Linewidth of c-axis plasma resonance in Josephson-coupled superconductors

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We derive equations which describe the interaction of the phase collective mode with vortex oscillations in multilayer superconductors with Josephson interlayer coupling. Using these dynamic equations for the phase difference between neighboring layers and pancake coordinates we calculate the linewidth of the c-axis plasma resonance in the vortex glass phase when a magnetic field is applied along the c axis. Three mechanisms contribute to the linewidth: interlayer tunneling of quasiparticles, inhomogeneous Josephson interaction in the presence of randomly positioned vortices (inhomogeneous broadening), and dissipation of the plasma mode into vortex oscillations. The phase collective mode is mixed with vortex oscillations in the linear approximation via the Josephson interaction when pancakes are positioned randomly along the c axis due to pinning and thermal fluctuations. Analyzing experimental data for the plasma resonance linewidth in a Bi-2:2:1:2 superconductor we conclude that in magnetic fields below 7 T the linewidth is determined mainly by inhomogeneous broadening. This leads to a nearly temperature-independent linewidth which is inversly proportional to the magnetic field. At higher fields or lower pinning the dissipation of the plasmon into vortex oscillations may become the dominant mechanism of line broadening. In this case the linewidth weakly depends on the magnetic field. [S0163-1829(96)05334-9]

I. INTRODUCTION

The highly anisotropic high-T_c superconductors may be considered as a stack of superconducting CuO_2 layers coupled by Josephson interactions.[1–3] The novel properties of these materials as compared with a single Josephson junction are associated with their multilayer structure and with the presence of pancake vortices[4] when a magnetic field is applied along the c axis. It was shown previously that Abrikosov vortices induced by such a field strongly suppress the interlayer maximum superconducting current by inducing random phase differences between layers in the presence of disorder in pancake positions along the c axis.[5,6] It was predicted in Ref. 7 that this effect leads to a decrease of the c-axis Josephson plasmon frequency with a magnetic field applied along the c axis because the plasma frequency squared is proportional to the maximum Josephson current. Recently, a sharp magnetoabsorption resonance was observed in the vortex state of the highly anisotropic layered superconductor Bi-2:2:1:2 by Tsui *et al.*[8,10,11] and Matsuda *et al.*[9] in the frequency range 30 – 90 GHz depending on the magnetic field and temperature. The field behavior of this resonance (decrease of frequency with the magnetic field applied along the c axis) as well as its angular dependence (sharp decrease of the resonance frequency near orientations of the strong magnetic field parallel to layers) was found to be in agreement with predictions of Refs. 7 and 12 for the Josephson plasmon in layered superconductors. Matsuda *et al.*[9] and Tsui[11] confirmed that this resonance is maximum when an ac electric field is oriented along the c axis. This observation provides strong evidence that the observed resonance is indeed the c-axis Josephson plasmon. A resonance with similar behavior was also observed in the Bi-2:2:0:1 superconductor by Maeda *et al.*[13] Thus, the plasma resonance found in Bi-2:2:1:2 and Bi-2:2:0:1 superconductors is the extension of the Josephson plasmon discovered in the single Josephson junction by Dahm *et al.*[15] to the multilayered system with Josephson interlayer coupling.

In a single Josephson junction the plasma mode is the charge oscillation between two superconductors forming the junction, with the current between the superconductors being the Josephson tunneling current. In layered superconductors, the plasma mode is a charge oscillation between the top and bottom layers of the sample, and the corresponding currents flow between all layers forming the crystal. For a single Josephson junction the effect of magnetic field on the plasma frequency has been observed for an orientation of the magnetic field parallel to the junction. However, plasma resonance was not studied in the situation when Abrikosov vortices penetrate through the junction. For this case only the effect of vortices on the critical current of junction was discussed.[16–18] The interplay between Abrikosov vortices and Josephson properties is an important new effect in multilayered superconductors: The coexistence of Abrikosov vortices and Josephson plasmon allows us to study the structure and dynamics of pancake vortex lattices in systems like Bi-2:2:1:2 by plasma resonance measurements.

The experimental studies[8–11] have established the following properties of the c-axis plasma resonance.

(a) Below the irreversibility line in the (B,T) plane the dependence of the plasma frequency Ω on the magnetic field $\mathbf{B}\|c$ and the temperature T has the form

$$\Omega^2(B,T)=A_1B^{-\mu}\exp(T/T_0), \tag{1}$$

where A_1 is a constant, $\mu\approx 0.7$–0.8 is temperature independent, and $T_0\approx 12.5$ K in fields 0.3–7 T and at temperatures 3–16 K; see Refs. 8, 10, and 11. Above the irreversibility line the power-law field dependence holds with $\mu\approx 0.9$–1 and Ω drops with temperature at a fixed magnetic field.[9] Thus the temperature dependence of the plasma frequency exhibits a cusp at the irreversibility line.

(b) At low temperatures, when the magnetic field $\mathbf{B}$ is tilted by an angle θ with respect to the ab plane, at high angles $\theta > 10°$ only the perpendicular component of the field, $B_z = B\sin\theta$, is effective, and Eq. (1) with $B=B_z$ describes the data. At θ smaller than 5° the plasma frequency decreases sharply as θ approaches zero in high magnetic fields above 2 T; see Refs. 10 and 11.

(c) In a magnetic field $\mathbf{B}\|c$ and below the irreversibility line, the relative resonance linewidth Γ/Ω is at most a weak function of magnetic field,[8] where Γ is the half width at half maximum.

Experimentally, the sample is glued to one end of the sapphire substrate and protrudes into the waveguide. A bolometer is glued to the other end of the substrate to monitor the temperature change in the sample due to microwave absorption.[8] With both temperature and microwave frequency fixed, the bolometric signal, which is proportional to the absorbed microwave power, is measured as a function of the applied field. For each field sweep, the resonance field B_0 is determined at the position of maximum absorption and ΔB_0 denotes the linewidth at half absorption maximum relative to the background. According to Eq. (1), the relative linewidth is $\Gamma/\Omega=\mu\Delta B_0/2B_0$. For the data published in Fig. 2 of Ref. 8 at $T=4.2$ K, Γ/Ω is weakly dependent on field and is 0.13 ± 0.023 for $\Omega/2\pi = 30-50$ GHz. The linewidth of the resonance with respect to a variable magnetic field at constant frequency was found to be almost temperature independent in the vortex glass state.[8] Matsuda *et al.*[9] found the relative linewidth ≈ 0.15–0.2 below the irreversibility line and they observed its increase with temperature above the irreversibility line in the vortex liquid state up to 0.5 at $T=63$ K. As for the resonance frequency, the relaxation rate has a cusp at the irreversibility line.[8]

The power law dependence of the plasma frequency on B at $\mathbf{B}\|c$ was explained in Ref. 19 assuming that the vortex lattice is strongly disordered along the c axis due to pinning in the vortex glass state or by thermal fluctuations in the liquid vortex state. Deviations of the pancake vortices from straight lines in equilibrium induce a nonzero phase difference $\varphi_{n,n+1}(\mathbf{r})$ between neighboring layers n and $n+1$ at coordinate $\mathbf{r}=(x,y)$. This phase difference suppresses the average interlayer Josephson energy and maximum possible interlayer superconducting current, $J_m = J_0\langle\cos\varphi_{n,n+1}(\mathbf{r})\rangle$. Here J_0 is the parameter which characterizes the strength of interlayer Josephson coupling, and $\langle\cdots\rangle$ means averaging over space and disorder. The suppression of the plasma frequency Ω by pancake vortices was described in Refs. 7 and 19 assuming that the plasma frequency is proportional to the maximum interlayer current:

$$\Omega^2=\frac{8\pi^2cs}{\epsilon_c\Phi_0}J_m=\frac{8\pi^2cs}{\epsilon_c\Phi_0}J_0\langle\cos\varphi_{n,n+1}(\mathbf{r})\rangle. \tag{2}$$

Here s is the interlayer spacing and ϵ_c is the high-frequency dielectric function for an electric field along the c axis. Equation (2) corresponds to averaging over the Josephson interaction. In this mean-field approach, pancake vortices are assumed to be fixed and effects of the inhomogeneity of Josephson interactions are not accounted for. The temperature dependence of the plasma frequency was explained in Ref. 19 by accounting for the effect of low-frequency thermal fluctuations of the phase difference (of thermally excited phase collective modes). These smooth out rapid changes of the phase difference produced by disordered pancake vortices and result in an increase of the average Josephson interaction and plasma frequency with temperature at low temperatures. Their effect on the plasma frequency may be compared with the effect of spin waves on the magnetic susceptibility of an antiferromagnet below the Néel temperature: Magnetic susceptibility increases because spin waves make the antiferromagnet softer.

The strong angular dependence of the plasma resonance in high magnetic fields was predicted in Ref. 12 as a result of interactions of pancake vortices, produced by the B_z component of the field, with the dense lattice of Josephson vortices, induced by the field component parallel to the ab plane. In this case pancake vortices form an almost hexagonal structure in the ab plane and a zigzag structure along the c axis. The zigzag structure of pancakes minimizes the Josephson energy and induces pinning for Josephson vortices. This results in a sharp enhancement of the plasma frequency when θ increases from zero and pancake vortices appear.

The relatively large linewidth of the c-axis plasmon observed in Refs. 8–11 is surprising. In a single Josephson junction the linewidth of the Josephson plasmon is associated with incoherent dissipative tunneling of quasiparticles. In a superconductor with a gap Δ in the quasiparticle spectrum the concentration of quasiparticles is exponentially small at low temperatures $T\ll\Delta$ and vanishes linearly with T for d-wave superconductors. Abrikosov vortices increase the number of quasiparticles linearly with B in s-wave superconductors and as $B^{1/2}$ in d-wave superconductors.[14] Thus we can imagine that a relatively large linewidth with a weak temperature and field dependence cannot be explained by an interlayer current of normal quasiparticles. Rather, it may be caused by the effect of pancake vortices on the Josephson interlayer interaction; randomly positioned, they induce inhomogeneity of the Josephson interaction which may lead to a significant broadening of the Josephson plasma resonance. In addition, plasma oscillations may mix with vortex oscillations and this effect also results in a broadening of the plasma resonance.

The main goal of this paper is to describe the phase collective mode in the presence of vortices beyond the mean-field approach to take into account the inhomogeneity of Josephson coupling and dynamic interaction between pancake vortices and the c-axis plasmon.

To study the dynamic effects of vortices we derive in Secs. II and III the equations for pancake coordinates and phase difference in the presence of vortices in the London regime, $B\ll H_{c2}$, and below the irreversibility line. These equations allow us to study all dynamic effects associated with phase difference variations in the presence of pancakes, including the field dependence of plasma resonance and

c-axis resistivity in the superconducting state. In Sec. IV we consider the effect of inhomogeneous Josephson interactions caused by randomly positioned vortices on the plasmon linewidth and formulate the condition when Eq. (2) is valid. In Sec. V we show that the phase collective mode and vortex oscillations around equilibrium positions are coupled in a linear approximation via a Josephson interaction if pancakes at equilibrium are displaced from straight lines due to pinning and thermal fluctuations. As a result, true collective modes in this case are mixed plasmon-vortex oscillations. This effect leads to additional line broadening of the c-axis plasma resonance. In Sec. VI we discuss the experimental data and show that in the fields studied the inhomogeneous broadening is dominant and it describes well the experimental data for the plasma resonance line shape. However, we argue that at higher fields, or in samples with weaker pinning, plasmon dissipation into vortex oscillations may become more important than inhomogeneous broadening.

II. STATIC EQUATION FOR THE PHASE DIFFERENCE AND FUNCTIONAL FOR PANCAKE COORDINATES

In this section, starting from the Lawrence-Doniach (LD) functional[1] for the superconducting order parameter phase, we present derivations which lead to the equation for the phase difference between neighboring layers at equilibrium in the presence of vortices,[21] and then we obtain the free energy functional with respect to vortex positions and phase differences. This allows us in the following to generalize these results to obtain a time-dependent equation for the phase difference, and the Lagrangian for pancake coordinates which accounts for the time-dependent phase difference. For a multilayered system without Josephson coupling the functional for pancake positions was obtained by Buzdin and Feinberg[20] and by Clem.[4] Such a functional in the presence of Josephson interlayer interaction was derived in Refs. 21 and 22 for the case of a single tilted vortex, and in the following we present such a functional for arbitrary positions of pancakes.

In the framework of the LD functional for the superconducting order parameter $\Psi_n(\mathbf{r})=|\Psi_n(\mathbf{r})|\exp[i\phi_n(\mathbf{r})]$ in the layer n, we assume the amplitude of the order parameter to be constant in space. Such an approach is invalid only in small areas inside the normal cores of the vortices. The radius of these cores is of the order of the superconducting correlation length ξ_{ab}. In magnetic fields $B\ll H_{c2}$ (in the London regime) the area of normal cores is negligible and an approximation of constant amplitude is adequate. We will show in the following those effects that are sensitive to spatial variation of the order parameter amplitude. In the approach of a fixed amplitude, the pancakes are pointlike ''particles'' which induce variation of the phase $\phi_n(\mathbf{r})$ in the surrounding space. The total change of $\phi_n(\mathbf{r})$ should be 2π along any closed contour in the layer surrounding a pancake vortex. The corresponding LD functional in terms of $\phi_n(\mathbf{r})$ and vector potential $\mathbf{A}(\mathbf{R})$ is

$$\mathcal{F}\{\phi_n(\mathbf{r}),\mathbf{A}(\mathbf{R})\}=E_0\sum_n\int d\mathbf{r}\left[\frac{1}{2}\left(\nabla\phi_n+\frac{2\pi}{\Phi_0}\mathbf{A}_n\right)^2+\frac{1}{\lambda_J^2}(1-\cos\varphi_{n,n+1})\right]+\int d\mathbf{R}\frac{B^2}{8\pi},$$

$$E_0=\frac{\Phi_0^2 s}{16\pi^3\lambda_{ab}^2}. \qquad (3)$$

Here $\mathbf{R}=(\mathbf{r},z)$—the z axis is perpendicular to the layers—layers are positioned at $z=ns$, $\nabla=\partial/\partial\mathbf{r}$, λ_{ab} is the London penetration depth for currents in the ab plane, $\lambda_J=\gamma s$ is the Josephson length, and γ is the anisotropy ratio. Further, $\mathbf{A}_n=(A_{nx},A_{ny})=[A_x(\mathbf{r},z=ns),A_y(\mathbf{r},z=ns)]$, $\mathbf{B}=\mathrm{curl}\mathbf{A}$, and the gauge-invariant phase difference between layers n and $n+1$ is

$$\varphi_{n,n+1}(\mathbf{r})=\phi_n(\mathbf{r})-\phi_{n+1}(\mathbf{r})-\frac{2\pi}{\Phi_0}\int_{ns}^{(n+1)s}dzA_z(\mathbf{r},z). \qquad (4)$$

Minimization of $\mathcal{F}$ with respect to $\phi_n(\mathbf{r})$ and $\mathbf{A}(\mathbf{R})$ yields the system of equations for phases and fields in equilibrium at given vortex positions. Varying $\mathcal{F}$ with respect to $\mathbf{A}$, we obtain

$$(\mathrm{curl}\,\mathrm{curl}\mathbf{A})_\alpha=\frac{4\pi}{c}j_\alpha=-\frac{\Phi_0 s}{2\pi\lambda_{ab}^2}\sum_n\left(\nabla\phi_n+\frac{2\pi}{\Phi_0}\mathbf{A}_n\right)_\alpha\delta(z-ns), \qquad (5)$$

$$(\mathrm{curl}\,\mathrm{curl}\mathbf{A})_z=\frac{4\pi}{c}j_z=-\frac{\Phi_0 s}{2\pi\lambda_{ab}^2\gamma^2}\sum_n f_{n,n+1}(z)\sin\varphi_{n,n+1}, \qquad (6)$$

where $\alpha=x,y$ and the factor $f_{n,n+1}(z)$ vanishes everywhere except for $ns<z<(n+1)s$, where it is unity. Minimization of $\mathcal{F}$ with respect to ϕ_n yields

$$\nabla^2\phi_n+\frac{2\pi}{\Phi_0}\nabla\mathbf{A}_n=\frac{1}{\lambda_J^2}(\sin\varphi_{n,n+1}-\sin\varphi_{n-1,n}). \qquad (7)$$

From Eqs. (5)–(7) the equation for the phase difference $\varphi_{n,n+1}$ was obtained in Refs. 21 and 23:

$$-\sum_m L_{nm}\nabla^2\varphi_{m,m+1}+\lambda_J^{-2}\sin\varphi_{n,n+1}=0, \qquad (8)$$

where the mutual inductance of layers, L_{nm}, is

$$L_{nm}=\int_0^{2\pi}\frac{dq}{2\pi}\frac{\cos(n-m)q}{2(1-\cos q)+s^2/\lambda_{ab}^2}=\frac{\lambda_{ab}}{s}\left(1-\frac{s}{\lambda_{ab}}\right)^{|n-m|}. \qquad (9)$$

Boundary conditions for Eq. (8) are determined by positions of topological singularities (vortices). The phases $\phi_n(\mathbf{r})$ are singular at the position of each pancake, $\mathbf{r}_{n\nu}$:

$$(\nabla_x\nabla_y-\nabla_y\nabla_x)\phi_n(\mathbf{r})=2\pi\sum_\nu \delta(\mathbf{r}-\mathbf{r}_{n\nu}). \tag{10}$$

Then the boundary condition for $\varphi_{n,n+1}(\mathbf{r})$ is

$$(\nabla_x\nabla_y-\nabla_y\nabla_x)\varphi_{n,n+1}(\mathbf{r})=2\pi\sum_\nu [\delta(\mathbf{r}-\mathbf{r}_{n\nu})-\delta(\mathbf{r}-\mathbf{r}_{n+1,\nu})]. \tag{11}$$

Note that for vortices placed along straight lines along the c axis ($\mathbf{r}_{n\nu}=\mathbf{r}_{n+1,\nu}$) the singularities in the phase difference are absent. To satisfy the boundary condition, Eq. (11), we present the phase difference as

$$\varphi_{n,n+1}(\mathbf{r})=\varphi^{(v)}_{n,n+1}(\mathbf{r})+\varphi^{(r)}_{n,n+1}(\mathbf{r}), \tag{12}$$

where $\varphi^{(v)}_{n,n+1}(\mathbf{r},\mathbf{r}_{n\nu})$ is the phase difference induced by pancakes at positions $\mathbf{r}_{n\nu}$ in the absence of a Josephson interaction (with infinite λ_J):

$$\varphi^{(v)}_{n,n+1}(\mathbf{r},\mathbf{r}_{n\nu})=\sum_\nu [f(\mathbf{r}-\mathbf{r}_{n\nu})-f(\mathbf{r}-\mathbf{r}_{n+1,\nu})],$$

$$f(\mathbf{r})=\arctan(x/y). \tag{13}$$

The function $\varphi^{(v)}_{n,n+1}(\mathbf{r})$ is singular at vortex positions $\mathbf{r}_{n\nu}$, while the function $\varphi^{(r)}_{n,n+1}(\mathbf{r})$ is regular everywhere and describes the effect of three-dimensional screening caused by interlayer Josephson currents.[21,22] Then Eq. (8) with the boundary condition (11) is equivalent to the equation

$$-\sum_m L_{nm}\nabla^2\varphi^{(r)}_{m,m+1}+\lambda_J^{-2}\sin[\varphi^{(v)}_{n,n+1}+\varphi^{(r)}_{n,n+1}]=0. \tag{14}$$

It is valid outside of vortex cores. Inside vortex cores we should take account that the Josephson parameter $J_0\propto\lambda_J^{-2}\propto|\Psi_n(\mathbf{r})||\Psi_{n+1}(\mathbf{r})|$ vanishes at the center of pancake, i.e., at $\mathbf{r}=\mathbf{r}_{n\nu}$ or $\mathbf{r}=\mathbf{r}_{n+1,\nu}$. This removes the singularity induced by $\varphi_{n,n+1}$ in the second term on the left-hand side. The solution of Eq. (14) for $\varphi^{(r)}_{n,n+1}(\mathbf{r})$ provides a minimum to the functional

$$\mathcal{F}_\varphi=E_0\int d\mathbf{r}\left\{\frac{1}{2}\sum_{mn} L_{nm}\nabla\varphi^{(r)}_{n,n+1}\cdot\nabla\varphi^{(r)}_{m,m+1}+\frac{1}{\lambda_J^2}[1-\cos(\varphi^{(v)}_{n,n+1}+\varphi^{(r)}_{n,n+1})]\right\}. \tag{15}$$

The free energy functional for pancake coordinates, for the gauge $A_z=0$, is[21]

$$\mathcal{F}_v(\mathbf{r}_{n\nu})=\sum_n \int d\mathbf{r}\left[-\frac{\Phi_0 s}{4\pi c}\mathbf{j}_n\cdot\nabla\phi_n+\frac{E_0}{\lambda_J^2}(1-\cos\varphi_{n,n+1})\right], \tag{16}$$

where $\varphi_{n,n+1}(\mathbf{r})$ is determined by Eq. (14) while $\nabla\phi_n$ and $\mathbf{j}_n$ are the solutions of Eqs. (5)-(7) with boundary condition (10). The solutions of the linear equations (5)–(7) can be expressed in terms of $\varphi_{n,n+1}(\mathbf{r})$ and the coordinates of pancakes as a sum of the contribution due to pancakes in the absence of Josephson coupling and that induced by the Josephson currents. The free energy functional of the system, Eq. (16), is a sum of these two contributions. Finally we obtain the functional for pancake coordinates as

$$\mathcal{F}_v(\mathbf{r}_{n\nu})=\mathcal{F}_{\rm em}(\mathbf{r}_{n\nu})+\mathcal{F}_J(\mathbf{r}_{n\nu})+\mathcal{F}_{\rm pin}(\mathbf{r}_{n\nu}). \tag{17}$$

Here $\mathcal{F}_{\rm em}(\mathbf{r}_{n\nu})$ is the functional which accounts for the two-dimensional energy of pancakes and includes also their electromagnetic interaction in different layers:[4,20]

$$\mathcal{F}_{\rm em}(\mathbf{r}_{n\nu})=\frac{1}{2}E_0\int d\mathbf{k}dq\times\sum_{n,m,\nu,\nu'}\frac{\exp[i\mathbf{k}\cdot(\mathbf{r}_{n\nu}-\mathbf{r}_{m\nu'})+iq(n-m)]}{k^2[1+\lambda_{ab}^{-2}(k^2+Q^2)^{-1}]}, \tag{18}$$

where $Q^2=(1-\cos q)/s^2$. The functional $\mathcal{F}_J(\mathbf{r}_{n\nu})$ accounts for the Josephson interaction of pancakes:

$$\mathcal{F}_J(\mathbf{r}_{n\nu})=\frac{E_0}{\lambda_J^2}\int d\mathbf{r}[1-\cos\varphi_{n,n+1}(\mathbf{r})]+\frac{E_0}{8\pi^2\lambda_J^2}\times\int d\mathbf{k}dq\,\frac{1+\lambda_{ab}^2Q^2}{\lambda_{ab}^2\gamma^2k^2}|[\sin\varphi_{n,n+1}(\mathbf{r})]_{\mathbf{k}q}|^2,$$

$$[\sin\varphi_{n,n+1}(\mathbf{r})]_{\mathbf{k}q}=\sum_n\int d\mathbf{r}\sin\varphi_{n,n+1}(\mathbf{r})\times\exp[-(i\mathbf{k}\cdot\mathbf{r}+iqn)], \tag{19}$$

where the function $\varphi_{n,n+1}(\mathbf{r},\mathbf{r}_{n\nu})$ is the solution of Eq. (14).

The contribution $\mathcal{F}_{\rm pin}(\mathbf{r}_{n\nu})$ accounts for pinning:

$$\mathcal{F}_{\rm pin}(\mathbf{r}_{n\nu})=\sum_{n,\nu}\int d\mathbf{r}V_{\rm pin}(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}_{n\nu}), \tag{20}$$

where $V_{\rm pin}(\mathbf{r})$ is the pinning potential for vortices; see Ref. 24.

The functional $\mathcal{F}_v$ can be obtained by minimization of the following total functional in terms of the variables $\varphi_{n,n+1}(\mathbf{r})$ and $\mathbf{r}_{n\nu}$:

$$\mathcal{F}\{\mathbf{r}_{n\nu},\varphi^{(r)}_{n,n+1}(\mathbf{r})\}=\mathcal{F}_{\rm em}(\mathbf{r}_{n\nu})+\mathcal{F}_{\rm pin}(\mathbf{r}_{n\nu})+\mathcal{F}_\varphi\{\mathbf{r}_{n\nu},\varphi^{(r)}_{n,n+1}(\mathbf{r})\}, \tag{21}$$

where the last term is defined by Eq. (15). This functional will be used in the following sections to obtain the Lagrangian of the system in terms of $\mathbf{r}_{n\nu}$ and $\varphi^{(r)}_{n,n+1}(\mathbf{r})$. Below the irreversibility line (in the vortex glass state) the equilibrium positions of vortices, $\mathbf{r}^{(0)}_{n\nu}$, are determined by the condition that this functional is minimum with respect to both variables.

III. TIME-DEPENDENT EQUATION FOR THE PHASE DIFFERENCE IN THE PRESENCE OF PANCAKE VORTICES

A. General equations

In our consideration of the dynamic effect of pancake vortices we follow the same procedure as for the static case. We generalize Eq. (14) to a time-dependent equation for $\varphi_{n,n+1}(\mathbf{r},t)$ which is similar to the sine-Gordon equation for a single junction but takes into account the presence of many layers as well as moving pancakes. Then we derive the Lagrangian for pancake coordinates which accounts for the time-dependent phase difference and provides the equation of motion for pancakes in the layered system with Josephson coupling. The equation for the time-dependent phase difference and equation of motion for pancakes form the coupled set of equations which describe the phase collective modes interacting with moving pancakes.

The time-dependent equation for the phase difference in the absence of pancake vortices and without accounting for relaxation of the superconducting order parameter was obtained in Refs. 25–27. This equation was derived by use of the Maxwell equations

$$\epsilon_{ab}\nabla\cdot\mathbf{E}_n+\frac{\epsilon_c}{s}(E_{z,n}-E_{z,n+1})=4\pi\rho_n, \quad (22)$$

$$\text{curl}\mathbf{B}(\mathbf{r},z,t)=\frac{\hat{\epsilon}}{c}\frac{\partial\mathbf{E}(\mathbf{r},z,t)}{\partial t}+\frac{4\pi}{c}\mathbf{j}(\mathbf{r},z,t), \quad (23)$$

where $\mathbf{E}_n$ is the electric field in the layer n, $E_{z;n,n+1}(\mathbf{r},t)$ is the average z component of the electric field between layers n and $n+1$, $\hat{\epsilon}$ is the high-frequency dielectric tensor with components ϵ_{ab} and ϵ_c for electric fields along the ab plane and along the c axis, respectively, and $\rho_n(\mathbf{r})$ is the average charge density in the layer n. Then the Maxwell equations are complemented by the constitutive equations for the current density $\mathbf{j}$. For the interlayer currents we have

$$J_{z,n,n+1}=J^{(s)}_{z,n,n+1}+J^{(n)}_{z,n,n+1}=J_0\sin\varphi_{n,n+1}+\sigma_c E_{z,n,n+1}, \quad (24)$$

with $J_0=2\pi cE_0/\Phi_0\lambda_J^2$ being the maximum Josephson supercurrent, $\varphi_{n,n+1}$ is the gauge-invariant phase difference given by Eq. (4), and σ_c is the c axis conductivity due to quasiparticles. For the in-plane current we have

$$\mathbf{J}_n=\mathbf{J}^{(s)}_n+\mathbf{J}^{(n)}_n=\frac{c\Phi_0}{8\pi^2\lambda_{ab}^2}\mathbf{Q}_n+\sigma_{ab}\mathbf{E}_n, \quad (25)$$

with the gauge-invariant phase gradient $\mathbf{Q}_n=-[\boldsymbol{\nabla}\phi_n+(2\pi/\Phi_0)\mathbf{A}_n]$ and σ_{ab} is the in-plane conductivity due to quasiparticles. From Eqs. (22) and (23) the continuity equation follows:

$$\frac{\partial\rho_n}{\partial t}-\nabla\cdot\mathbf{J}^{(s)}_n-\nabla\cdot\mathbf{J}^{(n)}_n-\frac{J_0}{s}(\sin\varphi_{n,n+1}-\sin\varphi_{n-1,n})$$
$$-\frac{1}{s}(J^{(n)}_{z;n,n+1}-J^{(n)}_{z;n-1,n})=0. \quad (26)$$

The Josephson relation

$$\frac{\partial\varphi_{n,n+1}(\mathbf{r},t)}{\partial t}=\frac{2es}{\hbar}E_{z;n,n+1}(\mathbf{r},t) \quad (27)$$

was used in Refs. 25–27. Equation (27) is valid at equilibrium when $\partial\phi_n(\mathbf{r},t)/\partial t=-(2e/\hbar)V_n(\mathbf{r},t)$; see Refs. 28 and 29 for more details. Here $V_n(\mathbf{r},t)$ is the scalar potential in the layer n.

From Eq. (26), for dynamic processes with characteristic frequencies $\omega\ll\omega_{ab}(\sigma_c/\sigma_{ab})^{1/2}\approx\omega_{ab}/\gamma$, the time-dependent equation for the phase difference is

$$\frac{1}{c_0^2}\hat{T}_c\frac{\partial}{\partial t}\varphi_{n,n+1}+\frac{1}{\lambda_J^2}\sin\varphi_{n,n+1}-\sum_m L_{nm}\nabla^2\varphi_{m,m+1}=0,$$

$$\hat{T}_c=\frac{\partial}{\partial t}+\Gamma_c, \quad (28)$$

where $c_0=cs/\lambda_{ab}\sqrt{\epsilon_c}$ plays the role of the Swihart velocity, $\omega_{ab}=c/\sqrt{\epsilon_{ab}}\lambda_{ab}$ is the in-plane plasma frequency, and $\Gamma_c=4\pi\sigma_c/\epsilon_c$. The dissipation of the phase collective mode in this approach is caused only by the interlayer current of normal quasiparticles as described by σ_c. The dissipation due to the in-plane current of quasiparticles is smaller by the parameter $(\omega/\omega_{ab})(\sigma_{ab}/\sigma_c)^{1/2}$. Equation (28) generalizes the sine-Gordon equation for a single junction to a multilayer system. It is valid if the relaxation time of the order parameter amplitude is much smaller than the characteristic time of the processes described by this equation. The Lagrangian $\mathcal{L}_\varphi$ and the dissipation function $\mathcal{R}_\varphi$ for the phase difference are

$$\mathcal{L}_\varphi=\frac{E_0}{2c_0^2}\sum_n\int d\mathbf{r}\left(\frac{\partial\varphi_{n,n+1}}{\partial t}\right)^2-\mathcal{F}_\varphi,$$

$$\mathcal{R}_\varphi=\frac{E_0\Gamma_c}{2c_0^2}\sum_n\int d\mathbf{r}\left(\frac{\partial\varphi_{n,n+1}}{\partial t}\right)^2. \quad (29)$$

The kinetic part of this Lagrangian is the energy of electric field in the system. The equation of motion (28) is obtained from the Lagrangian and the dissipation function as

$$\frac{d}{dt}\frac{\partial\mathcal{L}_\varphi}{\partial\dot{\varphi}_{n,n+1}}-\frac{\partial\mathcal{L}_\varphi}{\partial\varphi_{n,n+1}}+\frac{\partial\mathcal{R}_\varphi}{\partial\dot{\varphi}_{n,n+1}}=0. \quad (30)$$

A general equation, which accounts for relaxation of the superconducting order parameter in the framework of the time-dependent Ginzburg-Landau (TDGL) equation modified for the LD model, was derived in Ref. 32 in the absence of pancakes. In this approach the Josephson relation, Eq. (27), is replaced by the general relation

$$\frac{\partial\varphi_{n,n+1}(\mathbf{r},t)}{\partial t}=\frac{2es}{\hbar}E_{z;n,n+1}(\mathbf{r},t)+g_{n,n+1}(\mathbf{r},t). \quad (31)$$

Here $g_{n,n+1}(\mathbf{r},t)$ is the gauge-invariant time derivative of the phase difference:

$$g_{n,n+1}(\mathbf{r},t)=G_n(\mathbf{r},t)-G_{n+1}(\mathbf{r},t),$$

$$G_n(\mathbf{r},t)=\frac{\partial\phi_n(\mathbf{r},t)}{\partial t}-\frac{2e}{\hbar}V_n(\mathbf{r},t). \quad (32)$$

The function G_n can be determined by use of the TDGL equation for the superconducting phase:

$$4e\gamma_{\mathrm{GL}}G_n|\Psi_n|^2=\nabla\cdot\mathbf{J}_n^{(s)}+\frac{J_0}{s}(\sin\varphi_{n,n+1}-\sin\varphi_{n-1,n}),\tag{33}$$

where the parameter γ_{GL} characterizes the relaxation of the superconducting order parameter; see Refs. 32, 29, and 30 for more details. The second TDGL equation for the time evolution of $|\Psi_n(\mathbf{r},t)|$ is neglected in this London regime approach. The full dynamics can be derived, in terms of the gauge-invariant quantities $\varphi_{n,n+1}$, G_n, and $\mathbf{Q}_n$, from Eqs. (22)–(25) and (31)–(33). We obtain

$$\frac{\hat{T}_c}{c_0^2}\left(\frac{\partial\varphi_{n,n+1}}{\partial t}-g_{n,n+1}\right)+\frac{1}{\lambda_J^2}\sin\varphi_{n,n+1}-\sum_m L_{nm}\nabla^2\varphi_{m,m+1}=\sum_m L_{nm}W_m\{\mathbf{Q}_m,g_{m,m+1}\},\tag{34}$$

$$-\tilde{\gamma}_{\mathrm{GL}}(G_{n+1}|\Psi_{n+1}|^2-G_n|\Psi_n|^2)=W_n\{\mathbf{Q}_n,g_{n,n+1}\}+\frac{\hat{T}_c}{c_0^2}\hat{\delta}^2\left(\frac{\partial\varphi_{n,n+1}}{\partial t}-g_{n,n+1}\right),\tag{35}$$

$$\boldsymbol{\nabla}\cdot(\mathbf{Q}_{n+1}-\mathbf{Q}_n)+W_n\{\mathbf{Q}_n,g_{n,n+1}\}+\hat{\delta}^2\left[\frac{1}{\lambda_J^2}\sin\varphi_{n,n+1}+\frac{\hat{T}_c}{c_0^2}\left(\frac{\partial\varphi_{n,n+1}}{\partial t}-g_{n,n+1}\right)\right]=0,\tag{36}$$

with

$$W_n\{\mathbf{Q}_n,g_{n,n+1}\}=\frac{\hat{T}_{ab}}{\omega_{ab}^2}\left[\frac{\partial}{\partial t}\boldsymbol{\nabla}\cdot(\mathbf{Q}_{n+1}-\mathbf{Q}_n)-\nabla^2 g_{n,n+1}\right]=\frac{2\pi c\hat{T}_{ab}}{\Phi_0\omega_{ab}^2}\nabla\cdot(\mathbf{E}_{n+1}-\mathbf{E}_n),\tag{37}$$

the parameter $\tilde{\gamma}_{\mathrm{GL}}=4e\gamma_{\mathrm{GL}}8\pi^2\lambda_{ab}^2/c\Phi_0$, and we define the operators $\hat{T}_{ab}=\partial/\partial t+4\pi\sigma_{ab}/\epsilon_{ab}$ and $\hat{\delta}^2a_n=a_{n+1}-2a_n+a_{n-1}$. The only dependence on $\mathbf{Q}_n$ is through the scalar $d_{n,n+1}=\boldsymbol{\nabla}\cdot(\mathbf{Q}_{n+1}-\mathbf{Q}_n)$. Here Eq. (34) generalizes Eq. (28), while Eq. (35) comes from the TDGL equation (33) and Eq. (36) is another form of the continuity equation (26). Therefore, the full dynamics is described by Eqs. (34)–(36) for the coupled variables $\varphi_{n,n+1}$, $g_{n,n+1}$, and $d_{n,n+1}$. There are boundary conditions: For the phase variable $\varphi_{n,n+1}$ they are given by Eq. (11), but now with time-dependent positions of vortices, $\mathbf{r}_{n\nu}(t)$, and for $g_{n,n+1}$ they are given by

$$(\nabla_x\nabla_y-\nabla_y\nabla_x)g_{n,n+1}(\mathbf{r},t)=2\pi\frac{\partial}{\partial t}\sum_\nu[\delta(\mathbf{r}-\mathbf{r}_{n\nu}(t))-\delta(\mathbf{r}-\mathbf{r}_{n+1,\nu}(t))].\tag{38}$$

To satisfy the boundary conditions we can split again the total phase difference into two parts, Eq. (12), and the same for $g_{n,n+1}$:

$$g_{n,n+1}(\mathbf{r},t)=g_{n,n+1}^{(v)}(\mathbf{r},t)+g_{n,n+1}^{(r)}(\mathbf{r},t).\tag{39}$$

Note that the function $g_{n,n+1}^{(v)}(\mathbf{r},t)$ with singularities cancels when all the pancakes move with the same velocities. Equations (34)–(36), (11), and (38) have to be complemented with a set of equations providing the dynamics for $\{\mathbf{r}_{n\nu}(t)\}$. Or, alternatively, the full microscopic dynamics of $\mathbf{Q}_n(\mathbf{r},t)$ has to be given.

For low frequencies such that $(\omega/\omega_{ab})^2\ll 1$ and $\omega\sigma_{ab}/\omega_{ab}^2\ll 1$ one can obtain from Eq. (36)

$$d_{n,n+1}\approx-\left(1-\frac{\hat{T}_{ab}}{\omega_{ab}^2}\frac{\partial}{\partial t}\right)\left\{\hat{\delta}^2\left[\frac{1}{\lambda_J^2}\sin\varphi_{n,n+1}+\frac{\hat{T}_c}{c_0^2}\left(\frac{\partial\varphi_{n,n+1}}{\partial t}-g_{n,n+1}\right)\right]-\frac{\hat{T}_{ab}}{\omega_{ab}^2}\nabla^2 g_{n,n+1}\right\},\tag{40}$$

and then reduce the system of dynamical equations to Eqs. (34) and (35) with the effective

$$W_n\{\varphi_{n,n+1},g_{n,n+1}\}\approx-\frac{\hat{T}_{ab}}{\omega_{ab}^2}\left\{\frac{\partial}{\partial t}\hat{\delta}^2\left[\frac{1}{\lambda_J^2}\sin\varphi_{n,n+1}+\frac{\hat{T}_c}{c_0^2}\left(\frac{\partial\varphi_{n,n+1}}{\partial t}-g_{n,n+1}\right)\right]+\nabla^2 g_{n,n+1}\right\}.\tag{41}$$

In Ref. 32, the dynamical equations including the TDGL equation (33) have been studied in the absence of pancakes, where the amplitude of the superconducting order parameter $|\Psi_n|$ in Eq. (35) was taken as constant. In this case, the regular term $g_{n,n+1}^{(r)}$ was calculated from Eq. (35), for which the most important contribution is the second term on the right-hand side (the W_n term can be neglected). At frequencies $\omega\ll\omega_{ab}(\sigma_c/\sigma_{ab})^{1/2}$ the additional terms in Eq. (34), due to relaxation of the superconducting order parameter, may be neglected in comparison with that due to the interlayer current of quasiparticles if we use the standard estimate $\gamma_{\mathrm{GL}}\gtrsim\hbar\omega_{ab}^2/4e^2s^2|\Psi_n|^2\sigma_{ab}^{(n)}$ Thus Eq. (28) provides a quite accurate description of the phase collective mode at low frequencies. We note that all terms due to relaxation of the order parameter and in-plane current of quasiparticles vanish from the equation for a homogeneous plasma mode in the absence of pancakes. For this case dissipation of the plasmon is determined by the interlayer current of quasiparticles only.

B. Approximate solution

Now we analyze the time-dependent equations for the phase difference in the presence of vortices. In principle, besides Eqs. (34)–(36), we should account for spatial variations of the parameter λ_J^{-2} inside vortex cores. This will be done later. Here we plan to obtain an effective expression for $g_{n,n+1}$ in order to reduce the dynamical equations to the variables $\{\varphi_{n,n+1}(\mathbf{r},t)\}$ and $\{\mathbf{r}_{n\nu}(t)\}$ only. The regular part $g_{n,n+1}^{(r)}$ was calculated in Ref. 32. In the presence of pancakes, the singular part $g_{n,n+1}^{(v)}$ is the most relevant. In this case, the main contribution comes from the W_n term in Eq.

(35), i.e., the electric field $\mathbf{E}_n$ induced by the moving vortices. In general one can write from $g^{(v)}_{n,n+1}=G^{(v)}_{n+1}-G^{(v)}_n$ and using Eq. (32):

$$G^{(v)}_n(\mathbf{r},t)=\frac{\partial}{\partial t}\phi^{(v)}_n(\mathbf{r},t)-\frac{2e}{\hbar}V^{(v)}_n(\mathbf{r},t). \tag{42}$$

The scalar potential $V^{(v)}_n(\mathbf{r},t)$ can be calculated from the TDGL equation (33) using Eq. (42) and the gauge $\sigma_{ab}\nabla\cdot\mathbf{A}_n(\mathbf{r})+\sigma_c/s\int_{ns}^{(n+1)s}dzA_z(\mathbf{r},z)=0$:

$$4e\gamma_{\rm GL}\left[\frac{\partial}{\partial t}\phi^{(v)}_n-V^{(v)}_n\right]|\Psi_n(\mathbf{r},t)|^2+\sigma_{ab}\nabla^2V^{(v)}_n+\frac{\sigma_c}{s^2}[V^{(v)}_{n+1}+V^{(v)}_{n-1}-2V^{(v)}_n]=0. \tag{43}$$

In deriving Eq. (43) we neglected terms which have an additional small parameter proportional to the pancake velocities $\dot{\mathbf{r}}_{n\nu}$ (terms like $\partial V^{(v)}_n/\partial t$ in comparison with $\sigma_c V^{(v)}_n$). Equation (43) should be solved with the boundary condition that $V^{(v)}_n(\mathbf{r},t)$ is finite everywhere. For magnetic fields $B\ll H_{c2}$ the solution of Eq. (43) may be found as a superposition of solutions for single vortices. Then we write

$$G^{(v)}_n(\mathbf{r},t)=\sum_{m\nu}\frac{\partial}{\partial t}f(\mathbf{r}-\mathbf{r}_{n\nu})\delta_{nm}-\frac{2e}{\hbar}\mathcal{V}_{m-n}(\mathbf{r}-\mathbf{r}_{n\nu}), \tag{44}$$

where we have used Eq. (13) for $\phi^{(v)}_n$ and $\mathcal{V}_{m-n}(\mathbf{r}-\mathbf{r}_{n\nu})$ is the voltage induced in the plane m by a vortex moving in the plane n. For a single pancake vortex at $\mathbf{r}_{0\nu}$ in the layer $n=0$ we can calculate $\mathcal{V}_n(\mathbf{r}-\mathbf{r}_{0\nu})$ as

$$4e\gamma_{\rm GL}\left\{\frac{\partial}{\partial t}f[\mathbf{r}-\mathbf{r}_{0\nu}(t)]\delta_{n0}-\frac{2e}{\hbar}\mathcal{V}_n\right\}|\Psi_n(\mathbf{r},t)|^2+\sigma_{ab}\nabla^2\mathcal{V}_n+\frac{\sigma_c}{s^2}(\mathcal{V}_{n+1}+\mathcal{V}_{n-1}-2\mathcal{V}_n)=0. \tag{45}$$

The equation for the scalar potential in the case of a moving vortex line in an isotropic superconductor was discussed in Refs. 29 and 30. Equation (45) generalizes this equation to a pancake vortex in a layered superconductor. The function $|\Psi(\mathbf{r},t)|^2$ for a single vortex at point $\mathbf{r}_{0\nu}(t)$ in the layer $n=0$ has the form of a moving vortex,

$$|\Psi_n(\mathbf{r},t)|^2=|\Psi_n[\mathbf{r}-\mathbf{r}_{0\nu}(t)]|^2, \tag{46}$$

to lowest order in vortex velocity. Here $|\Psi_n(\mathbf{r})|$ is the order parameter amplitude at equilibrium. It vanishes at the center of a pancake, $r=0$, and tends to a constant value $|\Psi_\infty|$ far from the pancake center. The characteristic scale for this dependence is the superconducting correlation length ξ_{ab}. For $\partial f[\mathbf{r}-\mathbf{r}_{0\nu}(t)]/\partial t=-\dot{\mathbf{r}}_{0\nu}\nabla f[\mathbf{r}-\mathbf{r}_{0\nu}(t)]=-|\dot{\mathbf{r}}_{0\nu}|\sin\theta_{0\nu}/\rho_{0\nu}$ the solution for the scalar potential has the form $\mathcal{V}_n(\mathbf{r},t)=|\dot{\mathbf{r}}_{0\nu}|\sin\theta_{0\nu}V^*_n(\rho_{0\nu})$. Here $\boldsymbol{\rho}_{0\nu}=\mathbf{r}-\mathbf{r}_{0\nu}$ and $\theta_{0\nu}$ is the polar angle for coordinate $\boldsymbol{\rho}_{0\nu}$. With these definitions we can write

$$G^{(v)}_n(\mathbf{r},t)=-\sum_\nu|\dot{\mathbf{r}}_{n\nu}|\sin\theta_{n\nu}\left[\frac{1}{\rho_{n\nu}}+\frac{2e}{\hbar}\sum_m V^*_{m-n}(\rho_{n\nu})\right], \tag{47}$$

which explicitly shows the dependence of $G^{(v)}_n$ on vortex velocities. The equation for $V^*_n(\rho)$, with $\rho\equiv\rho_{0\nu}$, is

$$\frac{\partial}{\partial\rho}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho V^*_n)\right]+\Lambda_c^{-2}(V^*_{n+1}+V^*_{n-1}-2V^*_n)-\Lambda_{ab}^{-2}V^*_n=\frac{1}{\sigma_{ab}}\left\{-4e\gamma_{\rm GL}\frac{1}{\rho}|\Psi(\rho)|^2+\frac{8e^2\gamma_{\rm GL}}{\hbar}V^*_0(\rho)[|\Psi(\rho)|^2-|\Psi_\infty|^2]\right\}\delta_{n0}, \tag{48}$$

where $\Lambda_{ab}^2=\hbar\sigma_{ab}/8e^2\gamma_{\rm GL}|\Psi_\infty|^2$ and $\Lambda_c^2=s^2\sigma_{ab}/\sigma_c$. In the situation when the TDGL equation is valid (gapless superconductor with paramagnetic impurities) one obtains $\Lambda_{ab}=\xi_{ab}$; see Ref. 29. The solution for the scalar potential can be written in the form

$$V^{(v)}_n(\mathbf{r})=-\frac{\hbar}{2e}\frac{\partial}{\partial t}\vartheta_n(\mathbf{r}),$$
$$\vartheta_n(\mathbf{r})=\sum_{mv}\int_0^{|r-r_{mv}(t)|/\Lambda_{ab}}d\tilde{\rho}v^*_{n-m}(\tilde{\rho}), \tag{49}$$

where $v^*_n(\tilde{\rho})=2e\Lambda_{ab}V^*_n(\rho)/\hbar$ and $\tilde{\rho}=\rho/\Lambda_{ab}$. This gives the function $g^{(v)}_{n,n+1}(\mathbf{r})=\partial[\varphi^{(v)}_{n,n+1}(\mathbf{r})-\vartheta_{n,n+1}(\mathbf{r},t)]/\partial t$, where $\vartheta_{n,n+1}(\mathbf{r},t)=\vartheta_n(\mathbf{r})-\vartheta_{n+1}(\mathbf{r})$, which can be written as

$$\vartheta_{n,n+1}(\mathbf{r},t)=\varphi^{(v)}_{n,n+1}(\mathbf{r},t)+\sum_{m\nu}\int_0^{|\mathbf{r}-\mathbf{r}_{m\nu}(t)|/\Lambda_{ab}}d\tilde{\rho}v_{n-m}(\tilde{\rho}), \tag{50}$$

where the function $v_n(\tilde{\rho})=v^*_n(\tilde{\rho})-(1/\tilde{\rho})\delta_{n0}$ decays rapidly at distances $\tilde{\rho}\gg1$ far away from vortices. Actually, this function at $\tilde{\rho}\gg1$ obeys the equation

$$\frac{\partial}{\partial\tilde{\rho}}\left[\frac{1}{\tilde{\rho}}\frac{\partial}{\partial\tilde{\rho}}(\tilde{\rho}v_n)\right]-v_n-\frac{\Lambda_{ab}^2}{\Lambda_c^2}(v_{n+1}+v_{n-1}-2v_n)=0. \tag{51}$$

The solution of this equation for the Fourier transform $v(q)$ with respect to the integer variable n is the Bessel function $K_1[\lambda(q)\tilde{\rho}]$. It provides the asymptotic solution for $v_n(\tilde{\rho})$:

$$v_n(\tilde{\rho})=(\pi/2)^{1/2}\tilde{\rho}^{-1}\exp[-\tilde{\rho}(1+\Lambda_{ab}^2/\Lambda_c^2)]I_n(\tilde{\rho}\Lambda_{ab}^2/\Lambda_c^2), \tag{52}$$

where $I_n(x)$ is the Bessel function of imaginary argument. The function $v_n(\tilde{\rho})$ vanishes exponentially for large $\tilde{\rho}$ and for $\tilde{\rho}\Lambda_{ab}^2/\Lambda_c^2\ll1$ it drops with n as a power law, $I_n(x)\approx(x/2)^n$ for $x\ll1$. For $\tilde{\rho}\Lambda_{ab}^2/\Lambda_c^2\gg1$ the function v_n is almost n independent. Respectively, at distances far away from pancakes the function $\vartheta_{n,n+1}(\mathbf{r},t)$ approaches $\varphi^{(v)}_{n,n+1}(\mathbf{r},t)$ but differs from this function in the vicinity of vortices due to retardation in the time variation of the superconducting order parameter and superconducting currents in comparison with the moving center of the vortex (phase singularity). The effect of retardation becomes negligible far away from the vortex center. As a consequence, the functions $G^{(v)}_n(\mathbf{r})$ and $g^{(v)}_{n,n+1}(\mathbf{r})$ vanish exponentially far away from vortices. Note that the faster decrease of $G^{(v)}_n$

$\times(\mathbf{r})$ in comparison with that of $V_n^{(v)}(\mathbf{r})$ follows from Eq. (43). Here the left-hand side is proportional to $G_n^{(v)}(\mathbf{r})$ which includes $V_n^{(v)}(\mathbf{r})$, while the derivatives of $V_n^{(v)}(\mathbf{r})$ on the right-hand side drop faster than $V_n^{(v)}(\mathbf{r})$ itself. We will show in the following that the retardation effect, leading to a difference between $\vartheta_{n,n+1}(\mathbf{r},t)$ and $\varphi_{n,n+1}^{(v)}(\mathbf{r},t)$, gives rise to the dynamic vortex-plasmon interaction. Although we used the TDGL equation to obtain the function $\vartheta_{n,n+1}(\mathbf{r},t)$, our conclusion about the decay of the difference $\vartheta_{n,n+1}(\mathbf{r},t)-\varphi_{n,n+1}^{(v)}(\mathbf{r},t)$ at large distances from vortices is quite general, and only the scale of this decay, Λ_{ab}, is model dependent. Note that the retardation effect in the Josephson interaction may be neglected for $\hbar\partial\varphi_{n,n+1}/\partial t \ll \Delta$.

By the use of Eqs. (31), (32), and (49) we obtain the z component of the electric field:

$$E_{z;n,n+1}(\mathbf{r},t)=\frac{\hbar}{2es}\frac{\partial}{\partial t}[\varphi_{n,n+1}^{(r)}(\mathbf{r},t)+\vartheta_{n,n+1}(\mathbf{r},t)]. \tag{53}$$

Using the continuity equation and Eq. (53) with the known function $g_{n,n+1}(\mathbf{r},t)$ we obtain finally the equation for $\varphi_{n,n+1}^{(r)}(\mathbf{r},t)$:

$$\frac{1}{c_0^2}\hat{T}_c\frac{\partial}{\partial t}[\varphi_{n,n+1}^{(r)}+\vartheta_{n,n+1}]+\frac{1}{\lambda_J^2}\sin[\varphi_{n,n+1}^{(r)}+\varphi_{n,n+1}^{(v)}]-\sum_m L_{nm}\nabla^2\varphi_{m,m+1}^{(r)}=0, \tag{54}$$

where we have neglected $g_{n,n+1}^{(r)}$ and the W_n term in Eq. (34) which are of lower order for frequencies $\omega \ll \omega_{ab}(\sigma_c/\sigma_{ab})^{1/2}$. This equation corresponds to the Lagrangian

$$\mathcal{L}_\varphi=\frac{E_0}{2c_0^2}\sum_n\int d\mathbf{r}\left\{\frac{\partial}{\partial t}[\varphi_{n,n+1}^{(r)}+\vartheta_{n,n+1}]\right\}^2-\mathcal{F}_\varphi, \tag{55}$$

with $\mathcal{F}_\varphi$ as given in Eq. (15). The term $\vartheta_{n,n+1}$ originates from the in-plane electric field and the corresponding voltage induced in the normal cores of pancakes. It is nonzero if pancakes move differently in neighboring layers n and $n+1$.

To describe the excitation of the phase collective mode and vortices by an external homogeneous ac electric field $\mathcal{D}_z(t)$ oriented along the c axis, we add to the Lagrangian $\mathcal{L}_\varphi$ the term

$$\mathcal{L}_{\text{ext}}=-\int d\mathbf{r}dz\frac{E_z\mathcal{D}_z}{4\pi}=-\frac{\hbar}{8\pi e}\sum_n\int d\mathbf{r}\mathcal{D}_z(t)\frac{\partial}{\partial t}[\varphi_{n,n+1}^{(r)}(\mathbf{r})+\vartheta_{n,n+1}(\mathbf{r})]. \tag{56}$$

Such an approach is appropriate for a sample placed in a cavity acting as a capacitor. In the presence of an external electric field the equation for $\varphi_{n,n+1}^{(r)}(\mathbf{r},t)$ is:

$$\frac{1}{c_0^2}\hat{T}_c\frac{\partial}{\partial t}[\varphi_{n,n+1}^{(r)}+\vartheta_{n,n+1}]-\sum_m L_{nm}\nabla^2\varphi_{m,m+1}^{(r)}+\frac{1}{\lambda_J^2}\sin[\varphi_{n,n+1}^{(r)}+\varphi_{n,n+1}^{(v)}]=\frac{\hbar}{8\pi e E_0}\frac{\partial}{\partial t}\mathcal{D}_z(t). \tag{57}$$

At finite temperatures the Langevin force corresponding to the dissipation described by Γ_c should be added to the right-hand side. Equation (57) defines $\varphi_{n,n+1}^{(r)}(\mathbf{r})$ in the presence of the external electric field $\mathcal{D}_z(t)$ and moving vortices with coordinates $\mathbf{r}_{n\nu}(t)$ as described by the terms $\vartheta_{n,n+1}$ and $\varphi_{n,n+1}^{(v)}$. This is our main equation for the time-dependent phase difference. Note that the TDGL approach was used to define the function $\vartheta_{n,n+1}(\mathbf{r},t)$ in this equation. All other terms do not depend on the derivation of the function $g_{n,n+1}$. In a more general approach than the TDGL only the expression for the function $v_n(\mathbf{r},t)$ in $\vartheta_{n,n+1}(\mathbf{r},t)$ may be modified.

Next we determine the Lagrangian for the system in terms of $\varphi_{n,n+1}^{(r)}(\mathbf{r})$ and $\mathbf{r}_{n\nu}$ as

$$\mathcal{L}\{\varphi_{n,n+1}^{(r)}(\mathbf{r}),\mathbf{r}_{n\nu}\}=\mathcal{L}_0\{\mathbf{r}_{n\nu}\}+\frac{E_0}{2c_0^2}\sum_n\int d\mathbf{r}\times\left\{\frac{\partial}{\partial t}[\varphi_{n,n+1}^{(r)}(\mathbf{r})+\vartheta_{n,n+1}(\mathbf{r},\mathbf{r}_{n\nu})]\right\}^2-\mathcal{F}\{\varphi_{n,n+1}^{(r)}(\mathbf{r}),\mathbf{r}_{n\nu}\}+\mathcal{L}_{\text{ext}}\{\varphi_{n,n+1}^{(r)}(\mathbf{r}),\mathbf{r}_{n\nu}\}. \tag{58}$$

Here $\mathcal{L}_0(\mathbf{r}_{n\nu})$ is the two-dimensional dynamic part of this functional, which includes the Magnus force dynamic term and vortex mass term if necessary. Dissipation originating from the in-plane electric field and from relaxation of the order parameter induced by moving vortices is described phenomenologically by the vortex viscosity η in the dissipation function $\mathcal{R}_0\{\mathbf{r}_{n\nu}\}=(\eta/2)(\dot{\mathbf{r}}_{n\nu})^2$ for pancake $n\nu$.

IV. INHOMOGENEOUS BROADENING OF THE PLASMA RESONANCE

We consider now the solution for $\varphi_{n,n+1}^{(r)}(\mathbf{r},t)$ and vortex coordinates $\mathbf{r}_{n\nu}(t)$ in the presence of an oscillatory weak external field $\mathcal{D}_z(t)$. Then the phase collective modes are excited and in addition vortices oscillate near equilibrium positions $\mathbf{r}_{n\nu}^{(0)}$ due to phase variations. Now our goal is to obtain equations which describe small variations of the phase difference and small amplitude vortex oscillations. We denote pancake deviations by $\mathbf{u}_{n\nu}(t)=\mathbf{r}_{n\nu}(t)-\mathbf{r}_{n\nu}^{(0)}$ and expand $\varphi_{n,n+1}^{(v)}(\mathbf{r},t)$ in $\mathbf{u}_{n\nu}(t)$:

$$\varphi_{n,n+1}^{(v)}(\mathbf{r},t)=\sum_\nu [f(\mathbf{r}-\mathbf{r}_{n\nu}^{(0)})-f(\mathbf{r}-\mathbf{r}_{n+1,\nu}^{(0)})]+\tilde{\varphi}_{n,n+1}^{(v)}(\mathbf{r},t), \tag{59}$$

$$\tilde{\varphi}_{n,n+1}^{(v)}(\mathbf{r},t)=\sum_\nu \mathbf{D}(\mathbf{r}-\mathbf{r}_{n\nu}^{(0)})\cdot\mathbf{u}_{n\nu}(t)-\mathbf{D}(\mathbf{r}-\mathbf{r}_{n+1,\nu}^{(0)})\cdot\mathbf{u}_{n+1,\nu}(t), \tag{60}$$

where $\mathbf{D}(\mathbf{r})=(-y/r^2,x/r^2)$ for $a<r<\lambda_J$ and $\mathbf{D}(\mathbf{r})$ drops much faster with r for $r>\lambda_J$. We also expand $\vartheta_{n,n+1}(\mathbf{r},t)$ in $\mathbf{u}_{n\nu}(t)$:

$$\vartheta_{n,n+1}(\mathbf{r},t)=\sum_{m\nu}\mathbf{P}_n(\mathbf{r}-\mathbf{r}^{(0)}_{m\nu})\cdot\mathbf{u}_{m\nu}(t)-\mathbf{P}_{n+1}(\mathbf{r}-\mathbf{r}^{(0)}_{m+1,\nu})\mathbf{u}_{m+1,\nu}(t). \quad (61)$$

Here for large distances $|\mathbf{r}-\mathbf{r}^{(0)}_{m\nu}|\gg\Lambda_{ab}$ the function $\mathbf{P}_n(\mathbf{r}-\mathbf{r}^{(0)}_{m\nu})$ deviates exponentially small from the function $\mathbf{D}(\mathbf{r}-\mathbf{r}^{(0)}_{n\nu})\delta_{nm}$:

$$\mathbf{P}_n(\mathbf{r}-\mathbf{r}^{(0)}_{m\nu})=\mathbf{D}(\mathbf{r}-\mathbf{r}^{(0)}_{m\nu})\left[\delta_{nm}+\frac{|\mathbf{r}-\mathbf{r}^{(0)}_{m\nu}|}{\Lambda_{ab}}v_n\left(\frac{|\mathbf{r}-\mathbf{r}^{(0)}_{m\nu}|}{\Lambda_{ab}}\right)\right]. \quad (62)$$

We present the phase difference as

$$\varphi_{n,n+1}(\mathbf{r},\mathbf{r}_{n\nu},t)=\varphi^{(0)}_{n,n+1}(\mathbf{r})+\tilde{\varphi}^{(v)}_{n,n+1}(\mathbf{r},t)+\tilde{\varphi}^{(r)}_{n,n+1}(\mathbf{r},t), \quad (63)$$

where $\varphi^{(0)}_{n,n+1}$ is the phase difference at equilibrium when $\mathcal{D}_z=0$. It is determined by Eqs. (13) and (14) at $\mathbf{r}_{n\nu}=\mathbf{r}^{(0)}_{n\nu}$. The contribution $\tilde{\varphi}^{(v)}_{n,n+1}$ accounts directly for vortex motion and is given by Eqs. (59) and (60), while $\tilde{\varphi}^{(r)}_{n,n+1}$ accounts for the rest of the phase variations (it also includes a part due to vortex motion; see below). Both $\tilde{\varphi}^{(v)}_{n,n+1}$ and $\tilde{\varphi}^{(r)}_{n,n+1}$ are small in a weak external electric field. Thus we can expand in Eq. (57) and obtain a linear equation for $\tilde{\varphi}^{(r)}_{n,n+1}$:

$$\frac{1}{c_0^2}\hat{T}_c\frac{\partial}{\partial t}\tilde{\varphi}^{(r)}_{n,n+1}-\sum_m L_{nm}\nabla^2\tilde{\varphi}^{(r)}_{m,m+1}+\frac{1}{\lambda_J^2}[\cos\varphi^{(0)}_{n,n+1}(\mathbf{r})]\tilde{\varphi}^{(r)}_{n,n+1}=\frac{\hbar}{8\pi eE_0\epsilon_c}\frac{\partial}{\partial t}\mathcal{D}_z(t)-\frac{1}{\lambda_J^2}[\cos\varphi^{(0)}_{n,n+1}(\mathbf{r})]\tilde{\varphi}^{(v)}_{n,n+1}-\frac{1}{c_0^2}\hat{T}_c\frac{\partial}{\partial t}\vartheta_{n,n+1}. \quad (64)$$

The last two terms on the right-hand side describe the excitation of the phase collective mode due to vortex oscillations $\mathbf{u}_{n\nu}(t)$. The equation of motion for vortex displacements $\mathbf{u}_{n\nu}(t)$ which couple with phase oscillations, will be presented in the next section. After solving these two coupled equations for $\tilde{\varphi}^{(r)}_{n,n+1}$ and $\mathbf{u}_{n\nu}(t)$, we obtain the electric field $E_{z;n,n+1}(bfr,t)$ by the use of Eq. (53). From this we may derive the inverse dielectric function

$$1/\epsilon(\omega)=\langle E_z(\mathbf{r},\omega)\rangle/\mathcal{D}_z(\omega), \quad (65)$$

which describes the plasma resonance.

In this section we solve Eq. (64) with fixed vortices, when $\mathbf{u}_{n\nu}(t)=0$. Then the last two terms on the right-hand side of this equation are absent. We assume that the inhomogeneous part of $\tilde{\varphi}^{(r)}_{n,n+1}$, which is $\Theta_n(\mathbf{r},t)=\tilde{\varphi}^{(r)}_{n,n+1}(\mathbf{r},t)-\overline{\Theta}(t)$, is much smaller than the homogeneous part $\overline{\Theta}(t)=\langle\tilde{\varphi}^{(r)}_{n,n+1}(\mathbf{r},t)\rangle$. We use here the perturbation theory in the continuous spectrum; see Ref. 31. Averaging over space in Eq. (64) we obtain

$$\frac{1}{c_0^2}\hat{T}_c\frac{\partial\overline{\Theta}(t)}{\partial t}+\frac{b}{\lambda_J^2}\overline{\Theta}(t)+\frac{1}{\lambda_J^2}\langle[\cos\varphi^{(0)}_{n,n+1}(\mathbf{r})-b]\Theta_n(\mathbf{r},t)\rangle=\frac{\hbar}{8\pi eE_0}\frac{\partial}{\partial t}\mathcal{D}_z(t), \quad (66)$$

where $b=\langle\cos\varphi^{(0)}_{n,n+1}(\mathbf{r})\rangle$. The inhomogeneous part is determined by

$$\left[\frac{1}{c_0^2}\hat{T}_c\frac{\partial}{\partial t}+\frac{b}{\lambda_J^2}\right]\Theta_n(\mathbf{r},t)-\sum_m L_{nm}\nabla^2\Theta_m(\mathbf{r},t)=-\frac{1}{\lambda_J^2}[\cos\varphi^{(0)}_{n,n+1}(\mathbf{r})-b]\overline{\Theta}(t). \quad (67)$$

The solution of this equation in the Fourier representation $\omega,\mathbf{k},q$ with respect to $t,\mathbf{r},n$ is

$$\Theta(\omega,\mathbf{k},q)=\frac{\Omega_0^2[\cos\varphi^{(0)}_{n,n+1}(\mathbf{r})-b]_{\mathbf{k}q}}{\omega(\omega-i\Gamma_c)-\Omega^2(k,q)}\overline{\Theta}(\omega),\quad \Omega_0=\frac{c_0}{\lambda_J}, \quad (68)$$

$$\Omega^2(k,q)=b\Omega_0^2+\frac{c_0^2k^2}{2(1-\cos q)+s^2/\lambda_{ab}^2}. \quad (69)$$

Inserting this expression into Eq. (66) we obtain the solution for $\overline{\Theta}(\omega)$:

$$\overline{\Theta}(\omega)=-\frac{\hbar\Omega_0^2}{8\pi eE_0}\frac{i\omega}{\omega(\omega-i\Gamma_c)-[b-w(\omega)]\Omega_0^2}\mathcal{D}_z(\omega), \quad (70)$$

$$w(\omega)=\int\frac{d\mathbf{k}dq}{(2\pi)^3}\frac{\Omega_0^2F(\mathbf{k},q)}{\omega(\omega-i\Gamma_c)-\Omega^2(k,q)}, \quad (71)$$

where $F(\mathbf{k},q)$ is the Fourier component of the correlation function

$$F(\mathbf{r}-\mathbf{r}',n-m)=\langle[\cos\varphi^{(0)}_{n,n+1}(\mathbf{r})-b][\cos\varphi^{(0)}_{m,m+1}(\mathbf{r}')-b]\rangle. \quad (72)$$

The dispersion relation for the phase collective mode, Eq. (69), is valid for frequencies $\hbar\Omega(k,q)\ll\Delta$, and thus integration over $\mathbf{k}$ in Eq. (71) is restricted by $k<\Delta/c_0$.

Our next step is the calculation of the correlation function $F(\mathbf{r},n)$. We follow the approach used in Ref. 19 and consider the zero-temperature limit. The phase difference $\varphi^{(0)}_{n,n+1}(\mathbf{r})$ given by Eq. (14) differs from $\varphi^{(v)}_{n,n+1}(\mathbf{r})$ given by Eq. (13) because Josephson currents provide three-dimensional screening. The function $\varphi^{(0)}_{n,n+1}(\mathbf{r})$ induced by given close vortices $n\nu$ and $n+1,\nu$ falls rapidly at distances larger than λ_J from these two pancakes due to the Josephson interlayer current. For smaller distances the vortex contribution to $\varphi^{(0)}_{n,n+1}(\mathbf{r})$ and $\varphi^{(v)}_{n,n+1}(\mathbf{r})$ is practically the same. At high enough magnetic fields $B\gg B_J=\Phi_0/\lambda_J^2$ many vortices contribute effectively to the phase difference at a given point $\mathbf{r}$ since the phase difference $\varphi^{(v)}_{n,n+1}(\mathbf{r})$ for given close $\mathbf{r}_{n,\nu}$ and $\mathbf{r}_{n+1,\nu}$ is a slowly decreasing function of $\mathbf{r}$ at distances smaller than λ_J from these vortices; see text after Eq. (14). We note that in fields $B \gg B_J$ the intervortex distance $a=(\Phi_0/B)^{1/2} \ll \lambda_J$. According to the central theorem of the probability theory, the phase $\varphi^{(0)}_{n,n+1}(\mathbf{r})$ obeys a Gaussian distribution with $\langle \varphi^{(0)}_{n,n+1}(\mathbf{r})\rangle=0$. Therefore,

$$\langle \cos\varphi^{(0)}_{n,n+1}(\mathbf{r})\cos\varphi^{(0)}_{m,m+1}(0)\rangle = b^2\cosh\alpha_{nm}(\mathbf{r}), \quad (73)$$

where

$$\alpha_{nm}(\mathbf{r})=\langle \varphi^{(0)}_{n,n+1}(\mathbf{r})\varphi^{(0)}_{m,m+1}(0)\rangle = \int d\mathbf{r}_1 d\mathbf{r}_2 \widetilde{K}_{nm}(\mathbf{r}+\mathbf{r}_1,\mathbf{r}_2) f(\mathbf{r}_1) f(\mathbf{r}_2), \quad (74)$$

$$\widetilde{K}_{nm}(\mathbf{r}_1,\mathbf{r}_2)=\langle[\rho_n(\mathbf{r}_1)-\rho_{n+1}(\mathbf{r}_1)][\rho_m(\mathbf{r}_2)-\rho_{m+1}(\mathbf{r}_2)]\rangle. \quad (75)$$

Here we introduce the pancake density $\rho_n(\mathbf{r}) = \Sigma_\nu \delta(\mathbf{r}-\mathbf{r}_{n,\nu})$ in the layer n. Using the relations

$$\int d\mathbf{r}_1 \widetilde{K}_{nm}(\mathbf{r}_1,\mathbf{r}_2) = \int d\mathbf{r}_2 \widetilde{K}_{nm}(\mathbf{r}_1,\mathbf{r}_2) = 0 \quad (76)$$

and taking into account that for a disordered vortex state without long-range order $\widetilde{K}_{nm}(\mathbf{r}_1,\mathbf{r}_2)=\widetilde{K}_{nm}(\mathbf{r}_1-\mathbf{r}_2)$, we write

$$\alpha_{nm}(\mathbf{r}) = -\frac{1}{2}\int d\mathbf{r}_1 d\mathbf{r}_2 [f(\mathbf{r}-\mathbf{r}_1)-f(-\mathbf{r}_2)]^2 \widetilde{K}_{nm}(\mathbf{r}_1-\mathbf{r}_2). \quad (77)$$

As was mentioned in the Introduction, Tsui *et al.*[8,10,11] and Matsuda *et al.*[9] observed the power-law dependence of the plasma frequency on the magnetic field applied along the c axis for magnetic fields above 0.1 T. It was shown in Ref. 19 that this dependence may be explained in such a model of strong disorder in pancake positions along the c axis. In our model, correlations in the pancake positions in different layers are absent, $\langle \rho_n(\mathbf{r})\rho_m(\mathbf{r})\rangle = \langle \rho_n(\mathbf{r})\rangle^2 = a^{-2}$ for $n \neq m$. Therefore, we obtain $\widetilde{K}_{nn}(\mathbf{r})=2K(\mathbf{r})$, $\widetilde{K}_{n,n+1}(\mathbf{r}) = \widetilde{K}_{n-1,n}(\mathbf{r}) = -K(\mathbf{r})$, and $\widetilde{K}_{n,m}=0$ for $m \neq n,n+1$, and $n-1$. Here $K(\mathbf{r})=\langle \rho_n(\mathbf{r})\rho_n(\mathbf{r})\rangle - \langle \rho_n(\mathbf{r})\rangle^2$. Now we obtain

$$\alpha_{nn}(\mathbf{r}) = -\int d\mathbf{r}_1 d\mathbf{r}_2 [f(\mathbf{r}/2+\mathbf{r}_2/2+\mathbf{r}_1) - f(-\mathbf{r}/2+\mathbf{r}_2/2+\mathbf{r}_1)]^2 K(\mathbf{r}_2). \quad (78)$$

At $a \ll r \ll \lambda_J$, within logarithmic accuracy for large λ_J/r, we approximate

$$f(\mathbf{r}/2+\mathbf{r}_2/2+\mathbf{r}_1) - f(-\mathbf{r}/2+\mathbf{r}_2/2+\mathbf{r}_1) \approx [\nabla f(\mathbf{r}+\mathbf{r}_1) + \nabla f(-\mathbf{r}+\mathbf{r}_1)]\cdot \mathbf{r}_2. \quad (79)$$

Integration over $\mathbf{r}_1$ leads to the result

$$\int d\mathbf{r}_1 \{[\nabla f(\mathbf{r}+\mathbf{r}_1) + \nabla f(-\mathbf{r}+\mathbf{r}_1)]\cdot \mathbf{r}_2\}^2 = 4\pi r_2^2 \ln(\lambda_J/r), \quad (80)$$

and we obtain

$$\alpha_{nn}(r) = 2\alpha_{n,n+1}(r) = 4\mu \ln\frac{\lambda_J}{r},$$

$$\mu = \frac{\pi}{4}\int d\mathbf{r} r^2 K(\mathbf{r}). \quad (81)$$

The parameter μ is field independent because $K(\mathbf{r})$ oscillates on the scale a in the ab plane and decays with r on the same or larger scale. In the first case (strong disorder in the ab plane), independence of μ of B is obvious because there is only one characteristic length a in the system. In the second case, when $K(\mathbf{r})$ oscillates more rapidly than it decays, the relevant length in the integral is the period of oscillation only, and the integral practically does not depend on the length of decay. The situation here is similar to that for the integral

$$\int_0^\infty dx x^2 \sin kx \exp(-\delta x) = \frac{6}{(k^2+\delta^2)^{3/2}} \sin\left(3\arctan\frac{k}{\delta}\right), \quad (82)$$

which becomes δ independent in the limit of small δ.

Finally $F(\mathbf{r},0) \approx b^2(a/r)^{4\mu}$, while for $r \gg a$ and $F(\mathbf{r},0) \approx 1/2$ for $r \ll a$. The functions $F(\mathbf{r},n)$ at $n \neq 0$ are much smaller than $F(\mathbf{r},0)$ and may be neglected. In the same approach the relation $\Omega^2/\Omega_0^2 = b \approx (ae/\lambda_J)^{2\mu} = (B_J e^2/B)^\mu$, where $e=2.72$, was obtained in Ref. 19. Now the function $F(\mathbf{k},q)$ in Eq. (71) is

$$F(\mathbf{k},q) \approx a^2[\pi\mu/(2\mu-1)], \quad ka \ll 1,$$

$$F(\mathbf{k},q) \approx (2\pi)^{1/2} a^{1/2} k^{-3/2} \cos(ka - 3\pi/4), \quad ka \gg 1. \quad (83)$$

Integration over k and q in Eq. (71) yields

$$\mathrm{Im}w(\omega) = \frac{\Gamma^2_{\mathrm{inh}}}{\Omega_0^2} \approx \frac{\pi\mu}{2(2\mu-1)}\frac{a^2}{\lambda_J^2} \approx \frac{\pi\mu}{2(2\mu-1)}\frac{B_J}{B}. \quad (84)$$

The term $-i(\mathrm{Im}w)\Omega_0^2$ in the denominator in Eq. (70) for Θ describes the broadening of the plasma mode due to random spatial variations of the Josephson interaction in the presence of pancakes. It may be described also as a decay of the homogeneous phase collective mode into inhomogeneous phase collective modes. The rate of this decay is $\Gamma^2_{\mathrm{inh}}/2\Omega \ll \Omega$ for $a \ll \lambda_J$. Thus the use of perturbation theory with respect to $[\cos\varphi_{n,n+1}(\mathbf{r}) - \langle \cos\varphi_{n,n+1}(\mathbf{r})\rangle]$ is appropriate and the plasma frequency is determined by the average value of $\cos\varphi_{n,n+1}(\mathbf{r})$ according to Eq. (2).

The real part of w,

$$\mathrm{Re}w(\omega) = \frac{\mu}{2(2\mu-1)}\frac{a^2}{\lambda_J^2}\ln\frac{\Delta^2}{\Gamma\Omega}, \quad (85)$$

describes the shift of plasma frequency due to the inhomogeneity of the Josephson interaction. This shift is of the order $\Gamma^2_{\mathrm{inh}}/2\Omega$. Thus, neglecting pancake oscillations, we obtain the dielectric function

$$\frac{1}{\epsilon(\omega)} = \frac{1}{\epsilon_c}\frac{\omega^2}{\omega(\omega - i\Gamma_c) - \Omega^2 - i\Gamma^2_{\mathrm{inh}}}, \quad \Omega^2 = b\Omega_0^2. \quad (86)$$

We obtained Γ_{inh} at $T=0$. Below the irreversibility line Γ_{inh}^2 is anticipated to be almost temperature independent because it is determined mainly by short-range correlations of $\cos\varphi_{n,n+1}(\mathbf{r})$ which depend weakly on thermal phase fluctuations $[F(\mathbf{r},0)\approx 1/2$ at small $r]$.

The origin of inhomogeneous broadening may be explained also in the following way. Equation (64) without an external electric field is the Schrödinger equation for a ''particle'' moving along layers with kinetic energy determined by the matrix $L_{nm}>0$ in the random potential proportional to $\cos\varphi_{n,n+1}^{(0)}(\mathbf{r})$; see also Ref. 19. We denote the eigenvalues of this equation by E_α and eigenfunctions by $\Psi_{\alpha,n}(\mathbf{r})$. Then the dielectric function for an homogeneous external electric field is

$$\frac{1}{\epsilon(\omega)}=\frac{\omega^2}{\epsilon_c}\sum_{\alpha,m}\int d\mathbf{r}'\left\langle\frac{\Psi_{\alpha,m}^*(\mathbf{r})\Psi_{\alpha,m}(\mathbf{r}')}{\omega(\omega-i\Gamma_c)-\omega_\alpha^2}\right\rangle, \qquad (87)$$

$$\text{Im}\frac{1}{\epsilon(\omega)}=\frac{\pi\omega}{2\epsilon_c}\sum_{\alpha,m}\int d\mathbf{r}'\langle\Psi_{\alpha,m}^*(\mathbf{r})\Psi_{\alpha,m}(\mathbf{r}')\delta(\omega-\omega_\alpha)\rangle, \qquad (88)$$

where $\omega_\alpha^2=E_\alpha$ and we assume that Γ_c is small. We see that contributions to the plasma resonance come from those eigenstates (phase collective modes) which may be excited by a homogeneous external field (delocalized and weakly localized phase modes). Under the condition $a\ll\lambda_J$, the energies E_α of such modes are distributed near that determined by the averaged potential.

V. DISSIPATION OF A c-AXIS PLASMON INTO VORTEX OSCILLATIONS

In this section we present the equation for pancake oscillations coupled with phase variations. Then we solve the equation for $\overline{\Theta}(t)$ taking into account vortex motion to obtain the decay of plasma modes into vortex oscillations. In writing down the equation of motion for pancakes we will consider the case of strong pinning centers for simplicity and focus mainly on terms which describe the interaction of vortices with phase collective modes.

Using the Lagrangian (58) we obtain the equation for pancake deviations:

$$\eta\dot{\mathbf{u}}_{n\nu}+\alpha_M[\mathbf{n}\times\dot{\mathbf{u}}_{n\nu}]+\alpha_L\mathbf{u}_{n\nu}=\mathbf{F}_{n\nu}, \qquad (89)$$

$$\mathbf{F}_{n\nu}(t)=-\left[\frac{d}{dt}\frac{\partial(\mathcal{L}_\varphi+\mathcal{L}_{\text{ext}})}{\partial\dot{\mathbf{r}}_{n\nu}}-\frac{\partial\mathcal{L}_\varphi}{\partial\mathbf{r}_{n\nu}}+\frac{\partial\mathcal{R}_\varphi}{\partial\dot{\mathbf{r}}_{n\nu}}\right]. \qquad (90)$$

The pancake dynamics is given by the parameters α_L, η, and α_M. We replace all elastic moduli (i.e., vortex-vortex interaction terms) and effects of pinning by the Labusch parameter α_L. Such an approach is correct for strong identical pinning centers and it gives only an order of magnitude estimate for typical Bi-2:2:1:2 single crystals. For details on the Labusch parameter see Refs. 37 and 38 and references therein. The estimate for η is the Bardeen-Stephen expression $\eta=\Phi_0^2\sigma_{ab}^{(n)}/2\pi\xi_{ab}^2c^2$, where $\sigma_{ab}^{(n)}$ is the normal state conductivity; see Refs. 26 and 33–35 for more details. Here α_M is the Magnus force coefficient which is bounded by the hydrodynamic limit $\alpha_M\leq\pi\hbar n_s$, where n_s is the density of superconducting electrons.[34–36]

The interaction with the phase dynamics is given by the force $\mathbf{F}_{n\nu}(t)$. Taking into account that all the phase dependence on $\mathbf{r}_{n\nu}$ is in $\varphi_{n,n+1}^{(v)}$ and that the dependence on $\dot{\mathbf{r}}_{n\nu}$ is in $\vartheta_{n,n+1}$, we use $\partial/\partial\mathbf{r}_{n\nu}=(\partial\varphi_{n,n+1}^{(v)}/\partial\mathbf{r}_{n\nu})\partial/\partial\varphi_{n,n+1}^{(v)}$ and $\partial/\partial\dot{\mathbf{r}}_{n\nu}=(\partial\dot{\vartheta}_{n,n+1}/\partial\mathbf{r}_{n\nu})\partial/\partial\dot{\vartheta}_{n,n+1}$. We obtain from Eqs. (55), (56), and (90)

$$\begin{aligned}\mathbf{F}_{n\nu}(t)=-\frac{E_0}{s}\sum_m\int d\mathbf{r}\Bigg\{&\frac{1}{\lambda_J^2}\sin\varphi_{m,m+1}(\mathbf{r},\mathbf{t})\frac{\partial\varphi_{m,m+1}^{(v)}(\mathbf{r},t)}{\partial\mathbf{r}_{n\nu}}\\&+\left[\frac{1}{c_0^2}\hat{T}_c\frac{\partial}{\partial t}[\varphi_{m,m+1}^{(r)}(\mathbf{r},t)+\vartheta_{m,m+1}(\mathbf{r},t)]\right.\\&\left.-\frac{\hbar}{8\pi eE_0}\frac{\partial\mathcal{D}_z}{\partial t}\right]\frac{\partial\vartheta_{m,m+1}(\mathbf{r},t)}{\partial\mathbf{r}_{n\nu}}\Bigg\}. \end{aligned} \qquad (91)$$

The forces $\mathbf{F}_{n\nu}$ can be interpreted as Lorentz forces acting on pancakes due to different interlayer currents along the c axis, which give in-plane currents through the continuity equation (26). Our focus is in the forces $\mathbf{F}_{n\nu}$ acting on pancake $n\nu$ when the phase difference deviates from that at equilibrium. Then, after using Eqs. (63) and (64) we obtain

$$\begin{aligned}\mathbf{F}_{n\nu}(t)=\frac{E_0}{s}\sum_m\int d\mathbf{r}\Bigg\{&\sum_k L_{mk}\nabla^2\tilde{\varphi}_{k,k+1}^{(r)}(\mathbf{r},t)\frac{\partial\vartheta_{m,m+1}(\mathbf{r},t)}{\partial\mathbf{r}_{n\nu}}\\&+\frac{1}{\lambda_J^2}\cos[\varphi_{m,m+1}^{(0)}(\mathbf{r},t)]\tilde{\varphi}_{m,m+1}^{(r)}(\mathbf{r},t)\frac{\partial}{\partial\mathbf{r}_{n\nu}}\\&\times[\varphi_{m,m+1}^{(v)}(\mathbf{r},t)-\vartheta_{m,m+1}(\mathbf{r},t)]\Bigg\}. \end{aligned} \qquad (92)$$

With the help of Eqs. (60) and (61) this expression may be written as

$$\begin{aligned}\mathbf{F}_{n\nu}(t)=\frac{E_0}{\lambda_J^2 s}\sum_m\int d\mathbf{r}\Bigg\{&\mathbf{P}_m(\mathbf{r}-\mathbf{r}_{n\nu}^{(0)})\sum_k(L_{mk}-L_{m-1,k})\\&\times\lambda_J^2\nabla^2\tilde{\varphi}_{k,k+1}^{(r)}(\mathbf{r},t)+[\mathbf{D}(\mathbf{r}-\mathbf{r}_{n\nu}^{(0)})\delta_{nm}\\&-\mathbf{P}_m(\mathbf{r}-\mathbf{r}_{n\nu}^{(0)})][\cos\varphi_{m,m+1}^{(0)}(\mathbf{r})\tilde{\varphi}_{m,m+1}^{(r)}(\mathbf{r},t)\\&-\cos\varphi_{m-1,m}^{(0)}(\mathbf{r})\tilde{\varphi}_{m-1,m}^{(r)}(\mathbf{r},t)]\Bigg\}. \end{aligned} \qquad (93)$$

The pancake displacements caused by the force $\mathbf{F}_{n\nu}(\omega)$ are

$$u_{x;n\nu}(\omega)=\left[-F_{x;n\nu}(\omega)\frac{i\omega\eta+\alpha_L}{\mathcal{S}(\omega)}+F_{y;n\nu}(\omega)\frac{i\omega\alpha_M}{\mathcal{S}(\omega)}\right], \qquad (94)$$

$$\mathcal{S}(\omega)=(i\omega\eta+\alpha_L)^2-\omega^2\alpha_M^2, \qquad (95)$$

and the equation for $u_{x;n\nu}$ follows by replacing F_y for F_x and $-F_x$ for F_y. We use here the Fourier representation with respect to the time, $\mathbf{u}_{n\nu}(t)=\mathbf{u}_{n\nu}(\omega)\exp(i\omega t)$. Next we insert the solution for pancake displacements into the right-hand side of Eq. (64) and account for homogeneous collective mode only. The first term on the right-hand side of Eq. (92) vanishes for the homogeneous part of the plasma mode and thus may be omitted. The second term leads to a dynamic vortex-plasmon interaction which originates from regions near vortices only because the function $\varphi^{(v)}_{m,m+1}(\mathbf{r},t) - \vartheta_{m,m+1}(\mathbf{r},t)$ vanishes exponentially far away from vortex centers. This results in the additional contribution to the left hand side of Eq. (66):

$$\overline{\Theta}(\omega)\frac{E_0}{\lambda_J^4 s}\frac{i\omega\eta+\alpha_L}{\mathcal{S}(\omega)}\sum_{n\nu}\int\frac{dx'dy'}{\Lambda_{ab}^2}\Bigg\langle\cos\varphi^{(0)}_{n,n+1}(\mathbf{r})\times\Bigg\{v_0\left(\frac{|\mathbf{r}-\mathbf{r}^{(0)}_{n\nu}|}{\Lambda_{ab}}\right)v_0\left(\frac{|\mathbf{r}'-\mathbf{r}^{(0)}_{n\nu}|}{\Lambda_{ab}}\right)[\cos\varphi^{(0)}_{n,n+1}(\mathbf{r}') - \cos\varphi^{(0)}_{n-1,n}(\mathbf{r}')] - v_0\left(\frac{|\mathbf{r}-\mathbf{r}^{(0)}_{n+1,\nu}|}{\Lambda_{ab}}\right)v_0\left(\frac{|\mathbf{r}'-\mathbf{r}^{(0)}_{n+1,\nu}|}{\Lambda_{ab}}\right)\times[\cos\varphi^{(0)}_{n+1,n+2}(\mathbf{r}')-\cos\varphi^{(0)}_{n,n+1}(\mathbf{r}')]\Bigg\}\Bigg\rangle. \quad (96)$$

We note that random values $\varphi^{(0)}_{n,n+1}(\mathbf{r})$ and $\mathbf{r}^{(0)}_{n\nu}$ are weakly correlated because many vortices contribute to $\varphi^{(0)}_{n,n+1}(\mathbf{r})$,as was discussed above. Then we average $v_0(|\mathbf{r}-\mathbf{r}^{(0)}_{n\nu}|/\Lambda_{ab})v_0(|\mathbf{r}'-\mathbf{r}^{(0)}_{n\nu}|/\Lambda_{ab})$ and $\cos\varphi^{(0)}_{n,n+1}\times(\mathbf{r})\cos\varphi^{(0)}_{m,m+1}(\mathbf{r}')$ independently. Taking into account that $v_0(|\mathbf{r}|/\Lambda_{ab})$ is localized at distances Λ_{ab}, which are much shorter than the intervortex distance a, we obtain after summation over $\mathbf{r}^{(0)}_{n\nu}$

$$\Lambda_{ab}^{-2}\sum_\nu v_0(|\mathbf{r}-\mathbf{r}^{(0)}_{n\nu}|/\Lambda_{ab})v_0(|\mathbf{r}'-\mathbf{r}^{(0)}_{n\nu}|/\Lambda_{ab})=C\delta(\mathbf{r}-\mathbf{r}'), \quad (97)$$

where the numerical parameter C is of the order unity. The variation of the Josephson parameter J_0 inside vortex cores leads to an additional contribution to the function $v_0(\rho)$. This contribution has the same properties as $v_0(\rho)$: It is localized at distances $\xi_{ab}\approx\Lambda_{ab}$. This contribution results in the renormalization of the numerical parameter C. Then the term (96) may be written as

$$\overline{\Theta}(\omega)\frac{2CE_0}{\lambda_J^4 s a^2}\frac{i\omega\eta+\alpha_L}{\mathcal{S}(\omega)}\int d\mathbf{r}F(\mathbf{r},0)\delta(\mathbf{r}) = \overline{\Theta}(\omega)\frac{\mu CE_0}{(2\mu-1)\lambda_J^4 s}\frac{i\omega\eta+\alpha_L}{\mathcal{S}(\omega)}. \quad (98)$$

This additional term in Eq. (64) has an imaginary part due to vortex viscosity and it determines the dissipation rate Γ_v of plasma modes into vortex oscillations,

$$\frac{\Gamma_v}{\Omega}=\frac{\mu CE_0\Omega_0}{2(2\mu-1)s\lambda_J^2}\frac{\eta}{\alpha_L^2}\left(\frac{B}{B_J e^2}\right)^{\mu/2}, \quad (99)$$

and also the frequency shift $\Delta\Omega_v$,

$$\Delta\Omega_v=\frac{\alpha_L}{\eta}\frac{\Gamma_v}{\Omega}=\frac{\mu CE_0\Omega_0}{2(2\mu-1)s\lambda_J^2\alpha_L}\left(\frac{B}{B_J e^2}\right)^{\mu/2}.$$

Here we assume that pinning is strong, $\alpha_L/\Omega\gg\eta,\alpha_M$. Γ_v depends weakly on B, and both Γ_v/Ω and relative line shift $\Delta\Omega_v/\Omega$ increase with B in contrast with inhomogeneous broadening.

To conclude this section we discuss the possibility of exciting a plasma mode via vortex oscillations by applying an ac magnetic field. Let us assume that vortices oscillate homogeneously, $\mathbf{u}_{n\nu}(t)=\mathbf{u}(t)$, around their equilibrium positions under the effect of such a field. Then by the use of Eq. (60) we obtain on the right-hand side of Eq. (64) for $\tilde{\varphi}^{(r)}_{n,n+1}$ the term

$$-\frac{1}{\lambda_J^2}[\cos\varphi^{(0)}_{n,n+1}(\mathbf{r})]\tilde{\varphi}^{(v)}_{n,n+1} = -\frac{1}{\lambda_J^2}[\cos\varphi^{(0)}_{n,n+1}(\mathbf{r})]\sum_\nu[\mathbf{D}(\mathbf{r}-\mathbf{r}^{(0)}_{n\nu}) - \mathbf{D}(\mathbf{r}-\mathbf{r}^{(0)}_{n+1,\nu})]\cdot\mathbf{u}(t), \quad (100)$$

and a similar contribution originates from the term with $\vartheta_{n,n+1}$ in Eq. (64) by the use of Eq. (61). Both terms vanish after averaging over space. Thus homogeneous plasma modes cannot be excited by homogeneous coherent vortex oscillations; only inhomogeneous phase collective modes may be excited by an ac external magnetic field. Similarly, a plasma mode is coupled with inhomogeneous vortex oscillations only, and an ac electric field excites such vortex oscillations but not homogeneous ones.

VI. DISCUSSION AND EXPERIMENTAL DATA

First, we estimate parameters for single crystals Bi-2:2:1:2 relevant for our discussion. The anisotropy parameter $\gamma=420$ was extracted by Tsui *et al.*[10,11] from the angular dependence of the plasma frequency. The field dependence $\Omega(B)\approx\Omega_0(B_J e^2/B)^{\mu/2}$leads to $\gamma\approx250$ at $\lambda_{ab}=1700$ Å and $\epsilon_c=20$. Thus γ is in the interval (200–400) and $\Omega_0/2\pi\approx200$ GHz, while $B_J\approx100$ G. Note that from recent measurements[39] of the temperature variations of λ_c and λ_{ab} in the interval from 4.2 K to T_c the anisotropy ratio may be estimated as ≈250. These measurements show that temperature dependence of γ is at best weak.

Up to now information on the low-temperature dynamic parameters of vortices α_M and η for Bi-2:2:1:2 is absent. Measurements of the ab resistivity by Bulaevskii *et al.*[40] close to the flux-flow regime (at 70 K at high magnetic field and high currents) provide the estimate $\eta\leqslant2.3\times10^{-7}$ g/cm s. This value is about an order of magnitude smaller than the Bardeen-Stephen viscosity coefficient estimated using the normal state conductivity $\sigma^{(n)}_{ab}\approx2.5\times10^4\Omega^{-1}$ cm^{-1}, extrapolated from a temperature interval above T_c and by the use of the zero-temperature correlation length $\xi_{ab}=20$ Å. The Magnus force coefficient estimated using the relation $\alpha_M=\pi\hbar n_s$ is about 10^{-5} g/cm s. The result of Harris *et al.*[35] for Y-Ba-Cu-O with $T_c=60$ K at 13 K is that $\alpha_M\approx\eta\approx10^{-5}$ g/cm s with a tendency for α_M to increase and η to decrease on cooling (below 13 K the data are absent). The parameter α_L for Bi-2:2:1:2 was estimated by van der Beek *et al.*[37,38] from critical current measurements for single crystals. Its value was found to be in the interval (10^5–10^6) g/cm s^2. The critical current in samples used to study plasma resonance is unknown.

We obtained three contributions Γ_c, $\Gamma_{\rm inh}$, and Γ_v to the plasma linewidth Γ as given by Eqs. (84) and (99):

$$\Gamma = \Gamma_c/2 + \Gamma_{\rm inh}^2/2\Omega + \Gamma_v/2. \quad (101)$$

The contribution to the relative width $\Gamma_c/2\Omega = 2\pi\sigma_c/\epsilon_c\Omega$ due to quasiparticle interlayer tunneling is supposed to be much smaller than that determined by the upper limit for σ_c, given by $\sigma_c^{(n)}$, because the concentration of quasiparticles is small at low temperatures. The value $\sigma_c^{(n)} = 0.2\Omega^{-1}$ cm^{-1} obtained by Cho *et al.*[41] provides the upper limit $\Gamma_c/2\Omega < 0.25$. Taking into account that at low temperatures quasiparticles are formed mainly inside the normal cores, we anticipate that this contribution increases with B linearly in the case of s-wave pairing and as $B^{1/2}$ for d-wave pairing.[14] We also anticipate a strong temperature dependence for Γ_c. Certainly, these predictions do not agree with experimental data, and we conclude that this mechanism is ineffective in comparison with other mechanisms of line broadening when a magnetic field along the c axis is applied. However, in the absence of a magnetic field it is the only mechanism which determines the plasma linewidth.

The value $\Gamma_{\rm inh}$ determines the linewidth of the plasma resonance $\Gamma = \Gamma_{\rm inh}^2/2\Omega$ due to an inhomogeneous Josephson interaction in the presence of pancake vortices. For this mechanism, the line shape of resonance with respect to ω or B is determined by the function

$$\mathrm{Im}\frac{1}{\epsilon(\omega,B)} = \frac{A_1^{-2}C_1B^{-1}}{(B_0^{-\mu}-B^{-\mu})^2 + A_1^{-2}C_1^2B^{-2}},$$

$$C_1 \approx \frac{\pi\mu}{2(2\mu-1)}\Omega_0^2 B_J, \quad (102)$$

where $\mu \approx 0.8$, $\omega^2 = A_1 B_0^{-\mu}$, and $\Omega^2 = A_1 B^{-\mu}$. Figure 1 shows a series of absorption profiles obtained by Tsui *et al.*[8] for $\omega/2\pi =$ 30 – 50 GHz at T = 4.3 K. The solid lines are fitting curves to their data using Eq. (102) for the line shape in addition to a background that is approximated by a quadratic function in B:

$$R_s(\omega,B) = a_0 + a_1B + a_2B^2 + \frac{B^{-1}}{\alpha_1^2(B_0^{-\mu}-B^{-\mu})^2 + \alpha_2 B^{-2}}, \quad (103)$$

where a_i's, α_i's, and B_0 are the fitting parameters. The background is probably due to flux-flow resistivity. Matsuda *et al.*[9] show that it disappears in the zero-Lorentz-force configuration **J**$\|$ **H**$\|$ **c**, but is the most pronounced when **J**$\perp$(**H**$\|$ **c**), which proves the idea. Figure 2 shows absorption profiles obtained for a different sample at $T=$ 12.5, 16, and 20 K with $\omega/2\pi$ fixed at 54.4 GHz. The solid lines are fitting curves to the data using Eq. (103). The fitting parameters α_1 and α_2 are related to the line shape parameters C_1 and A_1 by $\alpha_2/\alpha_1^2 = C_1^2/A_1^2$. Using the dispersion relation $\omega^2 = A_1 B_0^{-\mu}$, A_1 can be determined independently to be 1.16×10^5 GHz^2 T$^{0.8}$. Therefore one may calculate C_1 from the curve-fit results. As Fig. 1 shows, $C_1 \approx (2.95\pm0.75)\times10^4$ (GHz)^2 T at all fields. Using Eq. (102), $\Omega_0 = c/\sqrt{\epsilon_c}\gamma\lambda_{ab}$, and $B_J = \Phi_0/\gamma^2 s^2$, we obtain $\gamma \approx 300$ in agreement with other estimates for this parameter. Γ remains temperature independent below the irreversibility line and increases with temperature above this line. We conclude that this mechanism explains the experimental data well for the plasma linewidth in magnetic fields $B \leqslant 7$ T and below the irreversibility line.

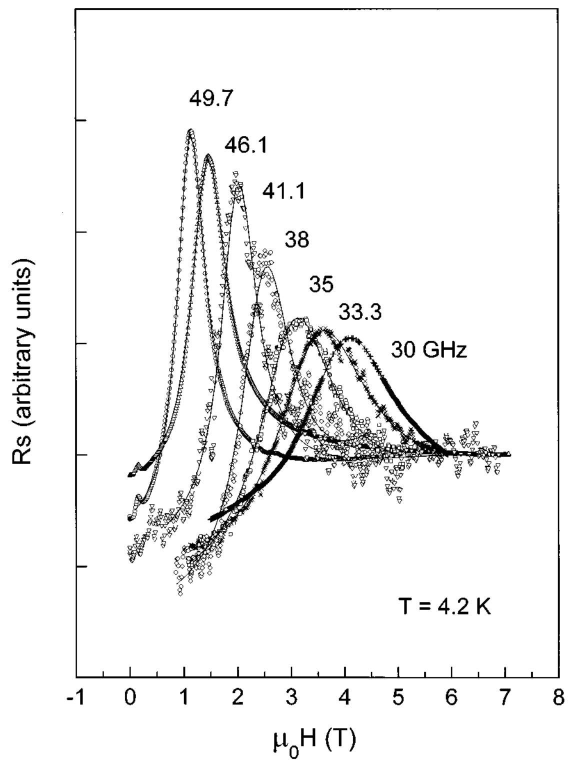

FIG. 1. Curve fits (solid lines) to data obtained in Ref. 8 for Bi-2:2:1:2 at $T=4.2$ K assuming inhomogeneous broadening with **B**$\|c$ (see text). Experimental data of different frequencies are represented by different symbols. The vertical scale of individual curves has been normalized and shifted with an offset for ease of comparison. The parameter C_1 extracted from the curve-fit result is 3.05, 3.7, 3.3, 2.6, 2.2, 2.8, and 2.7×10^4 (GHz)^2 T for $\omega/2\pi=30$, 33, 35, 38, 41.1, 46.1, and 49.68 GHz, respectively.

For the contribution Γ_v due to dissipation of a plasmon into vortex oscillations, we anticipate a significant increase with magnetic field, $\Gamma/\Omega \propto B^{\mu/2}$; see Eq. (99). We anticipate also a significant temperature dependence due to that of the viscosity coefficient η. Up to now we cannot estimate this contribution to the linewidth because η is unknown. However, the predicted field and temperature dependence is in obvious disagreement with experimental data.[8,10,9,11] We conclude that this mechanism is ineffective in the range of magnetic fields used to study the c-axis plasmon so far; $B \leqslant 7$ T. Assuming that the vortex contribution to the relative linewidth in this field interval is less than 20%, we obtain the upper limit for η/α_L^2 which is about 3.6×10^{-18} cm Hs^3/g. Noting that the line shift due to this mechanism should be small (otherwise the power-law dependence would be modified) we obtain a lower limit for α_L of about 10^5 g/cm Hs^2. These limits do not contradict the recent experimental data on α_L and η discussed above. We note that for higher magnetic fields or weaker pinning the coupling of the plasmon with vortex oscillations may become more effective. If so, the plasma frequency shift will change a power law dependence of $\Omega(B)$ and the relative linewidth will increase with magnetic field. Thus experiments in high magnetic fields can

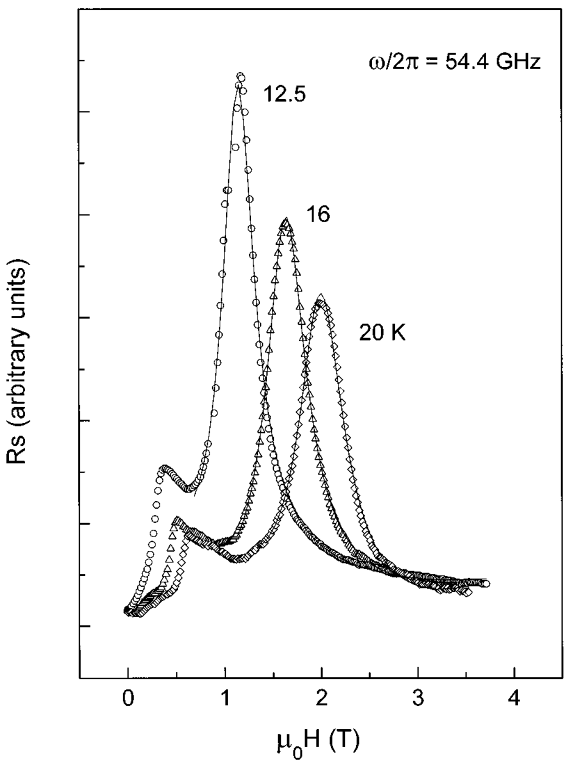

FIG. 2. Solid lines are fitting curves obtained by the use of Eq. (103) which assumes inhomogeneous broadening. Data (open symbols) are obtained for a different sample at $T = 12.5$, 16, and 20 K with the same microwave frequency 54.4 GHz. The vertical scale of each curve has been modified and shifted slightly for comparison. The parameter C_1 obtained from the curve-fit result is 1.67, 1.67, and 2.00×10^4 $(\mathrm{GHz})^2$ T for $T = 12.5$, 16, and 20 K, respectively.

uncover a vortex mechanism of plasma dissipation and provide information on vortex dynamics.

The important point is that when the dynamic pancake-plasmon interaction can be neglected, the plasma resonance sees an instantaneous picture of the vortex configuration, and its position and line shape depend on the static properties of the vortex lattice. This occurs because of the high frequency of the plasma resonance: Vortices can be considered as static during the period of plasma oscillation. In contrast, c-axis dc resistivity measurements correspond to zero frequency and they feel the dynamics of the vortex lattice over a long time interval. Thus, c-axis plasma resonance and dc resistivity measurements are complementary to each other and they both provide a complete picture of the behavior of a vortex lattice along the c axis.

In conclusion, we have obtained the Lagrangian for the coupled phase difference variations and vortex motion. From this Lagrangian we derived equations which describe oscillations of the phase difference coupled with pancake oscillations in the vortex glass phase. Using these equations we proved that the plasma frequency is determined by Eqs. (1) and (2) for the orientation of a magnetic field along the c axis.

We derived also the linewidth of the plasma resonance due to the interlayer tunneling of quasiparticles, arising from mixing of the homogeneous plasma mode with inhomogeneous phase collective modes in the presence of randomly positioned pancakes (inhomogeneous broadening), and due to the decay of the plasma mode into vortex oscillations.

We attributed the observed linewidth of the c-axis plasma resonance in magnetic fields below 7 T to inhomogeneous broadening. The positional disorder of pancake vortices induced by pinning leads to the possibility of exciting many phase collective modes by an homogeneous external ac electric field. This mechanism results in the practically temperature-independent line broadening which fits experimental data well.[8]

We argued that the mechanism of plasmon dissipation due to excitation of vortex oscillations does not show up in the magnetic fields studied experimentally so far, but may become effective at higher fields.

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