Interface roughening in driven magnetic systems with quenched disorder

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The kinetic roughening of a driven interface between spin-up and spin-down domains in a model with a nonconserved scalar order parameter and quenched disorder is studied numerically within a discrete time dynamics at zero temperature. Starting from a flat interface in a two-dimensional system the time evolution of the height correlation function is analyzed for different driving fields. It is found that the normal dynamic scaling ansatz is not sufficient to describe the observed behavior. The data are analyzed within a novel scaling ansatz. Arguments are given that the present model belongs to the Edwards-Wilkinson universality class with exponents, depending on the driving field, which is due to correlated effective noise. [S0163-1829(96)04237-3]

I. INTRODUCTION

The morphology and time evolution of interfaces in random media and of growing surfaces are problems of great current interest. For growing surfaces many problems can be mapped on equations like the Edwards-Wilkinson (EW) equation,[1] the Kardar-Parisi-Zhang (KPZ) equation,[2] or similar equations.[3–5] Recently, interest in the morphology and dynamics of interfaces in random media has increased considerably. Much work is devoted to problems like the immiscible displacement of viscous fluids in porous media[6,7] or pinning and roughening phenomena in magnetic systems[8,9] (see also Refs. 4 and 5). This second group of problems has, different from the first group, in general quenched (time-independent) disorder which results in a different roughness exponent α (defined below) in the range between 0.6 and 1.25 in $1+1$ dimensions. Thus it differs clearly from the KPZ value of 1/2. Different starting points for a solution like power law distributed noise amplitudes[10] are proposed but the situation is up to now still unclear.

The aim of this paper is to investigate the morphology and dynamics of the roughening process of a domain wall driven through a medium with quenched disorder. A main result of our work is that the corresponding exponents in the depinning phase depend on the strength of the driving field. An interpretation of this behavior in terms of an effective time-correlated noise is given.

II. MODEL

We study a model with nonconserved scalar order parameter (model A in the classification of Halperin and Hohenberg[11]) with Langevin dynamics at zero temperature. Thermal noise is neglected since it is believed to be irrelevant.[13,14] The Ginzburg-Landau-type Hamiltonian for this system with quenched disorder is given by

$$\mathcal{H}=\int d\mathbf{r}\left(-\frac{a}{2}\phi(\mathbf{r},t)^2+\frac{b}{4}\phi(\mathbf{r},t)^4+\frac{\tilde{J}}{2}[\nabla\phi(\mathbf{r},t)]^2 -[H+B(\mathbf{r})]\phi(\mathbf{r},t)\right), \tag{1}$$

where ϕ denotes a scalar order parameter, H a homogeneous driving magnetic field, and $B(\mathbf{r})$ a quenched random field and it is assumed that it has zero mean and that it is uncorrelated.

The dynamics of the model at zero temperature is defined by the relaxation equation

$$\gamma\frac{\partial\phi(\mathbf{r},t)}{\partial t}=-\frac{\partial\mathcal{H}}{\partial\phi(\mathbf{r},t)}, \tag{2}$$

with a relaxation time proportional to γ. This type of equation has been used in the past by various authors as a staring point for a derivation of local equation of motions for the position of an interface in the medium, i.e., for that position $\mathbf{r}(t)$ where $\phi(\mathbf{r},t)$ changes sign.[12] Although the gradient term of the EW equation[1] or the celebrated nonlinear term in the KPZ equation[2] is readily obtained within this approach, the resulting equations remain approximative. It is therefore of interest to study the full problem defined in Eq. (2). We consider quenched disorder but restrict ourselves to zero temperature. This makes Eq. (2) particular suitable for a description of field-driven interface movement in magnets at low temperatures.

In Ref. 15 we studied the field-driven interface dynamics of this model numerically using a discrete version of the Hamiltonian

$$\mathcal{H}=\sum_{\mathbf{l}}\frac{u_0}{4}(S_{\mathbf{l}}^4-2S_{\mathbf{l}}^2)-J\sum_{\mathbf{l},\mathbf{l}'}(S_{\mathbf{l}}S_{\mathbf{l}'}-z\,\delta_{\mathbf{l},\mathbf{l}'}S_{\mathbf{l}}^2) -\sum_{\mathbf{l}}(H+B_{\mathbf{l}})S_{\mathbf{l}}, \tag{3}$$

where $\mathbf{l}=(x,y)$, $\mathbf{l}'=(x',y')$ are nearest-neighbor pairs on a square lattice with lattice constant δ, z is the number of nearest neighbors, and $B(\mathbf{r})=B_{\mathbf{l}}$. The fields ϕ are scaled so that $a=b=u_0$ and $J=\tilde{J}/\delta^2$ and $S_{\mathbf{l}}$ denotes the scaled fields at discrete lattice points $\mathbf{l}$. The $S_{\mathbf{l}}$ may be termed as soft Ising spins at lattice points $\mathbf{l}$ with $-\infty<S_{\mathbf{l}}<\infty$. Due to this discretization, the relaxation equation is replaced by

$$\gamma\frac{\partial S_{\mathbf{l}}}{\partial t}=-\frac{\partial\mathcal{H}}{\partial S_{\mathbf{l}}}. \tag{4}$$

In the following we assume $J>0$, corresponding to ferromagnetic spin-spin coupling, and $u_0>0$, which is required in order that $\mathcal{H}$, Eq. (3), be bounded for $|S_{\mathbf{l}}|\to\infty$. The ground state is obtained as a time-independent solution of Eq. (4). Thus in the ground state the spin configuration is such that $\mathcal{H}$ is stationary with respect to variations of all spins $S_{\mathbf{l}}$. In the absence of a random field and external field it can be seen from Eq. (3) that the ground state is double degenerated with all spins having the value $S_{\mathbf{l}}=1$ or all spins having the value $S_{\mathbf{l}}=-1$ independent of the strength of the parameters u_0 and J.

Integration of Eq. (4) with the Euler scheme results in a set of difference equations

$$S_{\mathbf{l}}(\tau+\Delta\tau)=S_{\mathbf{l}}(\tau)+\Delta\tau\Bigg[-u[S_{\mathbf{l}}^2(\tau)-1]S_{\mathbf{l}}(\tau)+\sum_{\mathbf{l}'} S_{\mathbf{l}'}(\tau) -zS_{\mathbf{l}}(\tau)+h+b_{\mathbf{l}}\Bigg], \tag{5}$$

where magnetic fields h and $b_{\mathbf{l}}$ are measured in units of J, time τ is measured in units of γ/J, and $u=u_0/J$. Equation (5) is iterated starting from a vertical flat initial interface. At time $\tau=0$ all spins on the left-hand side of the interface located at $x=0$ are set to $S_{\mathbf{l}}=1$ and all spins at the right-hand side are set to $S_{\mathbf{l}}=-1$. The y direction where periodic boundary conditions are assumed is parallel to the interface. The linear system size L in this direction is chosen to be $L=400$ for all simulations if not specified otherwise.

Note that with open boundary conditions the interface width is larger than with periodic boundary conditions. However, the boundary effects introduced in this way are significant and can be neglected only in systems with a linear size large compared to the extension of the boundary layers. Such large systems cannot be simulated in a reasonable time so that periodic boundary conditions are a better choice for an investigation of the scaling behavior of the bulk. The extension of the system perpendicular to the interface was chosen in such a way that it does not play any role for the interface morphology. For more details see Ref. 15.

After a short time interval $\Delta\theta$ this sharp interface softens. Without random fields and driving field this time interval is of the order of $\Delta\theta\approx 10\Delta\tau$ and the width of the interface approaches its stationary value which is of the order of $\Delta x\approx 3$ for our choice of parameters (for a more detailed discussion of Δx see Sec. II). In this special case $H=B(\mathbf{r})=0$ and the assumed initial conditions, Eq. (2), can be solved exactly with the result that the scaled fields are given by $S(\mathbf{r})=\tanh(x/\Delta x)$ and $\Delta x=\sqrt{2J/u_0}$ is the intrinsic width of the interface in units of the lattice constant. For small driving fields this width is practically independent of the driving field. The interface remains flat and moves for $H>0$ with constant velocity towards $x=+\infty$, leaving the system in its ground state in which all spins are lined up with the external field. Introducing random fields the interface develops a rough structure. We are interested in this roughness on length scales large compared to the intrinsic surface width Δx since only on these length scales is scaling of the correlation function expected to work.

The random fields $b_{\mathbf{l}}$ are drawn with equal probability from an interval between $-p$ and p. If not specified, the parameters are chosen as $p=0.2$ and $u=0.9$. The time interval $\Delta\tau$ for the numerical integration was set to $\Delta\tau=0.1$ since previous investigations have shown that smaller time intervals do not change the results.

In our earlier work[15] we found that there exists a critical driving field h_C so that an interface gets trapped for driving fields $h<h_C$ and moves with a constant averaged velocity for fields larger than h_C where the average is over small time intervals. h_C depends on the strength of the random fields. Its value is $h_C\approx 0.015$ for the parameters given above. Note that h_C is smaller than the maximum of the random fields by more than a factor of 10.

If the driving field is set to zero, the magnetic moments close to the initially flat interface relax in such a way that the interface develops into a rough structure before all local moments are trapped in a metastable configuration. The dynamics of this roughening process and especially its scaling behavior have been analyzed in detail for an interface in a two-dimensional medium previously.[16] The roughening for $h>0$ will be analyzed in the present paper.

III. DYNAMIC SCALING ANALYSIS

For not too large values of p and for all applied driving fields h we found that the interface can be described by a single-valued function $x=\mathsf{h}(y,\tau)$; i.e., there are practically no overhangs or droplets. Here the position of the interface is defined as that point at which $S(x,y)$ as a function of x for fixed y changes sign.

As characteristics of the interface[18] we subsequently analyze the height correlation function which is defined as

$$C(r,\tau)=\langle[\mathsf{h}(y+r,\tau)-\mathsf{h}(y,\tau)]^2\rangle. \tag{6}$$

The angular brackets denote an average over all lattice sites at positions y as well as over different realizations of the quenched disorder. Another quantity of interest is the width of the interface,

$$w(L,\tau)=\langle[\mathsf{h}(y,\tau)-\langle\mathsf{h}(y,\tau)\rangle]^2\rangle^{1/2}, \tag{7}$$

which is related to $C(r,\tau)$ according to

$$w(L,\tau)^2=\frac{1}{2L}\sum_{r=1}^{L} C(r,\tau). \tag{8}$$

Equation (8) is exact for periodic boundary conditions. Because of this relation, we will concentrate in the following on the scaling behavior of the height correlation function.

In kinetic roughening phenomena dynamical scaling assumptions[18] are proposed. A central role in these scaling assumptions plays a time-dependent correlation length $\xi(\tau)$. For $r<\xi(\tau)$ the correlations do not grow further. Their saturation value is assumed to scale like

$$C(r\ll\xi,\tau)\propto r^{2\alpha} \tag{9}$$

with the roughness exponent α. For r larger than the time-dependent correlation length $\xi(\tau)$ the interface fluctuations are uncorrelated in space but they are still growing with time; their time evolution can be described by

$$C(r\gg\xi,\tau)\propto\tau^{2\beta}. \tag{10}$$

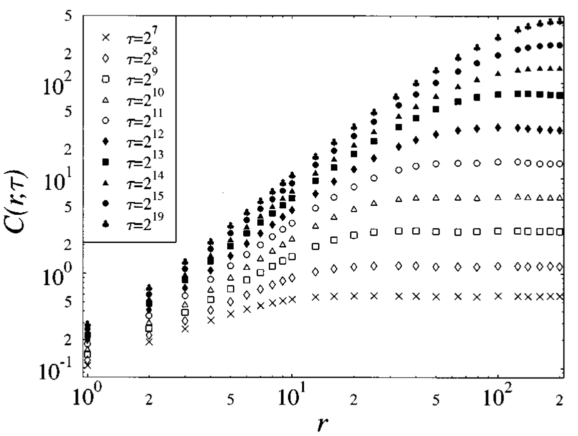

FIG. 1. The height correlation function $C(r,\tau)$ for $h=0.02$ at different times as indicated.

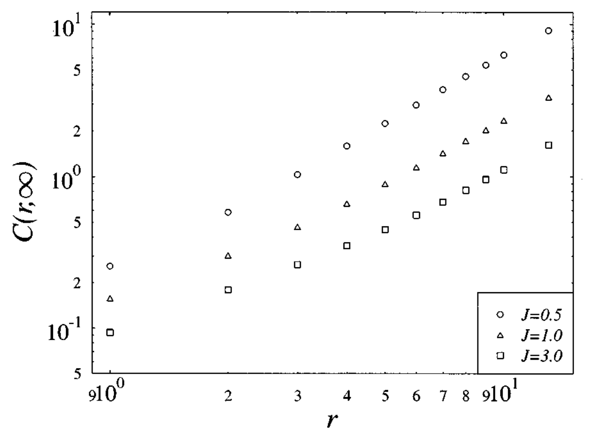

FIG. 2. The height correlation function for different values of the coupling constant J with $u=0.9/J$, $p=0.2/J$, and $h=0.05/J$ in the long-time limit.

The correlation length grows with time according to $\xi(\tau)=\tau^{1/z}$, where z is called the dynamic exponent. $\xi(\tau)$ may reach the system size, in which case a crossover to a finite-size behavior occurs. The scaling relations (9) and (10) are considered as the limiting behavior of a general dynamic scaling relation[18]

$$C(r,\tau)=[\xi(\tau)]^{2\alpha}g\left(\frac{r}{\xi(\tau)}\right). \tag{11}$$

In order that Eqs. (9) and (10) are recovered the scaling function $g(x)$ must have the properties $g(x)\approx\text{const}$ for $x\gg 1$ and $g(x)\propto x^{2\alpha}$ for $x\ll 1$ and $z=\alpha/\beta$.

Having introduced the usually assumed scaling assumptions we turn now to a discussion of our results. Figure 1 shows the spatial correlations of the height correlation function of a system in the depinning phase as a function of the distance r for different times on logarithmic scales. The data are obtained as averages over 200 runs with different random field configurations. An r-independent saturation is observed for large r in agreement with Eq. (10) and a linear dependence on $\ln(r)$ for small r in agreement with Eq. (9). However, in this latter case for driving fields close to the pinning transition, $\ln[C(r,\tau)]$ is still τ dependent; i.e., contrary to the scaling prediction, Eq. (9), the correlations are still growing also for very small times.

Such a behavior has also been observed in a recent study by Schroeder *et al.*[19] of the molecular beam epitaxy (MBE) model of Wolf and Villain.[20] In this study, $C(1,\tau)$ can be identified as the time evolution of the average step height. It grows as a power law $C(1,\tau)\propto\tau^{2\kappa}$ over a remarkable large time interval of about five decades and then seems to saturate, independent of system size. For small system sizes the saturation depends on it. One possibility to take this unusual behavior into account is a modification of the scaling relation (11) in the following way:

$$C(r,\tau)=\xi(\tau)^{2\alpha/\lambda}g\left(\frac{r}{\xi(\tau)}\right), \tag{12}$$

with the limiting forms

$$C(r\gg\xi(\tau),\tau)\propto\xi(\tau)^{2\alpha/\lambda} \tag{13}$$

and

$$C(r\ll\xi(\tau),\tau)\propto r^{2\alpha}\xi(\tau)^{2\alpha(1/\lambda-1)}. \tag{14}$$

If the correlation length $\xi(\tau)$ is small compared to the system size, it is expected to behave like a power law. If we keep the definition of the exponent β given in Eq. (10), we have to set

$$\xi(\tau)\propto\tau^{\lambda\beta/\alpha} \tag{15}$$

according to Eq. (13). The exponent appearing in Eq. (15) can be considered as dynamical exponent, $z=\alpha/\beta\lambda$, but other interpretations are possible.[19] From Eq. (13) one can see that the exponent κ introduced by Schroeder *et al.*[19] is related to λ according to $\lambda=1-\kappa/\beta$ and

$$C(r\ll\xi(\tau),\tau)\propto\tau^{2\kappa}r^{2\alpha}. \tag{16}$$

Equation (16) is capable of explaining the observed increase of $C(r,\tau)$ with time in the small-r region.

For $r\leqslant 3$ the data in Fig. 1 show a small upward turn; i.e., the r dependence of $C(r,\tau)$ is not powerlike for very small r. This has to do with the intrinsic width Δx of the interface due to the softening of the spins discussed in Sec. II. Changing the system parameters this width can be varied. It can be seen from Fig. 2 that the region where the small-r scaling breaks down is directly given by this intrinsic width.

To calculate the roughness exponent α we use the asymptotic behavior given in Eq. (14) for the largest τ available. For large τ, $\xi(\tau)$ saturates and the correlation function is proportional to $r^{2\alpha}$. It is shown in Fig. 3 in logarithmic scales for three different values of the driving field. The exponent α is obtained from a fit to a power law within the linear region starting at $r=3$. Since the linear region is rather small, we also fitted the data over the whole interval with a scaling function of the form

$$C(r,\infty)=A[\tanh(r/B)]^{2\alpha}, \tag{17}$$

following earlier work.[16,17] Within the error bars both methods give the same value for the roughness exponent. The solid lines show the final fit with Eq. (17). Good agreement with the data is obtained over the whole r interval.

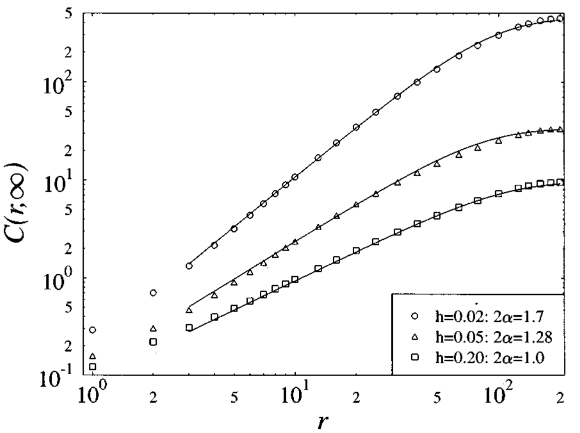

FIG. 3. Determination of the exponent α for different driving fields $h > h_C$.

To determine the exponents β and κ (or λ) we use the asymptotic forms (10) for large r ($r = L/2$ is the largest value available due to periodic boundary conditions) and (16) for fixed $r = 3$. Figure 4 shows $C(L/2,\tau)$ on logarithmic scales for three different values of the driving field. The solid lines are fits to the data for the whole τ interval using as an ansatz[16,17]

$$C(L/2,\tau) = [\Delta \tanh(\tau/\tau_0)]^{2\beta}. \tag{18}$$

The inset in Fig. 4 shows the correlation function $C(r,\tau)$ for $r = 3$ which is fitted to Eq. (16). The corresponding exponents β and $\lambda = 1 - \kappa/\beta$ together with the roughness exponent and the dynamical exponent z are shown in Fig. 5 for different values of the magnetic field.

Figure 6 shows the final scaling plot for $h = 0.02$ with a satisfactory data collapse. The same is true for other values of the magnetic field not shown here.

The results obtained so far are for systems of size $L = 400$. The saturation of the correlation function observed in Fig. 1 may be due to finite-size effects. Another possibility is the existence of an intrinsic length, above which a saturation takes place independent of system length. This was observed for $h = 0$, where the initially flat interface settles

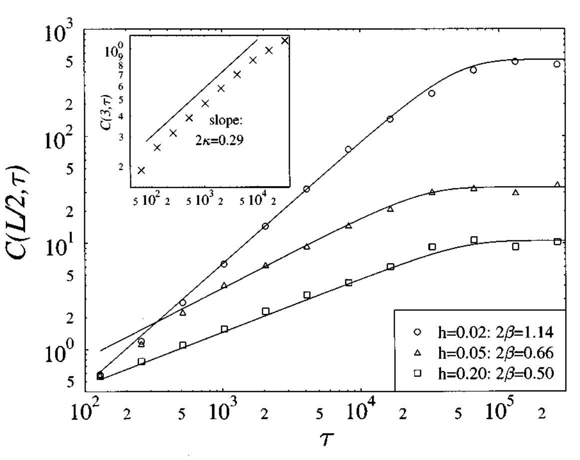

FIG. 4. Determination of the exponent β from the small time behavior of the height correlation function at $L/2 = 200$ and λ from $C(3,\tau)$ (inset) for $h = 0.02$.

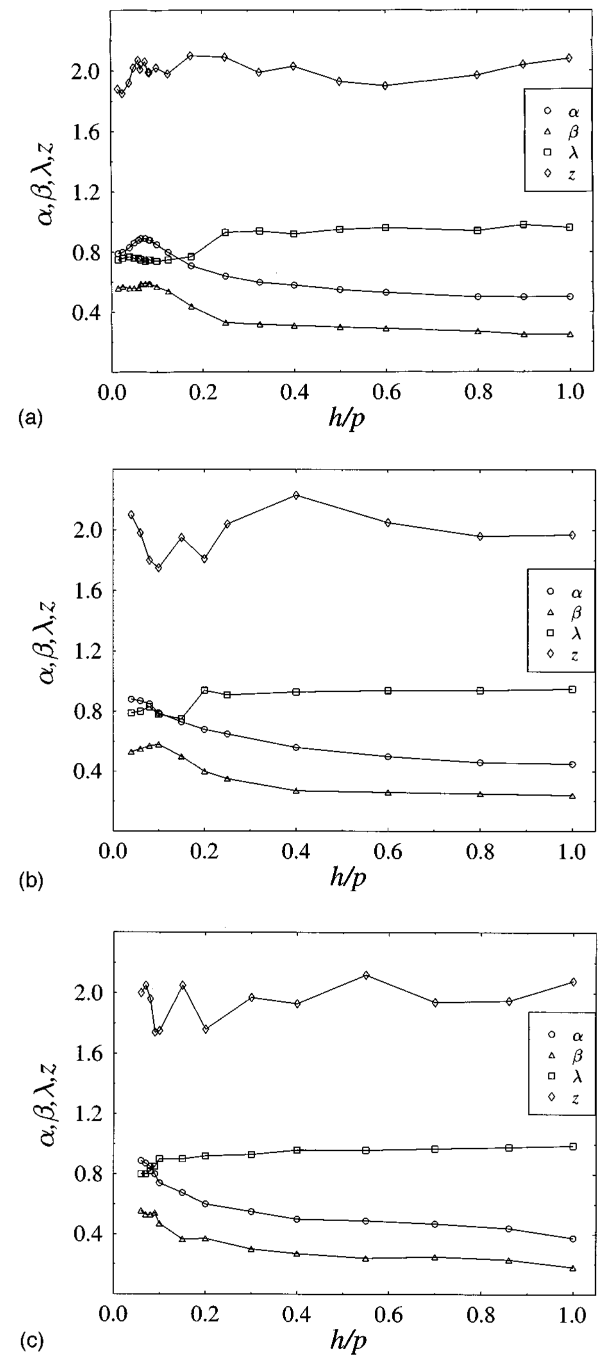

FIG. 5. The dependence of the exponents α, β, λ, and z on the rescaled driving field h/p for (a) $p = 0.2$, (b) $p = 0.1$, and (c) $p = 0.05$.

down in a metastable position,[16] and we have verified in the meantime that a similar behavior holds for every $h < h_C$. For moving interfaces, however, the saturation is due to finite-size effects. To prove this we consider eight $L = 400$ systems, each of which has a different distribution of random fields and which are all driven in a steady state at $\tau_s = 2^{19}$. Due to the periodic boundary conditions $\mathsf{h}_i(1) = \mathsf{h}_i(L+1)$, one can connect different systems so that $\mathsf{h}_i(L+1) = \mathsf{h}_{i+1}(1)$ with i denoting the different systems. Running the larger system for another 3×2^{17} time steps it is clearly seen from Fig. 7 that the correlation function for large r is still increasing

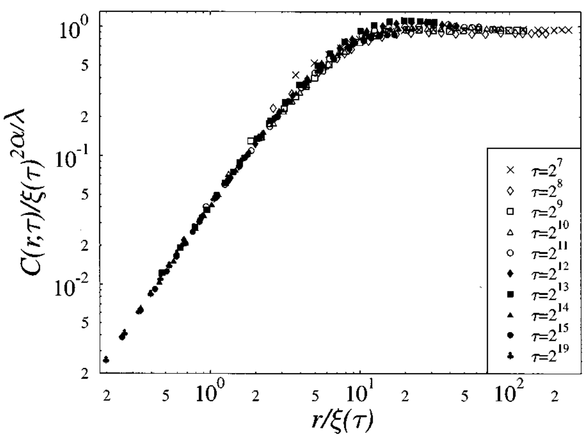

FIG. 6. Scaling plot of the height correlation function according to Eq. (14) for $r \geqslant 3$ and $h=0.02$.

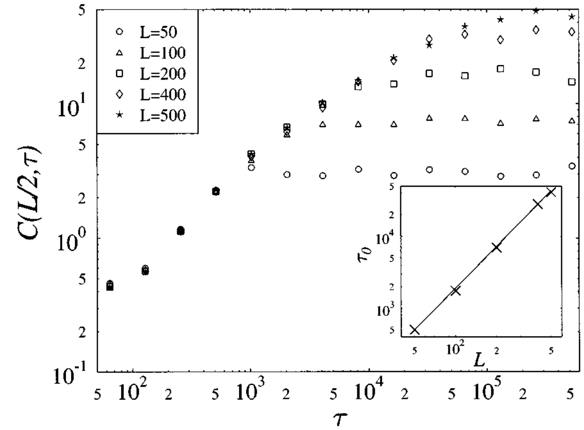

FIG. 8. Time dependence of $C(L/2,\tau)$ for different system sizes as indicated. The inset shows the size dependence of the time constant τ_0 ($h=0.05$).

which proves that the saturation observed in Fig. 1 is due to finite-size effects and not due to an intrinsic length scale. Note that the data in Fig. 7 are for one magnetic field only and that this very large system is still not driven into a steady state. Computer time limitations make it impossible to run these very large systems into steady states or to make these runs for different magnetic fields or random field configurations.

Having established that the saturation of the correlation function is due to finite-size effects we now turn to a discussion of the size dependence of the correlation function $C(r,\tau)$ which also will lead to an independent estimate of the exponent z. Finite-size effects manifest themselves in a modified $\xi(\tau)$ in Eq. (12) in such a way that $\xi(\tau)$ approaches L for large τ. This is achieved by writing $\xi(\tau)$ as

$$\xi(\tau)=\xi_0\left[T\left(\frac{\tau}{\tau_0}\right)\right]^{1/z}, \tag{19}$$

with $T(x)=x$ for $x \ll 1$ and $T(x)=1$ for $x \gg 1$ and $\xi_0 \propto L$. From the small-τ limit it follows that $\tau_0 \propto L^z$. Thus from the measurement of τ_0 by fitting $C(L/2,\tau)$ for different values of L the exponent z can be obtained. Results are shown in Fig. 8 for $h=0.05$. The corresponding exponent z is 1.94 ± 0.03 which is close to $z=2.09$ obtained from the relation $z=\alpha/\beta\lambda$.

Finally we note that not only does the correlation length become L dependent for large τ, the scaling function $g(x)$ appearing in Eq. (12) also gets an L dependence. This can be seen by studying the width of the interface. From the exact relation (8) it follows from Eq. (12) for $\xi(\tau) \ll L$ that

$$[w(L,\tau)]^2=\frac{1}{2L}\int_1^{L/\xi(\tau)} dx\, g(x)[\xi(\tau)]^{2\alpha/\lambda+1} \tag{20}$$

For $\xi(\tau) \ll L$, $g(x)$ can be replaced by its asymptotic value $g(x) \approx \text{const}$ and w^2 thus scales as $[w(L,\tau)]^2 \propto \tau^{2\beta}$. It has been verified that the exponent β deduced in this way agrees with β obtained from the correlation function. In the large time limit, on the other hand, $\xi(\tau \to \infty) \propto L$ so that

$$[w(L,\tau)]^2 \simeq L^{2\alpha/\lambda}\int_{1/L}^{1} dx\, g(x). \tag{21}$$

In Fig. 9 we show the scaling behavior for $w(L,\tau \to \infty)$. The exponent is given by 0.75 and therefore agrees very well with the value $\alpha=0.75$ obtained from the correlation func-

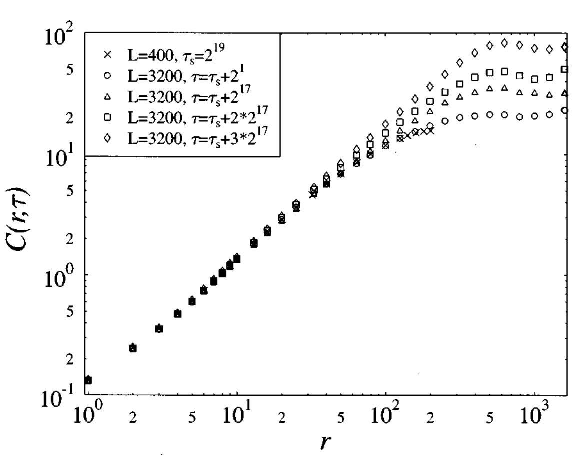

FIG. 7. The height correlation function $C(r,\tau \to \infty)$ for $L=400$ and $C(r,\tau)$ for a system with $L=3200$ for different times as indicated ($h=0.1$).

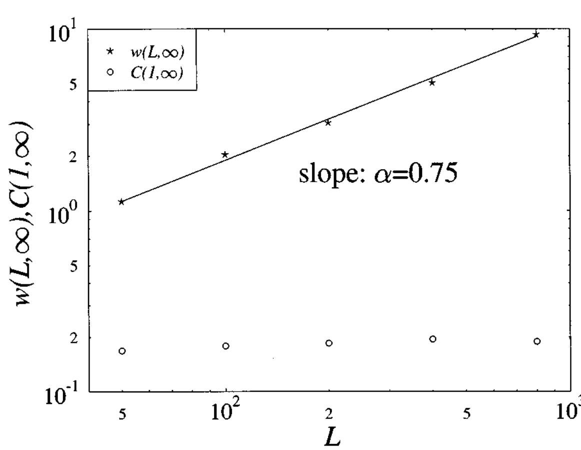

FIG. 9. Size dependence of the roughness $w(L,\tau \to \infty)$ and the average step height $C(1,\tau \to \infty)$ ($h=0.03$).

tion (for $h=0.03$). We conclude therefore that in the large τ limit $g(x)$ is proportional to $L^{2\alpha(1-1/\lambda)}$. This view is also supported by considering the average step height in the large τ limit. It is also shown in Fig. 9. Obviously, the step height is independent of L. Calculating the average step height from Eq. (12) we obtain $[\xi(\tau\rightarrow\infty)\rightarrow L]$

$$C(r=1,\tau\rightarrow\infty)\propto L^{2\alpha/\lambda}g(1/L). \tag{22}$$

Thus, only by inducing the above-mentioned factor does this quantity become independent of L.

IV. DISCUSSION

There is an obvious dependence of the exponents shown in Figs. 5(a)–5(c) on the strength of the driving field. For the larger field values h the exponent λ is practically one so that the usual scaling scenario, Eqs. (9) and (10), is valid. For smaller fields λ drops by more than 20% and both the exponents α and β rise considerably when h approaches the critical value h_C of the depinning transition. Increasing the driving field the velocity of the interface increases and the exponents α and β approach the values 0.5 and 0.25, respectively, for $p=0.2$. Changing the strength of the random fields a dependence of the exponents on p is also observed. At the depinning transition $h_C(p)$ the exponent α has its maximal value $\alpha\approx0.9$, independent of p which is close to the value obtained by Dong *et al.*[21] For increasing h, α decreases and tends towards different and rather small values for $h\gg h_C(p)$. As shown above the parameter u influences the intrinsic width Δx and the value of the critical field h_C but not the exponents. We have checked this in previous work[16] for $h=0$ and have verified this also for $h\neq0$.

Also shown in Figs. 5(a)–5(c) is the dynamical exponent z defined by $z=\alpha/\beta\lambda$. In this exponent the ratio of two quantities enters, α and β, which both decrease rather strongly by roughly a factor of 2 with increasing field. The third exponent which enters, λ, increases with increasing h only slightly above the depinning transition and then remains constant. Interestingly, z as defined above does not show this strong h dependence but stays constant, $z\approx2$. This behavior is observed for all values of p considered. The fluctuations observed in z are due to numerical uncertainties. They are rather small for $p=0.2$ where each data point has been obtained by averaging over about 200 realizations of the random field configurations but are larger for the other two p values where only 40 random field configurations are considered due to computer time limitations.

The decrease of the roughness with increasing driving field is easy to understand. Roughness occurs since some parts of the interface are trapped locally by the random fields for a short time interval; other parts move. These time intervals decrease with increasing driving fields and they get very large when h approaches h_C, resulting in a very rough structure. On the contrary the fast moving interface flattens; it has fairly large straight portions as can be seen in pictures of the interface obtained from the numerics and also quantitatively from the maximum value of the height correlation function $C(L/2,\tau\rightarrow\infty)\equiv C_m(h)$ (Fig. 10) and the average step height $C(1,\tau\rightarrow\infty)$ or $C(2,\tau\rightarrow\infty)$ (see inset of Fig. 10).

The driving field h cannot be increased too far, however,

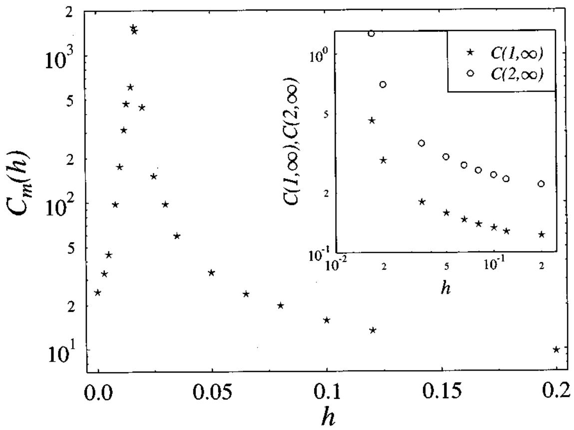

FIG. 10. The maximum value of the height correlation function $C_m(h)$. The inset shows the driving field dependence of $C(1,\tau\rightarrow\infty)$ and $C(2,\tau\rightarrow\infty)$ for $h>h_C$.

since for very large driving fields of the order of p or larger a different type of dynamics is observed. For these fields nucleation processes begin; i.e., isolated groups of spins in the sea of down spins turn over, forming islands of up spins, and the interface develops overhangs on all scales, meaning that the self-affine structure of the interface gets lost.

In the following we will give arguments in favor of the conjecture that the observed strong dependence of the exponents on the strength of the driving field can be understood in terms of an EW equation with correlations in the noise term. In Ref. 22 the EW model with an equation of motion for the interface position $\mathsf{h}(x,t)$,

$$\frac{\partial \mathsf{h}}{\partial t}=\nu\nabla^2\mathsf{h}+\eta(x,t), \tag{23}$$

was considered with noise correlations

$$\langle\eta(x,t)\eta(x',t')\rangle\sim D|x-x'|^{2\rho-d+1}|t-t'|^{2\theta-1}. \tag{24}$$

Dynamical scaling leads to[18]

$$\mathsf{h}(x,t)\sim t^{\beta}f(x/t^{1/z}), \tag{25}$$

where $f(y)$ has the limiting values $|f(y)|\sim\text{const}$ for $y\gg1$ and $f(y)\sim y^{\alpha}$ for $y\rightarrow0$. Equation (25) is meant to be valid in a statistical sense; it is the analog to Eq. (11). Equation (25) is invariant under the transformation $x\rightarrow ax$, $t\rightarrow a^z t$, $\mathsf{h}\rightarrow a^{\beta z}\mathsf{h}$. Applying a naive dimensional analysis to the EW equation with this transformation one obtains two relations between the exponents, $z=2$ and

$$2\rho-d+1+z(2\theta+1)-2z\beta=0, \tag{26}$$

where d is the embedding dimension.

In the present soft spin model the interface moves through a random medium. The analog to the noise term in Eq. (26) is then the random fields at the moving interface which play the role of an effective noise. It cannot be treated as uncorrelated since it is due to a local pinning-depinning process which depends on the very position of the interface. Such a behavior has been observed earlier by Havlin *et al.*[23] who

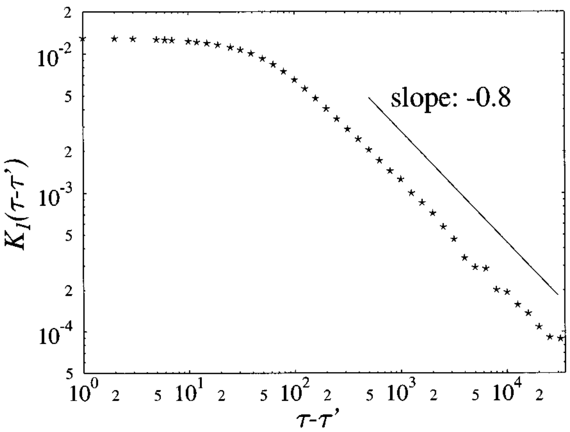

FIG. 11. Time correlations of the random fields for $h=0.02$.

reported that if the noise is the result of a stochastic process, it cannot be treated as random, but becomes correlated in space and/or time.

We have measured the correlations of the random fields at the interface,

$$\langle b(x,\, \mathsf{h}(x,\tau))b(x',\, \mathsf{h}(x',\tau'))\rangle = K_1(\tau-\tau')K_2(x-x'), \tag{27}$$

where $b(x,\, \mathsf{h}(x,\tau))$ denotes the random field at the interface position and the average is over space and time (for fixed distance $|x-x'|$ and time difference $|\tau-\tau'|$). We found that the spatial correlations are very weak if not absent, leading to $2\rho=1$ (for $d=2$). The reason is easy to understand: The interface gets trapped at random fields which are large and negative and it moves fast in a region where the random fields are large and positive. Looking at the interface at a certain instant of time the distribution of random fields at the interface is therefore centered at negative field values so that the spatial correlations are finite even for points far apart. The time correlations of the random fields at the interface are shown in Fig. 11 for $h=0.02$. For large time differences a power law behavior is observed with an exponent $2\theta-1=-0.8$. Inserting this exponent into the scaling relation, Eq. (26), we obtain $\beta=0.6$. This value is in fairly good agreement with the value obtained before, $\beta=0.57$. We consider this as strong evidence of the soft spin model to be in the EW universality class above the depinning transition with field-dependent exponents due to time-correlated noise. Further evidence for this conjecture comes from our observation that although the exponents α,β,λ depend on h and p, the exponent z defined above by $z=\alpha/\beta\lambda$ is much less dependent on these parameters and has a value of roughly 2 as is required by the scaling argument for the EW equation with correlated noise, Eq. (23).

Finally we would like to comment on the possibility that our system may be of the KPZ type. Of course, the nonlinear KPZ-type term[2] is very likely to be present but we think that it is not important, at least for the fast moving interfaces, in contrast to Ref. 24. This term arises from an expansion of $\sqrt{1+(\nabla\, \mathsf{h})^2}$, where $\mathsf{h}(y,t)$ denotes the position of the interface. The nonlinear term can be expressed in a discrete representation as[25]

$$(\nabla\, \mathsf{h})^2 = \tfrac{1}{4}[\mathsf{h}(y-1) - \mathsf{h}(y+1)]^2, \tag{28}$$

showing that its average is given by the heigh correlation function as

$$(\nabla\, \mathsf{h})^2\rangle = \tfrac{1}{4}C(r=2,\tau). \tag{29}$$

The inset of Fig. 10 shows the quantity $C(r=2,\ \tau\to\infty)$ for different values of the driving field. For $h>h_C$ it is very small compared to 1 and decreases monotonically for increasing h so that it can be neglected.

To further support this conjecture we follow the authors of Ref. 26 and impose a ‘‘tilt’’ of slope m on the interface. Due to the nonlinearity $(\nabla\, \mathsf{h})^2$ of the KPZ models, the average velocity has an m^2 dependence from the tilt m in contrast to the EW model which has a tilt-independent velocity. We have found that our model has no slope dependence of the velocity.

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