Interlayer fluxon interaction in Josephson stacks

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In a stack of Josephson tunnel junctions both attractive and repulsive interactions may exist between Josephson vortices (fluxons) allocated in different tunnel barriers. We report experimental observations of two modes of coherent fluxon motion in double-junction stacks. At the low-velocity mode, as well as in statics, vortices in neighboring Josephson layers repel each other. In contrast, the high-velocity mode corresponds to an attractive interlayer interaction between vortices of the same polarity. [S0163-1829(96)07233-5]

The interactions between vortices in layered superconductors and superconducting multilayers are presently a subject of intensive investigations. One of the stimulating reasons for such interest is the significance of this problem for very anisotropic high-T_c superconductors. A recently discovered intrinsic Josephson effect in Bi-Sr-Ca-Cu-O single crystals[1] indicated that some of these materials are essentially natural stacks of closely packed Josephson tunnel junctions. Josephson coupled layers exhibit properties which differ from that of isotropic superconductors. On the other side, their behavior is different from a single Josephson junction due to mutual interactions between the junctions in a stack.

In stacks, the thickness of superconducting layers is typically smaller than the London penetration depth λ_L. When a magnetic field is applied parallel to superconducting planes Josephson vortices can penetrate between the planes. Screening currents of a Josephson vortex extend over several layers and provide a strong coupling between vortices allocated in different junctions. While being very anisotropic, the quasistatic interaction between these Josephson vortices is similar to that of Abrikosov vortices in a bulk type-II superconductor; namely, it is always repulsive.

A Josephson vortex can be considered as the extreme case of an Abrikosov vortex without a normal core: The core of the Josephson vortex is virtually located in the tunnel barrier between superconducting electrodes. In general, the quasiparticle currents flowing in the normal core are responsible for the dissipation which occurs during the vortex motion in a superconductor. While the Abrikosov vortex motion is always *overdamped*, Josephson vortices in long tunnel junctions (often called fluxons or solitons) are well known for their *underdamped* dynamics and ballistic properties.[2] In high-quality Josephson tunnel junctions, the losses during Josephson vortex motion remain small due to low quasiparticle tunnel conductance. Under the influence of the bias current, fluxons can move along the Josephson junction with a velocity v close to the velocity $\bar{c}$, the maximum velocity of electromagnetic wave propagation in the junction. Under these conditions, the fluxon dynamics becomes relativistic with respect to $\bar{c}$ which may drastically change the nature of the mutual fluxon interaction. One example is the change from repulsion to attraction between fluxons of the same polarity, the so-called bunching phenomenon, theoretically predicted[3] and experimentally detected[4] in single-layer long Josephson junctions. Up to now, no experiments have been reported on the attraction between fluxons in multilayer Josephson junctions.

In this paper we report experiments with double-junction Nb-$(\mathrm{Al/AlO}_x\text{-Nb})_2$ Josephson stacks in a magnetic field applied parallel to superconducting planes. We observe two different types of coherent resonant modes in the current-voltage characteristics of the stacks. In both modes fluxons in two layers move coherently but their limiting velocity is clearly different. According to theory, in the low-velocity mode fluxons located in different Josephson layers are expected to repel each other. In contrast, for the high-velocity mode an attraction between moving fluxons of the same polarity takes place. Our observation of a stable high-velocity mode suggests that Josephson stacks can be operated in a superradiant state with in-phase oscillations in all layers.

Most existing theories for layered superconductors are based on the Lawrence-Doniach model[5] where stacked superconducting planes of zero thickness are coupled by the Josephson tunneling. Using this model, the main effort has been made to describe static magnetic flux structures in highly anisotropic superconductors.[6] More recently, Sakai *et al.*[7] developed a model for Josephson stacks which includes time dependence and has no limitation as to the thickness of the superconducting layers. According to that model,[7] two vertically stacked Josephson tunnel junctions with equal critical current densities j_c are described by a system of coupled perturbed sine-Gordon equations

$$\frac{\varphi^A_{xx}}{1-S^2}-\varphi^A_{tt}=\sin\varphi^A+\alpha_A\varphi^A_t+\gamma_A+\frac{S\varphi^B_{xx}}{1-S^2}, \quad (1)$$

$$\frac{\varphi^B_{xx}}{1-S^2}-\varphi^B_{tt}=\sin\varphi^B+\alpha_B\varphi^B_t+\gamma_B+\frac{S\varphi^A_{xx}}{1-S^2}. \quad (2)$$

Here $\varphi^A(x,t)$ and $\varphi^B(x,t)$ are the superconducting phase differences across the junctions A and B, respectively, and the subscripts indicate partial derivatives. The spatial coordinate x is normalized to the single-junction Josephson penetration depth λ_J, the time t to the inverse plasma frequency ω_0^{-1}, α_A and α_B are the dissipation coefficients due to quasiparticle losses, and γ_A and γ_B are the bias currents. The coupling parameter $S<0$ in Eqs. (1) and (2) is due to the inductive interaction between stacked junctions. It is associated with screening currents in superconducting electrodes which are shared by fluxons belonging to different layers. This parameter

$$S=-\left[\left(\frac{d}{\lambda_L}+\coth\frac{t}{\lambda_L}+\coth\frac{t_e}{\lambda_L}\right)\sinh\frac{t}{\lambda_L}\right]^{-1} \tag{3}$$

can be calculated directly from experimental data such as the barrier thickness d, the middle electrode thickness t, and the thickness of the top and bottom electrodes, t_e, which here for simplicity are assumed to be equal. Obviously, the coupling parameter S vanishes for $t \gg \lambda_L$.

Linearizing Eqs. (1) and (2) for the small-amplitude electromagnetic waves $\varphi^{A,B}(x,t)=\varphi_0^{A,B}\exp[i(kx-\omega t)]$ without perturbative terms ($\alpha^{A,B}=\gamma^{A,B}=0$) yields the dispersion relation $\omega^2=1+k^2/(1\pm S)$ which falls into two modes corresponding to different signs in front of S. These modes are characterized by the velocities

$$\overline{c_-}=\frac{\overline{c}}{\sqrt{1-S}}, \quad \overline{c_+}=\frac{\overline{c}}{\sqrt{1+S}}, \tag{4}$$

where $\overline{c}$ is the Swihart velocity (the maximum velocity of electromagnetic wave propagation) for the single-barrier junction; $\overline{c}=1$ in normalization units of Eqs. (1) and (2). In the linear approximation the lower ($\overline{c_-}$) and the higher ($\overline{c_+}$) velocities are characterized, respectively, by the *out-of-phase* ($\varphi_0^A=-\varphi_0^B$) and the *in-phase* ($\varphi_0^A=\varphi_0^B$) small-amplitude waves propagating in two junctions.[8]

For *linear* waves in two coupled Josephson junctions the splitting of velocities was predicted by Ngai[9] but only recently has it been observed experimentally using stacks.[10] Experiments showed that a fluxon chain allocated in one Josephson barrier [Fig. 1(a)] may move either in the $\overline{c_-}$ mode or in the $\overline{c_+}$ mode, depending on the applied bias current and the coupling between the junctions. Measurements of $\overline{c_-}$ and $\overline{c_+}$ for various electrode thicknesses[8] showed good agreement with the model by Sakai *et al.* Since a fluxon itself is a *nonlinear* wave, a puzzling question remains about possible mutual configurations of fluxon arrays coherently moving in neighboring tunnel barriers. For equal fluxon densities in two barriers, one may consider either the out-of-phase mutual configuration schematically shown in Fig. 1(b) or the in-phase configuration sketched in Fig. 1(c). In every case, dc voltages on both junctions arising from fluxon motion should be equal, but the Josephson oscillations would have a phase shift of either π or 0. Numerical simulations[7] showed that two fluxons of the same polarity moving in different junctions can form a stable bound state with identical phases in two junctions. This state has analytically been shown to be stable in the $\overline{c_+}$ mode.[11]

Existing theoretical studies[7,11] have been restricted to single-fluxon regimes which take place at zero magnetic field. In experiments, however, zero-field modes have been found to be rather unstable[10,12,13] and no stable bound states have been observed so far. In contrast, coherent fluxon motion in stacked junctions has been observed in an applied magnetic field[14] which corresponds to rather dense fluxon chains moving in the junctions. In order to illustrate two coherent (in-phase and out-of-phase) regimes for reasonably high fluxon density, in Fig. 2 we show the simulated current-

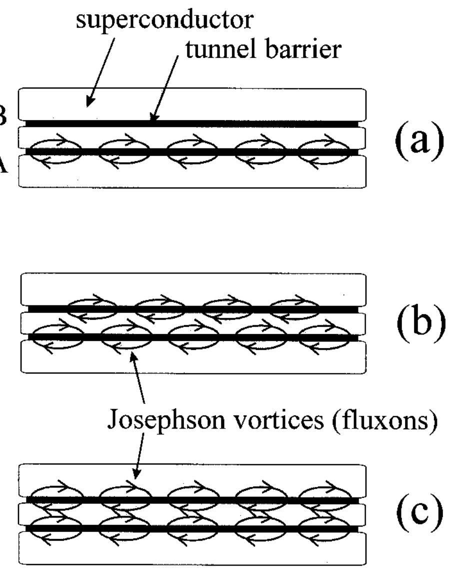

FIG. 1. Sketches of a cross section of two stacked long Josephson junctions. (a) If the critical current density j_c of the lower junction A is smaller than that of the top junction B, the fluxons first penetrate the junction A only. For nearly equal j_c's in A and B the fluxons may occupy both junctions in either an out-of-phase configuration (b), corresponding to repulsion, or an in-phase configuration (c), corresponding to mutual attraction between fluxons in different layers.

voltage (I-V) characteristics for a twofold stack of length $L/\lambda_J=\ell=5$ in the x direction. Equations (1) and (2) were solved numerically with periodic boundary conditions $\varphi^A(\ell)-\varphi^A(0)=2\pi N_A$ and $\varphi^B(\ell)-\varphi^B(0)=2\pi N_B$, assuming an equal number of fluxons, $N_A=N_B=3$, in the junctions. For the relativistic branch corresponding to the velocity $\overline{c_+}$ (lower inset in Fig. 2) we find a perfect *in-phase* locking: Fluxons in junctions A and B attract each other and the magnetic field profiles φ_x^A and φ_x^B cannot be distin-

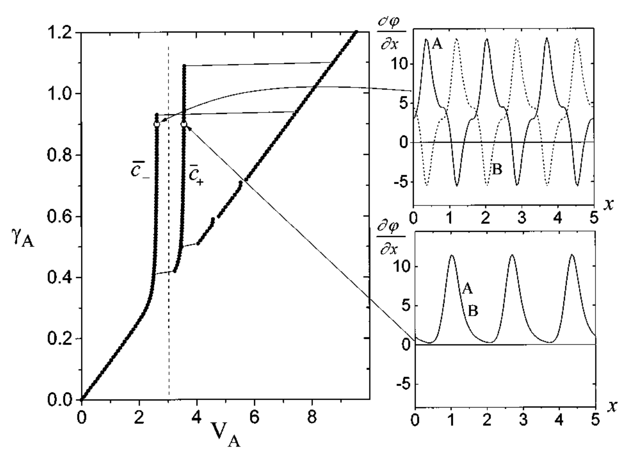

FIG. 2. Numerically calculated current-voltage (I-V) curve for a two-junction stack with coupling parameter $S=-0.3$. Simulations have been made for periodic boundary conditions with junctions biased in series ($\gamma_A=\gamma_B=\gamma$), for $\alpha_A=\alpha_B=\alpha=0.1$. The dashed line corresponds to the flux-flow step position in the uncoupled single junction with $S=0$. The two insets show instantaneous profiles of the magnetic fields φ_x^A (solid line) and φ_x^B (dashed line) in two points of the I-V curve indicated by arrows.

TABLE I. Parameters of investigated samples.

No.	t (nm)	L (μm)	δ	ΔV_- (μV)	ΔV_+ (μV)	ΔV_* (μV)	$-S$
1	90	200	0.03	27 ± 1	38 ± 1	55 ± 1	0.46
2	90	80	0.06	62 ± 2	86 ± 2	130 ± 3	0.46
3	140	400	0.16	15 ± 1	20 ± 2	44 ± 3	0.21
4	120	300	0.18	21 ± 1	26 ± 2	59 ± 3	0.26

guished. In the branch $\overline{c_-}$ (upper inset in Fig. 2) the *out-of-phase* locking is found; thus, individual fluxons repel each other as is usually expected for the static case. Further numerical simulations of the multifluxon case are presented elsewhere.[15]

Experimentally, we studied stacks of Nb-(Al/AlO$_x$-Nb)$_2$ Josephson tunnel junctions. In order to realize different coupling strengths, the thickness t of the intermediate Nb layer varied in the range from 35 nm to 140 nm. The London penetration depth for our sputtered Nb was $\lambda_L \approx 90$ nm at 4.2 K. A typical value for the Josephson penetration depth λ_J in single-barrier junctions was about 25 μm. The fabrication details are described elsewhere.[16] Here we present data for two series of samples with typical parameters listed in Table I. The parameter $\delta=(I_c^A-I_c^B)/I_c^B$ accounts for the difference between the critical currents of the junctions I_c^A and I_c^B. The I-V characteristics of stacked junctions have been measured in series.

By applying a magnetic field H parallel to superconducting planes, we find Fiske steps on I-V characteristics of the stacks. Fiske steps correspond to the resonances between the Josephson generation frequency and the frequency of one of the cavity modes of the junctions. By measuring the voltage spacing between neighboring Fiske steps, $\Delta V=\overline{c}\Phi_0/(2L)$, one can directly extract the characteristic velocity $\overline{c}$ of electromagnetic wave propagation in the junction. While measuring two junctions in series we were able to determine the contribution of a single junction by examining the part of the I-V curve above the gap voltage of one of the junctions (2.6 mV), which accounts for only one junction operated in the flux-flow state and another junction in the gap state. As shown in Fig. 3, two different flux-flow states are observed and can be clearly distinguished by their Fiske step voltage spacings ΔV_- and ΔV_+. These two voltage spacings account for the velocities $\overline{c_-}=2L\Delta V_-/\Phi_0$ and $\overline{c_+}=2L\Delta V_+/\Phi_0$, and are reported in Table I. Within experimental accuracy, the velocities coincide with that calculated from Eqs. (3) and (4).

When increasing the bias current from zero, both junctions can be switched simultaneously in the flux-flow state at $V=V_*$. In such a way, sweeping below the single-junction gap voltage, we observed a series of very sharp and regular resonances with the voltage spacing between them ΔV_* being substantially larger than either ΔV_- or ΔV_+. Additional measurement of the individual junction voltages using a contact to the middle electrode[14] indicated that these resonances correspond to both junctions simultaneously locked to the same dc voltage $V=V_*/2$. Moreover, the magnetic field dependence of the current amplitudes of the V_* resonances showed the coherence between the dynamic states of the two junctions in the stack.

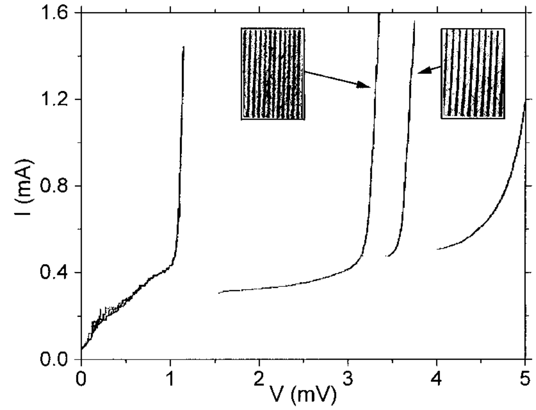

FIG. 3. Typical experimental I-V curve (sample No. 4) showing two distinctly different flux-flow resonances. These single-junction resonances are observed between the single-junction and the double-junction gap voltages, at about 3.5–3.8 mV. The insets show stored traces of the Fiske steps for each branch obtained by varying the magnetic field H. The voltage scale in the insets is expanded by a factor of 2.3. The different voltage spacings between the traces in the insets account for two wave propagation velocities $\overline{c_-}\approx 5.9\times10^6$ m/s and $\overline{c_+}\approx 7.3\times10^6$ m/s.

The very fact that the dc voltages on the junctions are equal is not sufficient to distinguish the two possible fluxon configurations shown in Figs. 1(b) and 1(c). The distinction between them can be made from the voltage spacing ΔV_* between sequential two-junction resonances. If both junctions in the stack lock to the same cavity resonance coherently, ΔV_* should be equal to the double of the singe-junction Fiske step voltage spacing. The out-of-phase fluxon locking shown in Fig. 1(b) should be characterized by the mode velocity $\overline{c_-}$, while the in-phase locking shown in Fig. 1(c) accounts for the velocity $\overline{c_+}$. Thus, the voltage spacing ΔV_* should be equal to either $2\Delta V_-$ or $2\Delta V_+$. Indeed, both these cases have been observed in our experiments. Figure 4 shows an example of stored traces of I-V curves ob-

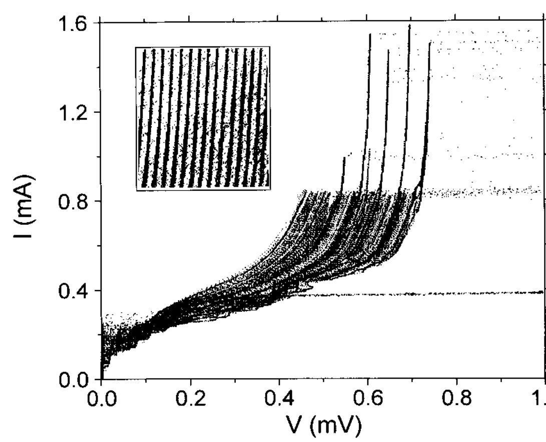

FIG. 4. Stored traces of I-V curves of stack No. 3 obtained in a continuously varying external magnetic field H. The sharp resonances correspond to the phase-locked $\overline{c_+}$ mode, which accounts for the in-phase fluxon state shown in Fig. 1(c). The inset shows the Fiske step voltage spacing of the single-junction $\overline{c_+}$ mode.

tained in continuously varying H. The sharp resonances between 0.5 and 0.8 mV have a voltage separation $\Delta V_{*}=2\Delta V_{+}$.

In stacks with a small spread of critical currents between the junctions δ and a rather large coupling constant $|S|>0.4$, as for samples No. 1 and No. 2, we observed very stable phase-locked states of the out-of-phase mode $\overline{c_{-}}$. As can be seen from Table I, for Nos. 1 and 2 we measured $\Delta V_{*}\approx 2\Delta V_{-}$. In samples like Nos. 1 and 2 we never succeed in detecting stable resonances of the $\overline{c_{+}}$ mode. In contrast, for stacks with an even larger spread of parameters between junctions but substantially weaker coupling ($|S|<0.3$), like samples No. 3 and No. 4, we observed stable phase-locked states of the in-phase mode $\overline{c_{+}}$. In Table I one can see that $\Delta V_{*}\approx 2\Delta V_{+}$ for Nos. 3 and 4.

The observation of the in-phase fluxon locking in stacks with weak coupling accounts for the pure relativistic (with respect to the Swihart velocity) nature of the mutual fluxon attraction in stacked junctions. Weak coupling is not sufficient here to stabilize the out-of-phase state corresponding to mutual fluxon repulsion in statics. In contrast, at high velocities fluxons of the same polarity belonging to different junctions tend to attract each other due to the Lorentz transformation of fluxon energy. The fluxon gets Lorentz contracted and the energy of the screening currents is drastically increased. For the fluxon configuration shown in Fig. 1(c) the screening currents in the middle electrode cancel. Thus, at sufficiently high velocity the attractive dynamic state of fluxons shown in Fig. 1(c) becomes energetically favorable. This leads to coherent in-phase oscillations in Josephson junctions embedded in the stack, which may be of considerable interest for possible applications.

One of the implications of experiments and analysis of phase locking in the low-T_c stacks presented here could be possible mutual phase locking and coherent radiation from intrinsic atomic-layer Josephson tunnel junctions in high-T_c single crystals. Single crystals of the intrinsically layered superconductor such as Bi-Sr-Ca-Cu-O (Ref. 1) can also be described by the inductively coupled stacked junction model. Considering many-layer stacks, Kleiner[17] predicted that one may obtain a standing-wave-like fluxon pattern in the direction normal to the superconducting planes (y direction). The excitation of standing waves in the y direction should appear as kinks in the I-V curve structure. Kleiner modes should lead to mutual phase shifts between Josephson oscillations in different layers and, thus, should be observed in radiation detection experiments. The lowest-velocity mode accounts for the out-of-phase oscillations in neighboring junctions, whereas the highest mode corresponds to the in-phase oscillations in all layers of the stack.

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