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Plasma instabilities in a steady-state nonequilibrium one-dimensional solid-state plasma of finite length

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We show theoretically that strong plasma mode generation is possible in a nonequilibrium steady-state quasi-one-dimensional bounded solid-state plasma, in which a nonequilibrium distribution is maintained by appropriate injection/extraction of carriers. We calculate the density response of realistic model systems using the random-phase approximation, determine the normal modes of the bounded carrier plasma, and show that strong plasma instabilities can be generated under suitable conditions. Such stimulated plasma oscillations could lead to sources of terahertz electromagnetic radiation. [S0163-1829(96)01135-6]

I. INTRODUCTION

Nonequilibrium plasmas can develop plasma oscillations, since for the excited particles of such plasmas this is one of the available energy dissipation channels.[1] This can be viewed as the result of plasma wave generation due to net downwards (in energy) single-particle transitions, arising from population inversion in the particle distribution. This effect can be used to generate or amplify electromagnetic radiation. When a constant current is applied to drive the plasma away from equilibrium, the resulting spontaneous plasma wave generation is called the current-driven plasma instability (CDPI). While CDPI's are well known in gaseous plasmas, to our knowledge their solid-state analogs have not yet been directly observed.[2,3] This is due to the fact that a direct transfer of energy from a current into a plasma mode requires that the carrier velocities exceed a certain threshold. This threshold velocity in solid-state systems is of the order of the Fermi velocity.[4–8] In uniform solid-state plasmas, even those at modulation-doped heterojunctions where scattering with phonons is strongly reduced, it is difficult to accelerate carriers to reach this very high threshold without generating strong plasma heating. In modulated lower-dimensional plasmas, such as quantum wires with a superposed periodic potential modulation along the length of the wire, the threshold velocity can be substantially reduced.[9] Unlike the unmodulated systems, the instability can then occur in the domain of essentially cold electron transport, and thus the driving fields do not enhance the dissipative collisions. Even though this scenario is quite promising, the generation of plasma waves could be possible only for low temperatures, rendering many applications of this phenomenon impractical.

In this paper we examine in detail an idea for achieving plasma wave generation, and subsequent decay of these plasma waves into electromagnetic radiation.[3] We consider a *bounded* solid-state plasma which can accommodate several eigenmodes. Such a system plays the role of a plasma mode resonator.[3] For the pump mechanism we assume an energetically selective injection and extraction process. By reducing the active size of the device below the corresponding mean free path for phonon scattering, one can overcome collisional losses while still maintaining electron-electron interactions, necessary for the development of collective effects. The charge-density oscillations of a bounded plasma couple directly to the electromagnetic radiation, and no additional coupling mechanism is thus necessary to convert the plasma wave energy into electromagnetic radiation.

II. THEORY

A model which simulates the conditions of the proposed idea consists of a quantum well confined by infinite potential walls located at $x=0$ and $x=L$. A given potential profile is assumed between the walls. We first consider a quasi-one-dimensional well. In the random-phase approximation (RPA) the density response of electrons to the external potential perturbation of the form $V_{\rm ext}(x;t)=V_{\rm ext}(x,\omega)\exp(-i\omega t)$ with frequency ω, is given by

$$\delta\rho(x;\omega)=\int dx'\,\chi_0(x,x';\omega)V_T(x';\omega), \tag{1}$$

where $\chi_0(x,x';\omega)$ is the single-electron susceptibility given by

$$\chi_0(x,x';\omega)=2\sum_{\varepsilon}\sum_{\varepsilon'}\frac{f_\varepsilon-f_{\varepsilon'}}{\varepsilon-\varepsilon'+\omega}\,\Psi_\varepsilon(x)\Psi_{\varepsilon'}(x)\Psi_\varepsilon(x')\Psi_{\varepsilon'}(x'). \tag{2}$$

ε and $\Psi_\varepsilon(x)$ are single-electron eigenvalues and eigenfunctions, respectively, corresponding to a chosen ground-state (static) potential inside the well. Here we use $\hbar=1$. The total dynamic potential is given by

$$V_T(x;\omega)=V_{\rm ext}(x;\omega)+\frac{e^2}{\kappa a}\int_{-a/2}^{a/2}dy\int_0^L dx'\,\delta\rho(x';\omega)\times[(x-x')^2+y^2]^{-1/2}, \tag{3}$$

where $V_{\rm ext}(x;\omega)$ is the external dynamic potential. The second term on the right is the Hartree potential. κ is the dielectric constant of the material. To avoid the Coulomb singularity of the Hartree term, we assume that the electron gas has a very small width a, and that the induced density does not change across this dimension. We next expand $\delta\rho$ and χ_0 in the Fourier series

$$\delta\rho(x;\omega)=\sum_{n=1}^{\infty} a_n(\omega)\sin(q_n x), \quad (4)$$

$$\chi_0(x,x';\omega)=\sum_{n=1}^{\infty}\sum_{n'=1}^{\infty} B_{nn'}(\omega)\sin(q_n x)\sin(q_{n'}x'), \quad (5)$$

where $q_n=n\pi/L$, and n is a positive integer. Substituting (4) and (5) into (2), we obtain

$$\sum_{n'=1}^{\infty}[\delta_{nn'}-B_{nn'}(\omega)v(q_{n'})]a_{n'}(\omega)=\sum_{n'=1}^{\infty}B_{nn'}(\omega)E_{n'}, \quad (6)$$

where

$$B_{nn'}(\omega)=\frac{4}{L^2}\int_0^L dx\int_0^L dx'\,\chi_0(x,x';\omega)\sin(q_n x)\sin(q_{n'}x'), \quad (7)$$

$$v(q_n)=\frac{2e^2L}{\kappa a|q_n|}\int_0^{|q_n|a/2}dz\; K_0(z), \quad (8)$$

$K_0(z)$ being the modified Bessel function of order zero, and

$$E_n=\int_0^L dx' V_{\rm ext}(x')\sin(q_n x'). \quad (9)$$

For $V_{\rm ext}(x)=V_0\sin(q_m x)$,

$$E_n=(V_0L/2)\delta_{nm}. \quad (10)$$

The solution of Eq. (6), for $a_n(\omega)$, is given symbolically by

$$a=[1-Bv]^{-1}BE. \quad (11)$$

Our formalism may be compared to that of Das Sarma and Lai,[10] who considered a quasi-one-dimensional (1D) (infinite) electron gas.

A normal mode of the system occurs when the charge-density oscillation response at a given frequency becomes large for an arbitrarily small external perturbation. A bounded system can have several normal modes, which can be determined by examining the charge-density response [Eq. (4)] as a function of complex ω. The sign of the imaginary part of the frequency, γ=Im(ω), if negative, indicates that the mode represents damped density oscillations (as a result of various losses in the system), and if positive it represents growing plasma oscillations (instability). When the electron distribution is out of equilibrium, its excess energy can lead to an instability. Note that, since the instability is a ''normal mode,'' albeit with a complex ω, its characteristics do not depend on the form of the external potential $V_{\rm ext}(x)$.

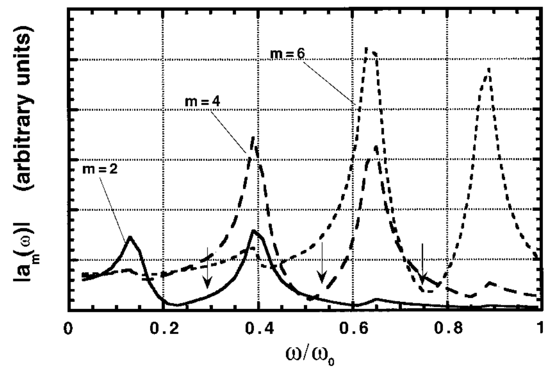

FIG. 1. $|a_m(\omega)|$ vs Re(ω) for three values of q_m (m=2, 4, and 6), for a simple well, with no internal potential. The frequencies are in units ω_0 with $\varepsilon_0=\hbar\omega_0$=21 meV. The Fermi energy is ε_F=21 meV. The arrows indicate the successively increasing eigenmode frequencies for a quasi-1D electron gas, for the corresponding q_m.

III. INSTABILITIES IN BOUNDED SYSTEMS

We first consider a simple well, with no internal potential, and an equilibrium electron distribution (which cannot lead to an instability). We obtain the mode structure using the formalism of Sec. II. Figure 1 shows $|a_m(\omega)|$ vs Re(ω) for three values of q_m (m=2, 4, and 6), Eq. (10). We normalize the distances by L_0=52 Å, and wave vectors by $1/L_0$. For the unit of energy we choose $\varepsilon_0=\hbar^2/2m^*L_0^2$=21 meV, with the effective mass for GaAs, $m^*=0.0665m_e$. The Bohr length in this medium is approximately 104 Å, and L_0 is taken to be half of that. We normalize the energy and frequencies by ε_0. In this calculation, in these reduced units we take L=70, a=1, ε_f=1 (i.e., L=3640 Å, a=52 Å, and ε_F=21 meV in the normal units), and γ=0.02 (or 0.53 meV). The response spectrum consists of a series of peaks, corresponding to standing plasma waves in the quasi-1D box. The two most prominent peaks in each curve represent the strongest plasmon resonances for a given q_n. The oscillator strength between various peaks changes, so that this pair of peaks moves toward higher frequencies for increasing q_n. This dispersion follows approximately the dispersion relation for a 1D plasmon,[10] shown by the arrows in Fig. 1. With increasing L or ε_F, the number of possible resonances in the box increases, and finally, for very large L, only one peak, which follows the dispersion of a one-dimensional electron-gas (1DEG) plasmon, will dominate the spectrum as expected.

Now we consider a scheme which gives rise to an instability. The electron energy spectrum in the well is discrete with $\varepsilon=\hbar^2n^2\pi^2/2m^*L^2$, where n is an integer. Levels below ε_F are occupied. When additional electrons are injected into levels located well above the Fermi level, strong downwards transitions will occur to the unoccupied states below, capable of generating plasmons, and an instability can occur. In order to maintain this population inversion in a steady state, it is necessary to extract carriers from the lower (unoccupied) band efficiently. To illustrate this effect, we inject electrons into a band of energies between ε_1=1.55 and ε_2=2.1. In terms of the notation of Sec. II f_ε=1 for $\varepsilon_1<\varepsilon<\varepsilon_2$ and for $\varepsilon<1$, and is zero otherwise. The corresponding $|a_m(\omega)|$ vs Re(ω) for γ=0.02 is shown by the dashed line in Fig. 2, for m=6. Comparison of Fig. 2 with the corresponding curve of Fig. 1 reveals an additional resonance at ω=0.52 (i.e., 10.9

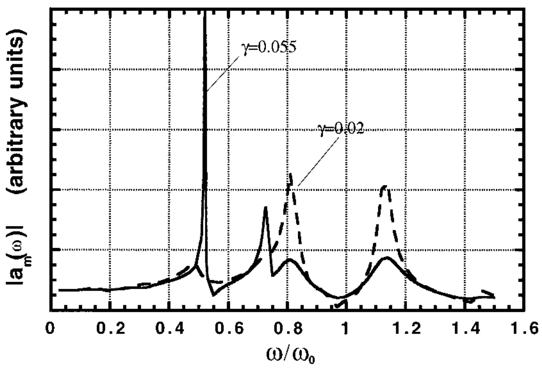

FIG. 2. $|a_m(\omega)|$ vs Re(ω) for $m=6$, for a simple well (as in Fig. 1), with additional carrier injection into the band of energies between 32.5 and 44 meV. The frequencies are in units ω_0, with $\varepsilon_0=\hbar\omega_0=21$ meV. Curves for two different γ's are shown.

meV) and $\omega=0.73$ (15.3 meV), while the double-peak structure (of Fig. 1) is shifted to higher frequencies as a result of the increased electron density in the system due to injection. By changing γ we can study the stability characteristics of these plasmon modes. We find that these features ($\omega=0.52$ and 0.73) represent unstable modes. For $\omega=0.52$ there is a pole at positive $\gamma=0.055$ (i.e., 1.2 meV), as shown by a dramatic enhancement of the density response and a reduction in its width at $\gamma=0.055$ (solid line in Fig. 2). A similar enhancement occurs for $\omega=0.73$ and $\gamma=0.056$. The double-peaked structure, on the other hand, represents stable modes, since their response becomes singular for vanishing γ. These unstable modes are the analogs of the current-driven acoustic (unstable) modes we found in uniform as well as modulated systems in our earlier work.[7,9] As in those cases, there is an energy gap in the distribution function in this scenario. The occurrence of two unstable modes, rather than one, is due to plasma wave reflections arising from the finite size of the system. This is a robust instability, well in excess of the dissipative collision rate in a typical system. The growth rate of the instability can be further enhanced by choosing higher energies and a larger width for the injected band.

Another scheme for generating plasma instabilities in bounded systems is to introduce a deep potential modulation inside an empty quantum well. With deep enough modulation, well-separated, narrow, minibands are formed. By injection into the second, or higher miniband, it is possible to obtain downwards transitions which generate plasmons in the injected carrier plasma. This scenario can be simulated by applying a periodically modulated potential of the form $A[1-\sin(2\pi x/d)]$, with period d, between the walls of our model system. The corresponding ground-state single-particle energies are shown in Fig. 3(a) for period $d=2\pi L_0=327$ Å, for different amplitudes of the potential modulation A, and show the formation of narrow band structures for sufficiently large valves of A. Selectively injecting carriers into the second band of ten levels [states 12–21; see Fig. 3(a)] for all cases, the γ for an unstable plasma mode vs A is shown in Fig.3(b). The growth rate γ also depends on N, the number of injected states. For the case of $A=1.2$ (i.e., 25.2 meV), the maximum $\gamma=0.079$ (i.e., 1.66 meV) occurs for $N=10$, when the second band is fully occupied. General features of this instability are the following: the energy gap

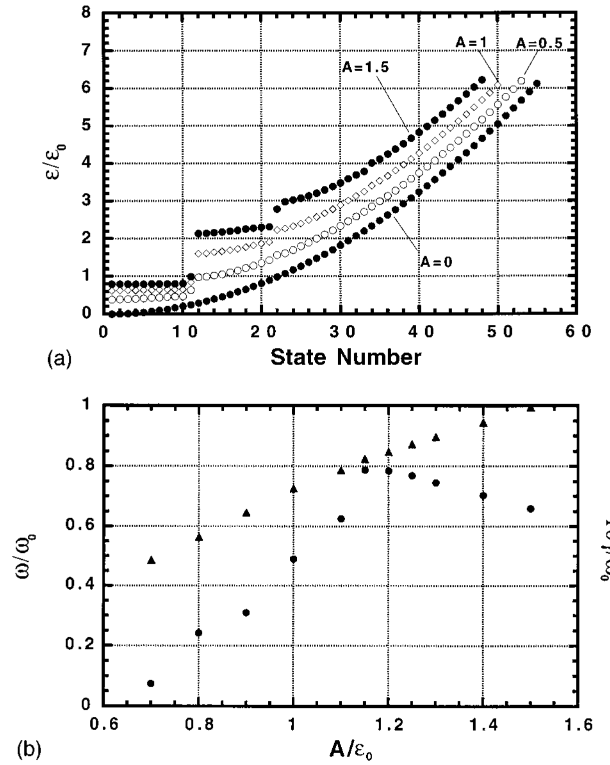

FIG. 3. (a) The energy vs state index. (b) Re(ω) vs A (triangles), and γ vs A (filled circles) for a square well with additional periodic potential modulation of the form $A\ [1-\sin(2\pi x/d)]$, with $d=327$ Å. For (b), carriers are selectively injected into the second band of ten levels (levels 12–21).

between the second and first minibands must be larger than the energy gap between the second and third minibands; then the mode frequency lies between these two gaps, and γ has a maximum at a certain injection level.

These calculations show the feasibility of achieving a strong instability (in both schemes above), with a growth rate far in excess of the dissipative collision rates, which typically do not exceed 0.5 meV for currently available samples at low temperatures. Thus devices based on these ideas could be practical. We note that the energies and growth rates can be scaled by changing the size (L), and by changing the strength of the modulation potential (A), for the second scheme. From the fabrication point of view, the first scheme has the advantage that it does not need any internal potential modulation, while the second does not require an occupied well; i.e., doping or other carrier generating schemes are not needed.

We point out that the instabilites considered here are largely insensitive to the dimensionality of the system, as long as a three-band population scenario is maintained through proper injection and extraction schemes, and should be essentially unchanged in ''fat'' wire systems. Such systems are easier to fabricate, and would allow a larger level of injection.

IV. DISCUSSION

In order to achieve the generation of growing plasma waves in a bounded plasma, we propose to employ a vertical

'' mesa'' structure, in which a quasi-1D electron plasma is confined in a quantum well, with injection from one side into a specified level, and extraction from the other side from specified levels. One possibility is to employ two resonant-tunneling energy filters, with proper energy levels. Another possibility involves periodical superlattices with proper arrangement of minibands so as to Bragg reflect, or transmit electrons in and extract out from desired levels of the quantum well (a similar arrangement was used by Faist *et al.*[11]).

Once the conditions for instability have been realized, a charge-density oscillation will develop in the active region. This effectively constitutes an oscillating dipole which will emit electromagentic radiation at the frequency of the plasma wave, if an efficient coupling scheme to free space can be introduced (see below).

As a first priority it is important to prove the existence of the instability. At the onset of the instability a measurable increase of the device current is expected, since another loss channel is opened. Another means of detecting the onset would be the observation of an absorption change, when sweeping an external frequency through the resonance.

The plasma wave amplitude will saturate at a given value determined through the onset of various loss mechanisms (including nonlinear effects), when they match the growth rate. The saturation level will determine the possible radiation power.

An estimate of the emission power can be given, assuming state-of-the-art tunneling injection structures. Assuming an injected current of 0.1 mA for a $1-\mu m^2$ A/cm^2 mesa structure, corresponding to a current density of 10^4 Å/cm^2, and a voltage drop of 1 mV across the confined structure, we can expect a radiation power of 10^{-9} W if we assume a quantum efficiency of 10^{-2}–10^{-3}. By arranging an array of devices which could in addition function as an antenna, power levels in the range of 10^{-3}–10^{-4} W are basically possible (for 10^6 structures).

The basic phenomenon discussed here might be realized in other experimental arrangements as well. In principle, the instability might develop in a superlattice of 2DEG, with potential modulation in the perpendicular direction as in our quasi-1D example. The advantage of such a system is that a much higher current can be employed. On the other hand, since there are not true minigaps in a superlattice of 2DEG, it will be necessary to have an extraction process which is faster than the filling-up rate due to electron-phonon scattering, so that the quasi-steady-state population arrangement necessary for the instability can be maintained. Recent work by Faist *et al.*[11] lends support to this possibility; it was shown that a local population inversion in a quantum cascade laser system, with no true minigap, was maintained in spite of strong electron-phonon scattering.

It is important to note that our mechanism relies on the collective-mode excitation of the system, to be contrasted with the single-particle, spontaneous emission mechanism of the quantum cascade lasers.[11] As a result, one can achieve a much stronger *stimulated* emission in which the collective mode plays the role of a Fabry-Perot resonator.

We note that, in general, competing processes, which cause nonradiative interminiband transitions (electron-phonon interactions, Auger processes, etc.) have longer time scales than the injection/extraction and plasma-field-induced transition rates, and therefore it should be possible to maintain the required carrier population arrangements to assure the instability.

Operation at temperatures above cryogenic levels is generally possible due to the collectivity of the macrocharge oscillations on which the device operation rests. Plasma effects can naturally extend this operational principle to domain of the microcharge oscillations. There is no ''in-principle'' restriction to cryogenic temperature operation for devices based on plasma (collective) effects. The phenomenon of plasma instability-induced stimulated emission as discussed in this paper utilizes the principle of microcharge oscillations at the collective level. Devices based on these ideas could operate at temperatures above cryogenic levels.

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