

Distribution of transmitted charge through a double-barrier junction

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The distribution function of transmitted charge through a double-barrier junction is studied at zero temperature and at small applied voltage. Both a semiclassical model, in which the transport is described by jump rates, and a quantum-mechanical model, which averages over resonant and nonresonant states, are used to determine the characteristic function of the transmitted electrons. It is demonstrated that for large times the logarithm of the characteristic function is equal within the two approaches. The charge distribution is in between a Gaussian and a Poisson distribution if both barriers have equal height and reduces to a Poisson distribution if one barrier is much higher than the other. [S0163-1829(96)06735-5]

I. INTRODUCTION

The nature of the current flow at low temperatures through mesoscopic structures has received a lot of attention during recent years. After initial focus on the conductance, which measures the average number of electrons transmitted in time, there has been an increasing interest in the noise power, a measure for the variance of the transmitted charge. At zero temperature, these current fluctuations are due to the discreteness of the electron charge. It has been found theoretically, that the zero-frequency shot-noise power P can be suppressed below its classical value characteristic for uncorrelated electron transport, $P_{\text{Poisson}} = 2eI$, with I the average current. This suppression is due to correlated electron transmission imposed by the Pauli principle.¹⁻⁵ Consequently, it has been shown that in a double-barrier junction P can be suppressed down to $\frac{1}{2}P_{\text{Poisson}}$,⁶⁻¹² depending on the relative height of the barriers. For a metallic, diffusive conductor various calculations yield that $P = \frac{1}{3}P_{\text{Poisson}}$.¹³⁻¹⁷ The shot-noise suppression in these two systems has been observed experimentally.¹⁸⁻²²

Recently, Levitov and Lesovik have gone one step further by studying the full *distribution function* of charge transmitted through a mesoscopic conductor.²³ This function gives the probability that a certain number of electrons are transmitted during a given time interval. Their quantum-mechanical analysis demonstrates that the attempts to transmit electrons are periodic in time, yielding a binomial distribution of transmitted electrons. On the basis of this result, Lee, Levitov, and Yakovets have calculated the charge distribution function for transport through a metallic, diffusive conductor.²⁴

In this paper, we derive the complete distribution of transmitted charge through a double-barrier junction, by two different methods: Firstly, we follow a semiclassical approach, in which phase is neglected but the Pauli principle is accounted for. Here, the electron transport is described by classical jump rates. Secondly, we take a quantum-mechanical approach, where we average the result of Ref. 23 over the distribution of transmission probabilities through the double-barrier system. In our calculations, we restrict ourselves to high tunnel barriers and assume that they are perfectly planar, so that intermode scattering is absent. Furthermore, we

neglect charging effects and we assume small applied voltage as well as zero temperature.

The linear-response conductance G of a double-barrier junction is given by

$$G = \frac{2e^2}{h} N \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2}, \quad (1)$$

with $\Gamma_i \ll 1$ the transmission probability through barrier $i = 1, 2$, and N the number of transverse modes at the Fermi energy. Interestingly, the two approaches to derive Eq. (1) are of a completely different nature.^{25,26} The semiclassical derivation consists essentially of the addition of the resistances of both junctions, whereas the quantum-mechanical derivation involves an average over resonant and nonresonant states. Physically, this averaging may correspond either to an applied voltage larger than the width of the resonance or to a summation over the modes in a multichannel conductor if the distance between the barriers is larger than the Fermi wavelength.

The role of the presence of phase coherence on the fluctuations in the current is still an intriguing issue.²⁷⁻²⁹ For example, the one-third suppression of the shot noise in a metallic, diffusive conductor was originally surmised to be of quantum-mechanical origin.¹³ However, later derivations through a semiclassical approach yielded a suppression by one-third as well.^{14,30,31} With respect to the shot-noise power in the double-barrier junction, a quantum-mechanical theory by Chen and Ting⁶ and a semiclassical theory by Davies *et al.*⁷ give identical results, namely,

$$P = \frac{\Gamma_1^2 + \Gamma_2^2}{(\Gamma_1 + \Gamma_2)^2} P_{\text{Poisson}}. \quad (2)$$

An additional aim of the present paper is to check to which extent this insensitivity to the presence of phase coherence applies also for the complete distribution of transmitted charge.

The quantity of interest is $P_n(t)$, denoting the probability that exactly n electrons have been transmitted during a time interval t . An alternative way to describe this distribution is through its characteristic function $\chi(\lambda, t)$. They are mutually related according to³²

$$\chi(\lambda, t) = \sum_{n=0}^{\infty} P_n(t) e^{in\lambda}, \quad (3a)$$

$$P_n(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\lambda e^{-in\lambda} \chi(\lambda, t). \quad (3b)$$

It is often more convenient to determine $\chi(\lambda, t)$ instead of $P_n(t)$. Normalization requires that $\chi(0, t) = 1$. Furthermore, it follows from Eq. (3a) that the average number of electrons transmitted during a time t is given by

$$\overline{n(t)} = \sum_{n=0}^{\infty} n P_n(t) = \lim_{\lambda \rightarrow 0} \frac{\partial}{i\partial\lambda} \chi(\lambda, t). \quad (4)$$

More generally, one can express the k th moment $\mu_k(t)$ of the distribution according to

$$\mu_k(t) \equiv \overline{n^k(t)} = \lim_{\lambda \rightarrow 0} \left(\frac{\partial}{i\partial\lambda} \right)^k \chi(\lambda, t). \quad (5)$$

We note that the average current is simply $I = e\mu_1(t)/t$, whereas the noise power is proportional to the variance of the number of transmitted electrons $P = 2e^2 \lim_{t \rightarrow \infty} \text{var}n(t)/t = 2e^2 \lim_{t \rightarrow \infty} [\mu_2(t) - \mu_1^2(t)]/t$. The logarithm of the characteristic function can be expanded as follows:

$$\ln \chi(\lambda, t) = \sum_{k=1}^{\infty} \frac{(i\lambda)^k}{k!} \kappa_k(t), \quad (6)$$

with $\kappa_k(t)$ the k th cumulant of the distribution. The moments and the cumulants have a direct polynomial relation,³² for example, $\kappa_1(t) = \mu_1(t)$, $\kappa_2(t) = \mu_2(t) - \mu_1^2(t)$.

The quantum-mechanical analysis by Levitov and Lesovik²³ has yielded the characteristic function of the charge through a single-channel conductor at zero temperature and at small voltage V ,

$$\chi(\lambda, t) = [(e^{i\lambda} - 1)T + 1]^{eVt/h}, \quad (7)$$

with T the transmission probability at the Fermi level through the conductor. The generalization to a spin-degenerate, multichannel conductor is given by²³

$$\chi_N(\lambda, t) = \prod_{l=1}^N [(e^{i\lambda} - 1)T_l + 1]^{2eVt/h}, \quad (8)$$

with T_l the l th transmission eigenvalue. From Eqs. (5) and (8) one can immediately derive the Landauer formula for the conductance and the formula for the shot-noise power,^{2,4}

$$G = \frac{2e^2}{h} \sum_{l=1}^N T_l, \quad (9)$$

$$P = \frac{4e^3 V}{h} \sum_{l=1}^N T_l (1 - T_l). \quad (10)$$

Note that Eq. (10) is only valid in the phase-coherent regime, whereas in the absence of phase coherence P is given by a different equation.³⁰ In Section II, it is demonstrated how the charge distribution through a single barrier can be derived in

a semiclassical approach. Section III repeats this analysis for the double-barrier junction. The quantum-mechanical calculation is given in Sec. IV, after which we conclude in Sec. V.

II. SIMPLE EXAMPLE: SINGLE-BARRIER JUNCTION

Let us illustrate our approach, by calculating the distribution of transmitted charge through a single-channel, single-barrier junction, with a transmission probability $\Gamma \ll 1$. The average current through the barrier $I = e^2 V \Gamma / h \equiv e \gamma$, with $\gamma = eV \Gamma / h$ the tunnel rate through the barrier. The probability $P_n(t)$ that n electrons have been transmitted in a time t obeys the master equation

$$\frac{dP_0(t)}{dt} = -\gamma P_0(t), \quad (11a)$$

$$\frac{dP_n(t)}{dt} = \gamma P_{n-1}(t) - \gamma P_n(t), \quad \text{if } n \geq 1, \quad (11b)$$

with the initial condition $P_n(0) = \delta_{0,n}$. Equation (11) can be solved straightforwardly by various means. Here, we adopt an approach that appears to be useful for the double-barrier junction. The solution of Eq. (11a) is $P_0(t) = \exp(-\gamma t)$. We define the waiting-time distribution $\psi(t) \equiv -dP_0(t)/dt$, denoting the probability density that an electron is transmitted immediately after having waited a time t ,

$$\psi(t) = \gamma e^{-\gamma t}. \quad (12)$$

We write

$$P_n(t) = G_n(t) - G_{n+1}(t), \quad (13)$$

where $G_n(t)$ denotes the probability that n or more electrons are transmitted during a time t . It can be calculated according to

$$G_n(t) = \int_0^t dt_1 \int_{t_1}^t dt_2 \cdots \int_{t_{n-1}}^t dt_n \psi(t_1) \times \psi(t_2 - t_1) \cdots \psi(t_n - t_{n-1}). \quad (14)$$

Since this is a convolution, one has for the Laplace transform

$$\tilde{G}_n(s) \equiv \int_0^{\infty} dt e^{-st} G_n(t) = \frac{1}{s} [\tilde{\psi}(s)]^n, \quad (15)$$

where the Laplace transform of Eq. (12) is given by

$$\tilde{\psi}(s) = \frac{\gamma}{s + \gamma}. \quad (16)$$

From Eqs. (13), (15), and (16), we find

$$\tilde{P}_n(s) = \frac{\gamma^n}{(s + \gamma)^{n+1}}, \quad (17)$$

yielding for the distribution in time

$$P_n(t) = \frac{(\gamma t)^n}{n!} e^{-\gamma t}. \quad (18)$$

This is the Poisson distribution, as one would expect for the uncorrelated electron transfers through the barrier. The characteristic function is given by

$$\chi(\lambda, t) = \exp[\gamma t(e^{i\lambda} - 1)]. \quad (19)$$

Indeed, in the limit $T = \Gamma \rightarrow 0$, Eqs. (7) and (19) coincide.

The generalization of Eq. (19) to the case of a spin-degenerate, multichannel conductor is straightforward upon the assumption that the transmission of the different channels is independent. Using the result that the characteristic function corresponding to the sum of independent variables is given by the product of the characteristic functions of the separate variables,³² one obtains

$$\chi_N(\lambda, t) = \exp[2N\gamma t(e^{i\lambda} - 1)]. \quad (20)$$

III. CLASSICAL APPROACH

We now study the double-barrier junction. The tunnel rate through barrier $i = 1, 2$ is $\gamma_i = eV\Gamma_i/h$. Due to the Pauli principle, the number of electrons in the double-barrier junction can be either 0 or 1.

Our analysis is similar to the single-barrier case. However, one now has to take into account two possible initial conditions at $t = 0$: either 0 electrons in the junction—a situation with probability $\gamma_2/(\gamma_1 + \gamma_2)$ —or 1 electron—probability $\gamma_1/(\gamma_1 + \gamma_2)$. The distribution of transmitted charge is thus given by

$$P_n(t) = \frac{\gamma_2}{\gamma_1 + \gamma_2} P_n^{(0)}(t) + \frac{\gamma_1}{\gamma_1 + \gamma_2} P_n^{(1)}(t), \quad (21)$$

where $P_n^{(j)}(t)$ starts from j electrons in the junction at $t = 0$. The probability that at least n electrons have been transmitted can be expressed as

$$\tilde{G}_n^{(0)}(s) = \frac{1}{s} [\tilde{\psi}_1(s)\tilde{\psi}_2(s)]^n, \quad (22a)$$

$$\tilde{G}_n^{(1)}(s) = \frac{1}{s} \tilde{\psi}_2(s) [\tilde{\psi}_1(s)\tilde{\psi}_2(s)]^{n-1}, \quad (22b)$$

with $\tilde{\psi}_i(s) = \gamma_i/(s + \gamma_i)$. From Eqs. (13), (21), and (22) we obtain

$$\tilde{P}_0(s) = \frac{s(\gamma_1 + \gamma_2) + \gamma_1^2 + \gamma_2^2 + \gamma_1\gamma_2}{(\gamma_1 + \gamma_2)(s + \gamma_1)(s + \gamma_2)}, \quad (23a)$$

$$\tilde{P}_n(s) = \frac{(\gamma_1\gamma_2)^n (s + \gamma_1 + \gamma_2)^2}{(\gamma_1 + \gamma_2)(s + \gamma_1)^{n+1}(s + \gamma_2)^{n+1}}, \quad \text{if } n \geq 1. \quad (23b)$$

The distribution function in time can be obtained through the inverse Laplace transform. In general, it yields a rather cumbersome expression. However, for the case of a symmetric double-barrier junction, $\gamma_1 = \gamma_2 \equiv \gamma$, one has

$$P_0(t) = \left(1 + \frac{\gamma t}{2}\right) e^{-\gamma t}, \quad (24a)$$

$$P_n(t) = \left[\frac{(\gamma t)^{2n-1}}{2(2n-1)!} + \frac{(\gamma t)^{2n}}{(2n)!} + \frac{(\gamma t)^{2n+1}}{2(2n+1)!} \right] e^{-\gamma t}, \quad \text{if } n \geq 1. \quad (24b)$$

For arbitrary γ_1 and γ_2 , we can evaluate from Eq. (23) the Laplace transform of the characteristic function

$$\tilde{\chi}(\lambda, s) = \frac{1}{\gamma_1 + \gamma_2} \left[\frac{(s + \gamma_1 + \gamma_2)^2}{(s + \gamma_1)(s + \gamma_2) - e^{i\lambda}\gamma_1\gamma_2} - 1 \right], \quad (25)$$

yielding for the characteristic function in time

$$\chi(\lambda, t) = \exp\left[-\frac{1}{2}(\gamma_1 + \gamma_2)t\right] \left\{ \cosh\left[\frac{1}{2}\beta(\lambda)t\right] + \left(\frac{\beta(\lambda)}{\gamma_1 + \gamma_2} - \frac{2\gamma_1\gamma_2(e^{i\lambda} - 1)}{(\gamma_1 + \gamma_2)\beta(\lambda)}\right) \sinh\left[\frac{1}{2}\beta(\lambda)t\right] \right\}, \quad (26)$$

with $\beta(\lambda) = \sqrt{(\gamma_1 + \gamma_2)^2 + 4\gamma_1\gamma_2(e^{i\lambda} - 1)}$. For a spin-degenerate, multichannel conductor we have

$$\chi_N(\lambda, t) = \exp[-N(\gamma_1 + \gamma_2)t] \left\{ \cosh\left[\frac{1}{2}\beta(\lambda)t\right] + \left(\frac{\beta(\lambda)}{\gamma_1 + \gamma_2} - \frac{2\gamma_1\gamma_2(e^{i\lambda} - 1)}{(\gamma_1 + \gamma_2)\beta(\lambda)}\right) \sinh\left[\frac{1}{2}\beta(\lambda)t\right] \right\}^{2N}. \quad (27)$$

Note that again we have made the assumption of independent channels, implying that the Pauli exclusion principle applies within each separate channel only. Physically, this situation corresponds to the absence of intermode scattering.

Equation (27) is the central result of our semiclassical analysis. Let us evaluate from Eqs. (5) and (27) the first two moments of the charge distribution. The average number of electrons transmitted during a time t is

$$\mu_1(t) = 2N \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2} t, \quad (28)$$

in agreement with Eq. (1). For the second moment we find

$$\begin{aligned} \mu_2(t) = & 4N^2 \frac{(\gamma_1\gamma_2)^2}{(\gamma_1 + \gamma_2)^2} t^2 + 2N \frac{\gamma_1\gamma_2(\gamma_1^2 + \gamma_2^2)}{(\gamma_1 + \gamma_2)^3} t \\ & + 4N \frac{(\gamma_1\gamma_2)^2}{(\gamma_1 + \gamma_2)^4} \{1 - \exp[-(\gamma_1 + \gamma_2)t]\}. \end{aligned} \quad (29)$$

The Fano factor, defined as the ratio of the variance to the average number of transmitted electrons, $r(t) \equiv [\mu_2(t) - \mu_1^2(t)]/\mu_1(t)$, follows from Eqs. (28) and (29),

$$r(t) = \frac{\gamma_1^2 + \gamma_2^2}{(\gamma_1 + \gamma_2)^2} + \frac{2\gamma_1\gamma_2(1 - e^{-(\gamma_1 + \gamma_2)t})}{(\gamma_1 + \gamma_2)^3 t}. \quad (30)$$

The Fano factor gives the relative magnitude of the current fluctuations. Indeed, for large t , $r(t)$ yields the shot-noise suppression according to Eqs. (1) and (2). In Fig. 1 we have plotted $r(t)$, for $\gamma_1 = \gamma_2 \equiv \gamma$. We find that $r(t)$ goes from 1 at small t , indicative of uncorrelated electron transmission, to $\frac{1}{2}$ at large t , indicative of a more correlated electron transmission. It is already within one percent of its final value at $\gamma t = 50$, which corresponds to an average number of 25 transmitted electrons for each channel.

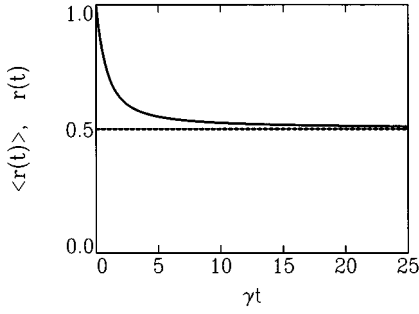


FIG. 1. The Fano factor $r(t)$, giving the ratio of the variance to the average number of transmitted electrons, versus time t for a symmetric double-barrier junction with tunnel rates $\gamma_1 = \gamma_2 = \gamma$. The solid line gives the result [Eq. (30)] of the semiclassical analysis and the dashed line the quantum result $\langle r(t) \rangle$, according to Eq. (37).

The cumulants of the charge distribution can be determined from the logarithm of Eq. (27)

$$\begin{aligned} \ln \chi_N(\lambda, t) = & N[\beta(\lambda) - (\gamma_1 + \gamma_2)]t + 2N \ln(1 + e^{-\beta(\lambda)t}) \\ & - 2N \ln 2 + 2N \ln \left[1 + \left(\frac{\beta(\lambda)}{\gamma_1 + \gamma_2} \right) \right. \\ & \left. - \frac{2\gamma_1\gamma_2(e^{i\lambda} - 1)}{(\gamma_1 + \gamma_2)\beta(\lambda)} \tanh\left[\frac{1}{2}\beta(\lambda)t\right] \right]. \end{aligned} \quad (31)$$

In general, it is cumbersome to derive the cumulants from this result. However, for large t , only the first term remains of importance.

There are various alternative methods to derive the above results. For example, one can use the approach outlined in the Appendix of Ref. 33.

IV. QUANTUM-MECHANICAL APPROACH

Whereas in a semiclassical picture the transmission probability through the double-barrier junction is just a constant, in a quantum-mechanical approach, the transmission probability T_l of the l th mode varies according to a Fabry-Perot type of formula,

$$T_l = \frac{\Gamma_1 \Gamma_2}{1 - 2\sqrt{(1-\Gamma_1)(1-\Gamma_2)}\cos\phi_l + (1-\Gamma_1)(1-\Gamma_2)}, \quad (32)$$

where ϕ_l is the phase accumulated in one round trip between the barriers. The distribution function $\rho(T) \equiv \langle \sum_l \delta(T - T_l) \rangle$ of the transmission probabilities through the system can be obtained from the assumption that the ϕ_l are uniformly distributed between 0 and 2π .³⁴ In the limit $\Gamma_1, \Gamma_2 \ll 1$ this implies

$$\rho(T) = \frac{N\Gamma_1\Gamma_2}{\pi(\Gamma_1 + \Gamma_2)} \frac{1}{\sqrt{T^3(T_{\max} - T)}}, \quad (33)$$

if $T \in [T_{\min}, T_{\max}]$, and $\rho(T) = 0$ otherwise, with $T_{\min} = 4\Gamma_1\Gamma_2 / [(\Gamma_1 + \Gamma_2)^2 + 4\pi^2]$ and $T_{\max} = 4\Gamma_1\Gamma_2 / (\Gamma_1 + \Gamma_2)^2$. The distribution function (33) is plotted in Fig. 2. Similar to a metallic, diffusive conductor,¹³ this distribution

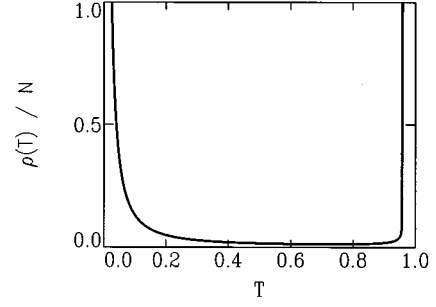


FIG. 2. The distribution of transmission probabilities through a double-barrier junction in a quantum-mechanical model, according to Eq. (33), for $\Gamma_1 = 0.02$ and $\Gamma_2 = 0.03$.

is bimodal, in the sense that the transmission probabilities are either close to $T_{\min} \approx 0$ or close to T_{\max} .

The ensemble average of a linear statistic over all possible transmission probabilities is given by $\langle \sum_l a(T_l) \rangle = \int_{T_{\min}}^{T_{\max}} dT \rho(T) a(T)$. It is convenient to switch variables from T to ν with $T = T_{\max} / (1 + \nu^2)$, so that $\rho(\nu) = \rho_0 \equiv N(\Gamma_1 + \Gamma_2) / 2\pi$ is uniform over the range $[0, \nu_{\max}]$. In practice, the upper limit ν_{\max} can be often replaced by infinity. The ensemble average for the m th power ($m \geq 1$) of the transmission probability is given by

$$\begin{aligned} \left\langle \sum_{l=1}^N T_l^m \right\rangle &= \rho_0 \int_0^\infty d\nu \left(\frac{T_{\max}}{1 + \nu^2} \right)^m \\ &= N \frac{(2m-2)!}{[(m-1)!]^2} \frac{(\Gamma_1 \Gamma_2)^m}{(\Gamma_1 + \Gamma_2)^{2m-1}}. \end{aligned} \quad (34)$$

Substituting this result into Eqs. (9) and (10), we recover for $\langle G \rangle$ and $\langle P \rangle$ the expressions given by Eqs. (1) and (2).

In order to obtain the ensemble average of all the cumulants of the distribution function, we average the logarithm of the characteristic function.²⁴ For the double-barrier junction, we obtain from Eq. (8)

$$\begin{aligned} \langle \ln \chi_N(\lambda, t) \rangle &= \rho_0 \frac{2eVt}{h} \int_0^\infty d\nu \ln \left[\frac{(e^{i\lambda} - 1)T_{\max}}{1 + \nu^2} + 1 \right] \\ &= N[\beta(\lambda) - (\gamma_1 + \gamma_2)]t, \end{aligned} \quad (35)$$

with $\beta(\lambda) = \sqrt{(\gamma_1 + \gamma_2)^2 + 4\gamma_1\gamma_2(e^{i\lambda} - 1)}$. This is the key result of the quantum-mechanical evaluation. Using Eq. (6), we find for the ensemble average of the first three cumulants

$$\langle \kappa_1(t) \rangle = 2N \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} t, \quad (36a)$$

$$\langle \kappa_2(t) \rangle = 2N \frac{\gamma_1 \gamma_2 (\gamma_1^2 + \gamma_2^2)}{(\gamma_1 + \gamma_2)^3} t, \quad (36b)$$

$$\begin{aligned} \langle \kappa_3(t) \rangle &= 2N \frac{\gamma_1 \gamma_2}{(\gamma_1 + \gamma_2)^5} t (\gamma_1^4 - 2\gamma_1^3 \gamma_2 + 6\gamma_1^2 \gamma_2^2 \\ &\quad - 2\gamma_1 \gamma_2^3 + \gamma_2^4). \end{aligned} \quad (36c)$$

Since $\langle \ln \chi_N(\lambda, t) \rangle$ is proportional to t , all the cumulants are linear in t as well. This implies for the Fano factor

$$\langle r(t) \rangle \equiv \frac{\langle \kappa_2(t) \rangle}{\langle \kappa_1(t) \rangle} = \frac{\gamma_1^2 + \gamma_2^2}{(\gamma_1 + \gamma_2)^2}, \quad (37)$$

which is constant in time and equal to the large- t value of Eq. (30) (see Fig. 1). If $\gamma_1 \gg \gamma_2$ (or vice versa), Eq. (35) reduces to a Poisson distribution [see Eq. (20)], as expected. For a symmetric double-barrier junction with $\gamma_1 = \gamma_2 \equiv \gamma$, expression (35) simplifies considerably:

$$\langle \ln \chi_N(\lambda, t) \rangle = 2N\gamma t(e^{i\lambda/2} - 1). \quad (38)$$

For the k th cumulant, we find from Eqs. (6) and (38)

$$\langle \kappa_k(t) \rangle = \frac{N\gamma t}{2^{k-1}}. \quad (39)$$

The charge distribution is thus somewhere between a Gaussian (where $\kappa_k = 0$ for $k \geq 3$) and a Poisson distribution (where $\kappa_k = 2N\gamma t$ for all k).

V. CONCLUSIONS

Let us make the comparison between the outcome of the two approaches, i.e., between the semiclassical result $\ln \chi_N(\lambda, t)$ from Eq. (31) and the quantum result $\langle \ln \chi_N(\lambda, t) \rangle$ given in Eq. (35), where we have averaged over the distribution of transmission probabilities. The semiclassical result yields a more complicated expression, however, the most important contribution, which is proportional to t , is *precisely* equivalent to the result (35). The other terms are either constant (do not depend on t) or vanish exponentially with t . Only on short time scales, corresponding to the transfer of a few electrons, we see sizeable differences between both approaches. We surmise that these differences at small t are not due to the neglect of the phase, but rather depend on the precise way the reservoirs are modeled in both approaches. The quantum-mechanical derivation is based on Eq. (8), which applies for arbitrary transmission eigenvalues between 0 and 1. Here, the number of electrons transmitted has a maximum value $n(t) = 2NeVt/h$. Our semiclassical calculation assumes independent tunnel events and is therefore only valid for small transmission probabilities. However, the number of electrons which can be transmitted $n(t)$ is not bounded. Even though this seems not to be very important for the case of high tunnel barriers, one may expect that it leads to differences on a small time scale. Therefore, we just draw conclusions from the comparison at larger t . Here we find that the two results are equal, and that, as a consequence, all the cumulants are also equal. This demonstrates that the

statistics of charge transport through a double-barrier junction does not reveal whether phase coherence is present or absent.

This insensitivity to the presence of phase coherence does not imply that phase breaking is not of influence in a real experiment. This depends on the physical process which destroys the phase coherence. For example, using the method given in Ref. 31, one can show that electron-electron scattering inside the double-barrier system, in which both phase coherence is destroyed and energy is redistributed among the electrons, increases the shot noise above the value of Eq. (2). However, the analysis in the present paper demonstrates that merely breaking the phase leaves the charge transport through the system unaffected. This is in contrast to the result of Ref. 29, in which incoherence is modeled by adding random phases to the wave function on each round trip. The authors find that this increases the shot noise, so that we conclude that their model is not equivalent to just destroying the phase.

It might be interesting to determine the role of inelastic processes inside the tunnel barrier on the charge distribution. We think that these effects can well be taken into account using the semiclassical analysis, whereas a complete quantum-mechanical derivation looks more complicated. Another extension of the work described in this paper would be to repeat the semiclassical analysis for a metallic, diffusive conductor, and compare the outcome with the quantum-mechanical derivation of Ref. 24.

In summary, we have derived the complete distribution of transmitted charge through a double-barrier junction at zero temperature and at low voltage. We have used a semiclassical approach on the basis of classical jump rates as well as a quantum-mechanical approach, in which the result of Levitov and Lesovik for an arbitrary mesoscopic conductor²³ is averaged over the distribution of transmission probabilities through the system. Our results are in precise agreement with previous values for the conductance and for the shot-noise power.^{6,7} Within both approaches, we have determined the logarithm of the characteristic function, which become equivalent at large times. It is found that for symmetric tunnel barriers, the charge distribution is between a Gaussian and a Poisson distribution.

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