Displaced squeezed number states of the phonon field in polar semiconductors

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Considering that the *Fröhlich continuum* model of polarons, in the *static* approximation, describes the electron-phonon interaction in polar semiconductors, and that the Hamiltonian of the generalized parametric oscillator represents the ion vibrations, we have studied a way of producing phonon-displaced squeezed number states. By the use of the evolution operator method, the exact wave function as well as the probability density are obtained. In order to see explicitly the wave function's squeezing property, we have given the analytical forms of the variances Δq and Δp. Dealing with the radiative recombination process, which occurs at imperfections, we have derived the phonon number distribution. A calculation of Mandel's Q parameter, which accounts for the kind of the distribution, and of the Huang-Rhys factor S, are also presented. An interesting result is obtained when we study the case of a simple driven harmonic oscillator, whose Q parameter imposes the value 0.5 on the factor S, in order to have a Poissonian distribution. Any deviation from this value yields sub- or super-Poissonian distributions which characterize the photoluminescence (PL) spectrum, as regards the sharpness of the PL lines and the number of phonons involved in the recombination process. With regard to the time dependence of the factor S and of the Hamiltonian representing the ion vibrations, unexpected values (in comparison with the time-independent case) for the Q parameter can be found. This behavior can affect the number of emitted phonons, defined by the kind of the phonon number distribution.
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I. INTRODUCTION

It is well known that squeezed states are nonclassical states, which were first introduced and studied in the field of quantum optics with the ultimate aim to obtain a reduced fluctuation in one field quadrature, at the expense of an increased fluctuation in the other,[1] leading to an increase in the signal-to-noise ratio in suitable experiments ranging from optical communication to the detection of gravitational radiation.[2,3]

Among the theoretical schemes that have been proposed to generate squeezed light is that of harmonic oscillator with a time-dependent frequency.[4–10] Most of the techniques that have been used are based on scattering experiments in nonlinear systems,[1,4,6,7] which can be described approximately by the Bogoliubov Hamiltonian.

Considerable attention has also been paid to generalizations of squeezed states, mainly to the displaced and squeezed number states.[11–15] Actually, as has been proven, these states can be generated by displacing the oscillator and changing its frequency.[11] More specifically, considering a time-dependent harmonic oscillator, prepared initially in the nth number state ($n=1,2...$) and driven by a transient, spatially uniform external force, we finally obtain the corresponding displaced squeezed number state. The case $n=0$ coincides exactly with the displaced squeezed vacuum state or simply squeezed state.[1]

An interesting quantity of a time-dependent harmonic oscillator is the set of time-dependent transition probabilities among the nth number states. The associated transition amplitudes are defined as corresponding matrix elements of the time evolution operators. Such amplitudes play an important role in the time-dependent formulation of molecular spectroscopic phenomena such as Raman scattering and absorption spectra.[16,17]

In the present paper we consider the case of polar doped semiconductors, such as CdTe, whose electronic quality can be assessed by photoluminescence (PL),[18,20,21] from the strength and sharpness of the spectral components. Using the adiabatic approximation, we can study the properties of the (boson) phonon field according to Refs. 18 and 19.

Assuming that the Hamiltonian which represents ion vibrations has the form (in the harmonic approximation) of the generalized parametric oscillator (in our analysis, we restrict ourselves to one vibrational mode and a basis of two ions) with $SU(1,1)$ algebraic structure,[22] we shall prove that the phonon field prepared initially in the nth number state ($n=1,2...$) and driven by an external force, evolves to a displaced squeezed number state, for appropriate values of the time-dependent parameters which appear in the Hamiltonian of the parametric oscillator.[23] The limited spreading Δq of the corresponding wave packets leads to the generation of localized[24] vibrational states. Localized phonon states are known in the literature[25,26] to arise due to the modifications which occur in the vibrational spectrum of the lattice in presence of isolated defects [see Eqs. (28) and (46)].

A study of polar semiconductors leads to taking into account the *Fröhlich continuum* model of polarons.[18,19,25,27] Presupposing a modification in the phonon dispersion spectrum, caused by the presence of time-dependent factors in the generalized parametric oscillator form, we give an expression for a charge-carrier–phonon interaction Hamiltonian, in the *static* approximation.[19] The effect of this interaction is to displace the equilibrium position of the ions, and its form has an $h(4)$ algebraic structure.[28]

Thus the Hamiltonian we are dealing with has the form[29]

$$H(t)=H_{\text{ion}}(t)+H_{\text{el-ph}}(t), \tag{1}$$

with

$$H_{\text{ion}}(t)=\omega(t)(a^+a+\tfrac{1}{2})+\xi(t)a^2+\xi^*(t)a^{+^2} \tag{2}$$

and

$$H_{\text{el-ph}}(t)=r(t)a+r^*(t)a^+. \tag{3}$$

It is worthwhile to stress that the use of the Fröhlich model of polarons assumes that the dominant contribution to the electron-phonon interaction is the polar interaction with long-wavelength longitudinal-optical (LO) phonons. It is also well known that the phonon number distribution among distinct number states, corresponding to the ion vibrational states before and after the electronic transitions at imperfections,[25] decides the shape of the emission (or absorption) line or band.

In the present work we are interested in radiative recombination processes starting from an initial state where the hole is bound to an acceptor impurity and the electron is either a free conduction electron or a donor-bound electron.[18] During electron-hole recombination, we assume that the phonon field evolves from an initial state described by a displaced squeezed phonon field to a state where the electron-hole recombination has taken place, i.e., a free LO-phonon field in a number state $|m\rangle$. In this regime, calculating the phonon number distribution, we obtain a form which accounts for the determination of the existing photoluminescence spectrum. Furthermore, based on the fact that the evolution operator corresponding to the Hamiltonian (3) is just the Weyl displacement operator multiplied by a phase factor,[1,11] we can obtain the explicit form of the Huang-Rhys factor,[30] appearing in the phonon number distribution.

Continuing to examine the nonclassical effects, we proceed to a calculation of Mandel's Q parameter,[1,31] finding the necessary condition to have sub-Poissonian, super-Poissonian, or simply Poissonian distribution, for both time-dependent and -independent harmonic oscillators. The above condition leads to the interesting result that for the case of the simple driven harmonic oscillator,[18] where there is no squeezing, the phonon number distribution is sub-Poissonian for Huang-Rhys factor values less than 0.5, and super-Poissonian for values larger than 0.5. This result determines the PL spectrum of CdTe,[18] as regards the number of phonons involved in the recombination process and the sharpness of the observed lines or bands.

The organization of the paper is as follows. In Sec. II we briefly review the evolution operator method, which is based on the $SU(1,1)\oplus h(4)$ algebraic structure of the Hamiltonian (1) and the Wei-Norman theorem.[28] We obtain the exact wave function, as well as the probability density, assuming that the phonon system is initially prepared in the nth number state. Also we prove that this wave function is a number state with respect to the operator $N(t)=A^+(t)A(t)$, where the operator $A(t)$ is related to the usual operator a by a Bogoliubov transformation plus a translation. By finding the coefficients of this transformation, we imply that the wave function we have obtained is a squeezed state.[1,9] In Sec. III A, by use of the Fröhlich continuum model of polarons, we give the explicit form of the Hamiltonian that represents the electron-phonon interaction in its modified version, due to the time dependence of the Hamiltonian that represents the lattice vibrations. Also in Sec. III B we cite the analytic form of the Huang-Rhys factor, which, as is known, is a measure of the charge-carrier–LO-phonon interaction for the time-independent case.[18,25,30] Section IV is concerned with nonclassical effects such as squeezing and sub-Poissonian statistics. In order to see explicitly the squeezing property of the phonon wave functions, which have already been calculated, we proceed to a computation of the variances Δq and Δp. Furthermore, we give the analytical form of Mandel's Q parameter, finding the necessary condition which characterizes the kind of distribution. We also study the special case of a time-independent driven oscillator, obtaining for the Huang-Rhys factor the value 0.5, in order to have a Poissonian distribution. Any value larger or less than 0.5 will lead to the appearance of super- or sub-Poissonian distributions, respectively. Finally we give the exact forms of the occupation probabilities both for displaced squeezed number and vacuum states. Section V is devoted to concluding remarks.

II. EVOLUTION OPERATOR METHOD—EXACT WAVE FUNCTION

Let us consider the Hamiltonian of the driven generalized parametric oscillator,[28] which is given by Eq. (1) or

$$H(t)=H_{\text{ion}}(t)+H_{\text{el-ph}}(t), \tag{4}$$

where

$$H_{\text{ion}}(t)=\frac{1}{2}\left[Z(t)\ \frac{p^2}{m}+\omega Y(t)(qp+pq)+X(t)m\omega^2q^2\right] \tag{5}$$

is the Hamiltonian of the generalized parametric oscillator,[22] and

$$H_{\text{el-ph}}(t)=\mu(t)q+\nu(t)p \tag{6}$$

is the driving term, where $X(t)$, $Y(t)$, $Z(t)$ $\mu(t)$, and $\nu(t)$ are in general nonsingular functions of time, and m is the reduced mass.

The relationship between Eqs. (1) and (4) is clearly demonstrated by the following equations;

$$\omega(t)=\frac{\hbar\omega}{2}\,[X(t)+Z(t)], \tag{7}$$

$$\xi(t)=\frac{\hbar\omega}{4}\,[X(t)-Z(t)-2iY(t)], \tag{8}$$

$$r(t)=\left(\frac{\hbar}{2m\omega}\right)^{1/2}\mu(t)-i\left(\frac{\hbar m\omega}{2}\right)^{1/2}\nu(t). \tag{9}$$

Taking advantage of knowing the ''unperturbed'' evolution operator [which corresponds to $H_{\text{ion}}(t)$], the explicit coordinate representations of the generators of the Weyl group and the one-dimensional Lorentz group, we express the evolution operator corresponding to $H(t)$ as[28]

$$U(t)=U_{\text{ion}}(t)U_I(t), \tag{10}$$

where $U_{\text{ion}}(t)$ and $U_I(t)$ satisfy the equations

$$i\hbar \frac{\partial U_{\text{ion}}}{\partial t} = H_{\text{ion}}(t) U_{\text{ion}}(t), \quad U_{\text{ion}}(0) = I, \tag{11}$$

$$i\hbar \frac{\partial U_I}{\partial t} = H_I(t) U_I(t), \quad U_I(0) = I, \tag{12}$$

where

$$H_I(t) = U^{+}_{\text{ion}}(t) H_{\text{el-ph}}(t) U_{\text{ion}}(t). \tag{13}$$

The ''unperturbed'' operator $U_{\text{ion}}(t)$ admits the Wei-Norman (WN) form

$$U_{\text{ion}}(t) = e^{\Lambda(t)} e^{\alpha(t) q^2} e^{b(t) q (\partial/\partial q)} e^{c(t)(\partial^2/\partial q^2)}, \tag{14}$$

where the WN characteristic functions $\Lambda(t)$, $\alpha(t)$, $b(t)$, and $c(t)$ are given in analytic form in Ref. 22. Replacing (14) in (13), with the help of (6), we obtain

$$H_I(t) = K(t) q - i N(t) \frac{\partial}{\partial q}, \tag{15}$$

where the functions $K(t)$ and $N(t)$ are given in Ref. 28.

Then, by Eq. (12) and, because of the $h(4)$ structure of $H_I(t)$, we obtain

$$U_I(t) = e^{h(t) q} e^{f(t)(\partial/\partial q)} e^{g(t)} \tag{16}$$

where

$$h(t) = -\frac{i}{\hbar} \int_0^t K(t') dt', \tag{17}$$

$$f(t) = -\frac{1}{\hbar} \int_0^t N(t') dt', \tag{18}$$

$$g(t) = \int_0^t h(t') \dot{f}(t') dt', \tag{19}$$

Once the evolution operator is known, we can find the exact wave function at any later time, supposing that we start with a number state at $t=0$,[11]

$$|\Psi(0)\rangle = |n\rangle; \tag{20}$$

that is, an eigenstate of the number operator $N = a^{+} a$,

$$N|n\rangle = n|n\rangle, \tag{21}$$

with

$$a = \frac{1}{2\hbar} \left(\sqrt{m\omega} q + i \frac{p}{\sqrt{m\omega}} \right). \tag{22}$$

The wave function at any later time with the help of (10), (14), (16), and (20) will be represented by

$$|\Psi(t)\rangle = U(t) |\Psi(0)\rangle \tag{23}$$

or

$$|\Psi(t)\rangle = \frac{1}{(2^n n!)^{1/2}} \left(\frac{m\omega}{\pi\hbar} \right)^{1/4} e^{\tilde{\Lambda}(t)} e^{\alpha(t) q^2} e^{-h(t) f(t)} e^{-[q e^{b(t)} + f(t)]^2 / 4c(t)} e^{[q e^{b(t)} + f(t) + 2h(t) c(t)]^2 / 4c(t)[1 + (2m\omega/\hbar) c(t)]}$$
$$\times \frac{\left[1 - \frac{2m\omega}{\hbar} c(t) \right]^{n/2}}{\left[1 + \frac{2m\omega}{\hbar} c(t) \right]^{(n+1)/2}} H_n \left[\left(\frac{m\omega}{\hbar} \right)^{1/2} \frac{q e^{b(t)} + f(t) + 2h(t) c(t)}{\left(1 - \frac{4m^2\omega^2}{\hbar^2} c^2(t) \right)^{1/2}} \right], \tag{24}$$

with

$$\tilde{\Lambda}(t) = \Lambda(t) + g(t). \tag{25}$$

Now we can define an operator $A(t)$ as[4,9,11]

$$A(t) = U(t,0) a U^{+}(t,0), \tag{26}$$

and it is easy to see that the wave function is a number state with respect to the operator $N(t) = A^{+}(t) A(t)$,

$$N(t) |\Psi(t)\rangle = n |\Psi(t)\rangle. \tag{27}$$

Using (10), (14), (16), and (22), it can be shown the operator a is related to the operator $A(t)$ by a Bogoliubov transformation plus a translation

$$A(t) = l_1(t) a + l_2(t) a^{+} + \beta(t), \tag{28}$$

with

$$|l_1(t)|^2 - |l_2(t)|^2 = 1 \tag{29}$$

and

$$l_1(t) = \tfrac{1}{2}(e^{b(t)} + e^{-b(t)}) + e^{-b(t)} \left(\frac{m\omega}{\hbar} c(t) - 2\alpha(t) c(t) \right) - \frac{\hbar}{m\omega} \alpha(t) e^{-b(t)}, \tag{30}$$

$$l_2(t) = \tfrac{1}{2}(e^{b(t)} - e^{-b(t)}) - e^{-b(t)} \left(\frac{m\omega}{\hbar} c(t) + 2\alpha(t) c(t) \right) - \frac{\hbar}{m\omega} \alpha(t) e^{-b(t)}, \tag{31}$$

$$\beta(t)=\left(\frac{m\omega}{2\hbar}\right)^{1/2}f(t)-\left(\frac{\hbar}{2m\omega}\right)^{1/2}h(t). \tag{32}$$

Also after some algebra we obtain the corresponding probability density, which has the following form:

$$|\Psi(t)|^2=\frac{1}{2^n n!}\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\frac{e^{b(t)}}{\left(1-\frac{4m^2\omega^2c^2(t)}{\hbar^2}\right)^{1/2}}\times\exp\left\{-\frac{m\omega}{\hbar}\frac{[qe^{b(t)}+f(t)+2h(t)c(t)]^2}{\left(1-\frac{4m^2\omega^2}{\hbar^2}c^2(t)\right)}\right\}\times H_n^2\left[\left(\frac{m\omega}{\hbar}\right)^{1/2}\frac{qe^{b(t)}+f(t)+2h(t)c(t)}{\left(1-\frac{4m^2\omega^2c^2(t)}{\hbar^2}\right)^{1/2}}\right], \tag{33}$$

with the evident condition

$$\int_{-\infty}^{+\infty}|\Psi(t)|^2dq=1. \tag{34}$$

III. FRÖLICH MODEL OF POLARONS

A. Electron-phonon interaction

According to Ref. 22, the classical equation of motion for the generalized parametric oscillator Eq. (5) has the form

$$\ddot{q}+\omega^2(t)q=0, \tag{35}$$

where

$$\omega^2(t)=\omega^2(XZ-Y^2)-\omega Z\frac{d}{dt}\left(\frac{Y}{Z}\right)+\frac{1}{2}\left(\frac{\ddot{Z}}{Z}-\frac{3}{2}\frac{\dot{Z}^2}{Z^2}\right). \tag{36}$$

The frequency $\omega(t)$ can be encountered as a modified version of the longitudinal-optical branch frequency ω,[21] due to the existence of the time-dependent factors $X(t)$, $Y(t)$, and $Z(t)$. As can easily be seen for the case of a simple harmonic oscillator, e.g., $X(t)=Z(t)=1$, $Y(t)=0$ coincides exactly with the usual frequency ω, e.g.,

$$\omega(t)=\omega. \tag{37}$$

By the use of the Fröhlich continuum model,[32] the Hamiltonian which represents the electron-phonon interaction in the *static* approximation,[19] taking into consideration Eq. (36), has the form

$$H_{\text{el-ph}}(t)=R(t)\Phi(\mathbf{k},\mathbf{r})\widetilde{A}(t)+R^*(t)\Phi^*(\mathbf{k},\mathbf{r})\widetilde{A}^+(t), \tag{38}$$

with

$$R(t)=\frac{i}{|k|}\left\{\frac{e^2\hbar\omega^2(t)}{2\varepsilon_0V\omega}\left[\frac{1}{\varepsilon(\infty)}-\frac{1}{\varepsilon(0)}\right]\right\}^{1/2}, \tag{39}$$

$$\Phi(\mathbf{k},\mathbf{r})=(e^{i\mathbf{k}\cdot\mathbf{R}}-1+e^{i\mathbf{k}\mathbf{r}_e}-e^{i\mathbf{k}\cdot\mathbf{r}_h}), \tag{40}$$

$$\widetilde{A}(t)=l_1(t)a+l_2(t)a^+. \tag{41}$$

Also, $\mathbf{R}$, $\mathbf{r}_e$, and $\mathbf{r}_h$ denote the positions of the acceptor center, the electron, and the hole, respectively.[19] V is the volume of the lattice, $\varepsilon(\infty)$ and $\varepsilon(0)$ are the high frequency and static dielectric constants, e is the charge of electron, $|k|$ denotes the measure of the phonon wave vector $\mathbf{k}$, and ε_0 is the electrical permittivity of free space.

With the help of Eq. (41), Eq. (38) takes the form

$$H_{\text{el-ph}}(t)=[l_1(t)R(t)\Phi(\mathbf{k},\mathbf{r})+l_2^*(t)R^*(t)\Phi^*(\mathbf{k},\mathbf{r})]a+[l_1^*(t)R^*(t)\Phi^*(\mathbf{k},\mathbf{r})+l_2(t)R(t)\Phi(\mathbf{k},\mathbf{r})]a^+. \tag{42}$$

B. Huang-Rhys factor

According to Eqs. (3), (9), and (38), we can obtain the following forms for the real functions appearing in Eq. (6):

$$\mu(t)=\left(\frac{m\omega}{2\hbar}\right)^{1/2}\{[l_1(t)+l_2(t)]R(t)\Phi(\mathbf{k},\mathbf{r})+[l_1^*(t)+l_2^*(t)]R^*(t)\Phi^*(\mathbf{k},\mathbf{r})\}, \tag{43}$$

$$\nu(t)=\frac{i}{(2\hbar m\omega)^{1/2}}\{[l_1(t)-l_2(t)]R(t)\Phi(\mathbf{k},\mathbf{r})+[l_2^*(t)-l_1^*(t)]R^*(t)\Phi^*(\mathbf{k},\mathbf{r})\}, \tag{44}$$

where m represents the reduced mass of the electron and hole in the static approximation.[19]

Now, based on the fact that the evolution operator which corresponds to the Hamiltonian (42) is the Weyl displacement operator,[11] according to Refs. 18 and 19 we can obtain the Huang-Rhys factor S as follows:

$$S(t)=|\beta(t)|^2, \tag{45}$$

where $\beta(t)$ is given by (32), and the functions $h(t)$ and $f(t)$ by (17) and (18), respectively.

IV. NONCLASSICAL EFFECTS

A. Squeezing effect

As is well known (see, for instance, Refs. 1, 4, 9, and 11) relation (29) implies that the wave function represents a displaced squeezed number state. To see its squeezing property explicitly, we will compute the variance of q and p. After a lengthy calculation, we obtain the following results, with respect to the wave function $|\Psi(t)\rangle$:

$$\Delta q=(n+\tfrac{1}{2})^{1/2}\left(\frac{\hbar}{m\omega}\right)^{1/2}|l_1(t)-l_2(t)| \tag{46}$$

or

$$\Delta q=(n+\tfrac{1}{2})^{1/2}W(t), \tag{47}$$

with

$$W(t)=\left(\frac{\hbar}{m\omega}\right)^{1/2}e^{-b(t)}\left(1-\frac{4m^2\omega^2c^2(t)}{\hbar^2}\right)^{1/2} \tag{48}$$

and

$$\Delta p=(n+\tfrac{1}{2})^{1/2}(\hbar m\omega)^{1/2}|l_1(t)+l_2(t)| \tag{49}$$

or

$$\Delta p=(2n+1)^{1/2}\hbar e^{b(t)}\left[\frac{m\omega/\hbar}{\left(1+\frac{2m\omega c(t)}{\hbar}\right)}-2\alpha(t)e^{-2b(t)}\right]^{1/2}\left\{1+\left[2\alpha(t)-\frac{m\omega}{\hbar}\frac{e^{2b(t)}}{\left(1+\frac{2m\omega c(t)}{\hbar}\right)}\right]W^2(t)\right\}^{1/2}. \tag{50}$$

As can be checked,

$$\Delta q\Delta p\geqslant\frac{\hbar}{2}(2n+1). \tag{51}$$

Thus, under appropriate values of the time-dependent parameters $X(t)$, $Y(t)$, and $Z(t)$ of the generalized parametric oscillator (5), squeezing in one of the quadrature variances can be obtained.

B. Sub-Poissonian statistics

We calculate the following averages in the state $|\Psi(t)\rangle$ [Eq. (24), with the help of Eq. (27)]:

$$\langle n\rangle=\langle\Psi(t)|a^+a|\Psi(t)\rangle=|l_1(t)|^2n+|l_2(t)|^2(n+1)+|\Gamma(t)|^2, \tag{52}$$

$$\begin{aligned}\langle n^2\rangle=&\langle\Psi(t)|(a^+a)^2|\Psi(t)\rangle\\=&|l_1(t)|^4n^2+2|l_1(t)|^2|l_2(t)|^2(2n^2+2n+1)\\&+2|l_1(t)|^2|\Gamma(t)|^2n+|l_2(t)|^4(n+1)^2\\&+2(n+1)|\Gamma(t)|^2|l_2(t)|^2+(2n+1)|L(t)|^2\\&+|\Gamma(t)|^4,\end{aligned} \tag{53}$$

where

$$L(t)=l_1^*(t)\Gamma(t)-l_2^*(t)\Gamma^*(t), \tag{54}$$

$$\Gamma(t)=l_2^*(t)\beta(t)-l_1(t)\beta^*(t). \tag{55}$$

The function that allows one to verify the occurrence, or not, of sub-Poissonian statistics of the phonon field (in accordance with the boson field of light), is given by Mandel's well-known Q parameter[1,31]

$$\begin{aligned}Q=&\frac{\langle(\Delta n)^2\rangle-\langle n\rangle}{\langle n\rangle}\\=&\frac{2|l_1(t)|^2|l_2(t)|^2(n^2+n+1)+|L(t)|^2(2n+1)}{|l_1(t)|^2n+|l_2(t)|^2(n+1)+|\Gamma(t)|^2}-1.\end{aligned} \tag{56}$$

The distribution of the phonon field is sub-Poissonian (super-Poissonian or simply Poissonian) if $Q<0$ ($Q>0$ or $Q=0$).

Of importance is the case of the driven harmonic oscillator, e.g., $X(t)=Z(t)=1$, or $Y(t)=0$. As can be seen, Mandel's Q parameter implies that

$$|\beta|^2(2n+1)\geqslant_{(<)}n+|\beta|^2. \tag{57}$$

Actually for $|\beta|^2=0.5$ we have a Poissonian distribution, and any value of $|\beta|^2$ less or larger than 0.5 will lead to a sub- or super-Poissonian distribution (although there is no squeezing). It is also worthwhile to emphasize that these distributions interpret the sharpness of the experimentally observed lines of the (PL) spectrum in CdTe,[18] and also determine the number of phonons involved in the recombination process.

The phonon number distribution for a transition between an initial state $|\Psi(t)\rangle$ [relation (24)] to a free state $|m\rangle$, after a lengthy calculation, takes the form

$$\begin{aligned}P_{nm}(t)=&|\langle m|\Psi(t)\rangle|^2=\frac{1}{2^nn!2^mm!}\frac{e^{b(t)}}{\left(1-\frac{4m^2\omega^2c^2(t)}{\hbar^2}\right)^{1/2}}\\&\times\exp\left[-\frac{m\omega}{\hbar}\frac{L^2(t)}{\left(1-\frac{4m^2\omega^2c^2(t)}{\hbar^2}\right)}\right]\\&\times\exp\left[\frac{1}{4}\left(\frac{\sigma^2(t)}{\rho(t)}+\frac{\sigma^{*^2}(t)}{\rho^*(t)}\right)\right]\frac{1}{\sqrt{\rho(t)}}|I(t)|^2,\end{aligned} \tag{58}$$

where $I(t)$ is given by the following relation:[33]

$$\begin{aligned}I(t)=&\sum_{\kappa_1=0}^{m}\sum_{\kappa_2=0}^{n}\binom{m}{\kappa_1}\binom{n}{\kappa_2}2^{k_1+k_2}\\&\times(-1)^{k_1}\left(\frac{\sigma(t)}{2\rho(t)}\right)^{k_1}\left[\frac{\left(\frac{m\omega}{\hbar}\right)^{1/2}[f(t)+2h(t)c(t)]-e^{b(t)}\frac{\sigma(t)}{2\rho(t)}}{\left(1-\frac{4m^2\omega^2c^2(t)}{\hbar^2}\right)^{1/2}}\right]^{\kappa_2}n!\left(\frac{1-\rho(t)}{\rho(t)-\chi^2(t)}\right)^{(m-n-\kappa_1+\kappa_2)/4}\\&\times\left\{2\left[\frac{1}{\rho(t)}[1+\chi^2(t)]-1\right]^{1/2}\right\}^{(m+n-\kappa_1-\kappa_2)/2}P_{|m+n-\kappa_1-\kappa_2|/2}^{|m-n-\kappa_1+\kappa_2|/2}\left[\frac{\chi(t)}{\sqrt{\rho(t)}}[1+\chi^2(t)-\rho(t)]^{-1/2}\right]\end{aligned} \tag{59}$$

[$P(x)$ denotes the Legendre polynomials] and

$$\chi(t)=\frac{1}{\sqrt{2}}\frac{|\sigma(t)|}{|\beta(t)|}, \tag{60}$$

$$\rho(t)=\frac{1}{2}-\frac{\hbar}{m\omega}\,\alpha(t)+\frac{e^{2b(t)}}{2\left(1+\frac{2m\omega c(t)}{\hbar}\right)}, \tag{61}$$

$$\sigma(t)=\frac{\sqrt{2}e^{b(t)}\beta(t)}{1+\frac{2m\omega c(t)}{\hbar}}, \tag{62}$$

$$L(t)=2h(t)c(t)+f(t). \tag{63}$$

In addition, the phonon number distribution for the usual squeezed states (e.g. $n=0$) has the form[34]

$$P_{n,0}(t)=\frac{1}{2^n n!}\frac{e^{b(t)}}{\left(1-\frac{4m^2\omega^2c^2(t)}{\hbar^2}\right)^{1/2}}\exp\left[-\frac{m\omega}{\hbar}\frac{L^2(t)}{\left(1-\frac{4m^2\omega^2c^2(t)}{\hbar^2}\right)}\right]\exp\left[\frac{1}{4}\left(\frac{\sigma^2(t)}{\rho(t)}+\frac{\sigma^{*2}(t)}{\rho^*(t)}\right)\right]\frac{1}{|\rho(t)|}\left[1-\frac{1}{\rho(t)}\right]^{n/2}$$
$$\times\left[1-\frac{1}{\rho^*(t)}\right]^{n/2}H_n\left[\frac{\sigma(t)}{2\rho(t)\left(1-\frac{1}{\rho(t)}\right)^{1/2}}\right]H_n^*\left[\frac{\sigma^*(t)}{2\rho^*(t)\left(1-\frac{1}{\rho^*(t)}\right)^{1/2}}\right]. \tag{64}$$

In the case where we have $X(t)=Z(t)=1$, $Y(t)=0$ (e.g., the simple driven harmonic oscillator), the distribution takes, as expected, the form of a Poissonian,[34]

$$P_{n,0}(t)=\frac{1}{n!}\,e^{-S(t)}S^n(t), \tag{65}$$

with

$$S(t)=\frac{|\sigma(t)|^2}{2}=|\beta(t)|^2, \tag{66}$$

and Mandel's Q parameter is equal to zero, as can easily be seen from relations (56) and (57).

V. CONCLUSION

In the present work we have studied the possibility of generating displaced squeezed number states of the phonon field, using (in the harmonic approximation) a driven time-dependent Hamiltonian with an $SU(1,1)\oplus h(4)$ algebraic structure. Using the Fröhlich continuum model of polarons, we have attributed the driving term of the above Hamiltonian to the electron-phonon interaction.

Assuming that the phonon field is initially prepared in the nth number state, we have obtained the exact form of the evolved wave function, using an algebraic operator technique that has been developed in our previous papers. As it can be proved, this wave function is a displaced squeezed number state, for appropriate values of the time-dependent parameters appearing in the $SU(1,1)$ part of the total Hamiltonian.

We are dealing with a radiative recombination process starting from an initial state where the hole is bound to an acceptor impurity, and the electron is either a free conduction electron or a donor-bound electron. In this regime we have calculated the exact phonon number distribution, for transitions between an initial state described by a displaced squeezed number state, to a state where the electron-hole recombination has taken place, e.g., a free LO-phonon field in a state $|m\rangle$. Furthermore, based on the fact that the evolution operator corresponding to the electron-phonon interaction Hamiltonian (Frohlich-type interaction) in the static approximation is the Weyl displacement operator, we have obtained the form of the Huang-Rhys factor (for one electron-hole case), which is time-dependent $S(t)$.

In order to determine the shape of the PL spectrum lines, we calculated Mandel's Q parameter, finding the necessary condition for observing sub-Poissonian, super-Poissonian, or simply Poissonian phonon-number distributions. Specifically, studying the case of a simple driven time-independent harmonic oscillator (although there is no squeezing), the condition $Q=0$, which insures Poissonian distribution, imposes the value 0.5 on the Huang-Rhys factor S. Any deviation from this value leads to the appearance of phonon-number-*squeezed* ($S<0.5$) or -*enhanced* ($S>0.5$) distributions. This result determines the form of the observed PL spectrum[18] for the case of CdTe, as regards the sharpness of the PL lines and the corresponding number of phonons involved in the recombination process. It is therefore evident that our results are in agreement with the experimental results for CdTe. Actually the authors of Ref. 18, studying the band at 1.54 eV which is not present in the spectrum of the undoped sample, and the usual band at 1.45 eV which is present in doped and undoped CdTe, deduced, by an overall fit of the measurements using a Gauss function, the Huang-Rhys factor $S=0.30\pm0.02$ for the band at 1.54 eV, and $S=1.3\pm0.1$ for the broadband at 1.45 eV (see Fig. 4 in Ref. 18).

In our analysis we predict the form of the bands, and provide insight for the interpretation of the experimental data in terms of sub-Poissonian and super-Poissonian phonon distributions. Giving the phonon number distribution for a radiative recombination process, between an initial state described by a simple squeezed state to a state where electron-hole recombination has taken place, we point out that in the case of a time-independent oscillator the distribution is always Poissonian for any value of the Huang-Rhys factor S.

As is easily understood from the above analysis, the initiation of time in the Hamiltonian describing the ion vibrations leads to the generation of phonon-displaced squeezed number states with limited spreading, for appropriate values of the parameters $X(t)$, $Y(t)$, and $Z(t)$ and to a time-dependent Huang-Rhys factor S. The time dependence of both $H_{\text{ion}}(t)$ and $S(t)$ can cause the existence of unexpected values (compared to the time-independent case) for Mandel's Q parameter, affecting the zero-phonon, one-phonon, and multiphonon processes, by means of a possible change in the distribution shape, as is expected for the time-independent case.

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