

Giant oscillations of acoustoelectric current in a quantum channel

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A theory of dc electric current induced in a quantum channel by a propagating surface acoustic wave (acoustoelectric current) is worked out. The first observation, to our knowledge, of the acoustoelectric current in such a situation was reported by J. M. Shilton *et al.*, *J. Phys. Condens. Matter* **8**, L337 (1996). The authors observed a very specific behavior of the acoustoelectric current in a quasi-one-dimensional channel defined in a GaAs-Al_xGa_{1-x}As heterostructure by a split-gate depletion—giant oscillations as a function of the gate voltage. Such a behavior was qualitatively explained by an interplay between the energy-momentum conservation law for the electrons in the upper transverse mode with a finite temperature splitting of the Fermi level. In the present paper, a more detailed theory is developed, and important limiting cases are considered. [S0163-1829(96)03335-8]

I. INTRODUCTION

In recent years much attention has been attracted by the interaction between surface acoustic waves (SAW's) and two-dimensional electron gases (2DEG's). Most experiments were performed in GaAs-Al_xGa_{1-x}As heterostructures, and effects linear in acoustic intensity—the sound attenuation and the change of sound velocity—were studied.¹⁻⁴ These works were aimed at investigating the linear response of a 2DEG to ac strain, and electric fields without electric contacts attached to the sample. In particular, very interesting experimental studies of these effects in the quantum Hall regime were carried out in the works.^{1,4}

The second class of studies deals with the so-called *acoustoelectric effect*, namely, a drag of 2D electrons by a traveling SAW.⁵⁻⁹ This effect is due to a transfer of momentum from the SAW to the electrons due to a SAW-electron interaction. As a result, a dc *acoustoelectric current* appears in a closed circuit. If the circuit is open, the SAW generates a dc *acoustoelectric voltage* across the sample. In the simplest case, the current (or the voltage) is proportional to the SAW *intensity* rather than to its amplitude. It is a simple nonlinear effect that can be also employed to study the 2DEG.

An observation of an acoustoelectric current through a quantum point contact was reported in Ref. 10. The authors observed a dc current in a quasi-one-dimensional channel defined in a GaAs-Al_xGa_{1-x}As heterostructure by a split-gate depletion. The conduction of such channels is quantized—its dependence on the applied gate voltage consists of a set of plateaus divided by sharp steps.^{11,12}

An important point is that a very specific behavior of the acoustoelectric current as a function of the applied gate voltage was observed. That is, the current is *not quantized*. On the contrary, it experiences *giant oscillations* as a function of gate voltage, having maxima near the steps between the plateaus of the conductance.

A semiquantitative explanation given in Ref. 10 attributes the oscillations to electrons in the upper transverse subband of the channel. When the bottom of this band is close to the Fermi surface (i.e., near the step of conductance), these electrons at the Fermi level have a low longitudinal velocity. At some value of the gate voltage this velocity can be close to the sound velocity w . Such electrons interact strongly with the wave because they move synchronously with the latter. At other gate voltages there are no electrons which are able to interact effectively, and the drag is not efficient.

In the following, we will discuss the acoustoelectric effect theoretically in more detail, and consider the most important parameters and limiting cases.

II. FORMULATION OF THE PROBLEM

Consider a ballistic point contact between two regions of a 2DEG. According to the Landauer formula, the conductance of such a junction is determined by transparencies T_n which correspond to different transverse modes

$$|nk\rangle = \chi_n(z, x) \exp\left(i \int^x k(\xi) d\xi\right), \quad (1)$$

the functions χ_n and the wave vector k being slowly dependent on x . Here we label k the wave vector at $x \rightarrow \infty$. We have

$$G = \frac{2e^2}{\pi\hbar} \int_{\epsilon_n}^{\infty} \sum_{n=1}^N d\epsilon_{nk} \left(-\frac{\partial f_0(\epsilon_{nk})}{\partial \epsilon_{nk}} \right) T_n(\epsilon_{nk}), \quad (2)$$

where $f_0(\epsilon_{nk})$ is the Fermi function, while $\epsilon_{nk} = \epsilon_n + \hbar^2 k^2 / (2m)$ is the electron energy for the n th transverse mode. Such a concept is based upon the assumption that electron thermalization takes place within the region $\sim \ell_{in}$ (inelastic relaxation length) near the contact. That allows one to reduce the problem to calculation of transmittance of non-equilibrium electrons.

One can imagine several sources of influence of the acoustic wave on the current through the contact. The first

one is a drag of the 2DEG in the leads. Such a drag produces a ‘‘phonon wind’’ in the leads which has been estimated by Kozub and Rudin¹³ on the basis of the model introduced by Streda.¹⁴ According to this model, one can introduce the electron-phonon collision integral *in the leads*, $\hat{I}_{e-ph}^{\text{drag}}(\epsilon)$, as a source of the force acting upon the electrons. Then the current through the contact is

$$j \propto \sum_n \int_{\epsilon_n}^{\infty} d\epsilon \hat{I}_{e-ph}^{\text{drag}}(\epsilon_{nk}) \left(-\frac{\partial f_0(\epsilon_{nk})}{\partial \epsilon_{nk}} \right) T_n(\epsilon_{nk}), \quad (3)$$

where $\hat{I}_{e-ph}^{\text{drag}}(\epsilon_{nk})$ is a smooth function, determined by the properties of the leads. Consequently, such a contribution has the form of a sum of steps similar to the conductance that is explicitly stated in Ref. 13.

However, the piezoelectric interaction between the SAW and the electron system in the leads is significantly screened by the 2DEG. Consequently, the steps are very small compared with effects observed in the experiment,¹⁰ where maxima of the acoustoelectric drag current rather than steps were observed.

As a result, to explain the observed experimental results we are left with the region of the quantum channel where the piezoelectric field is not screened too much. That is, we analyze the drag due to a momentum transfer from the SAW to the electron gas *inside the quantum channel*. The physical reason for the importance of this region in comparison with the much larger region of the 2DEG is the absence of effective screening. Such a physical picture implies that one has to consider SAW-electron interaction inside the channel rather than employ Büttiker-Landauer formalism.

III. GENERAL EXPRESSIONS FOR ACOUSTOELECTRIC CURRENT

In the presence of a harmonic acoustic wave with the frequency ω and wave vector \mathbf{q} the electrons acquire a perturbation

$$\mathcal{H}_{\text{int}} = U \sum_{nk, n'k'} [C_{nk, n'k'}(q) a_{nk}^\dagger a_{n'k'} + \text{H.c.}],$$

where

$$C_{nk, n'k'}(q) = \langle nk | \exp(iqx) | n'k' \rangle.$$

For simplicity, let us model the quantum point contact as a channel with uniform width d . Such an approximation is valid if the product of the channel’s length L times the SAW wave vector q is much greater than 1, $qL \gg 1$.¹⁵ We also assume that the inequality $qd \ll 1$ holds. The last assumption allows one to neglect the intermode transitions due to SAW, and to take into account only diagonal in the mode quantum numbers n, n' contribution.

In this approximation, $C = \beta_n(q) \delta_{nn'} \delta(k - k' - q)$, and we can consider the electrons of the n th mode as moving in the effective classical field

$$V_n(x) = \text{Re}[U\beta_n \exp(iqx - i\omega t)].$$

Having in mind low enough frequencies,

$$\hbar q \ll m\omega, p_F,$$

one can use the Boltzmann equation for the occupation number of the n th mode, $f_{nk}(x)$ (see, e.g., Ref. 16). This equation has the form

$$\left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial x} - \frac{1}{m} \frac{\partial V_n}{\partial x} \frac{\partial}{\partial v} + \hat{I} \right) f_{nk}(x) = 0,$$

where \hat{I} is the operator describing relaxation of the nonequilibrium distribution. We will specify its form below, taking into account two possible relaxation mechanisms: (i) impurity scattering, and (ii) escape from the channel.

Let us expand $f_{nk}(x)$ in powers of the amplitude U ,

$$f_{nk}(x) \equiv f_0[\epsilon_{nk} + V_n(x)] + f_1 + f_2, \quad (4)$$

where

$$f_1 = \text{Re}[f_{1\omega} \exp(iqx - i\omega t)]; \quad f_{1\omega} \propto U\beta_n. \quad (5)$$

The second-order part, $f_2 \propto |U\beta_n|^2$, is a sum of two items—a stationary part, and a part which varies in time with the frequency 2ω . We are interested only in the first part. The second part does not contribute to the dc current and will be omitted. From (4) we obtain

$$\begin{aligned} \hat{B}f_{1\omega} &= -i\omega U\beta_n \left(-\frac{\partial f_0(\epsilon_{nk})}{\partial \epsilon_{nk}} \right), \\ \hat{I}f_2 &= \left\langle \frac{\partial V_n(x)}{\partial x} \frac{\partial f_1}{\partial p} \right\rangle_t. \end{aligned} \quad (6)$$

Here we have introduced the operator

$$\hat{B}(q, \omega) \equiv \hat{I} + i(qv - \omega)$$

having the meaning of the operator of the linearized kinetic equation. Angular brackets mean the average over time t , $\langle \rangle_t \equiv (\omega/2\pi) \int_0^{2\pi/\omega} \dots dt$. Substituting f_1 and using the relationship

$$\langle \text{Re}[C_\omega \exp(i\omega t)] \text{Re}[D_\omega \exp(i\omega t)] \rangle_t = (1/2) \text{Re}(C_\omega D_\omega^*),$$

we obtain

$$f_2 = -\frac{|U\beta_n|^2 q \omega}{2} \hat{I}^{-1} \text{Re} \left\{ \frac{\partial}{\partial p} \left[\hat{B}^{-1} \left(-\frac{\partial f_0(\epsilon_{nk})}{\partial \epsilon_{nk}} \right) \right] \right\}. \quad (7)$$

As a result, we arrive at the following formal expression for the acoustoelectric current (taking into account the sum over spins):

$$\begin{aligned} j &= eU^2 q \omega \sum_n |\beta_n|^2 \text{Re} \int \frac{dp}{2\pi\hbar} v \hat{I}^{-1} \\ &\times \left\{ \frac{\partial}{\partial p} \left[\hat{B}^{-1} \left(-\frac{\partial f_0(\epsilon_{nk})}{\partial \epsilon_{nk}} \right) \right] \right\}. \end{aligned} \quad (8)$$

IV. ANALYSIS OF IMPORTANT LIMITING CASES

Equation (8) is the formal expression for the acoustoelectric current. To evaluate it one needs to specify the relaxation operator \hat{I} , i.e., to discuss sources of relaxation. The conventional way to treat relaxation in the quantum channel is to discuss scattering by individual defects described by scatter-

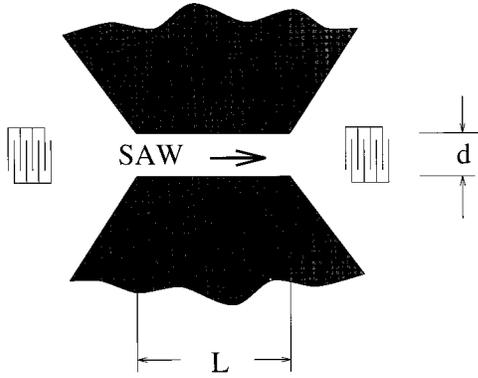


FIG. 1. Schematic shape of the quantum channel.

ing matrices. However, we want to emphasize that the problem of the acoustoelectric effect has an important specifics. Indeed, as we will demonstrate below, the effect is due to electrons with small momenta k . For these selected electrons the effective mean free path ℓ_{eff} can be much shorter than the mean elastic free path ℓ for a bulk 2DEG (see below). We believe that for realistic ballistic channels the inequalities $\ell > L > \ell_{\text{eff}}$ hold. As a result, one can use a relaxation rate approach, similar to the conventional one for bulk systems. The possibility for a nonequilibrium electron to escape the channel will be allowed for in a model way.

A. Relaxation rates

Here we discuss two sources of relaxation, namely, an elastic scattering within the channel (with the rate $1/\tau$) and an escape from the channel due to its finite length L (see Fig. 1), an effective rate being $|v_n|/L$. Here v_n is defined as the velocity for the n th mode at a given energy ϵ ,

$$\epsilon_{n,m} v_n / \hbar = \epsilon.$$

For lower transverse modes $v_n \sim v_F$, while for the upper one (with the number N) it can be small. On the other hand, because of low longitudinal electron velocity, the electrons of the upper mode are much more effectively scattered by impurities (see below). From this fact we come to an important conclusion: for the upper mode (which is responsible for the acoustoelectric effect near its maxima) the impurity scattering might be important even if the contact is ballistic. We would like to use this opportunity to emphasize once more that in mesoscopic systems an interplay between the impurity and other mechanisms of scattering is very dependent upon the problem in question (cf. with Ref. 17).

Introducing a correlation function of random impurity potential as

$$\mathcal{H}(\mathbf{r}) = \langle V(\mathbf{r})V(0) \rangle_{\text{im}},$$

and its matrix elements

$$K_{mk',nk} = \langle nk | \mathcal{H}(\mathbf{r}) | mk' \rangle,$$

we obtain

$$\frac{1}{\tau_n(k)} = \frac{2\pi}{\hbar} \sum_{mk'} |K_{mk',nk}|^2 \delta(\epsilon_{nk} - \epsilon_{mk'}). \quad (9)$$

If the impurity potential has a short range, then $|K_{mk',nk}|^2$ is independent of the arguments (mk',nk) . Consequently,

$$\frac{1}{\tau_n(k)} = \frac{2\pi |K|^2}{\hbar} \nu(\epsilon_{nk})$$

where $\nu(\epsilon)$ is the density of states per given spin,

$$\nu(\epsilon) = \frac{1}{\pi \hbar} \sum_{n=1}^N \frac{1}{|v_n|}.$$

Assuming that the channel behaves as a rectangular box with the thickness d in the transverse direction, we obtain $N \approx \sqrt{\epsilon_F / \epsilon^*}$, where $\epsilon^* = \hbar^2 / (8md^2)$.¹⁸

For a wide contact ($N \rightarrow \infty$) $\nu = \nu_b d$, where $\nu_b = m / (2\pi \hbar^2)$ is the 2D density of states. As a result, in a channel we arrive at a smooth part of the relaxation rate (which is of the order of the momentum relaxation rate in the bulk 2DEG, $1/\tau_b$) plus an oscillating term

$$\frac{2}{\pi \tau_b} \left(\frac{\epsilon^*}{\epsilon - \epsilon_N} \right)^{1/2}.$$

Finally, for all the levels except the highest one, one can put $1/\tau_b$ for the impurity relaxation rate, while for the highest one the appropriate estimate is

$$\frac{2}{\pi \tau_b} \frac{v^*}{|v_N|}, \quad (10)$$

where $v^* = \pi \hbar / md$. In high-mobility selectively doped structures, the scattering potential is smooth. Consequently, the matrix elements $K_{nk',nk}$ significantly increase with the decrease of k . As a result, impurity scattering for the upper mode is much stronger than for the lower ones,¹⁹ and expression (10) acquires an additional large factor $\alpha \sim |K_{N,k_w;N,-k_w} / K_{k_F,k'_F}|^2$, where $k_w \sim mw / \hbar$.

To take into account the finite length of the contact, we also introduce the relaxation rate $|v_n|/L$. As a result, an estimate for the relaxation operator of the upper level is

$$\hat{I} = \max \left(\frac{1}{\tau_b} \frac{v^*}{|v_N|}, \frac{|v_N|}{L} \right). \quad (11)$$

We observe that there is a border value v_c of v_N ,

$$v_c = v_F \left(\frac{\alpha L}{N \ell_b} \right)^{1/2}, \quad (12)$$

where both mechanisms make contributions of the same order of magnitude. Here ℓ_b is the mean free path in a bulk 2DEG (we assume $v_c \leq v_F$). At $|v_N| \leq v_c$ impurity relaxation becomes more important than finite size of the contact.

Expression (11) needs a more detailed discussion. Consider a ballistic one-dimensional pipe where the particles are subjected to a constant force F . As a result, they are accelerated as $v(t) = v_0 + (F/m)t$, and $x(t) = v_0 t + (F/2m)t^2$. At the time t , the distance between a given particle and a particle started after time θ is $\delta x = \theta [v_0 + (F/m)t]$. Consequently, the product of the velocity at the time t and the local density, $v(t)/\delta x(t) = 1/\theta$ remains constant, keeping constant the current density inside the pipe.

The total current is determined by the difference between the contributions of the particles with opposite directions of initial velocity. The difference is proportional to $(F/m)t \approx FL/(mv_0)$. Such a result can be reproduced by the order of magnitude by the assumption (11).

B. Estimates of the acoustoelectric effect

Now we are ready to make estimates. For simplicity, we model the relaxation operator by the following interpolation expression:

$$\hat{I} = \frac{1}{L} \frac{v_N^2 + v_c^2}{|v_N|}. \quad (13)$$

At $v_N \gg v_c$ this approximately describes escape from the channel, while at $v_N \ll v_c \ll v_F$ it is L independent and equivalent to the inverse life time for the electron with a given k due to elastic scattering. We have

$$v_N \hat{I}^{-1} = L \frac{v_N |v_N|}{v_N^2 + v_c^2}.$$

Integrating Eq. (8) by parts, and taking into account that

$$\frac{\partial(v_N \hat{I}^{-1})}{\partial p_N} = \frac{2L}{m} \frac{|v_N| v_c^2}{(v_N^2 + v_c^2)^2},$$

we arrive at the following expression for the oscillating part of the acoustoelectric current:

$$j = \frac{LeU^2\omega}{\pi\hbar} |\beta_N|^2 \int_{-\infty}^{\infty} dv_N F_\eta(v_N) F_T(v_N),$$

$$F_\eta(v_N) = \frac{\eta v_N^2 v_c^2}{(v_N^2 + v_c^2) [v_N^2 (v_N - w)^2 + \eta^2 (v_N^2 + v_c^2)^2]}, \quad (14)$$

$$F_T(v_N) = \frac{1}{4k_B T} \frac{1}{\cosh^2[m(v_N^2 - v_{NF}^2)/4k_B T]}.$$

Here $\eta = 1/qL$ has the meaning of the ratio between the acoustic wave length and the length of the contact, $w = \omega/q$ is the sound velocity, while $v_{NF} = v_N(\epsilon_F)$.

We observe that the expression (14) consists of the product of two functions. The function F_T has sharp maxima at $v_N = v_{NF}$ (see Fig. 3), the width being

$$\delta_T = \min \left[\frac{k_B T}{m v_N}, \left(\frac{2k_B T}{m} \right)^{1/2} \right].$$

The properties of F_η depend upon η and upon the ratio $\xi \equiv w/v_c$:

$$F_\eta(v_N) = \frac{1}{v_c^2} f \left(\frac{v_N}{v_c}, \frac{w}{v_c} \right),$$

$$f(x, \xi) = \frac{x^2}{1+x^2} \frac{\eta}{x^2(x-\xi)^2 + \eta^2(1+x^2)^2}. \quad (15)$$

The function $f(x, \xi)$ for $\eta = 0.1$ is shown in Fig. 2. Below we consider the important limiting cases.

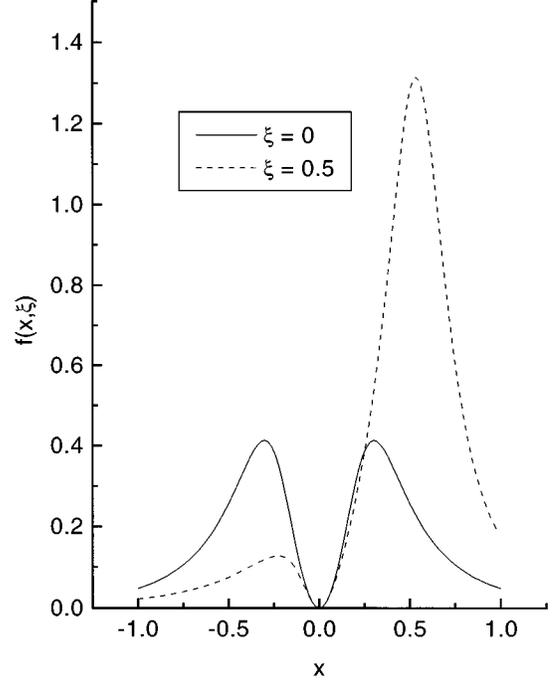


FIG. 2. The function $f(x, \xi)$ for $\eta = 0.1$.

1. Short waves, $\eta \ll 1$

At $\eta \ll 1$ $f(x, \xi)$ depends on the dimensionless parameter $\xi/\sqrt{\eta}$. The most realistic limiting case corresponds to the inequality

$$\xi = w/v_c \ll \sqrt{\eta}. \quad (16)$$

In this case the impurity scattering dominates for the important group of the electrons. If inequality (16) is met, one can set $\xi = 0$, and the integrand is symmetric. Then

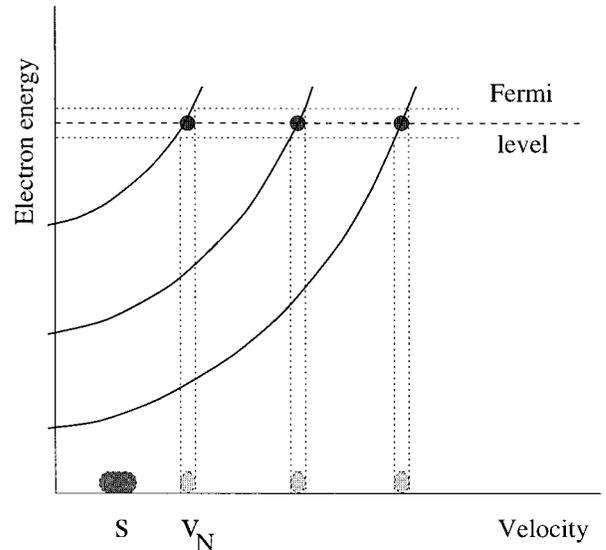


FIG. 3. Scheme of energy and momentum conservation laws for electron-SAW interaction.

$$F_\eta(v_N) = \frac{1}{v_c^2} f\left(\frac{v_N}{v_c}, 0\right),$$

$$f(x, 0) = \frac{x^2}{1+x^2} \frac{\eta}{x^4 + \eta^2(1+x^2)^2}. \quad (17)$$

The function $f(x, 0)$ has a maximum at $x = \pm\sqrt{\eta}$ with the peak value 0.5 and the width $\sim\sqrt{\eta}$. The shape of the oscillations of the acoustoelectric current depends upon the relationship between $v_c\sqrt{\eta}$ and δ_T . At $v_c\sqrt{\eta} \ll \delta_T$ one can replace

$$f(x) \rightarrow \frac{\pi\sqrt{\eta}}{2\sqrt{2}} \sum_{\pm} \delta(x \pm \sqrt{\eta}).$$

As a result,

$$j = \frac{eU^2\omega|\beta_N|^2}{\hbar v_F} \left(\frac{N\ell_b}{2q\alpha}\right)^{1/2} F_T(v_c\sqrt{\eta}). \quad (18)$$

In this case, we have a peak near $v_{NF}=0$ (i.e., exactly at the step), its shape being determined by the derivative of the Fermi function F_T .

At $v_c\sqrt{\eta} \gg \delta_T$ the function F_T behaves as

$$\delta[(m/2)(v_N^2 - v_{NF}^2)] = (mv_{NF})^{-1} \sum_{\pm} \delta(v_N \pm v_{NF}).$$

Consequently,

$$j = \frac{eU^2\omega N\ell_b}{\pi\alpha\hbar v_{NF}\epsilon_F} |\beta_N|^2 f\left(\frac{v_{NF}}{v_c}, 0\right). \quad (19)$$

Note that in both cases the current is independent of the channel's length L . This is natural, because we consider the situation where the intra channel impurity scattering is the most important relaxation mechanism.

At

$$1 \gg w/v_c \gg \sqrt{\eta},$$

$f(x, \xi)$ as a function of x is strongly asymmetric. It has two maxima, the most important being at $x \approx \xi$, the width being $\eta/\xi \ll \xi$. Near the maximum,

$$f(x, \xi) \approx \frac{\eta\xi^2}{\xi^2(x-\xi)^2 + \eta^2}. \quad (20)$$

If $\eta/\xi \ll \delta_T/v_c$, or $\delta_T \gg \alpha v_F^2/(wq\ell_b N)$ one can approximate $f(x, \xi)$ as

$$f(x, \xi) \approx \pi\xi\delta(x-\xi). \quad (21)$$

Consequently, to obtain an estimate for the acoustoelectric current one has to multiply Eq. (18) by $\xi/\sqrt{\eta} \approx (w/v_F)\sqrt{Nq\ell_b/\alpha}$. Otherwise, the estimate is given by Eq. (19) with the replacement of $f(x, 0)$ by $f(x, \xi)$ from Eq. (20).

The escape of nonequilibrium electrons becomes important only at $w \geq v_c$ —a condition which seems not very realistic. Then

$$f(x, \xi) \approx \frac{1}{\xi^2} \frac{\eta}{(x-\xi)^2 + \eta^2}.$$

As a result, the current appears proportional to the channel's length L .²⁰ Measurements of the L dependence of the acoustoelectric current may help to discriminate between different relaxation mechanisms.

2. Long waves, $\eta \gg 1$

If the wave length of the SAW is greater than the channel length, the function $f(x)$ can be approximated as

$$f(x) = \eta^{-1} \frac{x^2}{(1+x^2)^3}. \quad (22)$$

It has a maximum at $x = \pm 1/\sqrt{2}$ and width of the order 1. Consequently, in dimensional variables the width is of the order of v_c . It has to be compared to the difference

$$\delta_v = |v_N - v_{N-1}| \approx \sqrt{4N\epsilon^*/m} \approx v_F N^{-1/2}.$$

If

$$v_c \gg \delta_v \text{ or } L \gg \ell,$$

the oscillations are not pronounced. Conversely, at $L \ll \ell$ the oscillating part is pronounced. Again, its shape is determined by the relationship between the widths of the functions F_η and F_T . At $v_c \gg \delta_T$ the result is given by Eq. (19) with $f(x)$ taken from (22). In the opposite limiting case, $f(x)$ can be replaced by $(\pi/16\eta)\sum_{\pm}\delta(x \pm 1/\sqrt{2})$. Consequently, the shape is determined by the function F_T , like in Eq. (18).

V. DISCUSSION

Let us discuss qualitatively the picture of the acoustoelectric effect. The linear response of the electrons to the SAW with a given wave vector q is proportional to the effective ‘‘interaction time’’ $(qv - \omega)^{-1}$, during which an electron with the velocity v moves in an almost constant field. At small v , or near the resonance ($v = w$), this time diverges, and relaxation becomes important. In fact, the effect is proportional to $\text{Re}[(\bar{I} + i(qv - \omega))^{-1}]$. Consequently, to obtain an effective coupling both the electron velocity and the scattering rate have to be small.

In a homogeneous 2D system, the most important relaxation mechanism is disorder-induced scattering, the scattering rate being proportional to the 2D density of states. Thus, the electron-SAW coupling for such a system is determined by the product $q\ell$. In a point contact relaxation differs from the case of homogeneous 2DEG for the following reasons.

(i) The contact has a *finite length* L , the corresponding *escape* rate being $\sim |v|/L$. This rate decreases as $v \rightarrow 0$.

(ii) The density of states in a quantum point contact is an oscillating function of the energy, it diverges at thresholds corresponding to the filling of additional levels. Consequently, the disorder-induced relaxation rate for the upper mode increases at the threshold.

As a result, the total relaxation rate is a nonmonotonous function of the electron velocity. The analysis given above leads to the conclusion that coupling is optimal for electrons having their velocities in a relatively narrow range, the central velocity $s \sim v_c\sqrt{\eta}$ being small comparing to the Fermi velocity. Above we have estimated the position of the center and an effective width of the important region which is

shown in Fig. 3. On the other hand, electronic states can contribute to interaction only if their energies are in the vicinity of the Fermi level. Consequently, for a given mode number n the electron velocities have to belong to a narrow interval centered around some velocity v_{NF} . One can move these intervals by changing the gate voltage.

To obtain a nonvanishing contribution to the current, one of these regions has to overlap with the region centered around s (see Fig. 3), which is possible only for the upper level. As a result, the acoustoelectric current experiences *giant oscillations* as a function of the gate voltage.²¹ Indeed, in the plateau region for the conductance the velocity v_{NF} is large enough and cannot overlap with the hatched region near s . Then, as the system is driven to the step, v_{NF} decreases, and important regions start to overlap. Consequently, the current increases.

The fine structure of the peaks needs more careful discussion. The current theory can lead to quantitative results only when impurity relaxation is more important than the escape from the channel. In the opposite limiting case the model assumption (11) can provide only order-of-magnitude estimates. The exact results in such a situation depend both on the length and the shape of the channel.²¹

An important source of the fine structure of the peaks could be a gate-voltage-dependent screening of the coupling. This problem needs much more careful considerations. For a rough estimate, we can introduce a screening factor as

$$U^2 = \frac{U_0^2}{1 + g[\sigma(q, \omega)/s]^2},$$

where $\sigma(q, \omega)$ is the effective conductance, while g is a (small) geometry-dependent dimensionless factor. The effective conductance can be estimated as a sum of the contributions of lower transverse modes (which is a smooth function of the gate voltage), and of the contribution of the upper mode

$$\sigma = \sigma_{sm} + \sigma_{osc}.$$

The latter can be estimated near the peak of the drag current in the same way as for a long wire, because usually $L/s \gg \tau$. Considering a piece of wire with the length $\approx q^{-1}$, we obtain the estimate $C \sim q^{-1}$ for its capacitance (up to logarithmic terms), and $R \sim 1/q\sigma_1(q, \omega)$ for its resistance.

Here σ_1 is the 1D conductance calculated from the kinetic equation. As a result, the dimensionless screening parameter can be estimated as $(\omega RC)^{-1} \approx q\sigma_1/s$. Consequently, the screening parameter is

$$\begin{aligned} \sigma_{osc}(q, \omega)/s &= \frac{qe^2}{s\pi\hbar} \int dp (v\hat{B}^{-1}v) \left(-\frac{\partial f_0(\epsilon_{nk})}{\partial \epsilon_{nk}} \right) \\ &\sim \frac{e^2}{s\hbar} \frac{1}{qv_n\tau}. \end{aligned} \quad (23)$$

Far from the maximum of the acoustoelectric current this quantity appears small. However, near the maximum it can be of the order 1, leading to a decrease of the current. Probably this is the origin of the double-peak structure of the first peak of the acoustoelectric current, observed in Ref. 10. With the increase of the number of occupied modes the smooth part of screening increases. As a result, the current oscillations' amplitude must decrease, and the double-peak structure has to be less pronounced. Such a behavior is in a qualitative agreement with the experiment.¹⁰

Unfortunately, it is very difficult to give realistic estimates for the coupling constant connecting the intensity S of SAW and the amplitude U of the electron's potential energy. According to the experimental results,¹⁰ we believe that it is determined by piezoelectric interaction in the channel. Otherwise, under the conditions realized in Ref. 10 steps predicted in Ref. 13 would be observed rather than giant oscillations. However, the SAW in a layered structure has a complicated polarization structure, and only a rough estimate can be given

$$U_0 \sim \sqrt{\chi S e^2 / \omega}.$$

Here χ is piezoelectric coupling constant which can be determined by experiments on the change of sound velocity due to 2DEG. To check the presented theory quantitatively, it seems important to measure acoustic intensity independently.

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