m'}(\mathbf{k}, \theta) = -\frac{S}{2} I_{m,m'} [1 + \cos(\theta_m - \theta_{m'})], \quad m \neq m', \quad (8)$$

$$G_{m,m}(\mathbf{k}, \theta) = \frac{1}{4} \sqrt{2S(2S-1)} D_m \cos^2 \theta_m, \quad m \leq L, \quad (9)$$

$$G_{m,m}(\mathbf{k}, \theta) = \frac{1}{4} \sqrt{2S(2S-1)} D_m \sin^2 \theta_m, \quad m > L, \quad (10)$$

$$G_{m,m'}(\mathbf{k}, \theta) = \frac{S}{4} I_{m,m'} [1 - \cos(\theta_m - \theta_{m'})], \quad m \neq m', \quad (11)$$

$Z$  is the number of the nearest-neighbor sites in the  $(x, y)$  plane and

$$\gamma_k = \frac{1}{Z} \sum_{\delta} \exp(i\mathbf{k} \cdot \delta). \quad (12)$$

The LC spin configurations  $\{\theta_m\}$  are determined by

$$\delta U_0 / \delta \theta_m, \quad m = 1, 2, \dots, N. \quad (13)$$

In this case,  $H_1 = 0$  and the Hamiltonian becomes

$$H \simeq U_0 + H_2 + \dots. \quad (14)$$

### III. DIAGONALIZATION

From Eq. (5),  $H_2$  can be written in the matrix form as

$$H_2 = - \sum_{m,\mathbf{k}} \frac{1}{2} F_{m,m}(\theta, \mathbf{k}) + \sum_{\mathbf{k}} \tilde{\mathcal{A}}(\mathbf{k}) \mathcal{H}(\mathbf{k}) \mathcal{A}(\mathbf{k}), \quad (15)$$

where  $\tilde{\mathcal{A}}$  and  $\mathcal{A}$  are defined by

$$\tilde{\mathcal{A}}(\mathbf{k}) = (a_1^\dagger(\mathbf{k}), \dots, a_N^\dagger(\mathbf{k}), -ia_1(-\mathbf{k}), \dots, -ia_N(-\mathbf{k})), \quad (16)$$

$$\mathcal{A}(\mathbf{k}) = \begin{pmatrix} a_1(\mathbf{k}) \\ \vdots \\ a_N(\mathbf{k}) \\ -ia_1^\dagger(-\mathbf{k}) \\ \vdots \\ -ia_N^\dagger(-\mathbf{k}) \end{pmatrix}, \quad (17)$$

and

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} \frac{1}{2}\mathcal{F}(\mathbf{k}) & i\mathcal{G}(\mathbf{k}) \\ i\mathcal{G}(\mathbf{k}) & -\frac{1}{2}\mathcal{F}(\mathbf{k}) \end{pmatrix}. \quad (18)$$

$\mathcal{F}(\mathbf{k})$  and  $\mathcal{G}(\mathbf{k})$  in matrix  $\mathcal{H}(\mathbf{k})$  are two  $N \times N$  matrixes whose elements are defined by  $F_{m,m'}(\mathbf{k}, \theta)$  and  $G_{m,m'}(\mathbf{k}, \theta)$ , respectively.

The Hamiltonian  $H_2$  is a harmonic one so that it can be diagonalized by a Bogolyubov ( $U, V$ ) transformation. The coefficients must be layer number dependent: ( $\{U_l\}, \{V_l\}$ ). After the diagonalization, the Hamiltonian is transformed to

$$H = U'_0 + \sum_{l,\mathbf{k}} \epsilon_l(\mathbf{k}) \alpha_l^\dagger(\mathbf{k}) \alpha_l(\mathbf{k}) + \dots, \quad (19)$$

where

$$U'_0 = U_0 - \sum_{l,\mathbf{k}} \left[ \frac{1}{2} F_{l,l}(\theta, \mathbf{k}) - \lambda_l(\mathbf{k}) \right], \quad (20)$$

$$\epsilon_l(\mathbf{k}) = 2\lambda_l(\mathbf{k}). \quad (21)$$

$\{\lambda_l(\mathbf{k})\}$  are the eigenvalues of matrix  $\mathcal{H}(\mathbf{k})$ . The excitation energy  $\epsilon_l(\mathbf{k})$  is dependent on the spin configuration  $\{\theta_m\}$ . The minimum value of the excitation energy is defined as an energy gap:

$$\text{Min}[\epsilon_l(\mathbf{k})] = \Delta. \quad (22)$$

The gap  $\Delta$  must be a function of the external magnetic field  $h$ , spin configuration  $\{\theta_m\}$  and model parameters. From Eq. (13), one may obtain some spin configurations. One of them corresponds to the ground state with the lowest energy, others to the states which may be metastable or unstable. Every solution of Eq. (13) relates to a set of excitation energy spectrums  $\{\epsilon_l(\mathbf{k})\}$ . If the excitation energies are positive, it means that any excitation to destroy the state will cost energy so that the state is metastable. Otherwise, it is unstable. Thus, the stability of the configuration can be monitored by the energy gap  $\Delta(h)$ . In principle, the high order contributions can be considered based on a perturbation theory and they might give some corrections to the value of  $\Delta(h)$ . However, in this paper, we will only consider the quantum fluctuations under the harmonic approximation and believe that

the main physics picture will not be changed by the remainder interactions. The induced magnetization of the system can be calculated by

$$\begin{aligned} M &= \frac{1}{NN_{sm,\mathbf{R}}} \sum \langle 0 | S_m^z(\mathbf{R}) | 0 \rangle \\ &= \frac{1}{NN_{sm,\mathbf{R}}} \sum \langle 0 | \cos\theta_m S_m^z(\mathbf{R}) - \sin\theta_m S_m^x(\mathbf{R}) | 0 \rangle \\ &= \frac{1}{NN_{sm,\mathbf{k}}} \sum \cos\theta_m \left[ S - \sum_n (V_{m,n}(\mathbf{k}))^2 \right], \end{aligned} \quad (23)$$

where the ground state  $|0\rangle$  is defined by

$$\alpha_m(\mathbf{k})|0\rangle = 0. \quad (24)$$

If the temperature is not zero, we should study the magnon excitations at finite temperatures:  $\epsilon_m(\mathbf{k}, h, T)$ . Because of the interactions between magnons ( $H_4$ , etc.), the energy required to create an excitation will depend on the numbers of other excitation present ( $\langle n(\mathbf{k}) \rangle$ ) so that it will depend on the temperature. The renormalization of  $\epsilon_m(\mathbf{k}, h, T)$  by thermally excited spin waves have been discussed by many authors at low temperatures,<sup>15</sup> however, the calculation procedure is rather involved.

#### IV. METASTABLE STATE AND THE PHASE TRANSITION

In this section, we will study the metastable state and the coercive force. As a model, we discuss a system constructed by 10 recording layers plus two capping ones ( $L=2, N=12$ ). The parameters are reduced to

$$I_{m,m'}(\mathbf{R}, \mathbf{R}') = I, \quad (25)$$

$$\left( \frac{1}{2S} - 1 \right) D_m / I = \tilde{D}_1, \quad m=1, 2, \quad (26)$$

$$\left( \frac{1}{2S} - 1 \right) D_m / I = \tilde{D}_2, \quad m=3, \dots, 12, \quad (27)$$

$$h/SI = \tilde{h}. \quad (28)$$

$S$  is fixed by 1. We have done some numerical calculations for the systems with the following parameters:

$$\text{Model 1: } \tilde{D}_1 = 0.07, \quad \tilde{D}_2 = 0.2,$$

$$\text{Model 2: } \tilde{D}_1 = 0.25, \quad \tilde{D}_2 = 1.25.$$

Let us describe the picture we have found by the calculations. At the very beginning, we suppose that a very strong external magnetic field  $h$  is applied along the  $+z$  direction. Thus, all spins must be arranged along the  $+z$  direction. Only one trivial solution of Eq. (13),  $\{\theta_m = 0^\circ\}$ , is stable in this case since the spectrum  $\{\epsilon_l(\mathbf{k})\}$  are positive. Other possible solutions of Eq. (13), such as another trivial solution:  $\{\theta_m = 180^\circ\}$ , are unstable. When the applied field is decreased step by step, nontrivial solution of Eq. (13):  $\{\theta_m \neq 0^\circ\}$  will appear when the field  $h$  is less than a critical value  $h_c$ .<sup>8</sup> Let us denote this nontrivial solution as the type I

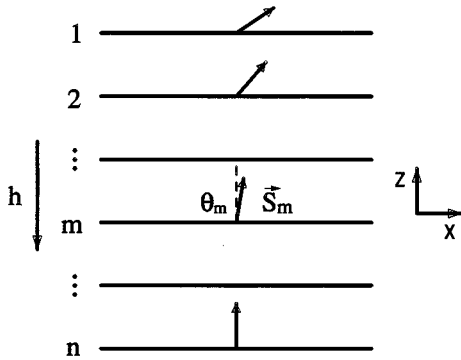


FIG. 1. Spin configuration of type I state for a magnetic coupled multilayer in an external magnetic field.

state and illustrate its spin configuration in Fig. 1. If we substitute that solution into the expression of  $\{\epsilon_i(\mathbf{k})\}$ , the energy gap  $\Delta_I(h)$  can be obtained by some numerical calculations. One can see that the state of type I will keep the metastability ( $\Delta_I(h) > 0$ ) until  $\Delta_I(h) = 0$ . However, there is another critical value of the field  $h_+^{\text{coer}} (> 0)$ . When  $h \leq h_+^{\text{coer}}$  another state (type II) shown in Fig. 2 starts to be metastable since its gap  $\Delta_{II}$  becomes positive. However, the system is initially at the state of type I. The transition from the state of type I to type II will cost energy because  $\Delta_I > 0$  even though the type II state may have a lower energy than the type I state. There is not any thermal fluctuation at the zero temperature so that the system will still keep on the type I state. If we decrease the field  $h$  more and more from the positive value to the negative one, there will be another critical field  $h_-^{\text{coer}} (< 0)$  where  $\Delta_I(h_-^{\text{coer}}) = 0$ . When  $h = h_-^{\text{coer}}$ , the state of type I will be unstable and will transit to the state of type II without costing any energy. The value of the critical field  $h_-^{\text{coer}}$  is the coercive force  $h^{\text{coer}}$ . It is similar when we apply the field along the  $-z$  direction at the beginning due to the symmetry reason.

As an example, Figs. 3 and 4 present the behaviors of the magnetization for model 2. Figure 3 shows the angle deviations of the first three layers in different applied field  $h$  and Fig. 4 the magnetization loop of the system. The coercive

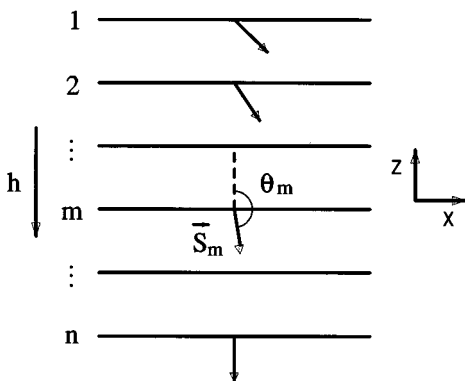


FIG. 2. Spin configuration of the type II state for a magnetic coupled multilayer in an external magnetic field.

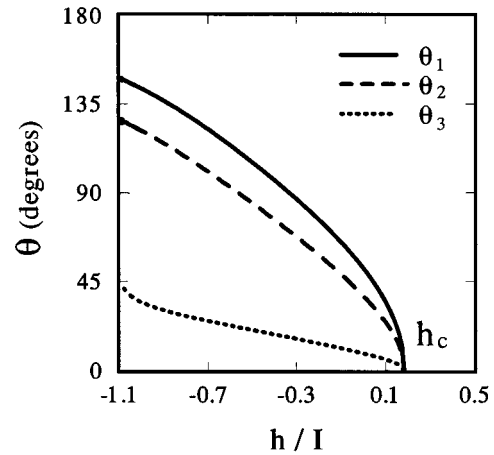


FIG. 3. Angle deviations  $\theta_m$  of the first three layers for model 2 in different applied field.

force  $h^{\text{coer}}$  is found to be 1.078. According to the Appendix, the coercive force of the system without capping  $h_0^{\text{coer}}$  is 2.5. Model 1 has also been studied, and the coercive forces  $h^{\text{coer}}$  and  $h_0^{\text{coer}}$  are 0.033 and 0.4, respectively.

The physics picture is the same if the temperature is not zero. One may find that the excitation gap  $\Delta(h, T)$  is a decreasing function of the temperature because of the interactions between magnons.<sup>15</sup> Thus the coercivity of a magnetic material is weakened by the thermal fluctuations.

We have also compared our method with the conventional ones. The coercive force is not defined explicitly in the old theories, but we understand that it should be the field at which the spin-up state is not stable. Based on this, we find that the coercive force of a single magnetic film with perpendicular easy-axis anisotropy ( $D$ ) at zero temperature is  $h_{\text{old}}^{\text{coer}} = 2SD$ .<sup>7</sup> However, from the Appendix, one may find that the coercive force of the same system is  $h^{\text{coer}} = (2S - 1)D$  in the quantum theory. The quantum fluctuation

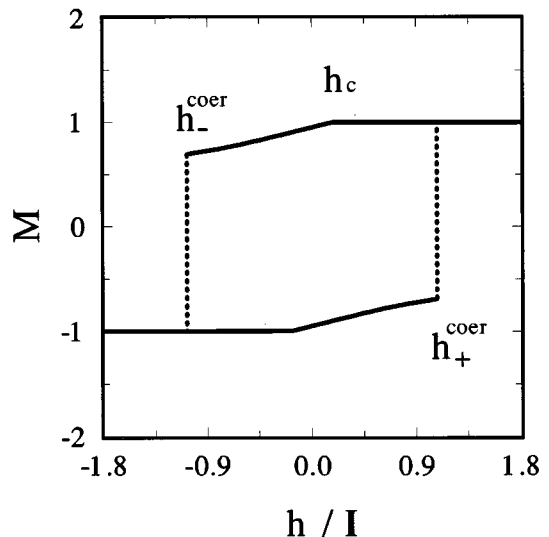


FIG. 4. Magnetization loop of model 2.

tuations make the coercivity of a system become lower. In the limit that  $S \rightarrow \infty$ , the quantum method recovers the results of the phenomenological one.

### V. CAPPING EFFECT

In practice, the recording media with higher field sensitivity is applicable for the devices. So it is very interesting to discuss what kind of capping layers can be used to obtain the most significant capping effect.

We have found that the capping effects are very different for the two models calculated in the last section. The deflation of the coercive force is almost 12 times for model 1 and about 2 times for model 2. Actually, such difference represents two kinds of the phase transitions. When the capping anisotropy  $\tilde{D}_1$  is greater than a critical value  $\tilde{D}_c$ , it was shown in Ref. 3 that the nonlinear equations can have non-trivial solutions at zero field other than two trivial solutions:  $\{\theta_m = 0^\circ\}$  and  $\{\theta_m = 180^\circ\}$ . After considering the quantum fluctuations, we find that there are two critical values  $\tilde{D}_c^1$  and  $\tilde{D}_c^2$  for each capping anisotropy parameter  $\tilde{D}_1$ .

(1) When  $\tilde{D}_c^1 < \tilde{D}_1 < \tilde{D}_c^2$ , we find that the magnon excitation gap  $\Delta$  is negative around the region  $h \sim 0$  so that the system cannot be described by the present method in harmonic approximation. More higher order contributions should be considered.

(2) When the parameter  $\tilde{D}_1$  is outside the region  $(\tilde{D}_c^1, \tilde{D}_c^2)$ , the excitation gap  $\Delta(h \sim 0)$  is always positive. The phase transitions for the cases of  $\tilde{D}_1 < \tilde{D}_c^1$  and  $\tilde{D}_1 > \tilde{D}_c^2$  are quite different. When  $\tilde{D}_1 < \tilde{D}_c^1$ , the spins will remain in their original configuration ( $\theta_m = 0^\circ$ ) until the critical point  $h_-^{\text{coer}}$  is reached. This kind of phase transition is called as the nonreconstructed phase transition. In another case of  $\tilde{D}_1 > \tilde{D}_c^2$ , the spins are reconstructed up to the critical field  $h_-^{\text{coer}}$ .

For example, the critical values of the capping anisotropy  $\tilde{D}_1$  are found to be  $\tilde{D}_c^1 = 0.085$  and  $\tilde{D}_c^2 = 0.113$  when  $\tilde{D}_2 = 0.2$ . The gap  $\Delta(\tilde{h})$  is shown in Fig. 5 for the system with  $\tilde{D}_1 = 0.05 < \tilde{D}_c^1$  and in Fig. 6 for the one with  $\tilde{D}_1 = 0.15 > \tilde{D}_c^2$ . Figure 5 shows that the gap  $\Delta(\tilde{h})$  decreases monotonically along with the increase of  $|\tilde{h}|$  for the nonreconstructed phase transition. However, the gap  $\Delta(\tilde{h})$  will first increase then decrease as  $|\tilde{h}|$  is enlarged for the reconstructed phase transition in Fig. 6.

The capping effects of the two kinds of transitions are presented in Fig. 7. In our calculation, the anisotropy of recording layers are fixed by  $\tilde{D}_2 = 0.2$ . In the figure, the parameter  $h_0^{\text{coer}}/h^{\text{coer}}$  is defined to express the capping effect. It is very interesting to point out that capping effect is almost a constant for the case of  $\tilde{D}_1 > \tilde{D}_c^2$  which is dominated by the reconstructed phase transition. However, the capping effect drastically varies for  $\tilde{D}_1 < \tilde{D}_c^1$  which corresponds to the nonreconstructed phase transition. Thus the greatest improvement of field sensitivity can be achieved when the capping anisotropy  $\tilde{D}_1$  is chosen to be a little smaller than the critical value  $\tilde{D}_c^1$ . Then, we calculate the two critical values of the capping anisotropy  $\tilde{D}_1$  as functions of the recording anisot-

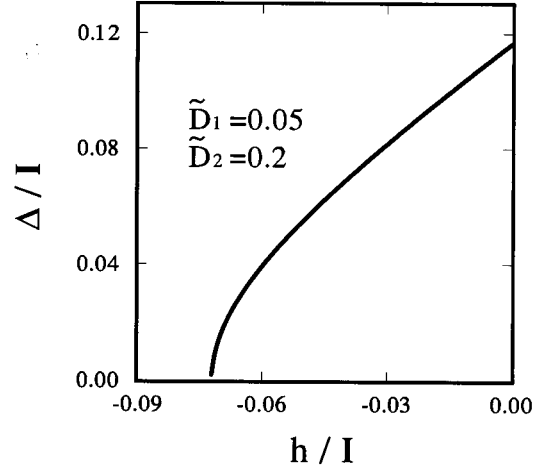


FIG. 5. Magnon excitation gap as a function of the negative external magnetic field in the case of nonreconstructed phase transition.

ropy  $\tilde{D}_2$ . The results are shown in Fig. 8. It is shown that there are three regions in the  $(\tilde{D}_1, \tilde{D}_2)$  plane which are denoted by I, II, and the unclear region, respectively. In region II,  $\tilde{D}_1 > \tilde{D}_c^2$  the improvement of the field sensitivity is not very large and is almost a constant; in region I,  $\tilde{D}_1 < \tilde{D}_c^1$  it varies drastically, and the largest capping effect can be achieved near the boundary in this area. The capping effect in the unclear region is not discussed in our present work.

We have also calculated the dependence of the layer number ( $L$ ) for the capping effect. It has been shown in Fig. 9 where the anisotropy parameters are  $\tilde{D}_1 = 0.01, \tilde{D}_2 = 0.2$ , and the number of the recording layers is 10. That corresponds to the case of nonreconstructed phase transition. From this figure, we can find that the capping effect will be significantly enhanced if the capping material is thicker.

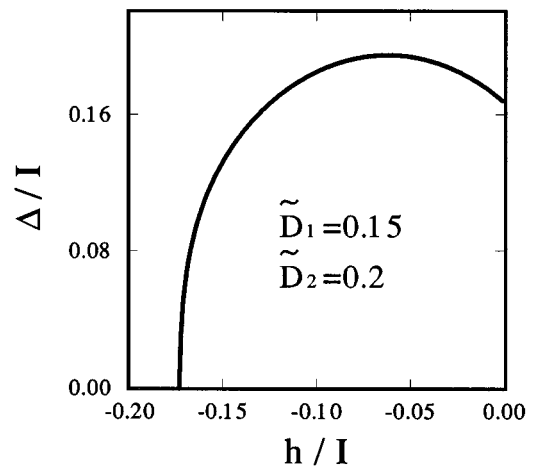


FIG. 6. Magnon excitation gap as a function of the negative external magnetic field in the case of reconstructed phase transition.

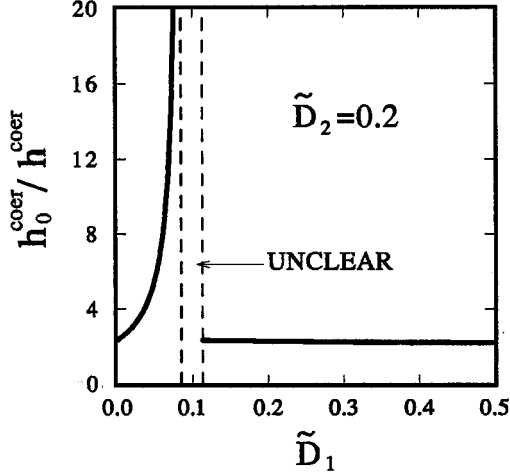


FIG. 7. Improvement of the field sensitivity as the function of capping anisotropy for the double-film system with recording anisotropy  $\tilde{D}_2=0.2$ .

## VI. SUMMARY

In this paper, we first present a model Hamiltonian for a double-film system which is a magnetic thin film with perpendicular easy-axis anisotropy capped by another one with in-plane easy-axis anisotropy, then transform the Hamiltonian by the LC and Bose transformations. The Hamiltonian is diagonalized under harmonic approximation.

A concept of the metastable state is proposed to study the magnetic coupled multilayers. We find that the magnon excitation gap can be used to determine the coercive force of the magnetic multilayer through the condition that the gap comes to zero. A general procedure for calculating the coercive force of a magnetic coupled thin film is described, numerical calculations are carried out for some typical systems. The coercive force of the double-film structure is found to decrease remarkably by means of the capping technique.

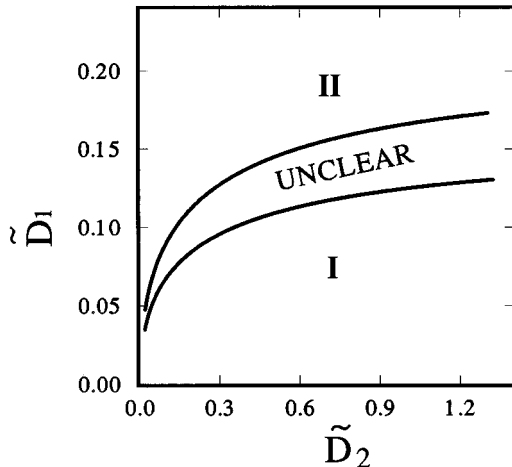


FIG. 8. Critical values  $\tilde{D}_c^1$  and  $\tilde{D}_c^2$  for the capping anisotropy as functions of the recording anisotropy  $\tilde{D}_2$ .

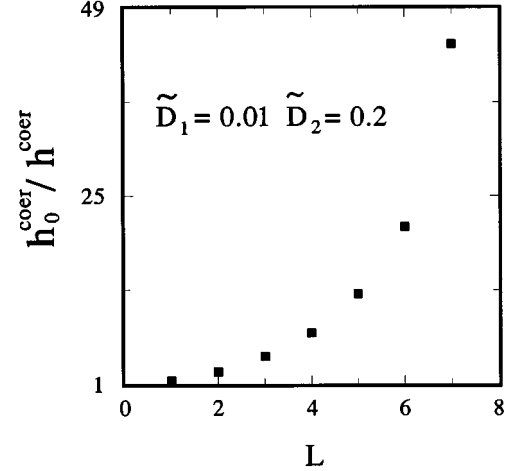


FIG. 9. Improvement of the field sensitivity as the function of layer numbers of the capping material.

It is found that there are two types of phase transition for the double-film structures. An expected capping effect can be achieved after selecting appropriate capping material. The capping effect will be enhanced if the capping anisotropy increases or the capping film is thickened for the case of capping materials with small anisotropy. However, when the capping anisotropy is larger than a critical value, the capping effect is rather trivial.

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## APPENDIX

In this appendix, we will derive the coercive force of an  $N$ -layer magnetic multilayer with perpendicular easy axis anisotropy.

Since there are no capping layers here, the solutions of Eq. (13) are just  $\theta_m=0^\circ$  (type-I state) and  $\theta_m=180^\circ$  (type-II state). Substitute the solution  $\theta_m=0^\circ$  into Eq. (13), the matrix  $\hat{\mathcal{H}}(\mathbf{k})$  are found to be

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} \frac{1}{2}\mathcal{F}(\mathbf{k}) & 0 \\ 0 & -\frac{1}{2}\mathcal{F}(\mathbf{k}) \end{pmatrix}, \quad (\text{A1})$$

where

$$F_{m,m}(\mathbf{k}, \theta) = I_{m,m}ZS(1 - \gamma_k) - D_m(2S - 1) + \sum_{m'} SI_{m,m'} + h, \quad (\text{A2})$$

$$F_{m,m'}(\mathbf{k}, \theta) = -SI_{m,m'}, \quad m \neq m'. \quad (\text{A3})$$

The diagonalization of the matrix  $\mathcal{H}(\mathbf{k})$  is equivalent to the diagonalization of the  $N \times N$  matrix  $\mathcal{F}(\mathbf{k})$  which can be further simplified as

$$\mathcal{F}(\mathbf{k}) = IS \cdot [Z(1 - \gamma_k) - 2\tilde{D}_2 - \tilde{h}] + \mathcal{W}, \quad (\text{A4})$$

where the elements of the matrix  $\mathcal{W}$  is defined by

$$W_{m,m}(\mathbf{k}, \theta) = \sum_{m'} SI_{m,m'}, \quad (\text{A5})$$

$$W_{m,m'}(\mathbf{k}, \theta) = -SI_{m,m'}, \quad m \neq m'. \quad (\text{A6})$$

The matrix  $\mathcal{W}$  has the lowest eigenvalue 0, so the lowest eigenvalue of matrix  $\mathcal{F}(\mathbf{k})$  must be  $IS \cdot [Z(1 - \gamma_k)$

$-2\tilde{D}_2 - \tilde{h}]$ . Then from Eqs. (21), (22), and (A1), the excitation gap can be easily derived as

$$\Delta(h) = h + (1 - 2S)D. \quad (\text{A7})$$

Thus it is easy to get the coercive force

$$|\tilde{h}_c^0| = 2\tilde{D}_2. \quad (\text{A8})$$

Actually, this result is not strange if one will compare it to a 2D easy axis planar ferromagnet or a 3D easy axis bulk ferromagnet. They yield the same result.

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