Dynamic magnetostatic interaction between amorphous ferromagnetic wires

J. Velázquez, C. García, M. Vázquez, and A. Hernando
Instituto de Magnetismo Aplicado, RENFE–Universidad Complutense de Madrid, and Instituto de Ciencia de Materiales, Consejo Superior de Investigaciones Científicas, P.O. Box 155, 28230 Las Rozas, Madrid, Spain

(Received 31 October 1995; revised manuscript received 25 April 1996)

The axial hysteresis loop of highly magnetostrictive amorphous wires shows a perfect square shape, with a single large Barkhausen jump due to its particular domain structure. When wires of this type are placed in proximity, the corresponding axial hysteresis loop exhibits a number of Barkhausen jumps equal to the number of wires, the form of the loop not being stable in time. We analyze this type of loop as resulting from magnetostatic interaction among the wires. The instability of the loop results from the lack of simultaneous magnetization reversal in each wire when the magnetic field is applied. This unstable behavior is interpreted considering the theory of chaos. [S0163-1829(96)03437-6]

I. INTRODUCTION

Amorphous wires have been a subject of deep study in the past decade owing to their outstanding magnetic properties, useful for technological applications, like magnetic sensors[1] (due to their soft magnetic character) and stress sensors[2] (by virtue of their magnetoelastic properties). Amorphous ferromagnetic wires are commonly prepared by means of the in-rotating-water quenching technique.[3] This procedure involves a very high cooling rate from the molten alloy, producing internal stresses that couple with the magnetic moments via the magnetostriction, giving rise to strong local magnetoelastic anisotropies.[4] The distribution of these anisotropies determines the magnetization process and the domain structure, in particular for those wires with high magnetostriction constant. These latter wires are largely magnetized through a single Barkhausen jump, giving rise to square hysteresis loops.[5] The axial magnetization process presents the following characteristics: (i) The demagnetized state cannot be reached but two well defined stable remanent states are found, (ii) upon application of an axial field anti-parallel to the remanent magnetization a small and reversible decrease of the magnetization is firstly detected and for a critical or switching field, the magnetization suddenly reverses its direction so that a square loop is observed, and (iii) if the applied field is further increased, the magnetization increases monotonously and reversibly up to its saturation along the field direction.

The magnetization process and the square loop can be roughly understood if we take into account the domain structure proposed for these wires:[6] a single-domain inner coaxial core with the magnetization axially oriented and a multidomain external shell where the magnetization is perpendicular to the axis of the wire. The magnetization reversal inside the inner core is then responsible for the square loop while the final reversible approach to saturation is determined by the continuous reduction of the transverse magnetization at the shell.

A refinement of that model has been proposed to account for the disappearance of that outstanding loop for wires shorter than a critical length.[7,8] It consists of including a closure domain structure at the ends of the wires, and consequently of the core, to reduce the otherwise quite strong magnetostatic energy. The nucleation of that closure domain structure requires on the other hand some magnetoelastic energy. The balance between magnetoelastic and magnetostatic energies determines the domain structure at the ends of the wire and, consequently, its behavior when an external magnetic field is applied. The influence of that magnetostatic energy in the reversal process was analyzed elsewhere.[9]

For example, in the case of Fe-rich amorphous wires the magnetostriction constant, λ, is high and positive ($\lambda = 3\times10^{-5}$). The order of magnitude experimentally determined[7,8,10] of the internal stresses, σ, is 10^2 MPa. Accordingly, the magnetoelastic energy density, E_σ, takes the order of magnitude of 10^3 J m^{-3} [$E_\sigma = (3/2)\lambda\sigma$]. On the other hand, an average magnetic energy density, E_H, due to the macroscopic demagnetizing field created by the magnetic poles is evaluated to be around 10 J m^{-3} ($E_H = \mu_0 N M_s^2$, where $\mu_0 = 4\pi\times10^{-7}$ H m^{-1} is the vacuum permeability, $M_s \sim 10^6$ A/m is the saturation magnetization, and $N \sim 10^{-5}$ is the demagnetizing factor). The relatively large intensity of the magnetoelastic anisotropy makes attaining a perfect compensation of stray fields very hard so that a final net magnetic poles density is expected to be distributed near the ends of the wire. When two wires are placed in proximity and an ac external magnetic field is applied, the stray fields couple the magnetizations of the wires affecting the magnetic state of each wire. The overall magnetization exhibits a nonperiodic evolution that leads to an unstable hysteresis loop.

The aim of this work is to analyze the characteristics of this interaction and its effect on the hysteresis loop corresponding to a number of magnetostatically coupled wires. It is also considered that a system of n coupled amorphous ferromagnetic wires submitted to an oscillating magnetic field can exhibits a chaotic dynamic behavior.

II. EXPERIMENT

As-cast amorphous wires of nominal composition $Fe_{77.5}Si_{7.5}B_{15}$, kindly supplied by Unitika Inc., have been used in the measurements, their diameter and length being 131 μm and 11.7 cm (or 31 cm), respectively. Axial hysteresis loop was measured by means of a conventional induc-

0163-1829/96/54(14)/9903(9)/$10.00

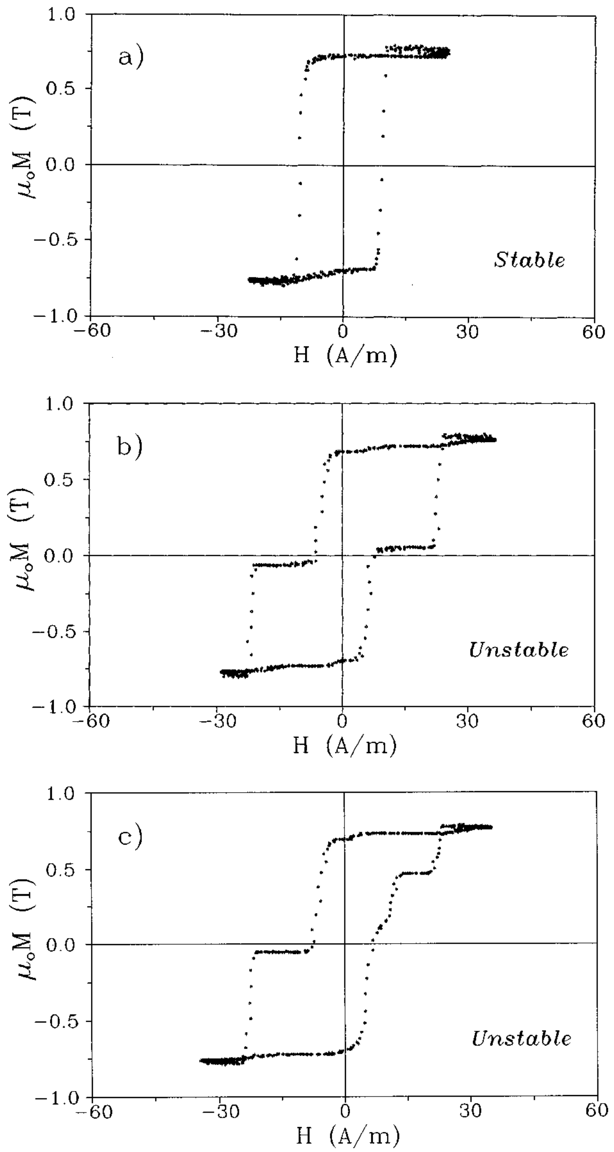

FIG. 1. Axial hysteresis loops for iron rich amorphous wires: (a) one wire (remanence, $\mu_0 M_r = 0.71$ T and coercive force, $H_c = 10.03$ A/m); (b) and (c) two wires ($\mu_0 M_r = 0.69$ T, $H_c = 7.16$ A/m).

tion method. The field is provided by a 32 cm long drive coil with a constant of 1856 A m^{-1}/A. Inside its middle part, a pickup coil is placed with 340 turns of copper wire, whose diameter is 0.1 mm, rolled in an albumin tube, being 1.5 mm in diameter and 8 cm in length.

The amorphous wires to be measured were placed inside a 1 mm diameter quartz tube holding the wires parallel and in contact. This tube is then placed inside the pickup coil. An ac axial magnetic field is applied by the drive coil with various amplitudes, frequencies and wave forms (sinusoidal and triangular). The voltage induced in the pickup coil was electronically integrated and displayed on a Philips PM 3335 digital oscilloscope together with the drive coil current. These two signals determine the hysteresis loops. The digital oscilloscope stores these signals for processing by a personal computer, using the HYPERPLT program for Fourier analysis.

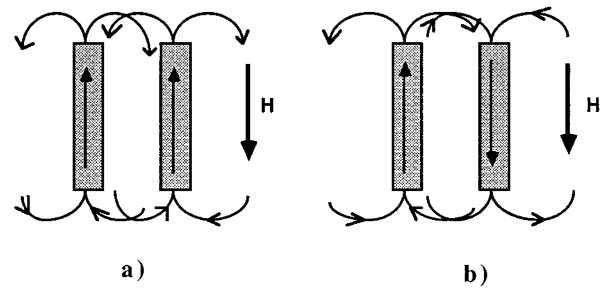

FIG. 2. Schematic view of the magnetostatic interaction between wires (see text).

III. RESULTS AND DISCUSSION

Figure 1(a) shows the hysteresis loop obtained for a single wire of length 11.7 cm. The frequency of the sinusoidal magnetic field was 40 Hz. A remanent magnetization of 0.71 T and a coercive force of 10.03 A/m was obtained. The observed magnetic transition is clearly a single Barkhausen jump. Furthermore, the loop is *stable* in time. Now, if a set of two wires is placed inside the quartz tube, it might be expected that the *induced signal from the wires would simply double*, and the loop keep its square shape. Nevertheless, as Fig. 1(b) shows the loop which contains two well defined jumps, with the remanent magnetization 0.69 T, nearly the same as for the single wire (the induced signal is double but the total volume of the samples is also double, then the remanent magnetization will be the same). The most important characteristic of this loop is its *unstable character* with the time, switching between those shown in Figs. 1(b) and 1(c), which differ only in their time of measurement. Similarly, with three, four, six, and eight wires, we observe three, four, six, and eight Barkhausen jumps. These jumps appear and disappear in a seemingly random way.

A. Magnetostatic interaction between wires

The results shown in Figs. 1(b) and 1(c) can be understood by taking into account a magnetostatic interaction between the two wires caused by the magnetic field of the poles at the ends of the wire. Consider a simple theoretical model. First, we simulate the square hysteresis loop of a single wire, assuming only two magnetization states, $+M$ and $-M$ ($\mu_0 M = 0.7$ T). The applied field is $H(t) = H_0 \sin(2\pi\nu t)$. The transition between these two states occurs when the external magnetic field reaches the value H_c. Denote by $M(H)$ the hysteresis loop of a single wire, by M_1 the magnetization state of wire 1 and by H_{12} the field created by the wire 1 at wire 2.

We assume

$$H_{12} = KM_1, \tag{1}$$

where K is dimensionless in the mks system. Similarly define $H_{21} = KM_2$. The hysteresis loop of each wire ($i = 1,2$) becomes

$$M_i = M_i(H - H_{ji}) \tag{2}$$

assuming H_{ji} has direction opposite the external field H, when M_j is parallel to H. Figure 2 shows this configuration. Initially, we have two wires (the drawing only represents the magnetization in the inner domain) magnetized in the same direction, the external magnetic field being applied in the opposite direction [Fig. 2(a)]. Increasing H, one of the two wires *reverses magnetization first* [Fig. 2(b)] and modifies with its own field the strength of the total magnetic field sensed by the wire that has not yet reversed magnetization. The total magnetization for the two wires, $M=(1/2)(M_1+M_2)$, is then determined by

$$M_1=M_1(H-H_{21})=M_1(H-KM_2), \tag{3}$$

$$M_2=M_2(H-H_{12})=M_2(H-KM_1). \tag{4}$$

Substituting Eq. (4) into Eq. (3)

$$M_1=M_1(H-K[M_2(H-KM_1)]). \tag{5}$$

Solutions to equations of the form $M_1=g(M_1)$ may be obtained by the fixed point iteration method,[11] M_2 following from Eq. (4) when M_1 is obtained. Figure 3 shows the results obtained for $H_c=3$ A/m, $H_0=10$ A/m, and $\nu=1$ Hz. Each loop corresponds to ten cycles. Figure 3(a) shows the result of $M(H)$ for a single wire. The loop for a set of two wires without interaction ($K=0$), is given in Fig. 3(b). For nonzero interaction (i.e., $K=1$), the loop is shown in Fig. 3(c). Its similarity to the experimental result observed in Fig. 1(b) is to be remarked. It is important to mention that the length of the horizontal section at $M=0$ is proportional to the value of K. Consequently, the experimental value of K could be evaluated from this length. The appearance of two well defined Barkhausen jumps is then a consequence of the nonsimultaneity in the magnetization reversal at each wire which results from the magnetic interaction between wires.

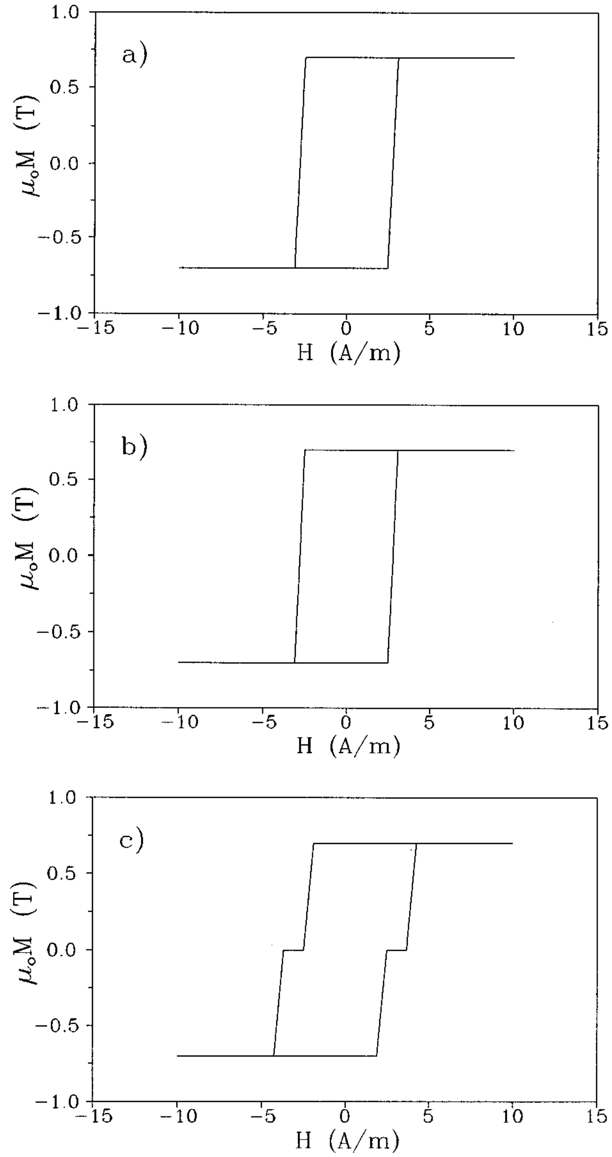

FIG. 3. Theoretical hysteresis loops: (a) one wire; (b) two wires without interaction ($K=0$); (c) two wires with interaction ($K=1$).

B. Why the wires do not reverse magnetization simultaneously

Magnetization reversal is achieved when the reversed magnetic field reaches the switching field, H^*. At temperatures different from absolute zero, the magnetic moments have thermal fluctuations. Like any macroscopic magnitude, the field H^* can be described by an average value, $\langle H^*\rangle$, and fluctuation $\langle(H^*-\langle H^*\rangle)^2\rangle$. These fluctuations are in principle a reason to cause one wire to randomly switch first.

But most probably, the answer seems to be related itself with the presence of closure domain at the ends. In finite wires, the reversal magnetization depends on the depinning of closure domains walls at the ends of the wire. The depinning in each end depends on the strength and direction of the applied magnetic field.[5] Anyway, it is reasonably impossible to count on the exactly equal domain structure at both ends as it has been experimentally shown.[12,13]

C. Why the hysteresis loop corresponding to some wires placed together are unstable

Figure 4 shows schematically the configuration for a set of three wires. In Fig. 4(a), all the wires are magnetized in the same direction. When a reverse magnetic field is applied, one of them reverses first the magnetization as shown in Fig. 4(b). When the magnetic field goes through another cycle a different configuration may result [Fig. 4(c)]. Intuitively, some relation must exist between the nonsimultaneousness of the Barkhausen jumps and the frequency and strength of the external magnetic field. Experimentally, we have found the following results independently of the number of wires: (i) An unstable loop becomes stable (but not square) for frequencies of the magnetic field above 500 Hz; (ii) for high field strengths the instability of the loop disappears.

If the applied magnetic field is large enough and the magnetization is in the opposite direction to the field, the high magnetic energy will cause all the wires to reverse simultaneously. The increase of the field frequency leads to a loss of squareness as the coercive field increases as a consequence of the increasing opposite field induced by the eddy currents.

We can conclude that there is a dynamical magnetostatic interaction between the amorphous wires arising from the magnetic state of each wire which arises from the nonsimultaneousness in the change of magnetization direction when an ac external magnetic field is applied.

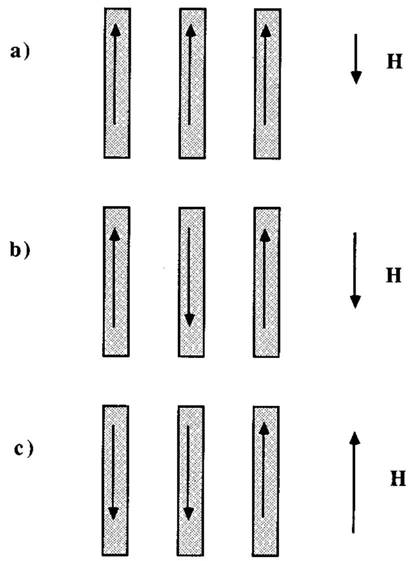

FIG. 4. Schematic view corresponding to no simultaneousness of the Barkhausen jumps.

D. Analysis of the instability of the loops from a point of view of chaos

The instability experimentally detected in the hysteresis loops of multiple coupled wires can be also simulated by using the theoretical model described above. Consider two wires with interaction $K=1$. The amplitude of the applied sinusoidal field is $H_0=10$ A/m and the coercive force is $H_c=3$ A/m. Figure 5 shows the results obtained for different values of the frequency of the field: $\nu=1.20$ Hz [Fig. 5(a)]; $\nu=1.94$ Hz [Fig. 5(b)]; $\nu=1.96$ Hz [Fig. 5(c)]. The loops shown in Fig. 5 correspond to ten cycles. Comparing these loops with the loop shown in Fig. 3(c), the shape is very similar but those in Fig. 5 show fluctuations. The fluctuations observed in these theoretical loops come from the different values of the magnetization in each wire (initial conditions) for a given value of the sinusoidal applied field, that is, there is no synchronicity between the frequency of the field and the period to build a hysteresis. The model used is only valid if the wire is bistable. Experimentally, the bistability is lost if the frequency is increased: in this case, when the instability disappears, the model is not useful. Since the behavior of the loop changes qualitatively with frequency, the frequency can be used as control parameter.

In hysteretic magnetic materials, the magnetization M is not uniquely determined by H. In the case of a single wire, for each value of H, there are two possible values of magnetization M. For a system composed of various interacting wires something more complex is expected around the coercivity. One possibility is to consider deterministic chaotic behavior in the temporal of the magnetization. Far from being equivalent to randomness, deterministic chaos describes a complexity in which exists a structure generated by a deterministic law.[14,15]

Deterministic chaos refers to the description of dynamical dissipative systems having asymptotic stability and local instability. Such a behavior is described in phase space (system in which each coordinate represents a degree of freedom of the dynamical system) by terms of strange attractors, geometrical structure that exhibits fractal properties, that is, autosimilarity at different scales.[16] In recent years, chaos has become a useful framework for explaining some experimental results, seemingly anomalous.

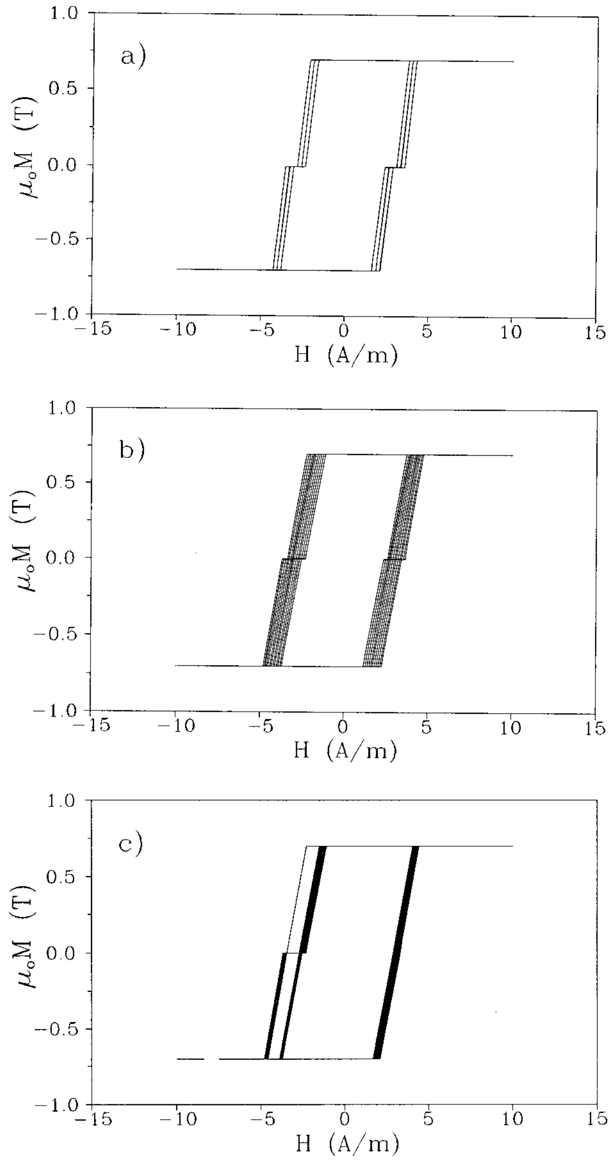

FIG. 5. Theoretical hysteresis loops for two interacting wires ($K=1$) as a function of the frequency, ν, of the applied magnetic field: (a) $\nu=1.20$ Hz; (b) $\nu=1.94$ Hz; (c) $\nu=1.96$ Hz.

Where chaos comes from in our system? The answer to this question is related to the depinning of the closure domains wall at the ends of the wire, that can be considered as a nonlinear process due to the nonlinear dependence of the density of energy of the wall with the radial coordinate (taking into account a cylindrical coordinate system) as a consequence of the radial variation of the internal stresses generated during the fabrication process of the wires.[4] In the depinning process, the closure domain wall can be considered as a forced nonlinear oscillator. We are trying to obtain a differential equation that permits to describe the process in order to analyze accurately the dynamical singularities of the system.

From a technical point of view, the behavior of a system is called chaotic if exhibits the so called ''*sensitive depen-*

dence to initial conditions,'' that is, the exponential divergence of temporal evolutions arbitrary close in the initial time. This property has two consequences: (i) the temporal evolution is determined by a continuous frequency spectrum and (ii) it exhibits fractal properties.[17] The fractality of an experimental signal is not easy to determine since it is necessary to use very long time of measurement in order to have large enough number of experimental points in every temporal scale so that the autosimilarity of the signals at different scales can be checked. Our experimental setup can store only four thousand points, which is generally not sufficient. On the other hand, the fractality of the attractor is not a good characterization of a chaotic behavior. Chaotic behavior can be characterized by calculating the Lyapunov exponents,[18] this permits to measure the sensitive dependence to initial conditions by computing the evolution with time of the difference of two magnetization which were close at one time. Experimentally, it is very difficult to do this due to the limit imposed by the experimental accuracy that characterizes the setup that measures and stores the data: the experimental accuracy limits drastically the possibility to obtain two signals sufficiently close in the initial time. On the other hand, it is not easy to obtain the Lyapunov exponent by considering a single signal.[19] Nevertheless, we are trying to improve the measuring conditions in order to obtain these exponents. We investigate the possibility of chaos by analyzing the frequency spectrum and the autocorrelation function, C, that denotes the average value in a temporal interval $[t_1,t_2]$ of the product of the signal $M(t)$, with the same signal at $t+\tau, M(t+\tau)$, formally:[17]

$$C(\tau)=\frac{1}{t_2-t_1}\int_{t_1}^{t_2} M(t)M(t+\tau)dt. \tag{6}$$

We use this function to study the complexity of $M(t)$. For $\tau=0$, the product $M(t)M(t+\tau)$ is always positive while for $\tau\neq 0$, the product $M(t)M(t+\tau)$ can be either positive or negative, depending on $t\in[t_1,t_2]$. In the former case, $C(0)$ will be positive having its maximum value and in the latter $C(\tau)$ can be positive or negative. If $M(t)$ involves many frequencies the product $M(t)M(t+\tau)$ changes sign even for very small values of τ so that $C(\tau)$ approaches to zero starting on a certain value τ_0. This means that $M(t)$ and $M(t+\tau_0)$ are not correlated. If $M(t)$ is periodic, $C(\tau)$ will be oscillating. The function $C(\tau)$ is normalized to unity, and the Wiener-Khintchine's theorem[17] ensures that the power spectrum of $M(t)$ is proportional to the Fourier transform of $C(\tau)$.

A useful, visual description of the dynamics of these systems is an attractor in phase space. The construction of such an attractor is not easy when we do not know all the degrees of freedom $(x,y,z,..)$ that describe the behavior of the system. In the present case, we have only a temporal signal $M(t)$. In order to determine the attractor we have used the technique due to Grassberger and Procaccia[20] (but originally developed by Packard, Ruelle, and Tackens[14,21]) by means of which a phase space whose coordinate axis are $x,y,z,...,$ can be replaced by an equivalent one with coordinates $M(t),M(t+T),M(t+2T),...,$ where T is a time that depends on τ_0 [time starting from $C(\tau)$ is zero] and on the embedding dimension of the attractor.[19] This technique uses the possibility

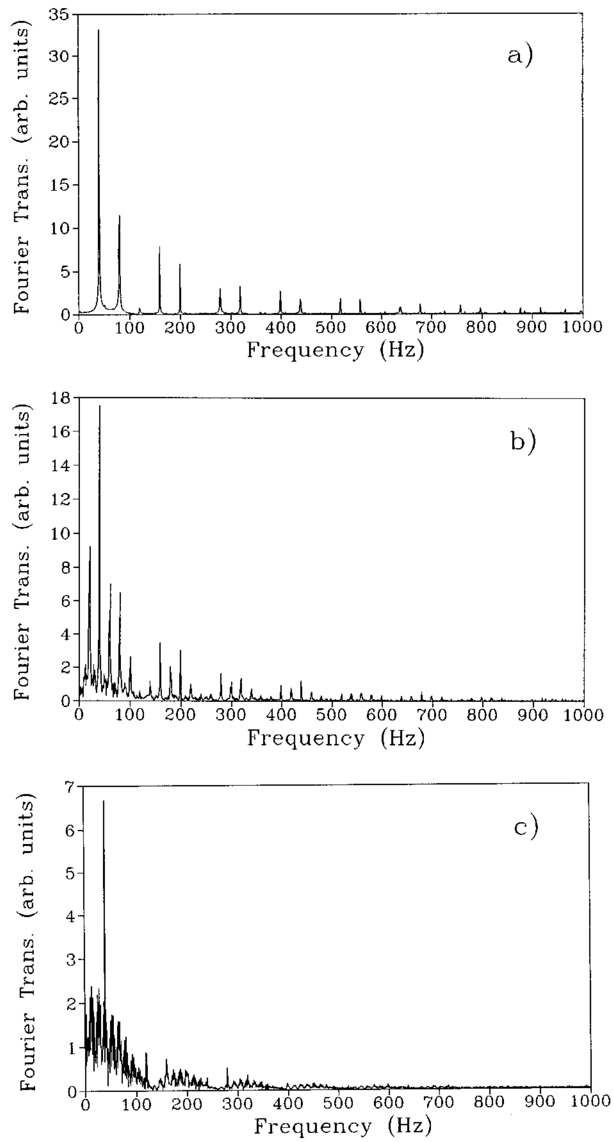

FIG. 6. Frequency spectrum of the temporal variation of the total magnetization of two wires, for maximum applied fields, H_{max}: (a) H_{max}=14.32 A/m; (b) H_{max}=13.91 A/m, and (c) H_{max}=13.30 A/m.

of reconstructing the dynamics of the system from the information contributed by a single variable, which in our case corresponds to the axial component of the total magnetization of the wires. Taking into account that the value of this variable depends on the rest of variables characterizing the system its dynamics can be studied. For sufficiently adequate value of T, the quantities $M(t),M(t+T),...,$ are linearly independent and it is possible to correctly reconstruct the phase space, or more precisely, its projection on a phase subspace having a smaller dimension. Due to the difficulty of calculating accurately the embedding dimension (that is the lowest integer larger than the fractal dimension of the attractor), we choose by inspection some value of T. Using this value, we can determine an attractor having some topological deformation in the phase space with regard to the real attractor. In the following we are going to apply these techniques to analyze the measured signals $M(t)$ coming from the total magnetization of multiple coupled wires.

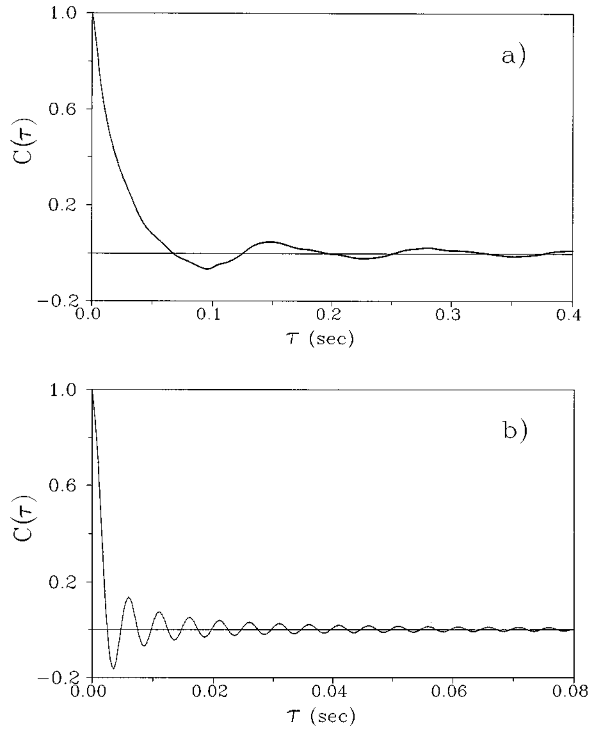

FIG. 7. Autocorrelation function for different frequencies, ν, and maximum applied field, H_{max}: (a) ν=40 Hz, H_{max}=12.51 A/m; (b) ν=200 Hz, H_{max}=17.77 A/m.

Analysis of the frequency spectrum

Consider two wires whose nominal composition is $Fe_{77.5}Si_{7.5}B_{15}$, of diameter 131 μm and length 31 cm. We measure the ac hysteresis loop by applying a triangular wave field whose frequency is 40 Hz. The strength of the field is measured by means of the intensity of current passing through the coil. The root mean square value of this intensity, I_{rms} is measured accurately using a multimeter FLUKE 8842A. Making use of the digital oscilloscope, the temporal variation of the magnetization for different values of I_{rms} is recorded. The Fourier transform of the signals allows us to study the involved frequencies. In Fig. 6(a) the spectrum for I_{rms}=4.455 mA (this value corresponds to a maximum field, peak to peak, H_{max}=14.32 A/m) is observed for which the loop is stable. The largest peak is at the fundamental frequency while the other peaks correspond to harmonics of this fundamental frequency. Decreasing the maximum value of the magnetic field (I_{rms}=4.326 mA $\rightarrow$ H_{max}=13.91 A/m) the loop becomes unstable. Figure 6(b) shows the new spectrum, that is more complex than that in Fig. 6(a). It shows clearly the presence of a duplication of period: it appears a new peak at 20 Hz, half of the fundamental frequency. There also appear peak of frequencies not present in Fig. 6(a). If we further decrease the magnetic field (I_{rms}=4.137 mA $\rightarrow H_{max}$=13.30 A/m) the spectrum becomes compact and dense. Note that when decreasing the value of the maximum applied magnetic field the height of peaks becomes smaller, being the available energy redistributed into the new frequencies.

We have numerically analyzed the relations between the frequencies. It is noticeable that when comparing Figs. 6(a) and 6(b), a new peak is observed in Fig. 6(b) between each

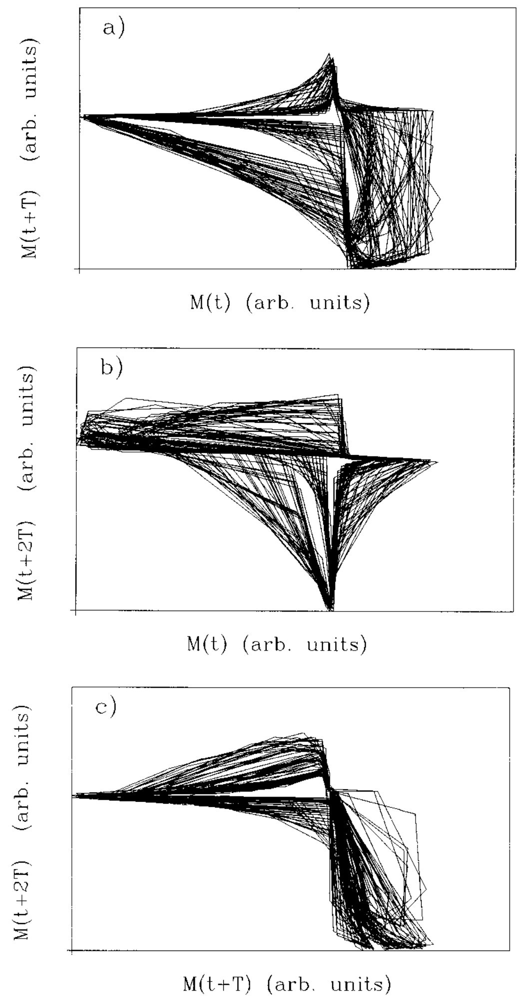

FIG. 8. Reconstruction of the attractor: (a) $M(t+T)$ versus $M(t)$, (b) $M(t+2T)$ versus $M(t)$, (c) $M(t+2T)$ versus $M(t+T)$, with T=0.005 sec and $M(t)$ the temporal variation of the magnetization corresponding to a set of two wires.

pair of peaks in Fig. 6(a). This suggests a quasiperiodic behavior. In quasiperiodicity, the frequencies that appear are linear combinations of fundamental frequencies and in the present case such fundamental frequencies can be ascribed to the oscillation of the magnetization in each wire at 40 Hz, the frequency of the field. The rest of frequencies that appear are linear combinations of this fundamental frequency. The frequency spectrum that appears in Fig. 6(c) is more complicated than in the previous cases. For frequencies smaller than 100 Hz, the spectrum is practically continuous, that is, peaks corresponding to all frequencies are detected and they cannot be determined simple by linear combinations of fundamental frequencies. Nevertheless, the peak corresponding to 40 Hz is the largest, that is, the system continues oscillating predominately at this frequency. It can be correlated to the fact that in spite of the instability of the loop, its shape is roughly kept.

Figure 6 suggests a route to chaos from quasiperiodicity at two frequencies when the strength of the applied magnetic field decreases. Nevertheless, the fully chaotic behavior is not reached (there is always a predominant frequency). This can be a case of *weak chaos*, i.e., where the chaos is localized in the phase space.

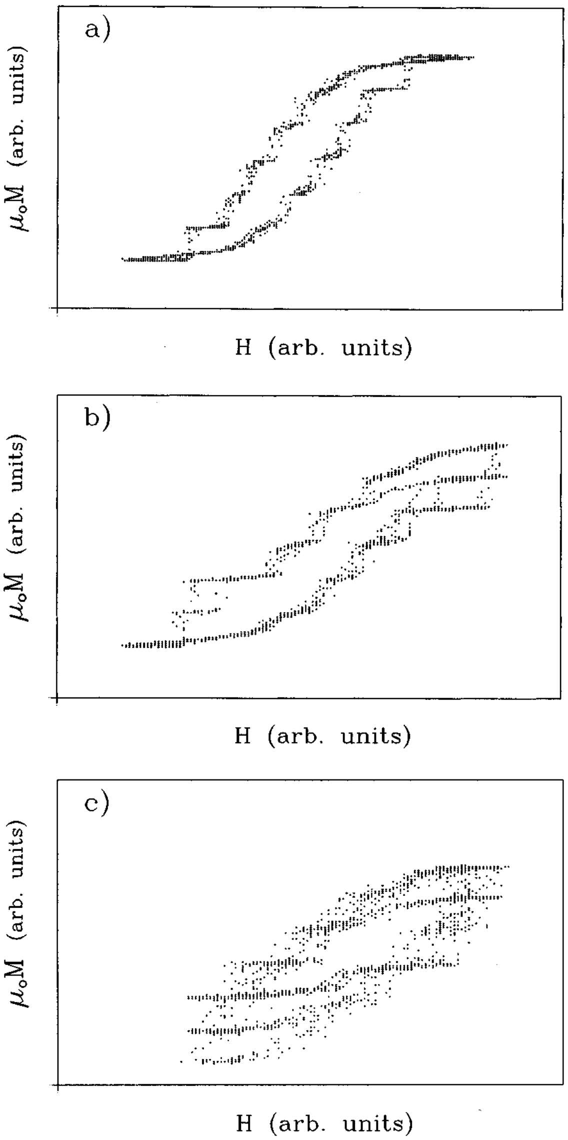

FIG. 9. Hysteresis loops for a set of six wires, measured at 40 Hz (sinusoidal wave form), for different values of the maximum applied field, H_{max}: (a) H_{max}=52.39 A/m; (b) H_{max}=37.11 A/m; (c) H_{max}=27.37 A/m.

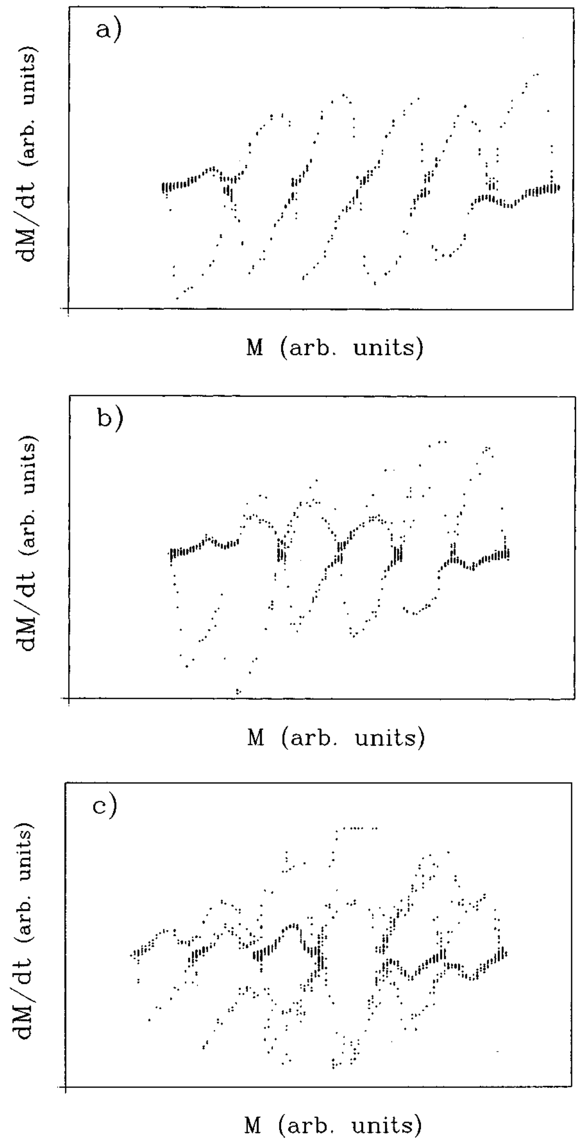

FIG. 10. $dM(t)/dt$ versus $M(t)$ for a set of six wires for different values of the maximum applied field, H_{max}. The frequency of the field is ν=40 Hz and the wave form is sinusoidal: (a) H_{max}=52.39 A/m; (b) H_{max}=37.11 A/m; (c) H_{max}=27.37 A/m.

Autocorrelation function

Figure 7 shows two autocorrelation functions for the signals corresponding to a set of two amorphous wires whose length is 11.7 cm. The applied magnetic field has triangular wave form. We have selected two different frequencies and maximum values of the field in order to induce instability or stability in the loop and in order to compare the corresponding autocorrelation functions.

Figure 7(a) corresponds to a frequency, v=40 Hz, and a field (peak to peak), H_{max}=12.51 A/m. In this case the hysteresis loop is unstable. The frequency spectrum is similar to that shown in Fig. 6(c). $C(\tau)$ is zero for τ_0=0.0675 sec. After a second zero crossing, $C(\tau)$ is substantially zero after 0.2 sec. This implies a nearly continuous frequency spectrum, a necessary condition for the presence of chaos. In a fully chaotic state (reached if the shape of the loop disappears) $C(\tau)$ will be zero for $\tau>\tau_0$ (taking τ_0 a value different that in the former case). Figure 7(b) shows the autocorrelation function for v=200 Hz and H_{max}=17.77 A/m. In this case, the hysteresis loop of the two wires is only slightly unstable, and the autocorrelation function is oscillating.

Reconstruction of the attractor

We now reconstruct the attractor for the conditions in which the loop is unstable. A set of two 11.7 cm long wires were used. The magnetic field has triangular wave form, with v=326 Hz and H_{max}=18.52 A/m. The frequency spectrum is similar to that of Fig. 6(c). Figure 8 display the image of the attractor in various perspectives. Figure 8(a) shows the value of magnetization, $M(t+T)$, in arbitrary units (the signal is taken directly from the digital oscilloscope), versus $M(t)$, taking T=0.005 sec. Figures 8(a) and 8(b) show $M(t+2T)$ versus $M(t)$ and $M(t+2T)$ versus $M(t+T)$, respectively. Notice that for a periodic or quasiperiodic evolution of $M(t)$, the lines in the attractor (that connect the points representing the state of the system for different values of t) must coincide.

E. Other studies

(i) Dynamic for a set of six wires. Chaos is an asymptotic effect, that is, it appears at a steady state when all transitory effects are over. In order to appreciate this, we consider a signal that involves many oscillation periods of the magnetic field. For this purpose, we record and store the signal in the digital oscilloscope using an adequate time scale. We measure the hysteresis loop for a set of six 11.7 cm long wires using a sinusoidal wave form magnetic field with a frequency of 40 Hz. The use of triangular or sinusoidal wave form does not alter substantially the characteristic of the loop. We record the signals corresponding to the total magnetization and the applied field. We plot one variable as a function of the other to obtain loops with two thousand points resolution. Figure 9(a) shows the loop for the maximum applied magnetic field $H_{\mathrm{max}}=52.39$ A/m. The loop is stable and six Barkhausen jumps are observed. If the field is decreased to $H_{\mathrm{max}}=37.11$ A/m, the loop becomes unstable [Fig. 9(b)]. This instability is more evident for $H_{\mathrm{max}}=27.37$ A/m [Fig. 9(c)]. It is clear that in consecutive oscillation periods of the magnetic field, the value of the magnetization is not the same.

By using the signal $M(t)$ and its derivative dM/dt, we can construct a kind of phase space. This is not the real one, but a topological deformation of this.[17] In this case, the splitting of the ''attractor'' can be demonstrated. For $H_{\mathrm{max}}=52.39$ mA [Fig. 10(a)] all the points lie on the same curve (that contains six undulations up and six down). For $H_{\mathrm{max}}=37.11$ A/m [Fig. 10(b)] there is a deformation of the figure and some points do not lie in the same curve for successive oscillation periods of the magnetization. The situation is even more complex for $H_{\mathrm{max}}=27.37$ A/m [Fig. 10(c)] where no clear undulations are detected.

(ii) Effect of low temperature, tensile stress, and change of composition. We have measured the possible influence of other variables in order to get further information on the complex behavior. No significant changes were detected by measuring at lower temperature (77 and 160 K) as well as under applied tensile stress of up to 150 MPa. On the other hand, the behavior observed in Fe-rich wires is not detected in the nonbistable Co-rich wires, that have a less value of the remanent magnetization (0.3 T for Co-rich and 0.7 for Fe-rich wires[2,22]) and a nearly zero magnetostriction, $\lambda=-0.08\times10^{-6}$.

IV. CONCLUSIONS

The hysteresis loop of magnetostically-coupled amorphous wires has been experimentally measured, the number of Barkhausen jumps being equal to the number of wires. For maximum applied fields near the coercive force of a single wire, the loop becomes unstable, the Barkhausen jumps appearing and disappearing in a seeming random way. The shape and instability of the loops have been analyzed from the point of view of a magnetostatic interaction between the wires and a nonsimultaneousness in the change of the direction of the magnetization due to small differences in the coercive force of each wire (arising from the depinning strength of closure domain walls at the ends of the wire). The total magnetization has a nonperidoc temporal variation, and the analysis of the Fourier spectrum suggests a quasiperiodic behavior and a transition to a nearly chaotic behavior (weak chaos) when the maximum magnetic field applied is slightly larger than the coercive force of the single wire. The exact nature of the dynamics involved is still to be determined more precisely. For this, it is necessary to improve the experimental measurements and develop a mathematical model more ambitious. The experimental results suggest that the origin of the instability lies mainly in the magnetically bistable character of the loops rather than to the magnetostatic interactions. Nevertheless the interaction among the different wires acts as an element enhancing the instability. This effect can be used for practical purposes (magnetic field sensor) and for a better understanding of the reversal magnetization process in amorphous ferromagnetic wires.

ACKNOWLEDGMENT

This work has been supported by Spanish CICYT under Project No. MAT 95-0273.

[1] M. Vázquez and A. Hernando, J. Phys. D **29**, 1 (1996).

[2] P. T. Squire, D. Atkinson, M. R. J. Gibbs, and S. Atalay, J. Magn. Magn. Mater. **132**, 10 (1994).

[3] T. Masumoto, I. Ohnaka, A. Inoue, and M. Hajiwara, Scripta Metall. **15**, 293 (1981).

[4] J. Velázquez, M. Vázquez, A. Hernando, H. T. Savage, and M. Wun-Fogle, J. Appl. Phys. **70**, 6525 (1991).

[5] M. Vázquez and D.-X. Chen, IEEE Trans. Magn. **31**, 1229 (1995).

[6] F. B. Humphrey, K. Mohri, J. Yamasaki, H. Kawamura, R. Malmhäll, and I. Ogasawara, in *Magnetic Properties of Amorphous Metals*, edited by A. Hernando, V. Madurga, M. C. Sánchez, and M. Vázquez (Elsevier Science, Amsterdam, 1987), p. 110.

[7] A. M. Severino, C. Gómez-Polo, P. Marín, and M. Vázquez, J. Magn. Magn. Mater. **103**, 117 (1992).

[8] M. Vázquez, C. Gómez-Polo, and D.-X. Chen, IEEE Trans. Magn. **28**, 3147 (1992).

[9] D. H. Smith, IEEE Trans. Magn. **MAG-3**, 350 (1967).

[10] J. Liu, R. Malmhäll, L. Arnberg, and S. J. Savage, J. Appl. Phys. **67**, 4238 (1990).

[11] R. L. Burden and J. D. Faires, *Numerical Analysis* (PWS, Boston, 1985), p. 45.

[12] T. Reininger, H. Kronmüeller, C. Gómez-Polo, and M. Vázquez, J. Appl. Phys. **73**, 5357 (1993).

[13] C. Gómez-Polo, M. Vázquez, and D.-X. Chen, Appl. Phys. Lett. **62**, 108 (1993).

[14] I. Stewart, *Does God play dice? The new mathematics of chaos* (Penguin Books Ltd., Harmondswoth, 1989), Chap. 7, p. 189.

[15] D. Ruelle, *Chance and Chaos* (Princeton University Press, Princeton, 1991), Chap. 10.

[16] N. Packard, J. Crutchfield, J. Farmer, and R. Shaw, Phys. Rev. Lett. **45**, 712 (1980).

[17] P. Bergé, Y. Pomeau, and Ch. Vidal, *L'ordre dans le chaos. Vers una approche deterministe de la turbulence* (Hermann éditeurs des sciences et des arts, Paris, 1992), p. 109; Chap. 6, Sec. 2; Chap. 3, Sec. 2.3; Chap. 4, Sec. 5.

[18] R. Cuerno, A. F. Rañada, and J. J. Ruiz-Lorenzo, Am. J. Phys. **60**, 76 (1992).

[19] H. D. Abarbanel, R. Brown, J. J. Sidorowich, and L. S. Tsimring, Rev. Mod. Phys. **65**, 1359 (1993); **65**, 1331 (1993).

[20] P. Grassberger and I. Procaccia, Phys. Rev. Lett. **50**, 346 (1983).

[21] F. Takens, *Lecture Notes in Mathematics* (Springer, Berlin, 1981), Vol. 898.

[22] M. Takajo, J. Yamasaki, and F. B. Humphrey, IEEE Trans. Magn. **29**, 3484 (1993).