Emission of spin waves by a magnetic multilayer traversed by a current

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The interaction between spin waves and itinerant electrons is considerably enhanced in the vicinity of an interface between normal and ferromagnetic layers in metallic thin films. This leads to a local increase of the Gilbert damping parameter which characterizes spin dynamics. When a dc current crosses this interface, stimulated emission of spin waves is predicted to take place. Beyond a certain critical current density, the spin damping becomes negative; a spontaneous precession of the magnetization is predicted to arise. This is the magnetic analog of the injection laser. An extra dc voltage appears across the interface, given by an expression similar to that for the Josephson voltage across a superconducting junction. [S0163-1829(96)00237-8]

I. INTRODUCTION

In metallic ferromagnets, the spins **s** of itinerant $4s$ conduction electrons are coupled to the spins $\mathbf{S}(\mathbf{r})$ of $3d$ magnetic electrons by the s-d exchange interaction $-2J_{sd}\mathbf{s}\cdot\mathbf{S}(\mathbf{r})$:

$$V_{sd}=g\mu_B\mathbf{s}\cdot\mathbf{H}_{sd}(\mathbf{r}),$$

$$\mathbf{H}_{sd}=-2J_{sd}\langle\mathbf{S}(\mathbf{r})\rangle/g\mu_B, \qquad (1)$$

where g is the gyromagnetic ratio and μ_B is the Bohr magneton. Also, J_{sd} is the s-d exchange integral, and $\mathbf{H}_{sd}(\mathbf{r})$ is the intra-atomic s-d exchange field acting on **s**. The transverse quantum fluctuations of $\mathbf{H}_{sd}$ are neglected in Eq. (1). For simplicity, we treat the $3d$ spins **S** as localized.

Scattering events between spin waves and itinerant electrons, caused by the isotropic exchange V_{sd}, are generally believed to be rare or nonexistent[1] in bulk ferromagnets below the Curie point. In most of the earlier work,[2] which treated V_{sd} by the first Born approximation, a sizable scattering probability was usually predicted, but this is probably illusory. Actual electron-magnon scattering in bulk metals is probably mediated by the smaller anisotropic exchange interaction[1,3] instead.

On the other hand, electrical-resistance measurements versus temperature in magnetic Fe/Cr multilayers[4] indicate the existence of intense electron-magnon scattering. This has been ascribed[5] to V_{sd} and the thermal excitation of localized spin-wave modes at the interface between Fe and Cr layers.

The purpose of the present paper is to show that a large electron-magnon coupling exists at an interface between normal and ferromagnetic layers, even without localized spin-wave modes. In the bulk, electron states have all the time needed to "adapt" themselves to the existing spin wave,[6] at minimal energy cost. This opportunity does not exist for an electron entering a ferromagnet through a sharp interface. In addition, we predict an emission of coherent spin waves when the interface is traversed by a dc current.

II. SINGLE ELECTRON AT AN INTERFACE

Recently,[7] we calculated the electron states in a sandwich composed [Fig. 1(a)] of two ferromagnetic layers F_1, F_2, separated by a normal layer N, in the case where the magnetic spins $\mathbf{S}_1$, $\mathbf{S}_2$ in F_1, F_2 are at an oblique angle θ. In N, we use a frame (x,y,z) where x is normal to the N-F_2 interface, and z parallel to $\mathbf{S}_1$ [Fig. 1(a)]. The origin of x is at the $N-F_2$ interface. $\mathbf{S}_1$ and $\mathbf{S}_2$ are assumed uniform over F_1 and F_2. Also, $\mathbf{S}_1$ is assumed parallel to the interface, although this is not essential. We consider a conduction electron injected from F_1 into N, with expectation $\langle\mathbf{s}\rangle$ parallel to z, i.e., a "spin-up" electron in N:

$$\psi=\left[e^{ik_x^Nx}\begin{vmatrix}A\\0\end{vmatrix}+e^{-ik_x^Nx}\begin{vmatrix}B\\C\end{vmatrix}\right]e^{i(k_y^Ny+k_z^Nz)}. \qquad (2)$$

Here, B, C are the spin-up and spin-down amplitudes in N caused by reflection at the N-F_2 interface, and $\mathbf{k}_N$ is the wave vector in N.

In F_2, we use the same frame (x,y,z) to describe the spatial motion of the electron. In Ref. 7, we assumed $\mathbf{S}_2$ to be parallel to the plane (y,z) of the interface; we now consider the more general case of arbitrary $\mathbf{S}_2$ direction, given [Fig. 1(a)] by the polar angles (θ,ϕ) in the (x,y,z) frame. The electron wave transmitted into F_2 can be written in the form
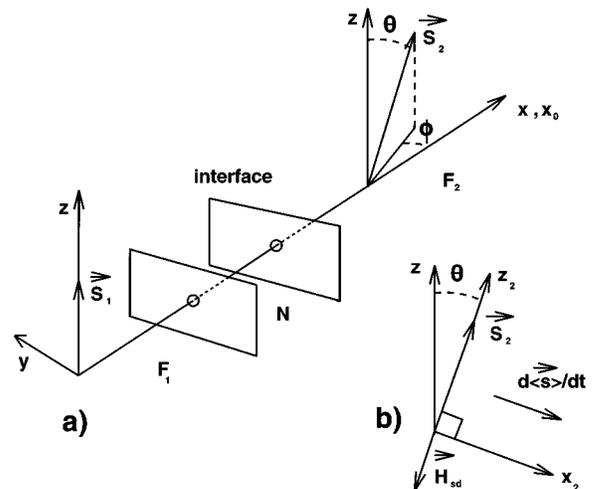
FIG. 1. (a) Coordinate system x,y,z, and polar angles θ,φ giving the orientation of localized spin $\mathbf{S}_2$ in layer F_2. (b) Coordinate system x_2,y_2,z_2 in layer F_2, with the z_2 axis parallel to $\mathbf{S}_2$, and the x_2 axis in the $(z,\mathbf{S}_2)$ plane.

$$\psi = D e^{i\mathbf{k}_\uparrow \cdot \mathbf{r}} \begin{vmatrix} e^{-i\varphi/2}\cos(\theta/2) \\ e^{i\varphi/2}\sin(\theta/2) \end{vmatrix} + E e^{i\mathbf{k}_\downarrow \cdot \mathbf{r}} \begin{vmatrix} -e^{-i\varphi/2}\sin(\theta/2) \\ e^{i\varphi/2}\cos(\theta/2) \end{vmatrix}. \tag{3}$$

Here, the two spin states correspond to $\langle \mathbf{s} \rangle$ parallel and anti-parallel to $\mathbf{S}_2$, respectively. Hence, $\mathbf{k}_\uparrow$ and $\mathbf{k}_\downarrow$ are the spin-up and spin-down wave vectors. And D and E are the spin-up and spin-down electron amplitudes, in a frame (x_2,y_2,z_2) with z_2 parallel to $\mathbf{S}_2$ and x_2 in the $(\mathbf{S}_2,z)$ plane [Fig. 1(b)]. The (x,y,z) frame was called (x_N,y_N,z_N) in Ref. 7, and the x_2,y_2,z_2 axes correspond to $-y_2,x_2,z_2$ in the special case $\varphi = -\pi/2$ of Ref. 7. The boundary conditions of continuity of ψ and $d\psi/dx$ at $x=0$ give

$$D = 2A e^{i\varphi/2}\cos(\theta/2)/(1+k_x^\uparrow/k_x^N),$$
$$E = -2A e^{i\varphi/2}\sin(\theta/2)/(1+k_x^\downarrow/k_x^N). \tag{4}$$

These values are consistent with the ones in Ref. 7 for the case $\varphi = -\pi/2$. From Eqs. (3) and (4), we can calculate[7] the local expectation of the spin components of one conduction electron along the x_2 and y_2 axes, at a space location at a distance $x_0 > 0$ from the $N-F_2$ interface

$$\langle s_{x2} \cdot \delta(\mathbf{r}-\mathbf{r}_0)\rangle = \mathrm{Re}[e^{i(k_x^\uparrow - k_x^\downarrow)x_0}E^*D]$$
$$= -2|A|^2 \frac{f(x_0)\sin\theta}{(1+k_x^\uparrow/k_x^N)(1+k_x^\downarrow/k_x^N)}$$
$$\times \cos[(k_x^\uparrow - k_x^\downarrow)x_0].$$

$$\langle s_{y2} \cdot \delta(\mathbf{r}-\mathbf{r}_0)\rangle = \mathrm{Re}[ie^{i(k_x^\uparrow - k_x^\downarrow)x_0}E^*D]$$
$$= 2|A|^2 \frac{f(x_0)\sin\theta}{(1+k_x^\uparrow/k_x^N)(1+k_x^\downarrow/k_x^N)}$$
$$\times \sin[(k_x^\uparrow - k_x^\downarrow)x_0]. \tag{5}$$

This is consistent with Eq. (4) of Ref. 7, taking into account the exchange of the x_2 and y_2 axes. At $x_0 = 0$, $\langle \mathbf{s}\cdot\delta(\mathbf{r}-\mathbf{r}_0)\rangle$ is parallel to the (z_2,x_2) plane.

Equations (5) predict[7] that the local $\langle \mathbf{s}\rangle$ components along x_2 and y_2 have spatial oscillations of wavelength $2\pi/|k_x^\uparrow - k_x^\downarrow|$ as a function of the distance x_0 from the $N-F_2$ interface. The reason for these oscillations[7] is that the electron spin precesses around the s-d exchange field $\mathbf{H}_{sd}$ [Eq. (1)] as it moves in F_2 away from the $N-F_2$ interface.

The effect on ψ of electron scattering by solute atoms and phonons in F_2 may be simulated approximately by multiplying the first and second term of Eq. (3) by damping factors $\exp(-k_\uparrow x_0/\Lambda_\uparrow k_x^\uparrow)$ and $\exp(-k_\downarrow x_0/\Lambda_\downarrow k_x^\downarrow)$, respectively. Here, $\Lambda_\uparrow$ and $\Lambda_\downarrow$ are the spin-up and spin-down mean free paths in F_2. In turn, this leads to the existence of the correction factor $f(x_0)$, introduced *a posteriori*[7] into Eq. (5):

$$f(x_0) = \exp\left[-\left(\frac{k_\uparrow}{\Lambda_\uparrow k_x^\uparrow} + \frac{k_\downarrow}{\Lambda_\downarrow k_x^\downarrow} \right)x_0 \right]. \tag{6}$$

The effect of this factor is to attenuate the density $\langle s_{x2}\cdot\delta(\mathbf{r}-\mathbf{r}_0)\rangle$ strongly at distances x_0 from the interface larger than $\Lambda_\uparrow$ or $\Lambda_\downarrow$.

Equation (3) is the "coherent" part of ψ, and Eqs. (5) and (6) are the corresponding spin density. There is also an incoherent part of ψ, where the electron has a diffusive, random-walk motion inside F_2. The electron enters the incoherent part at the first scattering event in F_2. Because of the random direction of motion, the phases of the spin-up and spin-down amplitudes $\psi_\uparrow$ and $\psi_\downarrow$ of the incoherent part are largely uncorrelated in space. As a result, transverse components such as $\langle s_{x2}\cdot\delta(\mathbf{r}-\mathbf{r}_0)\rangle = (1/2)\mathrm{Re}(\psi_\uparrow^*\psi_\downarrow)$ do not have regular spatial oscillations in the incoherent part, only random short-range fluctuations around an average of zero. On the other hand, the longitudinal component $\langle s_{z2}\cdot\delta(\mathbf{r}-\mathbf{r}_0)\rangle$ (in the x_2,y_2,z_2 frame) is $(1/2)(|\psi_\uparrow|^2 - |\psi_\downarrow|^2)$ and independent of phases. Therefore, it is usually not zero.

From the exchange torque exerted by $\mathbf{H}_{sd}$, we can find the rate of change of component $\langle s_{x2}\rangle$ of $\langle \mathbf{s}\rangle$, using Eqs. (5) and (6):

$$\hbar \frac{d\langle s_{x2}\rangle}{dt} = -g\mu_B \langle s_{y2}\rangle \cdot H_{sd}^{z2} = -g\mu_B H_{sd}^{z2} \int\int\int_{x=0}^{x=\infty} dV \langle s_{y2}\cdot\delta(\mathbf{r}-\mathbf{r}_0)\rangle$$
$$= -g\mu_B H_{sd}^{z2} L_y L_z 2|A|^2 \frac{\sin\theta}{(1+k_x^\uparrow/k_x^N)(1+k_x^\downarrow/k_x^N)} \frac{1}{k_x^\uparrow - k_x^\downarrow} \tag{7}$$

where L_y and L_z are the sample dimensions along y and z, and we assume $\Lambda_\uparrow,\Lambda_\downarrow \gg 1/|k_x^\uparrow - k_x^\downarrow|$. The effect of $1/\Lambda_\uparrow, 1/\Lambda_\downarrow$ is to make the integral converge at $x_0 = \infty$. Equation (7) shows that only a region of F_2 of thickness $\simeq 1/|k_x^\uparrow - k_x^\downarrow|$ near the interface contributes appreciably to the total torque on the electron spin. By the same method, one can show that $d\langle s_{y2}\rangle/dt \simeq d\langle s_{z2}\rangle/dt \simeq 0$ in the same frame (x_2,y_2,z_2). Thus, $d\langle \mathbf{s}\rangle/dt$ is a vector parallel to the x_2 axis [Fig. 1(b)], so that Eq. (7) also gives its magnitude $|d\langle \mathbf{s}\rangle/dt|$. Finally, we can use the relation $(\hbar^2/2m)((k_x^\uparrow)^2 - (k_x^\downarrow)^2) = -2\mu_B H_{sd}^{z2}$ to

eliminate H_{sd}^{z2}. With $g=2$, Eq. (7) becomes

$$\left| \frac{d\langle \mathbf{s}\rangle}{dt} \right| = L_y L_z |A|^2 \frac{|v_x^\uparrow + v_x^\downarrow|}{(1+k_x^\uparrow/k_x^N)(1+k_x^\downarrow/k_x^N)}|\sin\theta|. \tag{8}$$

Here, $\mathbf{v}_\uparrow$ and $\mathbf{v}_\downarrow$ are the spin-up and spin-down Fermi velocities in F_2.

We use a fictitious normalization volume V_N,[7] located mostly in N but including the $N-F_2$ interface. Normalization gives $|A|^2 = 1/V_N$.

Slonczewski has already predicted[8] a rate of change of $\langle \mathbf{s} \rangle$ similar to Eq. (8) near an interface, in a somewhat different manner. As in our case his $d\langle \mathbf{s} \rangle/dt$ has the effect of bringing $\langle \mathbf{s} \rangle$ closer to $\mathbf{S}_2$ in direction.

III. SPIN-FLIP TIME NEAR THE INTERFACE

Instead of the frame (x_2, y_2, z_2), we now use again the original frame (x, y, z), more appropriate in connection with spin waves. The vector $d\langle \mathbf{s} \rangle/dt$ has a projection [Fig. 1(b)] on that fixed z axis, given by

$$\frac{d\langle s_z \rangle}{dt} = -\left| \frac{d\langle \mathbf{s} \rangle}{dt} \right| \sin\theta. \tag{9}$$

The measured value of a spin component such as s_z can only be $\pm 1/2$. Therefore, for the average $\langle s_z \rangle$ to change in time, the electron must sometimes flip its spin along z. The total spin-flip rate, from up to down, is

$$\frac{dn_{\uparrow\downarrow}}{dt} = -\frac{d\langle s_z \rangle}{dt} \Delta n_\uparrow. \tag{10}$$

Here, $\Delta n_\uparrow$ is the number of such spin-up electrons assumed present on a particular element dS of the Fermi surface in N. We define an electron spin-flip time $\tau_{\uparrow\downarrow}$ at that point of the Fermi surface by

$$\frac{dn_{\uparrow\downarrow}}{dt} = \frac{\Delta n_\uparrow}{\tau_{\uparrow\downarrow}}. \tag{11}$$

By combining Eqs. (8)–(11), we obtain finally

$$\frac{1}{\tau_{\uparrow\downarrow}} = L_y L_z |A|^2 \frac{v_x^\uparrow + v_x^\downarrow}{(1 + k_x^\uparrow/k_x^N)(1 + k_x^\downarrow/k_x^N)} \sin^2\theta. \tag{12}$$

We assign $1/\tau_{\uparrow\downarrow} = 0$ to states where $k_x^\uparrow$ or $k_x^\downarrow$ is imaginary.

IV. SPIN-WAVE RELAXATION TIME

So far, we assumed that only spin-up electrons enter F_2 through the interface. This was sufficient for our definition and determination of the spin-flip time $\tau_{\uparrow\downarrow}$ [Eq. 12]. Now, we consider a more realistic situation where electrons of both spins enter F_2. In that general case, we must pay attention to the energies $\epsilon_\uparrow$ and $\epsilon_\downarrow$ of the two states involved in the quantum transition. Energy conservation implies (Fig. 2)

$$\varepsilon_\downarrow - \varepsilon_\uparrow = \hbar\omega. \tag{13}$$

Here, $\hbar\omega$ is the energy quantum (magnon) of a spin wave of angular frequency $\omega > 0$. To have a spin wave in F_2 means that the localized spins $\mathbf{S}_2$ are precessing clockwise around the fixed axis z [Fig. 1(a)], at a rate $\omega = -d\phi/t$. For simplicity, we assume the spin-wave wavelength very large, corresponding to the uniform precession present in ferromagnetic resonance. This will be discussed further in Sec. VIII. Since we treated $\mathbf{S}_2$ as a classical object until now, magnons did not enter our formalism explicitly. Because of conservation of the total angular momentum along z, the electron must flip from up to down as a magnon is annihilated, and vice versa.
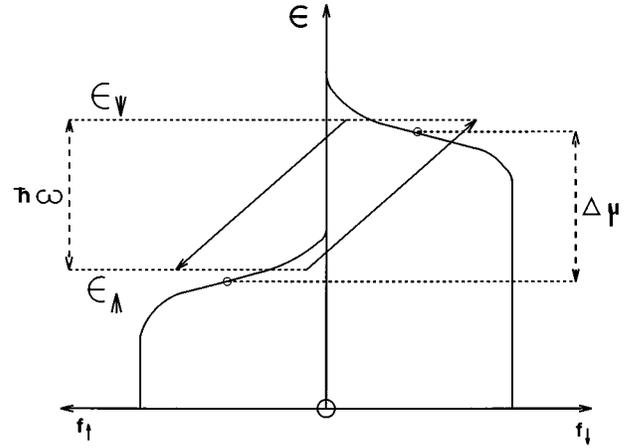
FIG. 2. Occupation numbers $f_\uparrow \leq 1$, $f_\downarrow \leq 1$ of spin-up and spin-down states, as a function of electron energy ε. The spin-up Fermi level is shifted by an energy $\Delta\mu$ with respect to the spin-down Fermi level, for $k_x^N > 0$. Two oblique solid lines show electron spin-flip transitions between states of energy $\varepsilon_\uparrow, \varepsilon_\downarrow$ with $\varepsilon_\downarrow - \varepsilon_\uparrow = \hbar\omega$. Here, $\hbar\omega$ is the magnon energy, and ω the spin-wave frequency.

Therefore, we always have $\varepsilon_\downarrow > \varepsilon_\uparrow$ (Fig. 2), in agreement with Eq. (13). In addition, if n_m is the total number of magnons in F_2,

$$\frac{dn_m}{dt} = -\frac{dn_{\uparrow\downarrow}}{dt}. \tag{14}$$

We generalize Eq. (11), in the form

$$\frac{dn_{\uparrow\downarrow}}{dt} = \int_{-\infty}^{+\infty} d\varepsilon_\uparrow \frac{D_\uparrow}{2\overline{\tau}_{\uparrow\downarrow}} f_\uparrow(\varepsilon_\uparrow)[1 - f_\downarrow(\varepsilon_\uparrow + \hbar\omega)]$$
$$- \int_{-\infty}^{+\infty} d\varepsilon_\downarrow \frac{D_\downarrow}{2\overline{\tau}_{\uparrow\downarrow}} f_\downarrow(\varepsilon_\downarrow)[1 - f_\uparrow(\varepsilon_\downarrow - \hbar\omega)], \tag{15}$$

where $1/\overline{\tau}_{\uparrow\downarrow}$ is some average of $1/\tau_{\uparrow\downarrow}$ over the active half of the Fermi surface, with $k_x^N > 0$, in N. Also, $D_\uparrow = D_\downarrow = D_N/2$ are the N densities of states for spin up and down, and $f_\uparrow, f_\downarrow$ the average occupation numbers of spin-up and spin-down states. The $(1 - f_\uparrow), (1 - f_\downarrow)$ factors take into account the exclusion principle for the final states. We put a factor of 2 in the denominator because only the half of the Fermi surface with $k_x^N > 0$, and the corresponding halves of $D_\uparrow$ and $D_\downarrow$, contribute to $dn_{\uparrow\downarrow}/dt$. Only electrons on that half have crossed the interface.

We assume the spin-up and spin-down Fermi levels possibly to be shifted (Fig. 2) by amounts $\Delta\mu_\uparrow, \Delta\mu_\downarrow$ from their equilibrium value μ_0. Thus, if f_0 is the Fermi function at temperature T,

$$f_\uparrow(\varepsilon_\uparrow) = f_0(\varepsilon_\uparrow - \mu_0 - \Delta\mu_\uparrow),$$
$$f_\downarrow(\varepsilon_\downarrow) = f_0(\varepsilon_\downarrow - \mu_0 - \Delta\mu_\downarrow). \tag{16}$$

Then Eq. (15) becomes, after defining $\Delta\mu = \Delta\mu_\uparrow - \Delta\mu_\downarrow$,

$$\frac{dn_{\uparrow\downarrow}}{dt} = \frac{D_N}{4\overline{\tau}_{\uparrow\downarrow}}(\Delta\mu + \hbar\omega). \tag{17}$$

This result holds even at finite temperature.

Each magnon has an angular momentum of $-\hbar$ along z. Therefore, if $|\theta| \ll 1$ rad:

$$n_m = S_2(1 - \cos\theta)n_2 \simeq (S_2 n_2 \sin^2\theta)/2, \qquad (18)$$

where S_2 is the magnitude of $\mathbf{S}_2$ and n_2 the number of atoms in F_2. We combine Eqs. (12), (14), (17), and (18), and define the spin-wave relaxation time, τ_m:

$$\frac{1}{\tau_m} = -\frac{1}{n_m}\frac{dn_m}{dt}$$
$$= \left(\frac{D_N}{V_N}\right)\left(\frac{V_2}{n_2}\right)\frac{\Delta\mu + \hbar\omega}{2L_2^x S_2}$$
$$\times \left[\overline{\frac{v_x^\uparrow + v_x^\downarrow}{(1 + k_x^\uparrow/k_x^N)(1 + k_x^\downarrow/k_x^N)}}\right], \qquad (19)$$

where $V_2 = L_2^x L_y L_z$ is the volume of F_2, and L_2^x is the thickness of F_2 along x. The reason why L_2^x appears in the denominator is that the enhanced electron-magnon scattering is a surface effect. Equation (19) is valid as long as $L_2^x \gg \Lambda_\uparrow$, $\Lambda_\downarrow$. The horizontal bar indicates an averaging over the $k_x^N > 0$ half of the Fermi surface. Note that τ_m is related to the ferromagnetic resonance linewidth ΔH by $\gamma \Delta H \simeq 1/\tau_m$. Note also that Eq. (19) does not contain $\sin\theta$ anymore.

When $\Delta\mu + \hbar\omega$ is positive, dn_m/dt is negative and proportional to n_m, corresponding to dominant spin-wave absorption. If $\Delta\mu \neq 0$, the constant term $\Delta\mu$ gives deviations from the usual relation $1/\tau_m \propto \omega$ associated with Gilbert damping. On the other hand, at zero current, the electrons are in equilibrium and we have $\Delta\mu = 0$. Then Eq. (19) predicts $1/\tau_m \propto \omega$, consistent with Gilbert damping. The dimensionless Gilbert parameter α is

$$\alpha = \frac{1}{2\omega\tau_m} = \left(\frac{D_N}{V_N}\right)\left(\frac{V_2}{n_2}\right)\frac{\hbar}{4L_2^x S_2}\left[\overline{\frac{v_x^\uparrow + v_x^\downarrow}{(1 + k_x^\uparrow/k_x^N)(1 + k_x^\downarrow/k_x^N)}}\right]. \qquad (20)$$

We use $D_N/V_N = 11.4 \times 10^{46}$ J^{-1} m^{-3} for a free-electron metal similar to copper, and $n_2/V_2 = 9.14 \times 10^{28}$ m^{-3} as for nickel, $S_2 \simeq 0.5$ as for Ni$_{80}$Fe$_{20}$. Also, we assume $v_x^\uparrow \simeq v_x^\downarrow = 1 \times 10^6$ m/s and $L_2^x = 3$ nm, and $k_x^\uparrow, k_x^\downarrow \simeq k_x^N$. Then, Eq. (20) gives $\alpha \simeq 0.011$. This is at least comparable to the experimental value $\alpha \simeq 0.004$ for bulk Ni$_{80}$Fe$_{20}$, which arises from anisotropic s-d exchange.[1] This indicates a significant enhancement of Gilbert damping near the $N-F_2$ interface.

V. STIMULATED EMISSION OF SPIN WAVES

When an electric current with spin-up and spin-down densities $j_x^\uparrow, j_x^\downarrow$ is flowing across the $N-F_2$ interface, the Fermi surfaces for spin up and spin down in N are shifted in k space by amounts $\Delta k_x^\uparrow$ and $\Delta k_x^\downarrow$ along x,

$$\Delta k_x^\uparrow = \frac{-2j_x^\uparrow m}{e n_e^N \hbar}; \quad \Delta k_x^\downarrow = \frac{-2j_x^\downarrow m}{e n_e^N \hbar}; \qquad (21)$$

where n_e^N is the total number of electrons per unit volume in N, and e, m are the electron charge and mass, respectively.

Electronlike carriers are assumed. These shifts produce shifts $\Delta\mu_\uparrow$, $\Delta\mu_\downarrow$ of the local Fermi level at a given point of the Fermi surface:

$$\Delta\mu_\uparrow = \hbar\Delta k_x^\uparrow v_x^N; \quad \Delta\mu_\downarrow = \hbar\Delta k_x^\downarrow v_x^N, \qquad (22)$$

where $\mathbf{v}_N$ is the Fermi velocity in N. For simplicity, we assume F_1 and F_2 to be made of the same material, with $|\theta| \ll 1$ rad. Also, we assume N to be much thinner than a spin-diffusion length. Then,[9] $j_x^\uparrow$ and $j_x^\downarrow$ are the same in N as in F_1 and in F_2, where their ratio[10] was $\alpha_1 = j_x^\uparrow/j_x^\downarrow = \sigma_1^\uparrow/\sigma_1^\downarrow$. Here, $\sigma_1^\uparrow$, $\sigma_1^\downarrow$ are the spin-up and spin-down conductivities in F_1 far from any interface. Then, Eqs. (21) and (22) give, with $j_x = j_x^\uparrow + j_x^\downarrow$ as the total current density and $\mathbf{k}_N$ as the Fermi wave vector in N

$$\Delta\mu = \Delta\mu_\uparrow - \Delta\mu_\downarrow = -2\left(\frac{\alpha_1 - 1}{\alpha_1 + 1}\right)j_x \frac{\hbar k_x^N}{e n_e^N}. \qquad (23)$$

With $n_e^N = 8.5 \times 10^{28}$ m^{-3} and $k_x^N \le k_N = 1.36 \times 10^{10}$ m^{-1} for copper, $\alpha_1 \gg 1$, and $j_x = +1 \times 10^{11}$ A/m^2 achievable[11] in dc or with current pulses, Eq. (23) gives $\Delta\mu = -1.31 \times 10^{-4}$ eV. This is to be compared to $\hbar\omega \simeq 0.41 \times 10^{-4}$ eV for $\omega/2\pi = 10$ GHz. The $|\Delta\mu|$ value above is a maximum, and the $|\Delta\mu|$ average over a half Fermi surface would be somewhat smaller. We see, however, that $\Delta\mu + \hbar\omega$ may become negative in Eq. (19), leading to negative $1/\tau_m$. Then, dn_m/dt is positive and proportional to n_m, reflecting stimulated emission of spin waves. There is no spontaneous emission, since $\mathbf{S}_2$ has no quantum fluctuations in our formalism.

Note that the critical current density where $\Delta\mu + \hbar\omega = 0$, and spin-wave emission starts, is proportional to ω, by Eqs. (19) and (23). Thus, low-frequency spin waves are easiest to excite.

There is some degree of analogy between this spin-wave emitting diode and an injection laser. We suggest the name SWASER (spin-wave amplification by stimulated emission of radiation) for this device. It is through a Fermi-level difference $\Delta\mu = \Delta\mu_\uparrow - \Delta\mu_\downarrow < 0$ (Fig. 2) that the electrons are "pumped up." This sign of $\Delta\mu$ at $k_x^N > 0$ requires the correct sign $j_x > 0$ [see Eq. (23)], if $\alpha_1 > 1$ as in Ni$_{80}$Fe$_{20}$.[10]

The current also causes shifts $\Delta\mu_\uparrow$, $\Delta\mu_\downarrow$ of the opposite sign on the other half $k_x^N < 0$ of the Fermi surface, but these are inactive in F_2, as these electrons do not flip their spin in F_2. These shifts may be active for F_1, after the electrons cross N. In Eqs. (19) and (20), the positive additional term caused by anisotropic s-d exchange in the bulk[1] has been neglected.

Slonczewski[8] has predicted a current-induced precession somewhat similar to ours. However, he treats a tunneling junction, and his predicted exchange torques are definite functions of the voltage across the junction and of the band structures of the ferromagnets; see his Eq. (5.4). On the other hand, our theory involves ordinary conduction processes in metals, and the predicted net exchange torques depend on j_x, and on the conductivity ratio α_1, i.e., on the spin-up and spin-down mean free paths in F_2 or F_1; see our Eqs. (19) and (23). And the quantity $\Delta\mu$ in these equations has no simple relation to the total voltage across the interface. Finally, there does not seem to be any equivalent in Slonczewski's work[8] of our prediction of enhanced Gilbert damping near the interface even at $j_x = 0$ [Eq. (20)].
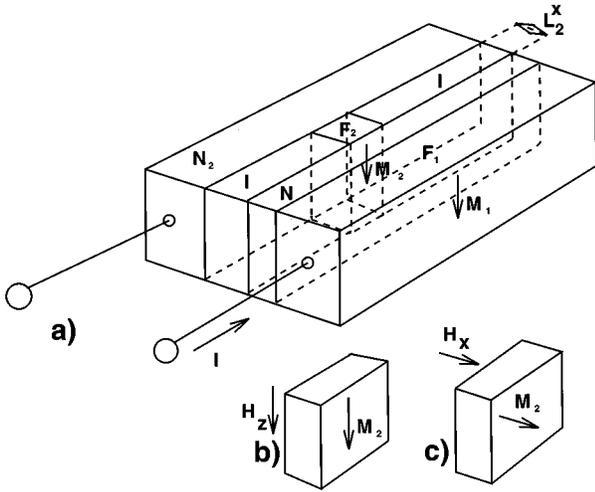
FIG. 3. (a) Possible configuration for a SWASER experiment, where layer F_2 is a rod with square cross section. The magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$ of F_1 and F_2 must be parallel if the conductivity ratios α_1, α_2 of F_1, F_2 are both larger than one. (b) Case with F_2 in the shape of a plate with in-plane field H_z. (c) Case with plate F_2 and field H_x normal to layer plane.

VI. EXPERIMENTAL CONFIGURATIONS

We show in Fig. 3(a) a possible configuration for a SWASER. Layer F_2 is patterned in the shape of a rod with square cross section, with its length parallel to the $N-F_2$ interface. The insulating layer I forces the current to flow through F_2. The second normal-metal layer N_2 returns this current to one of the two current leads. The other lead is connected to F_1. In the absence of spin waves, the magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$ of F_1 and F_2 must be parallel if the conductivity ratios α_1 and α_2 are both larger than one. $Ni_{80}Fe_{20}$ is a good material for F_1 and F_2, as it has a large conductivity ratio,[10] a narrow ferromagnetic-resonance linewidth, and a small Gilbert parameter. Because of Eq. (19), the F_2 thickness L_2^x must be minimized. On the other hand, the F_1 thickness should preferably be at least one spin-diffusion length $\simeq 1$ μm, to insure the value of α_1.[9] For the same reason, the metal in N_2 should have a very short spin-diffusion length, obtained with Mn or Pt solutes. And the condition $L_y, L_z \gg \Lambda_\uparrow, \Lambda_\downarrow$ must hold.

This rod shape for F_2 has the advantage of giving circularly polarized spin waves. But it is difficult to manufacture, as the F_2 thickness (and therefore the width) L_2^x must be kept small in Eq. (19), i.e., $L_2^x \simeq 1-10$ nm. In addition, its ferromagnetic-resonance frequency[12] near zero field is $\omega \simeq \gamma M_S/2$, where $\gamma \simeq 1.76 \times 10^{11}$ rad/s T is the gyromagnetic ratio and M_S is the saturation magnetization. In $Ni_{80}Fe_{20}$, this yields a rather too high value of $\hbar\omega$, equal to 0.6×10^{-4} eV. And the condition $L_y \gg \Lambda_\uparrow, \Lambda_\downarrow$ would not be satisfied.

For these reasons, we show in Fig. 3(b) an alternate flat shape for F_2. This gives $\omega \simeq \gamma[\mu_0 H_z(\mu_0 H_z + M_S)]^{1/2}$, where H_z is[12] an external in-plane field, and μ_0 is the permeability of the vacuum. Thus, smaller and tunable ω values are achievable, but the spin-wave polarization is elliptical.

Finally, a large external field $\mu_0 H_x > M_S$ could be applied[12] normal to the same F_2 plate [Fig. 3(c)]. Then $\omega \simeq \gamma(\mu_0 H_x - M_S)$, also tunable down to low values, but with circular polarization. Now, $\mathbf{M}_1$ and $\mathbf{M}_2$ are parallel to

j_x, but this is immaterial as long as they are kept parallel to each other by the field.

One technical problem is the magnetostatic coupling between F_1 and F_2. It leads to energy losses in F_2 unless F_1 and F_2 are both precessing, in phase with each other.

Akhiezer, Baryakhtar, and Peletminskii[13] and Coutinho Filho, Miranda, and Rezende[14] have suggested that an electron flow would cause amplification of spin waves, even in bulk samples. We believe (see Sec. I) that bulk samples are not good for that purpose.

VII. VOLTAGE ACROSS THE INTERFACE

The energy needed to create magnons must come from the dc current flowing through the sample. Hence, in addition to the usual ohmic voltage, a voltage δV must exist across the interface, given by energy conservation

$$j_x \delta V L_y L_z = \hbar \omega \frac{dn_m}{dt}. \tag{24}$$

We combine Eqs. (18), (19), (23), and (24), and obtain

$$\delta V \simeq \sin^2\theta \left(\frac{\alpha_1 - 1}{\alpha_1 + 1} \right) \left\{ \frac{3}{2v_N} \left[\overline{\frac{v_x^\uparrow + v_x^\downarrow}{(1 + k_x^\uparrow/k_x^N)(1 + k_x^\downarrow/k_x^N)}} \right] \right\} \frac{\hbar}{e} \omega. \tag{25}$$

We assume $\omega/2\pi \simeq 10$ GHz, $\alpha_1 \gg 1$, $\sin\theta \approx 0.5$, and note that the round bracket is probably of order unity. Then $\delta V \simeq 10$ μV. The form $\delta V \propto \hbar\omega/e$ of Eq. (25) resembles the Josephson voltage across a superconducting junction. It also resembles the predicted ''ferro-Josephson'' voltage across a precessing magnetic domain wall.[15] Note that the voltage persists in the absence of the current, if the spin wave is excited with an external microwave.

VIII. SPIN-WAVE COHERENCE

So far, we have only considered spin waves of near-zero wave number q. Actually, the electrons interact equally with spin waves of a wide range of q values, leading to possibly very incoherent spin-wave emission in our SWASER. In the following, we suggest how the lowest spin-wave mode could be selected, and coherence achieved.

Consider the configuration of Fig. 3(b) with a flat F_2 made of $Ni_{81}Fe_{19}$. In very thin films ($L_2^x \lesssim 10$ nm), the lowest-energy spin waves have $\mathbf{q}$ in the in-plane y or z directions. Assuming spin pinning[12] only at the boundary planes normal to y and z, the two lowest modes correspond to $n=1$ and $n=2$, where n is the number of half-wavelengths within L_y or L_z. Assuming $L_y = L_z = 0.5$ μm, difficult but not impossible to achieve, and an in-plane field $\mu_0 H = 0.03$ T, we find a magnon-energy difference $\hbar(\omega_2 - \omega_1)$ which is 6% of $\hbar\omega_1$ itself. Then, using a current such that $\Delta\mu + \hbar\omega_1 = 0$, $\Delta\mu + \hbar\omega_2$ would still be appreciably positive in Eq. (19); only the $n=1$ mode would be emitted, leading to very coherent spin waves. Spin pinning at the boundaries normal to y and z could be realized[12] through a slight diffusion of oxygen from I into F_2 where they touch.

An interesting paper by J. C. Slonczewski[16] covers somewhat similar ideas. Most of the remarks at the end of Sec. V

apply to this paper, too. For example, his drive torques depend on a ratio P of tunneling densities of states [his Eq. (11)], while ours depend on a ratio α_1 of conductivities, i.e., of mean free paths in F_1 [our Eq. (23)].

[1] V. Korenman and R. E. Prange, Phys. Rev. B **6**, 2769 (1972); R. E. Prange and V. Korenman, J. Magn. Reson. **6**, 274 (1972).

[2] D. L. Mills, A. Fert, and I. A. Campbell, Phys. Rev. B **4**, 196 (1971).

[3] L. Berger, J. Phys. Chem. Solids **38**, 1321 (1977); V. Kambersky, Can. J. Phys. **48**, 2906 (1970).

[4] J. E. Mattson, Mary E. Brubaker, C. H. Sowers, M. Conover, Z. Qiu, and S. D. Bader, Phys. Rev. B **44**, 9378 (1991).

[5] S. Zhang and P. M. Levy, Phys. Rev. B **43**, 11 048 (1991).

[6] L. L. Hirst, Phys. Rev. **141**, 503 (1966).

[7] L. Berger, IEEE Trans. Magn. **MAG-31**, 3871 (1995).

[8] J. C. Slonczewski, Phys. Rev. B **39**, 6995 (1989).

[9] T. Valet and A. Fert, Phys. Rev. B **48**, 7099 (1993).

[10] A. Fert and I. A. Campbell, J. Phys. F **6**, 849 (1976).

[11] C. Y. Hung and L. Berger, J. Appl. Phys. **63**, 4276 (1988). Magnetic random-access memories and magnetoresistive reading heads operate with dc current densities between 10^{10} and 10^{11} A/m^2; see A. V. Pohm and C. S. Comstock, *ibid.* **69**, 5760 (1991), for example.

[12] A. H. Morrish, *The Physical Principles of Magnetism* (Wiley, New York, 1965), pp. 545, 546, 588–592.

[13] A. I. Akhiezer, V. G. Baryakhtar, and S. V. Peletminskii, Phys. Lett. **4**, 129 (1963).

[14] M. D. Coutinho Filho, L. C. M. Miranda, and S. M. Rezende, Phys. Status Solidi B **57**, 85 (1973).

[15] L. Berger, Phys. Rev. B **33**, 1572 (1986).

[16] J. C. Slonczewski, J. Magn. Magn. Mater. (to be published).