

## Persistent currents in coupled mesoscopic rings

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We have analyzed the nature of persistent currents in open coupled mesoscopic rings. Our system is comprised of two ideal loops connected to an electron reservoir. We have obtained analytical expressions for the persistent current densities in two rings in the presence of a magnetic field. We show that the known even-odd parity effects in isolated single loops have to be generalized for the case of coupled rings. We also show that when the two rings have unequal circumferences, it is possible to observe opposite currents (diamagnetic or paramagnetic) in the two rings for a given Fermi level. [S0163-1829(96)03829-5]

### I. INTRODUCTION

It was predicted by Büttiker, Imry, and Landauer<sup>1</sup> that an equilibrium persistent current flows in an ideal one-dimensional mesoscopic ring threaded by a magnetic flux  $\phi$ . Persistent current flows in a ring as a response to magnetic field which destroys the time-reversal symmetry and is periodic in magnetic flux, with a period  $\phi_0$ ,  $\phi_0$  being the elementary flux quanta ( $\phi_0 = hc/e$ ). At zero temperature the amplitude of persistent current is given by  $ev_f/L$ , where  $v_f$  is the Fermi velocity and  $L$  is the circumference of the ring. For spinless electrons, the persistent current can be either diamagnetic or paramagnetic depending upon whether the total number of electrons present is odd or even, respectively.<sup>2,3</sup> This behavior of the persistent current is also known as the parity effect. The existence of persistent currents in mesoscopic rings has subsequently been confirmed by several experiments.<sup>4-6</sup>

Persistent currents occur in both open and isolated closed systems.<sup>7-13</sup> A simple open system<sup>7</sup> is a metallic ring connected to an electron reservoir, characterized by the chemical potential  $\mu_1$ . Several effects related to persistent currents can arise in open systems which have no analog in closed or isolated systems. Recently we have also shown<sup>13-15</sup> that large circulating currents can arise in open mesoscopic rings in the presence of a transport current, in the absence of magnetic field. This is purely a quantum effect, and is related to the property of current magnification in the loop.

So far theoretical treatments of persistent currents deal with single rings (open and closed systems) threaded by magnetic flux. Studies in a closed ring have been extended to include finite-temperature effects, multichannel rings, disorder, spin-orbit coupling, and electron-electron effects.<sup>2,3,16-19</sup> In our present work we study persistent currents in coupled mesoscopic rings. Specifically, we consider two normal one-dimensional (single-channel) coupled rings connected through a one-dimensional ideal wire, as shown in Fig. 1. The connecting lead makes contact with the left and right rings at junctions  $J_1$  and  $J_2$ , respectively. This lead, in turn, is connected to an external electron reservoir characterized by a chemical potential  $\mu$  via another ideal lead making a connection at point  $X$ . The circumference of the left and right rings are  $l_1$  and  $l_2$ , respectively. The distances  $J_1X$  and  $J_2X$  are  $l_3$  and  $l_4$ , respectively. The electron reservoir acts as a

source as well as a sink of electrons, and by definition there is no phase relationship between the absorbed and emitted electrons. Electrons emitted by the reservoir propagate along the lead, are partially reflected by the junction point  $X$ , and are partially transmitted along the loop via repeated scatterings at junction points  $X$ ,  $J_1$ , and  $J_2$ . Electrons in the loops will eventually reach the reservoir after some time delay. This gives rise to a finite lifetime broadening for the electron states of the coupled rings. Scattering processes in the coupled rings are elastic. Only the reservoir acts as an inelastic scatterer. There is a complete spatial separation between the sources of elastic and inelastic scattering. Our present analysis concerns noninteracting spinless electrons. In the presence of an external uniform magnetic field  $B$ , the magnetic fluxes through the left and right rings are given by  $\alpha_1 = Bl_1^2/4\pi$  and  $\alpha_2 = Bl_2^2/4\pi$ , respectively;  $\alpha_1$  and  $\alpha_2$  are not independent quantities as the magnetic fluxes in both rings arise from the same applied uniform magnetic field  $B$ . We have obtained analytical expressions for the persistent current densities in both rings in the presence of a magnetic field. We show that persistent currents in the two rings are very sensitive to the geometric features (such as lengths  $l_1$ ,  $l_2$ ,  $l_3$ , and  $l_4$ ) of the system. Even though we have obtained an analytical expression for the general case, we restrict ourselves to a case where  $l_3 = l_4$ . When the rings are of the same size the magnitude and sign of the persistent currents are the same in both the rings (due to symmetry). We observe that if the distance between the rings,  $l = (l_3 + l_4)$ , is much larger than the circumference of the two identical rings, the known even-odd parity effect (known for isolated rings) breaks down.<sup>20</sup> Strictly speaking even-odd parity effects have to be

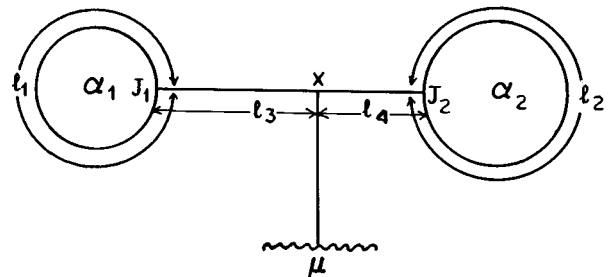


FIG. 1. Two metal loops connected to an electron reservoir with chemical potential  $\mu_1$ .

discussed in the context of isolated rings, wherein one considers the problem of persistent currents in the framework of the canonical ensemble. In our present problem, the system being coupled to a reservoir, a detailed treatment should invoke the grand canonical ensemble (the number of electrons in the system depends on the magnetic flux). However, by analyzing the nature of persistent current densities as a function of Fermi energies and quasienergy level structures as a function of magnetic flux in open systems, one can readily infer the nature of even-odd parity effect that should be observable in an isolated system of coupled rings.

In the second case we have considered rings of unequal circumferences. In such a situation at a given Fermi level it is possible to observe diamagnetic current in one of the rings, and simultaneously paramagnetic current in the other ring. In Sec. II we give a brief account of theoretical treatment, and Sec. III is devoted to results and conclusions.

## II. THEORETICAL TREATMENT

In this section we derive expressions for persistent current in the left and the right rings for the general case when  $l_3 \neq l_4$ . For this we consider a noninteracting electron system. Our system is considered as a free-electron network, i.e., the quantum potential  $V$  throughout the network is as-

sumed to be identically zero. The scattering arises solely due to junctions (or geometric scattering) at  $J_1$ ,  $J_2$ , and  $X$ . For scattering at the junctions we do not assume any specific form of scattering matrix, instead the junction scattering matrix follows from principles of quantum mechanics. We use the Griffiths boundary condition (current conservation) and the single valuedness of the wave function at the junctions.<sup>21</sup> Following exactly the same procedure as used before,<sup>11-15</sup> one can readily derive analytical expressions for the persistent current densities ( $dJ/dk$ ) (i.e., persistent current density in the small wave vector interval  $k$  and  $k+dk$ ) in the left ( $dJ_L/dk$ ) and right ( $dJ_R/dk$ ) loops,<sup>7</sup> which are given by

$$\begin{aligned} dJ_L/dk = & -(ek\hbar/2\pi m)256 \sin[\alpha] \sin[kl_1] \{ \sin[k(l_2-l_4)] \\ & - 3 \sin[k(l_2+l_4)] - 2 \sin[\beta-kl_4] \\ & + 2 \sin[\beta+kl_4] \}^2 / (\Omega_1^2 + \Omega_2^2), \end{aligned} \quad (1)$$

$$\begin{aligned} dJ_R/dk = & -(ek\hbar/2\pi m)256 \sin[\beta] \sin[kl_2] \{ \sin[k(l_1-l_3)] \\ & - 3 \sin[k(l_1+l_3)] - 2 \sin[\alpha-kl_3] \\ & + 2 \sin[\alpha+kl_3] \}^2 / (\Omega_1^2 + \Omega_2^2). \end{aligned} \quad (2)$$

Here  $\Omega_1$  and  $\Omega_2$  are given by

$$\begin{aligned} \Omega_1 = & 2\{-3 \cos[k(l_1-l_2-l_3-l_4)] - \cos[k(l_1+l_2-l_3-l_4)] + 9 \cos[k(l_1-l_2+l_3-l_4)] + 3 \cos[k(l_1+l_2+l_3-l_4)] \\ & + \cos[k(l_1-l_2-l_3+l_4)] + 3 \cos[k(l_1+l_2-l_3+l_4)] - 3 \cos[k(l_1-l_2+l_3+l_4)] - 9 \cos[k(l_1+l_2+l_3+l_4)] \\ & - 4 \cos(\alpha-\beta-kl_3-kl_4) - 4 \cos(\alpha+\beta-kl_3-kl_4) + 6 \cos(\beta-kl_1-kl_3-kl_4) + 2 \cos(\beta+kl_1-kl_3-kl_4) \\ & + 6 \cos(\alpha-kl_2-kl_3-kl_4) + 2 \cos(\alpha+kl_2-kl_3-kl_4) + 4 \cos(\alpha-\beta+kl_3-kl_4) + 4 \cos(\alpha+\beta+kl_3-kl_4) \\ & - 2 \cos(\beta-kl_1+kl_3-kl_4) - 6 \cos(\beta+kl_1+kl_3-kl_4) - 6 \cos(\alpha-kl_2+kl_3-kl_4) - 2 \cos(\alpha+kl_2+kl_3-kl_4) \\ & + 4 \cos(\alpha-\beta-kl_3+kl_4) + 4 \cos(\alpha+\beta-kl_3+kl_4) - 6 \cos(\beta-kl_1-kl_3+kl_4) - 2 \cos(\beta+kl_1-kl_3+kl_4) \\ & - 2 \cos(\alpha-kl_2-kl_3+kl_4) - 6 \cos(\alpha+kl_2-kl_3+kl_4) - 4 \cos(\alpha-\beta+kl_3+kl_4) - 4 \cos(\alpha+\beta+kl_3+kl_4) \\ & + 2 \cos(\beta-kl_1+kl_3+kl_4) + 6 \cos(\beta+kl_1+kl_3+kl_4) + 2 \cos(\alpha-kl_2+kl_3+kl_4) + 6 \cos(\alpha+kl_2+kl_3+kl_4)\}, \end{aligned} \quad (3)$$

$$\begin{aligned} \Omega_2 = & 4\{-3 \sin[k(l_1-l_2-l_3-l_4)] - \sin[k(l_1+l_2-l_3-l_4)] + 3 \sin[k(l_1-l_2+l_3+l_4)] + 9 \sin[k(l_1+l_2+l_3+l_4)] \\ & - 4 \sin(\alpha-\beta-kl_3-kl_4) - 4 \sin(\alpha+\beta-kl_3-kl_4) + 6 \sin(\beta-kl_1-kl_3-kl_4) + 2 \sin(\beta+kl_1-kl_3-kl_4) \\ & + 6 \sin(\alpha-kl_2-kl_3-kl_4) + 2 \sin(\alpha+kl_2-kl_3-kl_4) + 4 \sin(\alpha-\beta+kl_3+kl_4) + 4 \sin(\alpha+\beta+kl_3+kl_4) \\ & - 2 \sin(\beta-kl_1+kl_3+kl_4) - 6 \sin(\beta+kl_1+kl_3+kl_4) - 2 \sin(\alpha-kl_2+kl_3+kl_4) - 6 \sin(\alpha+kl_2+kl_3+kl_4)\}, \end{aligned} \quad (4)$$

where  $\alpha=2\pi\alpha_1/\phi_0$ ,  $\beta=2\pi\alpha_2/\phi_0$ , and  $\phi_0=hc/e$  are the elementary flux quanta. The wave vector of an electron is denoted by  $k$ , and is related to the energy  $E$  of an electron by a simple relation  $E=\hbar^2k^2/2m$ . Since we are considering the case wherein the magnetic field  $B$  is due to the same source, consequently the fluxes ( $\alpha$  and  $\beta$  are written in a dimensionless form) piercing through the two loops are related by the following relation (i.e.,  $\alpha$  and  $\beta$  are dependent variables):

$$\alpha = ((l_1^2/l_2^2)\beta). \quad (5)$$

For the above case from Eqs. (1)–(5) one can readily verify that persistent current densities are antisymmetric in  $B$ , or that the persistent currents in two loops change sign on the reversal of magnetic field ( $B \rightarrow -B$ ). Henceforth we rescale the current densities in the dimensionless form and denote  $dj_L = (dJ_L/dk)(2m\pi/\hbar ek)$  and  $dj_R = (dJ_R/dk)(2m\pi/\hbar ek)$ . We have also rescaled all the lengths with respect to the length  $l_1$  of the left-hand loop. The wave vector is written in the dimensionless form as  $kl_1$ .

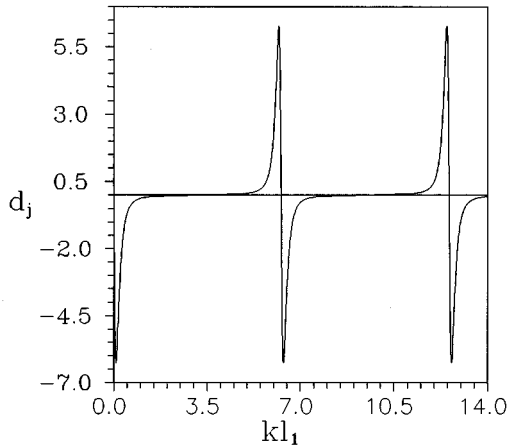


FIG. 2. Plot of circulating current vs  $kl_1$  for a fixed value of  $\alpha=0.2$ . For this case  $l_2/l_1=1.0$  and  $l_3/l_1=l_4/l_1=0.5$ .

### III. RESULTS AND DISCUSSIONS

We would like to point out that our expression for the persistent current densities obtained in Eqs. (1) and (2) are quite general and valid even for the case where fluxes enclosed by two rings  $\alpha_1$  and  $\alpha_2$  are independent variables. This case corresponds to a situation in which the enclosed magnetic fluxes in the left and right rings may arise, respectively, from two independent magnetic-field sources. However, in our present detailed analysis we have not considered this case. If the two rings are identical ( $l_1=l_2$ ) we notice that the magnitude of the persistent current densities in the left and right rings are unequal. This follows from the fact that there is an asymmetry in the system. This asymmetry arises because of the junction scattering point  $X$ , which is not placed at a symmetrical position with respect to the position of the two rings ( $l_3 \neq l_4$ ). Henceforth we restrict our discussion further to the case  $l_3=l_4$  (symmetrical situation). For this special case when  $l_1=l_2$ , the magnitude and direction of the persistent current are the same in both the rings.

In Fig. 2 we have plotted the persistent current density  $d_{jL}$  as a function of the dimensionless wave vector  $kl_1$  for a fixed value of  $l_2/l_1=1$ ,  $l_3/l_1=l_4/l_1=0.5$ , and  $\alpha=0.5$ . Since in this particular case the system is symmetric about the junction  $X$ , we expect that currents in the left or right rings will be the same. As one varies  $kl_1$  the persistent currents oscillate between diamagnetic and paramagnetic behaviors. In our problem the coupled rings are connected to a reservoir, which, in turn, leads to finite lifetime broadening of the electron states in the system, and as a consequence the persistent current shows a broadened feature as a function of  $kl_1$  compared to an isolated system. The amplitude extrema in persistent current occur approximately at the values of  $kl_1=2\pi(n+\alpha_1/\phi_0)$ , where  $n=0, \pm 1, \pm 2$ , etc., which correspond to the allowed states in a single isolated loop of length  $l_1$ . The observed small deviation from values of  $kl_1$  for an isolated ring follows from the fact that there is a coupling between the rings and additional scatterings at  $J_1, J_2$ , and  $X$ .

In Fig. 3 we have plotted persistent current densities as a function of  $\alpha$  for a fixed value of  $kl_1=6.0$ . Other parameters are the same as used for Fig. 2. We notice that results obtained in Figs. 2 and 3 are qualitatively the same as one observes in a single loop of length  $l_1$  connected to an elec-

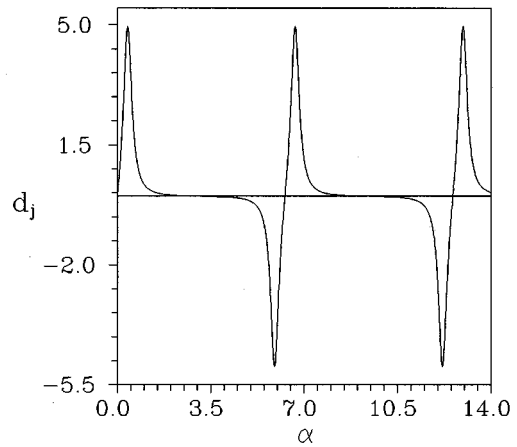


FIG. 3. Plot of circulating current vs  $\alpha$  for a fixed value of  $kl_1=6.0$ . For this case  $l_2/l_1=1.0$  and  $l_3/l_1=l_4/l_1=0.5$ .

tron reservoir.<sup>7</sup> It is also to be noted that the simple periodicity observed in Figs. 2 and 3 is due to the fact that all lengths ( $l_1, l_2, l_3$ , and  $l_4$ ) are simple rational multiples of each other; otherwise we would have obtained a complicated structure in the behavior of persistent current as a function of  $kl_1$  as well as  $\alpha$ .

In Fig. 4 we have plotted the total persistent current (in dimensionless form)  $I/I_0$  as a function of  $\alpha$  for a fixed value of Fermi wave vector  $k_f=6\pi/l_1$ . The total persistent current can be obtained by integrating Eqs. (1) or (2) up to the Fermi wave vector. Here  $I_0=e\hbar k_f/ml_1=ev_f/l_1$ ,  $v_f$  is the Fermi velocity, and we have taken  $l_2/l_1=1.0$ ,  $l_3/l_1=l_4/l_1=0.5$ . As expected for symmetric rings of equal circumference, the persistent current is periodic in  $\alpha$  with period  $2\pi$ . It should also be noted that the absolute peak magnitudes of the persistent current are much less than  $ev_f/l_1$ , expected for isolated rings. This is because the presence of inelastic scattering (arising in the reservoir) leads to the broadening of energy states in the loop and, consequently, the amplitude of the persistent current is smaller as compared to the closed or isolated systems.

From now on we discuss the case when the length of the

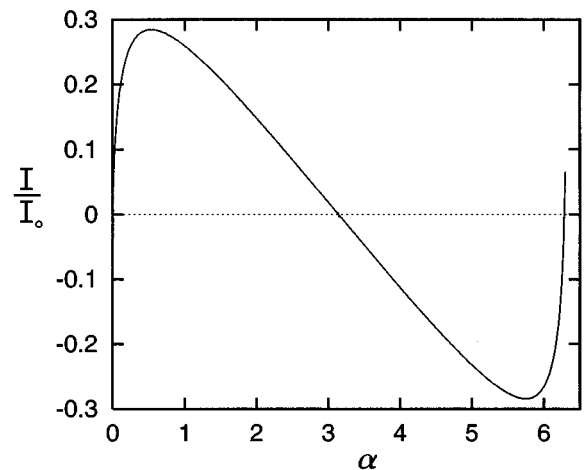


FIG. 4. Plot of total persistent current  $I/I_0$  vs  $\alpha$ . For this case  $l_2/l_1=1.0$ ,  $l_3/l_1=l_4/l_1=0.5$ , and  $k_f=6\pi/l_1$ .

connecting lead ( $l_3+l_4$ ) is much larger than the circumference  $l_1$  of the loops. We have taken both loops to be of equal circumference. We show that in this simple case the even-odd parity effect known for isolated rings breaks down and the parity effect is modified in accordance with the length ratio  $(l_3+l_4)/l_1$ . In the absence of magnetic field, an isolated single loop has eigenstates corresponding to wave vector  $k=2\pi n/l_1$ , whereas an isolated connecting wire of length  $l_3+l_4$  has eigenstates with  $k=n\pi/(l_3+l_4)$  ( $n=0, \pm 1, \pm 2$ , etc.). Therefore for the length  $(l_3+l_4) > l_1$ , energy levels in the isolated lead are more closely spaced than the energy levels in the loop. These closely spaced energy levels leak into the loops (hybridized with the states within the loop) in a connected ring system, and consequently additional quasibound states arise which have energies lying between the states of the isolated ring. Naturally the energies of these states of the coupled system will be shifted from either of those of the separate lead and the ring due to the coupling (perturbation). In the presence of a magnetic field such an additional state contributes to the persistent current diamagnetically or paramagnetically depending on whether it is near a diamagnetic or paramagnetic state of the isolated loop (in the presence of a magnetic field). These states basically owe their existence to the resonant states in the isolated lead, and their contribution to the magnitude of persistent current is small compared to the contribution of persistent current from the states near the resonant states of the isolated loops. Thus a situation can arise a system of coupled loops [with  $(l_3+l_4) \gg l_1$ ] such that first  $N_1$  states are diamagnetic and the next  $N_1$  states will be paramagnetic ( $2N_1$  is the number of resonant states, of the lead, lying between the two successive levels of the isolated ring), and this process repeats as we go to higher states. In a single isolated loop, for spinless electrons, it is well known<sup>2,3</sup> that current in a loop is diamagnetic or paramagnetic depending on whether the number of particles is odd or even, respectively (even-odd parity effect). Now for coupled mesoscopic rings this simple even-odd parity effect is altered, and instead we have the first  $N_1$  levels contributing to a diamagnetic current and the next  $N_1$  levels contributing a paramagnetic current, where  $N_1$  depends on the ratio  $(l_3+l_4)/l_1$ . This is true for the case of two identical loops. The parity effect will have different meaning if  $l_1 \neq l_2$  (nonidentical loops). For this case the underlying concepts will become slightly complicated, as we have to discuss parity effects in the left and right loops separately as they carry different currents for any given state, which will be discussed below. In Fig. 5 we have plotted the persistent current  $d_j$  as a function of  $kl_1$ , for the case when  $(l_3+l_4)/l_1=2$ ,  $l_2/l_1=1$ , and for a fixed value of  $\alpha=1.2$ . For this situation we have two additional states of the connecting lead (lying between eigenstates of the isolated loops), which leak into the loops. We clearly observe from Fig. 5 that as we vary  $kl_1$  we obtain the first two peaks which are diamagnetic, then the later two peaks which are paramagnetic, and then the sequence repeats.

In our problem we basically solve a scattering problem wherein electrons are injected into the system from the reservoir, and then reflected back to the reservoir. From a scattering matrix structure one can obtain information about quasibound states. This can be achieved by looking at the poles in a complex  $kl_1$  plane of the complex reflection amplitude.

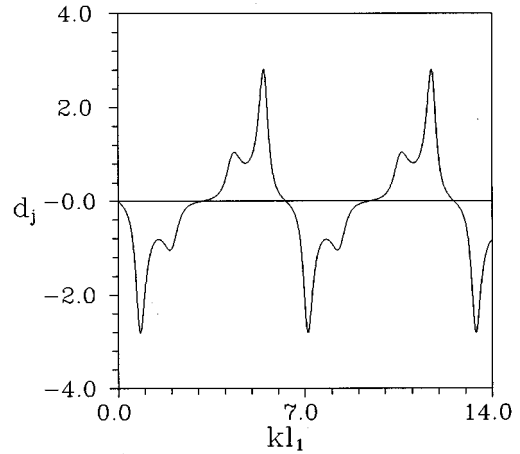


FIG. 5. Plot of circulating current vs  $kl_1$  for a fixed value of  $\alpha=1.2$ . For this case  $l_2/l_1=1.0$  and  $l_3/l_1=l_4/l_1=1.0$ .

The real part of the poles ( $R$ ) gives the wave-vector values of the resonant states, whereas the imaginary part gives the information about the lifetime of these states. In Fig. 6 we have plotted the real part  $R$  of these complex poles as a function of  $\alpha$ . All parameters used here are the same as in Fig. 5. We clearly observe that additional states (in the present case 2) appear within the intervals of  $kl_1$  values of isolated loops. Moreover, one can readily notice that the first two resonant states carry a diamagnetic current (as their slopes with respect to the magnetic flux are positive<sup>2,3</sup>), the next two carry paramagnetic currents, and so on. As expected, on general grounds values of  $R$  are periodic in flux  $\alpha_1$  with a period  $\phi_0$ . In Fig. 7 we have plotted  $d_j$  versus  $kl_1$  for the case  $(l_3+l_4)/l_1=10.0$ , and for a fixed value of  $\alpha=1.2$ , while  $l_2/l_1=1.0$ . It is clear from this figure that the first six states carry a diamagnetic current, the next six states carry a paramagnetic current, and so on. Figures 5 and 7 clearly indicate that the known even-odd parity effect in an isolated ideal ring breaks down for a system of coupled rings and, moreover, the emergence of the additional parity effect discussed above is sensitive to the length ratio  $(l_3+l_4)/l_1$ .

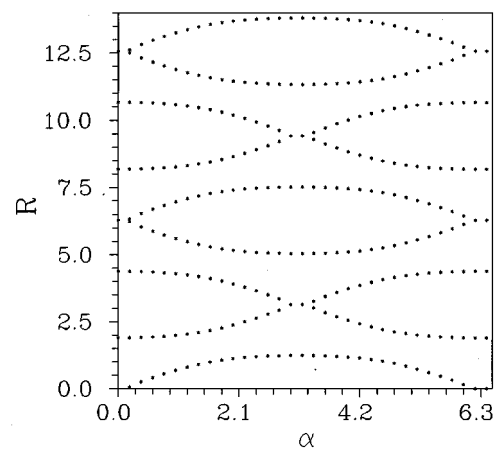


FIG. 6. The plot of real part  $R$  of the complex poles in the  $kl_1$  plane of the reflection amplitude as a function of  $\alpha$  for  $l_2/l_1=1.0$  and  $l_3/l_1=l_4/l_1=1.0$ .

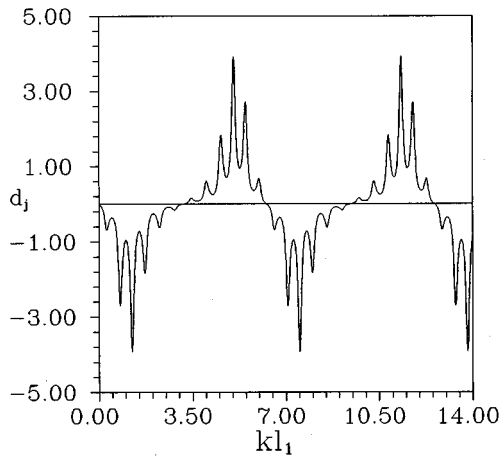


FIG. 7. Plot of circulating current vs  $kl_1$  for a fixed value of  $\alpha=1.2$ . For this case  $l_2/l_1=1.0$  and  $l_3/l_1=l_4/l_1=5.0$ .

We further consider the case for which the loops are not identical, in that their circumferences are different. In such a situation one has to discuss the persistent currents in the right and left loops separately. Consider a situation where  $l_2 > l_1$ . Naturally resonant states in the right loop are more closely spaced than those in the left loop. There will be mixing between these states due to the coupling. However, it is possible that as one varies the wave vector  $kl_1$ , the persistent current in the right loop oscillates between diamagnetic and paramagnetic behavior much more rapidly than the persistent current in the left-hand loop; i.e., in a given interval of  $kl_1$ , the persistent current does not change sign for the case of left-hand loop, whereas in the same interval the persistent current in the right-hand loop changes sign several times. We can have a situation where for a given state ( $kl_1$ ) current in the left and right loops have either the same sign or different (i.e., the current in left loop are diamagnetic whereas the current in the right loop is paramagnetic). This is illustrated in Fig. 8, where the dotted lines and the solid lines indicate persistent current in the right ( $dj_R$ ) and left ( $dj_L$ ) loops, respectively. For the above case we have taken  $l_2/l_1=4$  and

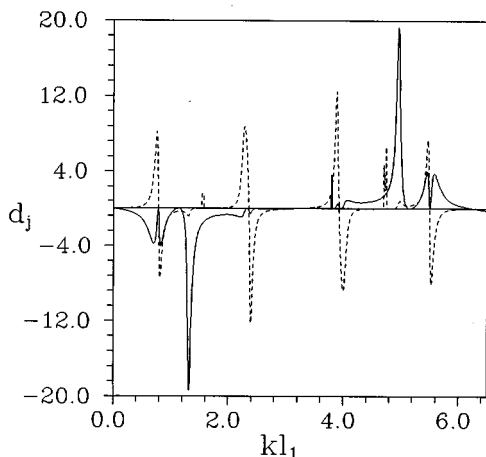


FIG. 8. The persistent current as a function of  $kl_1$  in the left loop (solid line) and the right loop (dashed lines) for a fixed value of  $\alpha=1.2$ . For this case  $l_2/l_1=4$  and  $l_3/l_1=l_4/l_1=0.5$ .

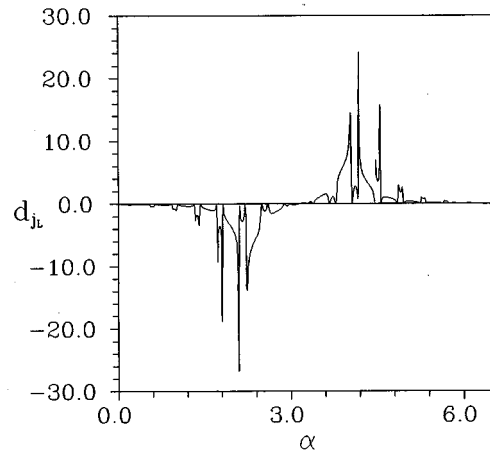


FIG. 9. Plot of persistent current in the left loop as a function of  $\alpha$  for a fixed value of  $kl_1=2.2$ . For this case  $l_2/l_1=4$  and  $l_3/l_1=l_4/l_1=0.5$ .

( $l_3+l_4$ )/ $l_1=1$ . In Figs. 9 and 10 we have plotted persistent currents as a function of  $\alpha$  for a fixed value of  $kl_1=2.2$  for the right and the left loops, respectively. The other parameters are the same as those used in Fig. 8. From Figs. 9 and 10 we notice that  $dj_L$  and  $dj_R$  are periodic in  $\alpha_1$  with a period  $\phi_0$ . We would like to mention that this is so because for our case we have considered a commensurate ratio  $l_2/l_1=4$ . In general if we choose the ratio to be incommensurate (or irrational),  $dj_L$  or  $dj_R$  will have much larger values of the period with respect to  $\alpha$ . It should be kept in mind that as one varies  $\alpha_1$  (the flux through the left ring) by  $\phi_0$ , the flux through the right ring ( $\alpha_2$ ) changes by an amount  $16\phi_0$  (since  $l_2=4l_1$ ). It follows from this that as  $\alpha$  is varied from 0 to  $2\pi$  the persistent current density in the left loop changes sign once, while the persistent current density in the right loop changes the sign 16 times. This fact also can be noticed from Fig. 11, where we have plotted total persistent current  $I/I_0$  versus  $\alpha$ , the physical parameters are kept the same as for Fig. 8. The dotted line and the solid line indicate the total persistent current in the right and left rings, respec-

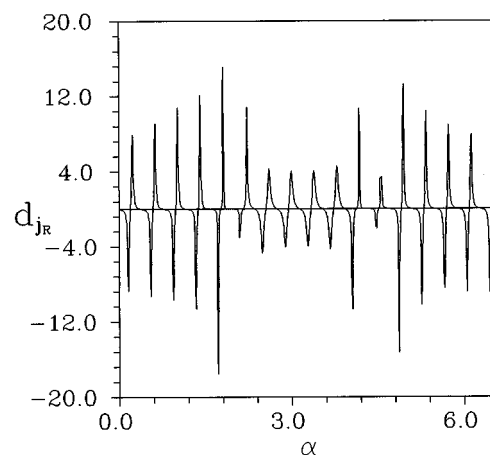


FIG. 10. Plot of persistent current in the right loop as a function of  $\alpha$  for a fixed value of  $kl_1=2.2$ . For this case  $l_2/l_1=4$  and  $l_3/l_1=l_4/l_1=0.5$ .

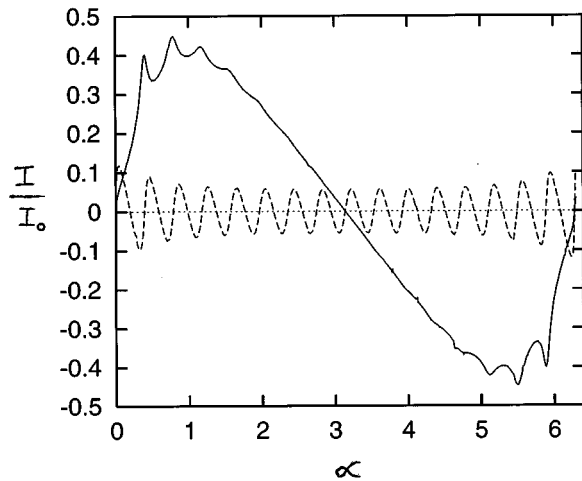


FIG. 11. Plot of total persistent current  $I/I_0$  vs  $\alpha$ . The solid lines and dashed lines are for left and right, respectively. For this case  $l_2/l_1=4.0$ ,  $l_3/l_1=l_4/l_1=0.5$ , and  $k_f=2\pi/l_1$ .

tively. We have taken the Fermi wave vector to be  $k_f=2\pi/l_1$ .

It is well known that for a simple case of an isolated single loop (or a single hole in the sample) the persistent current carried by the  $n$ th state of energy  $E_n$  is given by  $I_n = -(1/c)\partial\epsilon_n/\partial\phi$ , where  $\phi$  is the flux piercing through the

loop (or hole). In the present case of multiply connected nonidentical rings one cannot infer the values of persistent current in the individual rings from the above definition. To calculate the persistent current in the presence of magnetic field in each loop of the system of coupled rings, one has to calculate the quantum-mechanical wave function in each ring explicitly, and from that one can calculate the currents.

In our analysis we have mostly discussed the persistent current densities  $dj$  in the small wave vector interval  $k$  and  $k+dk$ . However, experimentally it is the total persistent current generated by all the conducting electrons in the system that can be observed. This is calculated by integrating the persistent current densities up to the Fermi wave vector  $k_f$  using Eqs. (1) and (2) (see Figs. 4 and 11). In conclusion, we have studied the nature of persistent currents in an open mesoscopic coupled ring system in the presence of magnetic field. Throughout we have considered simple commensurate ratios of  $l_1/l_2$  and  $(l_3+l_4)/l_1$ . For coupled identical rings one observes different parity effects. The parity effect depends on the ratio  $(l_3+l_4)/l_1$  of the connecting lead length to the circumference of the rings. In the case of nonidentical loops, for a given state it is possible that the persistent current in one loop is diamagnetic whereas in the other it may be paramagnetic or diamagnetic. Moreover all these effects are very sensitive to the length ratio involved in the system, as the problem is inherently quantum mechanical in nature, where interference effects dominate.

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