In-plane paraconductivity and fluctuation-induced magnetoconductivity in biperiodic layered superconductors: Application to $YBa_2Cu_3O_{7-\delta}$

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(Received 25 September 1995; revised manuscript received 26 April 1996)

The existing mean-field-like calculations of the different direct and indirect order parameter fluctuation (OPF) contributions to the in-plane paraconductivity $\Delta\sigma_{ab}$ and fluctuation-induced magnetoconductivity $\Delta\tilde{\tilde{\sigma}}_{ab}$ are extended here to layered superconductors with two different interlayer distances and different strengths of the tunneling couplings between adjacent layers (the so-called bilayered, or biperiodic layered, superconductors). The calculations are performed for magnetic fields H in the weak limit, applied perpendicular to the superconducting layers, and at temperatures near but above the $H=0$ mean-field transition temperature, T_{c0}. We obtain final explicit expressions and find that the effects of the layer biperiodicity may be summarized through an effective number N_e of independent fluctuating superconducting layers per unit cell length, already encountered also in our recent calculations of the fluctuation-induced diamagnetism $\Delta\chi_{ab}$ in biperiodic layered superconductors. Our study includes some limiting cases of the indirect contributions associated with the density of states (DOS) fluctuations, which have been recently proposed for $\Delta\sigma_{ab}$ and $\Delta\tilde{\tilde{\sigma}}_{ab}$ for single periodic layered superconductors. As an application, we use then our theoretical results to analyze the paraconductivity and the fluctuation-induced magnetoconductivity recently measured in the a direction (nonaffected by the presence of CuO chains) of untwinned $YBa_2Cu_3O_{7-\delta}$ crystals. This analysis shows that the approaches based on the conventional Lawrence-Doniach (i.e., single layered) model cannot explain simultaneously and quantitatively the intrinsic $\Delta\sigma_a$, $\Delta\tilde{\tilde{\sigma}}_a$, and $\Delta\chi_{ab}$ in $YBa_2Cu_3O_{7-\delta}$ crystals, even when the DOS contributions are considered. In contrast, when the double periodicity of this layered superconductor is taken into account, it is possible to explain consistently and at a quantitative level all such experimental data in the reduced temperature region above T_{c0} bounded by, approximately, 2×10^{-2} and 10^{-1}, which is expected to correspond to the mean-field region without high temperature and nonlocal effects. In the resulting biperiodic scenario, the indirect (i.e., Maki-Thompson and DOS) contributions to the in-plane $\Delta\sigma_a$ and $\Delta\tilde{\tilde{\sigma}}_a$ are negligible, confirming our earlier findings which suggested unconventional, pair-breaking, wave pairing. Moreover, the direct OPF effects have a dimensionality (two-dimensional–three-dimensional) crossover in the mean-field region. Our results strongly suggest also that to understand at a quantitative level the OPF effects on any in-plane or bulk physical observable in layered superconductors with various superconducting layers per unit cell length, it is crucial to take into account the influence of such a multiperiodicity. [S0163-1829(96)09829-3]

I. INTRODUCTION

Due to their short coherence length amplitudes in all directions, high-T_c and layered nature, the high-temperature copper oxide superconductors (HTSC) present important order parameter fluctuation (OPF) effects at easily accessible temperature distances from their superconducting transition.[1–3] One of the magnitudes best adapted, from both the theoretical and the experimental point of view, to study the OPF effects in HTSC is the so-called in-plane fluctuation-induced conductivity above the superconducting transition, which in presence of an external magnetic field, H, applied perpendicular to the ab (CuO_2) superconducting layers, may be defined as

$$\Delta\sigma_{ab}(\epsilon,H)\equiv\sigma_{ab}(\epsilon,H)-\sigma_{abB}(\epsilon,H), \qquad (1.1)$$

where $\epsilon\equiv(T-T_{c0})/T_{c0}$ is the reduced temperature, T_{c0} is the mean-field superconducting transition temperature at zero applied magnetic field, $\sigma_{ab}(\epsilon,H)$ is the measured in-plane electrical conductivity, and $\sigma_{abB}(\epsilon,H)$ is the so-called in-plane background electrical conductivity (the normal conductivity above the superconducting transition if the thermal fluctuations were absent). The usefulness of $\Delta\sigma_{ab}(\epsilon,H)$ is strongly enhanced, mainly in the case of HTSC, by the fact that it may be decomposed in two additive parts which may be calculated and measured separately: the zero-magnetic-field fluctuation-induced conductivity, $\Delta\sigma_{ab}(\epsilon,H=0)$, currently called paraconductivity and henceforth noted simply as $\Delta\sigma_{ab}(\epsilon)$, and the so-called fluctuation-induced magnetoconductivity, $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$, which in terms of $\sigma_{ab}(\epsilon,H)$ and $\sigma_{abB}(\epsilon,H)$ may be defined as

$$\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)\equiv[\sigma_{ab}(\epsilon,H)-\sigma_{ab}(\epsilon,0)]-[\sigma_{abB}(\epsilon,H)-\sigma_{abB}(\epsilon,0)], \qquad (1.2)$$

so that we have for $\Delta\sigma_{ab}(\epsilon,H)$,

$$\Delta\sigma_{ab}(\epsilon,H)=\Delta\sigma_{ab}(\epsilon)+\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H). \qquad (1.3)$$

The central point here is that the available experimental results in HTSC clearly indicate that the background in-plane magnetoconductivity, $\sigma_{abB}(\epsilon,H)-\sigma_{abB}(\epsilon,0)$, measured in the normal region well above T_c, is always very small, orders of magnitude smaller than $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$ measured near T_c.[4–10] This result strongly suggests, therefore, that $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$ may be approximated, even through the transition, as

$$\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)\approx\Delta\tilde{\sigma}_{ab}(\epsilon,H)\equiv\sigma_{ab}(\epsilon,H)-\sigma_{ab}(\epsilon,H=0). \tag{1.4}$$

i.e., as the difference between two directly measurable magnitudes, without any dependence on a never well settled background. However, for applied magnetic fields in the weak limit (see also below) the fluctuation-induced magnetoconductivity measured in HTSC has been found to be also orders of magnitude smaller than the zero-field paraconductivity.[4–10] Therefore, the two contributions to the total fluctuation-induced in-plane paraconductivity, $\Delta\sigma_{ab}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$, are complementary from both the theoretical and the experimental point of view and their simultaneous study may provide useful information on the OPF effects above T_c in the HTSC.

In the case of the HTSC, the correct comparison of the theoretical predictions for $\Delta\sigma_{ab}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$ with the experimental data needs also a proper account of the layered structure of these materials. Usually, this is made by means of the Lawrence and Doniach (LD) model of layered superconductors,[11] which just considers a stack of superconducting layers with only one interlayer distance and the same strength of the tunneling interlayer coupling between all the adjacent superconducting planes. Quite exhaustive calculations for both $\Delta\sigma_{ab}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$ within the LD model have been already presented by several authors.[11–17] However, the copper oxide superconductors have in general various superconducting layers per unit cell length, with different interlayer distances and different strengths of the interlayer tunneling couplings. So, it is important to extend the calculations of $\Delta\sigma_{ab}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$ indicated above to the case of multiperiodic layered superconductors, with different interlayer distances and tunneling coupling strengths. In fact, previous analysis of the fluctuation-induced diamagnetism for a (weak) magnetic field applied perpendicular to the superconducting layers, $\Delta\chi_{ab}(\epsilon)$, measured in different HTSC, have shown that the multiperiodicity effects play an important role in the understanding of $\Delta\chi_{ab}(\epsilon)$ at a quantitative level.[18–20] Therefore, one must expect a similar conclusion for the case of $\Delta\sigma_{ab}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$. For the zero-magnetic-field in-plane paraconductivity, $\Delta\sigma_{ab}(\epsilon)$, some important (although quite partial and with few echo on experimentalists) theoretical results on both the direct, or Aslamazov-Larkin (AL), and on the anomalous Maki-Thompson (MT) contributions to $\Delta\sigma_{ab}(\epsilon)$ in multiperiodic layered superconductors have been already published, first by Maki and Thompson[13] and later [for the direct contribution, $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$] by Klemm.[21] Also, some of the results of Maki and Thompson for $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$ have been confirmed by Baraduc and Buzdin[22] for a particular limit of the parameters involved. To our knowledge, however, the fluctuation-induced in-plane magnetoconductivity, $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$, has not yet been calculated until now in a multiperiodic layered superconductor. Also, the relationships among $\Delta\sigma_{ab}(\epsilon)$, $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$, and $\Delta\chi_{ab}(\epsilon)$, which will provide very useful tests of consistency in comparing the theory with the experimental data, have been studied until now only for $\Delta\chi_{ab}(\epsilon)$ and $\Delta\sigma_{ab}(\epsilon)$ for layered superconductors with a single periodicity.[10,23,24]

The main aim of this paper is twofold. First, we present the first calculation of the two contributions to $\Delta\sigma_{ab}(\epsilon,H)$, the in-plane paraconductivity and the fluctuation-induced in-plane magnetoconductivity, in a biperiodic layered superconductor. Then, we use these theoretical results to analyze the paraconductivity and the fluctuation-induced magnetoconductivity recently measured in the *a* direction (nonaffected by the presence of CuO_2 chains) in untwinned $YBa_2Cu_3O_{7-\delta}$ crystals.[10,24] This analysis is performed consistently with the previous analysis of the fluctuation-induced diamagnetism measured in the same crystals.[19,20] For that, in Sec. II we introduce some preliminaries concerning the theoretical model. Then, in Sec. III we calculate the AL and MT contributions to the zero-magnetic-field paraconductivity, $\Delta\sigma_{ab}(\epsilon)$. Here we also extend to multiperiodic layered superconductors the relationship between the direct contributions to $\Delta\sigma_{ab}(\epsilon)$ and $\Delta\chi_{ab}(\epsilon)$. In Sec. IV, the orbital Aslamazov-Larkin (ALO) and the orbital Maki-Thompson (MTO) contributions to $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$ in a biperiodic layered superconductor are obtained. To perform our calculations, first we find relationships (to our knowledge unnoticed up to now) among the AL contribution to the zero-magnetic-field paraconductivity, $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$, and the rest of the different contributions arising in $\Delta\sigma_{ab}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$. We apply then these results to obtain explicit expressions for $\Delta\sigma_{ab}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$ in the single-periodicity and biperiodic layered cases. Also, these relationships allow us to summarize the main effects introduced by the biperiodicity by using a single quantity, $N_e(\epsilon)$, the so-called effective number of independent fluctuating planes. A discussion of the physical meaning of $N_e(\epsilon)$, that give us also some clarifying insights on the physical differences between the order parameter fluctuations in single-periodicity and biperiodic layered superconductors, is presented in Sec. V. In Sec. VI, we briefly consider other additional contributions to $\Delta\sigma_{ab}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$ in biperiodic layered superconductors: The Zeeman terms and the contributions associated with the fluctuations of the normal quasiparticle density of states (DOS), these last contributions having been recently proposed by Dorin and co-workers[17] for layered superconductors with one single periodicity. Finally, in Sec. VII we compare these different theoretical results for biperiodic layered superconductors with available experimental data on $\Delta\sigma_a(\epsilon)$, $\Delta\tilde{\sigma}_a(\epsilon,H)$, and $\Delta\chi_{ab}(\epsilon)$ obtained in high quality Y-123 single crystals.[10,19,24] The implications of this comparison on $\Delta\sigma_a(\epsilon)$ and $\Delta\tilde{\sigma}_a(\epsilon,H)$ in the Y-123 superconductors as well as on various general aspects of the OPF effects above T_c in HTSC will be also briefly commented in this section and in the conclusions.

II. PRELIMINARY REMARKS: THE MODEL

To calculate $\Delta\sigma_{ab}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$ in a biperiodic layered (also called bilayered) superconductor, we will assume the usual Lawrence-Doniach (LD) mean-field free energy functional for layered superconductors, as generalized by Klemm to the case of several superconducting layers in a layer periodicity length:[21]

$$\Delta F[\Psi]=\sum_{n=-\infty}^{\infty}\sum_{j=1}^{N}\int d^2\mathbf{r}\left\{a_0\epsilon|\Psi_{jn}|^2+\sum_{\mu=x,y}\frac{\hbar^2}{2m_{ab}}\times\left|\left(\partial_\mu-\frac{2ie}{\hbar}A_\mu\right)\Psi_{jn}\right|^2+a_0\gamma_j|\Psi_{jn}-\Psi_{j+1,n}|^2\right\}. \tag{2.1}$$

As schematized in Fig. 1, in this equation N is the number of superconducting layers in the unit cell length $s=d_1+\cdots d_N$, and γ_j are the tunneling coupling constants between the (j,n) and $(j+1,n)$ layers, where the index (j,n) stands for the $j=1,\ldots,N$ superconducting layer of the nth cell of length s. We also use in Eq. (2.1) the values $(j,n)=(N+1,n)$ for the $(1,n+1)$ layer and we consider in each (j,n) plane a superconducting wave function $\Psi_{jn}(x,y)$. As we are interested only in the Gaussian fluctuation regime, in Eq. (2.1) we have neglected terms in powers higher than $|\Psi|^2$. Also, m_{ab} is the in-plane effective mass of the superconducting pairs (we neglect the possible in-plane anisotropy), $\hbar$ and e are, respectively, the reduced Plank constant and the electron charge, and a_0 is a normalization constant relating m_{ab} to the corresponding correlation length through $\xi_{ab}(\epsilon)=\xi_{ab}(0)\epsilon^{-1/2}=\hbar/(2m_{ab}a_0\epsilon)^{1/2}$. In Eq. (2.1), a weak magnetic field, H, applied perpendicular to the superconducting planes has been introduced by means of the usual gauge-invariant substitution $\partial_\mu\rightarrow\partial_\mu-2ie\hbar^{-1}A_\mu$ in the $H=0$ free energy expression. We used the gauge choice $\mathbf{A}=(-\mu_0Hy,0,0)$, where $\mathbf{A}$ is the electromagnetic vector potential and μ_0 is the vacuum permeability. This choice simplifies $\Delta F[\Psi]$ eliminating the magnetic-field dependence of the tunneling couplings.

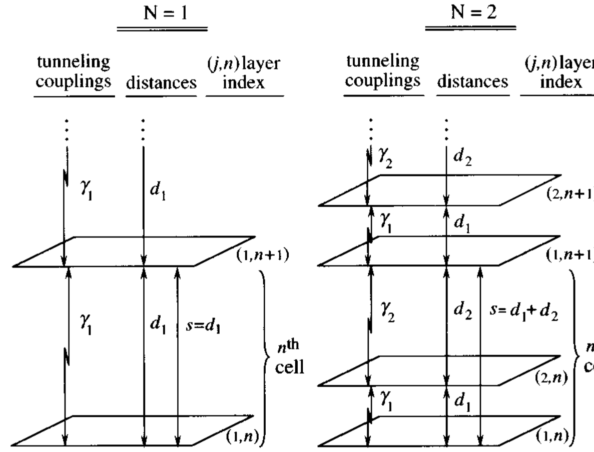

FIG. 1. Schematic view of the superconducting layers in layered superconductors with one $(N=1)$ or two $(N=2)$ layers per unit cell length (called in this work single and, respectively, biperiodic layered superconductors), with a summary of our notation. The index (j,n) denotes the $j=1,\ldots,N$ superconducting layer of the nth cell of length $s=d_1+\cdots d_N$. The tunneling coupling constant between the $(j+1,n)$ and the (j,n) layers is noted γ_j.

The z-direction spectrum of fluctuations resulting from the above $\Delta F[\Psi]$ functional is composed by N different branches, ω_{jk_z}, with $j=1,\ldots,N$ and $-\pi/s<k_z<\pi/s$.[21] For the cases $N=1$ and 2, these ω_{jk_z} are given by

$$\omega_{jk_z}=\begin{cases}2\gamma_1(1-\cos k_zs) & (\text{for } N=1),\\ \gamma_1+\gamma_2+(-1)^{j+1}\sqrt{\gamma_1^2+\gamma_2^2+2\gamma_1\gamma_2\cos k_zs} & (\text{for } N=2).\end{cases} \tag{2.2}$$

Such a spectrum allows one to obtain the c-direction Ginzburg-Landau (GL) correlation length via its usual relationship with the effective mass of the superconducting pairs, $m_c=a_0^{-1}\hbar^2(\partial^2\omega/\partial k_z^2)^{-1}_{\omega_{\min}}$. This corresponds to $\xi_c(\epsilon)=\xi_c(0)\epsilon^{-1/2}$, where the amplitude $\xi_c(0)$ may be given in terms of the model parameters as

$$\xi_c(0)=\begin{cases}s\left(\dfrac{1}{\gamma_1}\right)^{-1/2} & (\text{for } N=1),\\ \dfrac{s}{\sqrt{2}}\left(\dfrac{1}{\gamma_1}+\dfrac{1}{\gamma_2}\right)^{-1/2} & (\text{for } N=2).\end{cases} \tag{2.3}$$

Other aspects of this multilayered model may be seen in the original paper of Klemm[21] and also in Ref. 20.

III. PARACONDUCTIVITY

Two different kinds of contributions to the in-plane paraconductivity may be considered.[2,3,25–27] First, there is the *direct* or Aslamazov-Larkin (AL) contribution, just reflecting the conductance of the short-lived thermally activated Cooper pairs. In addition, the appearance above T_{c0} of such Coopper pairs may also change the responses to external fields of the normal carriers of the material. The latter produce the so-called *indirect* contributions of the thermal fluctuations. In this section, we consider the direct AL contribution, noted $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$, and the indirect anomalous Maki-Thompson contribution, noted $\Delta\sigma_{ab\mathrm{MT}}$. The total in-plane paraconductivity will be then given by

$$\Delta\sigma_{ab}(\epsilon)=\Delta\sigma_{ab\mathrm{AL}}(\epsilon)+\Delta\sigma_{ab\mathrm{MT}}(\epsilon). \tag{3.1}$$

We note that also other indirect contributions to $\Delta\sigma_{ab}(\epsilon)$, as the ones produced by the fluctuations of the normal carrier density of states (DOS), have been recently proposed by Dorin and co-workers for single layered superconductors.[17] A discussion of such possible additional indirect contributions in the case of biperiodic layered superconductors is deserved to Sec. VI.

A. The Aslamazov-Larkin contribution

To obtain $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$, we apply to $\Delta F[\Psi]$ given by Eq. (2.1) the procedure proposed by Abrikosov in Ref. 28 (based on the standard GL-like Schmid's formalism[29]). We obtain then $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$ in terms of ω_{jk_z} as

$$\Delta\sigma_{ab\mathrm{AL}}(\epsilon)=\frac{e^2}{32\pi\hbar}\sum_{j=1}^{N}\int_{-\pi/s}^{\pi/s}\frac{dk_z}{\epsilon+\omega_{jk_z}}. \tag{3.2}$$

The importance of this expression is twofold. First, it allow the direct calculation of explicit expressions for $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$, as we discuss below. But, in addition, we will see that this expression allows us to demonstrate various useful relationships between $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$ and other fluctuation-induced contributions to $\Delta\sigma_{ab}(\epsilon)$, $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$, and $\Delta\chi_{ab}(\epsilon)$, this last being the fluctuation-induced diagmagnetism for a weak magnetic field applied perpendicularly to the *ab* planes. In fact, in the case of $\Delta\chi_{ab}(\epsilon)$ such a relationship may be directly obtained by just noting that Eq. (3.2) involves the same summation-integration as the Eqs. (7) and (10) of our previous calculation of $\Delta\chi_{ab}(\epsilon)$ in Ref. 20. So, we get (in mksa units)

$$\frac{\Delta\chi_{ab}(\epsilon)/T}{\Delta\sigma_{ab\mathrm{AL}}(\epsilon)}=\frac{16\mu_0 k_B\xi_{ab}^2(0)}{3\pi\hbar}=2.79\times10^5\xi_{ab}^2(0). \tag{3.3}$$

Note that this relationship (already proposed in Ref. 23 for the $N=1$ case) will apply to the measured in-plane paraconductivity only if the indirect contributions to $\Delta\sigma_{ab}(\epsilon)$ are negligible (see later).

From Eq. (3.2) $\Delta\sigma_{ab}(\epsilon)$ may now be obtained: for one single periodicity (i.e., $N=1$), we get the well-known LD result[11]

$$\Delta\sigma_{ab\mathrm{AL}}^{N=1}(\epsilon)=\frac{A_{\mathrm{AL}}}{\epsilon}\left(1+\frac{B_{\mathrm{LD}}}{\epsilon}\right)^{-1/2}. \tag{3.4}$$

Here A_{AL} and B_{LD} are the AL paraconductivity amplitude[25] and, respectively, the Lawrence-Doniach parameter,[11] given by

$$A_{\mathrm{AL}}\equiv\frac{e^2}{16\hbar s},\quad B_{\mathrm{LD}}\equiv\left(\frac{2\xi_c(0)}{s}\right)^2. \tag{3.5}$$

For a biperiodic layered superconductor (i.e., for $N=2$), Eq. (3.2) lead to

$$\Delta\sigma_{ab\mathrm{AL}}^{N=2}(\epsilon)=N_e(\epsilon)\Delta\sigma_{ab\mathrm{AL}}^{N=1}(\epsilon), \tag{3.6}$$

where $N_e(\epsilon)$, henceforth called the effective number of independent fluctuating superconducting layers, is given by

$$N_e(\epsilon)=\left(\frac{\frac{1}{4}+c_1\beta+c_2\beta^2+c_1^2\beta^3}{1+\tilde{c}_1\beta+\tilde{c}_2\beta^2+c_1^2\beta^3}\right)^{-1/2}. \tag{3.7}$$

Here β is a shorthand for $\beta\equiv B_{\mathrm{LD}}/\epsilon$ and $c_1,c_2,\tilde{c}_1,\tilde{c}_2$ are coefficients with dependence only on the tunneling-coupling ratio, γ_1/γ_2, between adjacent layers, as

$$c_1=\frac{1}{2}\frac{(\gamma_1/\gamma_2+1)^2}{\gamma_1/\gamma_2},\quad \tilde{c}_1=2c_1+1,$$
$$c_2=c_1^2+\frac{1}{2}c_1,\quad \tilde{c}_2=c_1^2+2c_1. \tag{3.8}$$

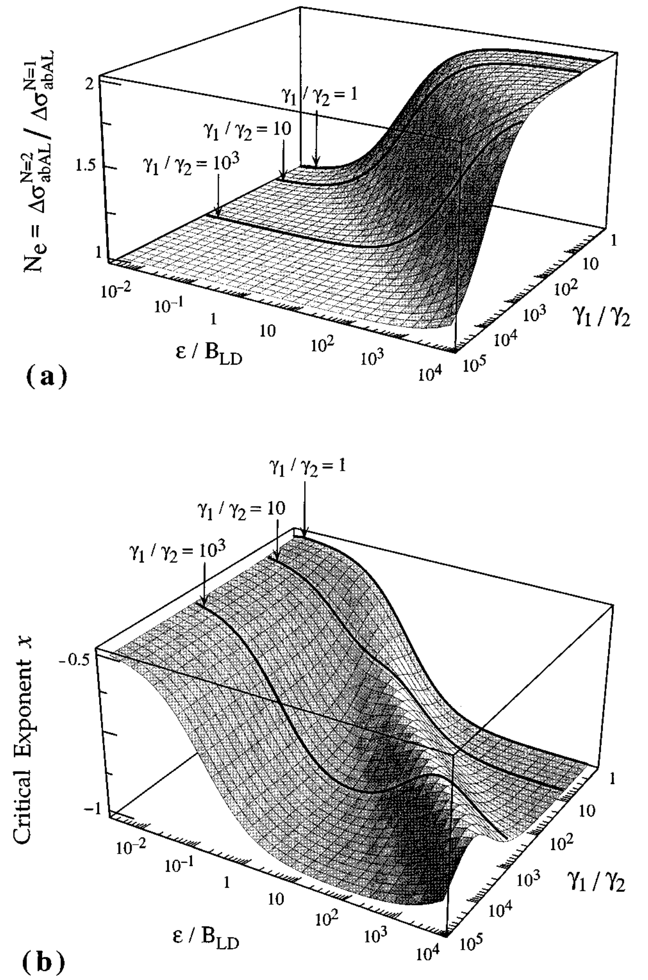

FIG. 2. (a) Effective number, N_e, of independent fluctuating superconducting planes per unit cell length, s, and (b) mean-field critical exponent, x [as defined in Eq. (5.2)], in a biperiodic layered superconductor as a function of the reduced temperature in units of the Lawrence-Doniach crossover parameter, B_{LD}, and for different values of the relative tunneling coupling strength, γ_1/γ_2, between adjacent superconducting layers. As discussed in Sec. V, the plot reveals two combined crossovers: the $N_e=1$ to 2 crossover, and the dimensional 3D–2D crossover (i.e., $x=-1/2$ to -1). The limit for one single periodicity layered superconductors ($N=1$) corresponds to $\gamma_1/\gamma_2\to\infty$ with $B_{\mathrm{LD}}\neq0$. See main text for details.

A plot of N_e is shown in Fig. 2(a), and a discussion of its physical meaning will be presented in Sec. V. Note that Eqs. (3.5)–(3.8) include the results proposed by Maki and Thompson in Ref. 13 for biperiodic layered superconductors. This may be seen by using Eq. (2.3) and identifying the microscopic parameters K and G used by Maki and Thompson as $K=\gamma_1$ and $G=\gamma_2$. The expression of Baraduc and Buzdin[22] for $\Delta\sigma_{ab\mathrm{AL}}^{N=2}(\epsilon)$, valid only for $\gamma_1\gg\gamma_2$, may also be easily obtained as a particular case of our results.

B. The Maki-Thompson contribution

To calculate $\Delta\sigma_{ab\mathrm{MT}}(\epsilon)$ in the case of a multiperiodic layered superconductor, we apply the same procedure already used for layered superconductors with a single periodicity by Hikami and Larkin.[12] We get

$$\Delta\sigma_{ab\mathrm{MT}}(\epsilon)=\frac{2}{\epsilon-\epsilon_\phi}\int_{\epsilon_\phi}^{\epsilon}d\epsilon'\,\Delta\sigma_{ab\mathrm{AL}}(\epsilon'),\qquad(3.9)$$

where ϵ_ϕ is the pair-breaking parameter (see below) and $\Delta\sigma_{ab\mathrm{AL}}(\epsilon')$ is given again by the summation-integration appearing in Eq. (3.2). Therefore, we see that $\Delta\sigma_{ab\mathrm{MT}}(\epsilon)$ may be written in terms of the AL contribution, in the form of a mean value of $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$ over the reduced temperatures ϵ_ϕ and ϵ. This relationship (to our knowledge unnoticed up to now) is valid for all the multiperiodicities of the layering, as well as for the three-dimensional (3D) and 2D limits. The pair-breaking parameter ϵ_ϕ appearing in Eq. (3.9) is given by[12,17]

$$\epsilon_\phi\equiv\frac{\tau}{\tau_\phi}\left[\Psi\left(\frac{1}{2}\right)+\frac{\hbar}{4\pi k_B\tau T}\,\Psi'\left(\frac{1}{2}\right)-\Psi\left(\frac{1}{2}+\frac{\hbar}{4\pi k_B\tau T}\right)\right],\qquad(3.10)$$

where τ is the scattering time, τ_ϕ is the phase pair-breaking time (with $\tau_\phi\geqslant\tau$, according to the Matthiessen rule[30]), Ψ is the digamma function, and Ψ' its derivative. In the dirty and clean limits (i.e., $\tau T\ll1$ and, respectively, $\tau T\gg1$, in $k_B/\hbar$ units), Eq. (3.10) may be reduced to[12,16,17]

$$\epsilon_\phi=\frac{\pi\hbar}{8k_BT\tau_\phi}\quad\text{(dirty limit)},\qquad(3.11a)$$

$$\epsilon_\phi=\frac{\pi\hbar}{8k_BT\tau_\phi}\,\frac{7\zeta(3)\hbar}{2\pi^3k_BT\tau}\quad\text{(clean limit)},\qquad(3.11b)$$

where $\zeta(x)$ is the Riemann function and $\zeta(3)=1.202$. In view of Eq. (3.9), $\Delta\sigma_{ab\mathrm{MT}}(\epsilon)$ for $N=1$ and 2 may be obtained from the corresponding expressions for $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$. For $N=1$, we get

$$\Delta\sigma_{ab\mathrm{MT}}^{N=1}(\epsilon)=\frac{2A_{\mathrm{AL}}}{\epsilon-\epsilon_\phi}\ln\left(\frac{\epsilon}{\epsilon_\phi}\left(\frac{1+\sqrt{1+B_{\mathrm{LD}}/\epsilon}}{1+\sqrt{1+B_{\mathrm{LD}}/\epsilon_\phi}}\right)^2\right),\qquad(3.12)$$

and for a biperiodic layering

$$\Delta\sigma_{ab\mathrm{MT}}^{N=2}(\epsilon)=\frac{4A_{\mathrm{AL}}}{\epsilon-\epsilon_\phi}\ln\left(\frac{\epsilon}{\epsilon_\phi}\,\frac{\sqrt{1+2c_1B_{\mathrm{LD}}/\epsilon}+\sqrt{1+2c_1B_{\mathrm{LD}}/\epsilon+2c_1(B_{\mathrm{LD}}/\epsilon)^2}}{\sqrt{1+2c_1B_{\mathrm{LD}}/\epsilon_\phi}+\sqrt{1+2c_1B_{\mathrm{LD}}/\epsilon_\phi+2c_1(B_{\mathrm{LD}}/\epsilon_\phi)^2}}\right).\qquad(3.13)$$

Equations (3.12) and (3.13) include the results proposed for $N=1$ by Hikami and Larkin in Ref. 12 and for $N=2$ by Maki and Thompson in Ref. 13 (in the last case, by doing the same identifications $K=\gamma_1$ and $G=\gamma_2$ mentioned in the previous subsection). Moreover, note as well that Eq. (3.9) shows that the biperiodicity effects on $\Delta\sigma_{ab\mathrm{MT}}(\epsilon)$ are also accounted by N_e.

IV. FLUCTUATION-INDUCED MAGNETOCONDUCTIVITY

It is now well established in single-periodic layered superconductors that, for a weak magnetic field applied perpendicular to the layers, the fluctuation-induced in-plane magnetoconductivity, $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$, may be approximated as[12–15]

$$\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)=\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{ALO}}(\epsilon,H)+\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{MTO}}(\epsilon,H),\qquad(4.1)$$

where $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{ALO}}(\epsilon,H)$ and $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{MTO}}(\epsilon,H)$ are the so-called orbital-Aslamazov-Larkin (ALO) and, respectively, orbital-Maki-Thompson (MTO) contributions to the magnetoconductivity, produced by the coupling of the magnetic field with the orbital motion of the electrical carriers of the material. In Eq. (4.1), we have not considered the so-called Zeeman fluctuation-induced terms, which reflect the Zeeman splitting effects, due to the coupling of the magnetic field with the carriers' spin, on the AL and MT contributions to the paraconductivity. In the HTSC, these last effects are negligible for weak magnetic fields applied perpendicular to the superconducting layers.[5–10] Also, as before for the $H=0$ case, we are not going to take here into account the possible DOS contributions. We will include, however, an appropriate discussion of these different contributions in Secs. VI and VII. So, the central aim of this section is to present calculations of $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{ALO}}(\epsilon,H)$ and of $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{MTO}}(\epsilon,H)$ in a biperiodic layered superconductor with a magnetic field applied in the c direction. The magnetic field is assumed in the so-called weak limit, given by[31,32]

$$h\equiv\frac{2e\mu_0\xi_{ab}^2(0)H}{\hbar}=\frac{H}{H_{c2}^{\parallel c}(0)}\ll\epsilon,\qquad(4.2)$$

where $H_{c2}^{\parallel c}(0)$ is the amplitude (for $T=0$ K) of the upper critical magnetic field parallel to the c direction and h is the so-called reduced magnetic field applied in the c direction. The possible high magnetic-field effects[31,32] which may appear (already in the modelization presented in Sec. II) if the condition $h\ll\epsilon$ is not verified will be not discussed here.

A. The orbital Aslamazov-Larkin contribution

To calculate the orbital Aslamazov-Larkin (ALO) magnetoconductivity in a layered superconductor with several periodicities, we will use again the procedure proposed by Hikami and Larkin to calculate $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$ in a single-periodicity layered superconductor.[12] So, we consider first $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{ALO}}^{(q_z)}(\epsilon,H)$, the part of the ALO magnetoconductivity arising from the in-plane spectrum of fluctuations for a given z-direction fluctuational state q_z. This part simply coincides with the ALO contribution for a 2D film of thickness unity and at a reduced temperature $\epsilon+\omega_{q_z}$, where ω_{q_z} is the fluctuational spectrum of the z-direction state q_z.[33,34] Then, we sum over all the q_z. In the multiperiodic layered case, as shown in Sec. II, such a fluctuational spectrum corresponds to the ω_{jk_z} given by Eq. (2.2), and the q_z sum corresponds to the summation over $j=1,\ldots,N$ and integration of $k_z/2\pi$ over $-\pi/s\leqslant k_z\leqslant\pi/s$. We obtain then

$$\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{ALO}}(\epsilon,H)=\frac{-h^2e^2}{64\pi\hbar}\sum_{j=1}^{N}\int_{-\pi/s}^{\pi/s}\frac{dk_z}{(\epsilon+\omega_{jk_z})^3}. \quad (4.3)$$

In this expression, we have retained only terms at the second order in the reduced magnetic field h, in accordance with our previous assumption on its smallness [see Eq. (4.2)]. By using in Eq. (4.3) derivation with respect to ϵ, we obtain a relationship between the AL in-plane paraconductivity and the ALO in-plane magnetoconductivity with a weak perpendicular magnetic field,

$$\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{ALO}}(\epsilon,H)=\frac{-h^2}{4}\frac{\partial^2\Delta\sigma_{ab\mathrm{AL}}(\epsilon)}{\partial\epsilon^2}. \quad (4.4)$$

Such a relationship, expressing $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{ALO}}(\epsilon,H)$ in terms of $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$, is the equivalent of Eq. (3.9) relating $\Delta\sigma_{ab\mathrm{MT}}(\epsilon)$ and $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$. It allows us to obtain the explicit expressions for $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{ALO}}(\epsilon,H)$ by using the ones obtained above for $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$. Also, as it was the case of the Eq. (3.9) for $\Delta\sigma_{ab\mathrm{MT}}(\epsilon)$, Eq. (4.4) clearly indicates that the effects of a biperiodic layering, that appear on $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$ through N_e, are transmitted to $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{ALO}}(\epsilon,H)$ via that relationship. For $N=1$, Eqs. (3.4) and (4.4) lead to

$$\Delta\tilde{\tilde{\sigma}}^{N=1}_{ab\mathrm{ALO}}(\epsilon,H)=\frac{-h^2}{\epsilon^2}\left[\frac{1+B_{\mathrm{LD}}/\epsilon+3/8(B_{\mathrm{LD}}/\epsilon)^2}{2+4B_{\mathrm{LD}}/\epsilon+2(B_{\mathrm{LD}}/\epsilon)^2}\right]\times\Delta\sigma^{N=1}_{ab\mathrm{AL}}(\epsilon), \quad (4.5)$$

which is the result first proposed for $N=1$ by Hikami and Larkin.[12] By using Eqs. (3.6) and (4.4), we obtain for the $N=2$ case,

$$\Delta\tilde{\tilde{\sigma}}^{N=2}_{ab\mathrm{ALO}}(\epsilon,H)=\frac{-h^2}{\epsilon^2}\left[4\left(\frac{P_1}{P_0}\right)^2-\left(\frac{P_1}{P_0}+\frac{Q_1}{Q_0}\right)^2\times-\frac{P_2}{P_0}+\frac{Q_2}{Q_0}\right]\Delta\sigma^{N=2}_{ab\mathrm{AL}}(\epsilon), \quad (4.6)$$

where P_0, P_1, P_2 and Q_0, Q_1, Q_2 are polynomials on $\beta\equiv B_{\mathrm{LD}}/\epsilon=(2\xi_c(\epsilon)/s)^2$, with coefficients depending only on γ_1/γ_2 through the quantities c_1, c_2, $\tilde{c}_1$, and $\tilde{c}_2$:

$$P_0=1+(1+4c_1)\beta+4(c_1+c_2)\beta^2+4(c_1^2+c_2)\beta^3+4c_1^2\beta^4,$$

$$P_1=(1/4+c_1)\beta+2(c_1+c_2)\beta^2+3(c_1^2+c_2)\beta^3+4c_1^2\beta^4,$$

$$P_2=(1/4+c_1)\beta+3(c_1+c_2)\beta^2+6(c_1^2+c_2)\beta^3+10c_1^2\beta^4,$$

$$Q_0=4+4\tilde{c}_1\beta+4\tilde{c}_2\beta^2+4c_1^2\beta^3,$$

$$Q_1=2+3\tilde{c}_1\beta+4\tilde{c}_2\beta^2+5c_1^2\beta^3,$$

$$Q_2=3+6\tilde{c}_1\beta+10\tilde{c}_2\beta^2+15c_1^2\beta^3. \quad (4.7)$$

B. The orbital Maki-Thompson contribution

To calculate orbital-Maki-Thompson magnetoconductivity, $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{MTO}}(\epsilon,H)$ in a layered superconductor with several periodicities, we will proceed in a quite similar way as we have done above for $\Delta\sigma_{ab\mathrm{MT}}(\epsilon)$ and for $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{ALO}}(\epsilon,H)$. We first consider $\Delta\tilde{\tilde{\sigma}}^{(q_z)}_{ab\mathrm{MTO}}(\epsilon,H)$, the MTO contribution due only to the in-plane fluctuational energy, which is given by the result valid for 2D films of thickness unity at reduced temperatures $\epsilon+\omega_{q_z}$ and with pair breaking parameter $\epsilon_\phi+\omega_{q_z}$.[12,34,35] Then, we sum over all the z-direction fluctuational states q_z, given for the multiperiodic layered superconductors by Eq. (2.2). At lowest h^2 order, we obtain then for $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{MTO}}(\epsilon,H)$,

$$\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{MTO}}(\epsilon,H)=\frac{e^2}{96\pi\hbar}\frac{h^2}{\epsilon-\epsilon_\phi}\sum_{j=1}^{N}\left(\int_{-\pi/s}^{\pi/s}\frac{dk_z}{(\epsilon+\omega_{jk_z})^2}-\int_{-\pi/s}^{\pi/s}\frac{dk_z}{(\epsilon_\phi+\omega_{jk_z})^2}\right). \quad (4.8)$$

As before for $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{ALO}}(\epsilon,H)$, differentiation with respect to ϵ and ϵ_ϕ leads us to a direct relationship between the AL zero-field paraconductivity and the orbital Maki-Thompson magnetoconductivity in the weak magnetic-field limit,

$$\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{MTO}}(\epsilon,H)=\frac{-1}{3}\frac{h^2}{\epsilon-\epsilon_\phi}\left(\frac{\partial\Delta\sigma_{ab\mathrm{AL}}(\epsilon)}{\partial\epsilon}-\frac{\partial\Delta\sigma_{ab\mathrm{AL}}(\epsilon_\phi)}{\partial\epsilon_\phi}\right). \quad (4.9)$$

Through these dependences, we may get now explicit expressions for $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{MTO}}(\epsilon,H)$ by simply using the $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$ results. For $N=1$, we obtain

$$\Delta\tilde{\tilde{\sigma}}^{N=1}_{ab\mathrm{MTO}}(\epsilon,H)=\frac{h^2}{\epsilon-\epsilon_\phi}\left[\left(\frac{2+B_{\mathrm{LD}}/\epsilon}{1+B_{\mathrm{LD}}/\epsilon}\right)\frac{\Delta\sigma^{N=1}_{ab\mathrm{AL}}(\epsilon)}{6\epsilon}-\left(\frac{2+B_{\mathrm{LD}}/\epsilon_\phi}{1+B_{\mathrm{LD}}/\epsilon_\phi}\right)\frac{\Delta\sigma^{N=1}_{ab\mathrm{AL}}(\epsilon_\phi)}{6\epsilon_\phi}\right], \quad (4.10)$$

a result first proposed by Hikami and Larkin.[12] For $N=2$, we obtain

$$\Delta\tilde{\tilde{\sigma}}^{N=2}_{ab\mathrm{MTO}}(\epsilon,H)=\frac{h^2}{\epsilon-\epsilon_\phi}\left[\left(\frac{R_1(\epsilon)}{R_0(\epsilon)}-\frac{S_1(\epsilon)}{S_0(\epsilon)}\right)\frac{\Delta\sigma^{N=2}_{ab\mathrm{AL}}(\epsilon)}{6\epsilon}-\left(\frac{R_1(\epsilon_\phi)}{R_0(\epsilon_\phi)}-\frac{S_1(\epsilon_\phi)}{S_0(\epsilon_\phi)}\right)\frac{\Delta\sigma^{N=2}_{ab\mathrm{AL}}(\epsilon_\phi)}{6\epsilon_\phi}\right], \quad (4.11)$$

where R_0, R_1, S_0, and S_1 are given by

$$R_0(\epsilon)=1+\tilde{c}_1\beta+\tilde{c}_2\beta^2+c_1^2\beta^3,$$

$$R_1(\epsilon)=2+3\tilde{c}_1\beta+4\tilde{c}_2\beta^2+5c_1^2\beta^3,$$

$$S_0(\epsilon)=1/4+(1/4+c_1)\beta+(c_1+c_2)\beta^2+(c_1^2+c_2)\beta^3+c_1^2\beta^4,$$

$$S_1(\epsilon)=(1/4+c_1)\beta+2(c_1+c_2)\beta^2+3(c_1^2+c_2)\beta^3+4c_1^2\beta^4. \quad (4.12)$$

Here we have used again the notation $\beta\equiv B_{\mathrm{LD}}/\epsilon$.

V. EFFECTIVE NUMBER OF INDEPENDENT FLUCTUATING PLANES IN A BIPERIODIC LAYERED SUPERCONDUCTOR

In view of the results of the preceding sections, the main relevant quantity determining the effects of the layer biperiodicity on the in-plane paraconductivity and fluctuation-induced magnetoconductivity (for weak magnetic fields) is the coefficient $N_e(\epsilon/B_{LD},\gamma_1/\gamma_2)$, given by the Eq. (3.7). First of all, such N_e directly summarizes [see Eq. (3.6)] the $N=2$ effects on the direct AL contribution to the paraconductivity, $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$. Added to that, we have shown that the rest of the until here considered contributions to the paraconductivity and fluctuation-induced magnetoconductivity [i.e., $\Delta\sigma_{ab\mathrm{MT}}(\epsilon)$, $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{ALO}}(\epsilon,H)$, and $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{MTO}}(\epsilon,H)$] may be related to such AL direct contribution, and so N_e determines the biperiodicity effects also in these contributions. In fact, the same conclusion will be valid as well for the Zeeman contributions to $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$, discussed in Sec. VI below. Moreover, our previously published calculations[20] of the fluctuation-induced diamagnetism for weak magnetic fields applied perpendicular to the ab planes, $\Delta\chi_{ab}(\epsilon)$, shows that the same $N_e(\epsilon/B_{LD},\gamma_1/\gamma_2)$ relates as well $\Delta\chi_{ab}^{N=2}(\epsilon)$ to $\Delta\chi_{ab}^{N=1}(\epsilon)$ [note here Eq. (3.3)]. Finally, in Ref. 20 we have shown that N_e indeed summarizes the biperiodicity effects already in the effective, or thermodynamic, GL free energy, $\langle\Delta F\rangle$, that gives rise to such a fluctuation-induced diamagnetism, so that

$$\langle\Delta F\rangle^{N=2}=N_e(\epsilon)\langle\Delta F\rangle^{N=1}. \tag{5.1}$$

All these results strongly suggest that, in analogy with the equipartition of the free energy by uncoupled degrees of freedom, N_e may be seen as an effective number of independent fluctuating planes in a unit cell. This physical meaning of N_e is confirmed by its limiting values ($1\leqslant N_e\leqslant 2$) and by its behavior with respect to the parameters of the system. Such a behavior has been summarized in Fig. 2(a), where N_e has been shown as a function of ϵ/B_{LD} and of γ_1/γ_2. In this figure, and in what follows, we consider only values of γ_1/γ_2 higher or equal to one, $\gamma_1/\gamma_2\geqslant 1$, because our final expressions remain invariant if we interchange the γ_1 and γ_2 values.

To characterize the differences between the order parameter fluctuations (OPF) configurations of the $N=2$ and 1 layered superconductors, it will be also useful to introduce, in addition to N_e, the mean-field critical exponent, x, of the direct in-plane paraconductivity, defined as the slope of $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$ in a log-log representation, i.e.,

$$x(\epsilon)\equiv\left(\frac{\partial\ln\Delta\sigma_{ab\mathrm{AL}}(\epsilon)}{\partial\ln\epsilon}\right)_{\gamma_1/\gamma_2}. \tag{5.2}$$

The exponent x takes values bounded by the pure 3D and 2D results ($x=-1/2$ and, respectively, $x=-1$), so informing about the dimensional behavior of the OPF. In Fig. 2(b) we show a plot of x as a function of ϵ/B_{LD} and of γ_1/γ_2. The results presented in Figs. 2(a) and 2(b) clearly illustrate one of the main differences between the $N=2$ and the single-periodicity (or $N=1$) layered superconductors: Whereas the exponent crossover between the 3D limit ($x=-1/2$) and the 2D limit ($x=-1$) appears in both cases, the N_e crossover between $N_e=1$ and 2 is present for $N=2$, but it is absent in the single periodicity layered superconductors, with $N=1$. This last difference affects directly the amplitude of $\Delta\sigma_{ab}(\epsilon)$ and of $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$ and it is, therefore, crucial in comparing with the experimental data (see Sec. VII).

The differences between the $N=1$ and 2 cases concern also the possible OPF limiting behaviors. In the conventional $N=1$ case, these limiting behaviors are associated with the relationship between $\xi_c(\epsilon)$ and s and, therefore, as it is well known, there are three possibilities (each of them with a different dimensional exponent, x, but which indeed lead always to $N_e=1$).

(i) $\xi_c(\epsilon)\gg s$, where $x=-1/2$, and where the OPF are 3D.
(ii) $\xi_c(\epsilon)\approx s$, that corresponds to the 2D–3D crossover.
(iii) $\xi_c(\epsilon)\ll s$, where $x=-1$ and the OPF are 2D.

However, in the $N=2$ case, the situation is slightly more complex, and the different possible behaviors will be given by the different combinations of the values of $\xi_c(\epsilon)/s$ and γ_1/γ_2 (or, equivalently, by the different possible combinations of x and N_e). For instance, in contrast to the $N=1$ case, for $N=2$ it is possible to have three different 2D behaviors.

(i) The case characterized by $x=-1$ and $N_e=2$, in which each superconducting layer undergoes fluctuations completely uncorrelated to the other layers.

(ii) The case characterized by $x=-1$ and $N_e=1$, in which the system is composed by superconducting sheets, each one fully uncorrelated to the others, but each sheet being in turn composed by two layers so strongly correlated between them that each bilayer may be considered as a single 2D superconducting plane without internal structure.

(iii) The case characterized by $x\approx-1$ and N_e crossing over the values 1 and 2. Here the system presents again uncorrelated superconducting bilayers, but now composed by two superconducting layers with some correlation between them (intermediate between the former cases). In this last case, each bilayer presents a frustrated internal 3D structure, and in consequence the value of the exponent differs something from the 2D value ($x=-1$), as it can be seen in the Fig. 2. To our knowledge, this mesoscopic effect in biperiodic layered superconductors has not been tested experimentally up to now.

Finally, let us inspect here the influence of γ_1/γ_2 on $\Delta\sigma_{ab}^{N=2}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}^{N=2}(\epsilon,H)$ [and on $\Delta\chi_{ab}^{N=2}(\epsilon)$]. For that, we first note that the results of Secs. III and IV [and of Ref. 20 for $\Delta\chi_{ab}(\epsilon)$] have shown that all the dependences of these physical quantities on γ_1/γ_2 arise through N_e, which may take the two limiting values [if $\xi_c(0)\neq 0$]

$$N_e=1,\quad \text{for}\ \ \gamma_1/\gamma_2\to\infty, \tag{5.3}$$

and [by making $\gamma_1/\gamma_2=1$ in Eq. (3.7) and factorizing the resulting polynomials],

$$N_e=2\left(\frac{1+\dfrac{B_{LD}}{\epsilon}}{1+4\dfrac{B_{LD}}{\epsilon}}\right)^{-1/2},\quad \text{for}\ \ \gamma_1/\gamma_2=1. \tag{5.4}$$

In the first limit ($\gamma_1\gg\gamma_2$), the tunneling interaction between the more coupled superconducting planes is so strong that these planes are, in all the mean-field-like region, equivalent to a single layer without internal structure. It is then easy to check that in this case the expressions for $\Delta\sigma_{ab}^{N=2}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}^{N=2}(\epsilon,H)$ [and also for $\Delta\chi_{ab}^{N=2}(\epsilon)$], given by Eqs. (3.6), (3.13), (4.6), (4.11), and (3.3), coincide with the $N=1$ expressions with the same unit cell length $s=d_1+d_2$ [given by Eqs. (3.4), (3.12), (4.5), (4.10), and (3.3)]. Note that, in fact, this is the limit used until now, without other justification than its simplicity, by most of the authors in analyzing the OPF effects on different observables in the multiperiodic layered high-T_c cuprates.[3–9,36,37]

In the opposite limit (i.e., for $\gamma_1=\gamma_2$), the biperiodic system will become equivalent to a layered superconductor with a single periodicity, but now with $s/2$ as characteristic length, instead of the unit cell length s. It is easy to see, by using Eq. (5.4), that in this last case $\Delta\sigma_{ab}^{N=2}(\epsilon,H)$ is again given by the $N=1$ expressions, but with $s/2$ instead of s in the expressions of $A_{\rm AL}$ and $B_{\rm LD}$ [i.e., in Eq. (3.5)]. In other words, in these γ_1/γ_2 limits, the $N=2$ expressions for $\Delta\sigma_{ab}(\epsilon,H)$ [and $\Delta\chi_{ab}(\epsilon)$] may be written as for $N=1$, but substituting the unit cell length, s, by an effective interlayer distance, d_e, given by

$$d_e=\begin{cases} s, & \text{for } \gamma_1/\gamma_2\to\infty, \\ s/2, & \text{for } \gamma_1=\gamma_2. \end{cases} \tag{5.5}$$

VI. OTHER CONTRIBUTIONS TO THE PARACONDUCTIVITY AND TO THE FLUCTUATION-INDUCED MAGNETOCONDUCTIVITY

A. Zeeman contributions

We have neglected so far the effects of the Zeeman splitting on the fluctuation-induced magnetoconductivity, first introduced by Aronov, Hikami, and Larkin[14] and Thompson.[15] As it is now well established,[5–10] these contributions are negligible in the HTSC for weak magnetic fields applied perpendicular to the layers. However, these contributions are important for H applied parallel to the layers. Therefore, for completeness, we are going to calculate here, for a biperiodic layered superconductor, these terms. This may be easily done by taking into account that, as first showed in Refs. 14, 15, and 38, the Zeeman effect produced on $\Delta\tilde{\tilde{\sigma}}(\epsilon,H)$ by an external field may be summarized by means of a shift of the critical temperature given by

$$\epsilon_Z(H)\equiv\epsilon+7\zeta(3)\left(\frac{\mu_B H}{2\pi k_B T_c}\right)^2, \tag{6.1}$$

where $\zeta(x)$ is the Riemann function [with $\zeta(3)=1.202$] and $\mu_B=e\hbar/2m_{e^-}$ is the Bohr magneton. The Zeeman contributions are then given by

$$\Delta\tilde{\tilde{\sigma}}_{ab\rm ALZ}(\epsilon,H)=\Delta\sigma_{ab\rm AL}(\epsilon_Z(H))-\Delta\sigma_{ab\rm AL}(\epsilon), \tag{6.2a}$$

$$\Delta\tilde{\tilde{\sigma}}_{ab\rm MTZ}(\epsilon,H)=\Delta\sigma_{ab\rm MT}(\epsilon_Z(H))-\Delta\sigma_{ab\rm MT}(\epsilon), \tag{6.2b}$$

where $\Delta\sigma_{ab\rm AL}(\epsilon)$ and $\Delta\sigma_{ab\rm MT}(\epsilon)$ are the $H=0$ expressions obtained in Sec. III. One may expand the above expressions in powers of the weak magnetic field up to order h^2, in accordance with the $h\ll\epsilon$ condition stated in Eq. (4.2). We obtain then for the ALZ contribution in the single periodicity ($N=1$) superconductors,

$$\Delta\tilde{\tilde{\sigma}}_{ab\rm ALZ}^{N=1}(\epsilon,H)=\frac{-7\zeta(3)}{2\epsilon}\left(\frac{\mu_B H}{2\pi k_B T_c}\right)^2\frac{2+B_{\rm LD}/\epsilon}{1+B_{\rm LD}/\epsilon}\times\Delta\sigma_{ab\rm AL}^{N=1}(\epsilon), \tag{6.3}$$

a result already proposed by Aronov, Hikami, and Larkin in Ref. 14. For biperiodic layered superconductors ($N=2$), we find

$$\Delta\tilde{\tilde{\sigma}}_{ab\rm ALZ}^{N=2}(\epsilon,H)=\frac{-7\zeta(3)}{2\epsilon}\left(\frac{\mu_B H}{2\pi k_B T_c}\right)^2\left(\frac{R_1(\epsilon)}{R_0(\epsilon)}-\frac{S_1(\epsilon)}{S_0(\epsilon)}\right)\times\Delta\sigma_{ab\rm AL}^{N=2}(\epsilon). \tag{6.4}$$

In the case of the MTZ contribution, we obtain, for all the values of N,

$$\Delta\tilde{\tilde{\sigma}}_{ab\rm MTZ}(\epsilon,H)=\frac{-7\zeta(3)}{\epsilon-\epsilon_\phi}\left(\frac{\mu_B H}{2\pi k_B T_c}\right)^2\times(\Delta\sigma_{ab\rm MT}(\epsilon)-2\Delta\sigma_{ab\rm AL}(\epsilon)), \tag{6.5}$$

a result which coincides when $N=1$ with the expression already proposed by Thompson[15] for single periodicity layered superconductors. Finally, let us stress here that these expressions hold for H perpendicular or parallel to the ab superconducting planes.

B. Density of states contributions

Our above calculations of the MT contributions to $\Delta\sigma_{ab}(\epsilon,H)$ are based on the grounds of the standard first-order perturbative diagrammatic techniques that Maki and Thompson applied in their original works,[26,27] assuming a Fermi-liquid BCS-like behavior of the electrical carriers. Quite recently, however, the effects introduced by some further perturbative orders were considered by several authors, mainly addressed by the study of the striking experimental features of the paraconductivity in the c direction, $\Delta\sigma_c(\epsilon,H)$, of the HTSC compounds.[17,39–41] In what concerns the in-plane paraconductivity and fluctuation-induced magnetoconductivity, the same calculations have been introduced by Dorin and co-workers for layered superconductors with a single periodicity.[17] Here, we will just extend these last results to two particular limits, characterized by $\gamma_1=\gamma_2$ and $\gamma_1\gg\gamma_2$, of the biperiodic layered superconductors. We will see in the next section that these two limits are particularly useful in analyzing the experimental data measured in Y-123 crystals.

In the $N=1$ case, the new contributions to the in-plane paraconductivity may be written as[17]

$$\Delta\sigma_{ab\rm MTR}(\epsilon)=4\kappa_{\rm MTR}A_\sigma\ \ln\left(\frac{\epsilon}{4}\left(1+\sqrt{1+B_\sigma/\epsilon}\right)^2\right), \tag{6.6a}$$

$$\Delta\sigma_{ab\rm DOS}(\epsilon)=4\kappa_{\rm DOS}A_\sigma\ \ln\left(\frac{\epsilon}{4}\left(1+\sqrt{1+B_\sigma/\epsilon}\right)^2\right), \tag{6.6b}$$

where $\Delta\sigma_{ab\rm MTR}(\epsilon)$ and $\Delta\sigma_{ab\rm DOS}(\epsilon)$ are the so-called regular Maki-Thompson and, respectively, density of states

(DOS) contributions. This last contribution may be related to the variation, produced by the appearance of the Cooper pairs, of the density of states of the normal carriers. In Eq. (6.6), the constants A_σ and B_σ are given by

$$A_\sigma = A_{\mathrm{AL}}, \quad B_\sigma = B_{\mathrm{LD}} \quad (\text{if } N=1), \tag{6.7}$$

and κ_{MTR} and κ_{DOS} are parameters depending on the relaxation time of the normal carriers, τ, as

$$\kappa_{\mathrm{MTR}} \equiv \frac{\Psi'\left(\frac{1}{2}+\frac{\hbar}{4\pi k_B \tau T}\right) - \Psi'\left(\frac{1}{2}\right) - \frac{\hbar}{4\pi k_B \tau T}\Psi''\left(\frac{1}{2}\right)}{\pi^2\left[\Psi\left(\frac{1}{2}\right) + \frac{\hbar}{4\pi k_B \tau T}\Psi'\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2}+\frac{\hbar}{4\pi k_B \tau T}\right)\right]}, \tag{6.8a}$$

$$\kappa_{\mathrm{DOS}} \equiv \frac{\Psi'\left(\frac{1}{2}+\frac{\hbar}{4\pi k_B \tau T}\right) - \frac{\hbar}{2\pi k_B \tau T}\Psi''\left(\frac{1}{2}\right)}{\pi^2\left[\Psi\left(\frac{1}{2}\right) + \frac{\hbar}{4\pi k_B \tau T}\Psi'\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2}+\frac{\hbar}{4\pi k_B \tau T}\right)\right]}, \tag{6.8b}$$

where $\Psi(x)$, $\Psi'(x)$, and $\Psi''(x)$ are the digamma function and its derivatives. In the clean and dirty limits these expressions may be reduced to[17]

$$\kappa_{\mathrm{MTR}} = 0.3455, \quad \kappa_{\mathrm{DOS}} = 0.691 \quad (\text{dirty limit}), \tag{6.9a}$$

$$\kappa_{\mathrm{MTR}} = 0.5865, \quad \kappa_{\mathrm{DOS}} = 9.384\left(\frac{k_B \tau T}{\hbar}\right)^2 \quad (\text{clean limit}). \tag{6.9b}$$

In accordance with Eq. (5.5), the regular-MT and DOS contributions to the in-plane paraconductivity in a biperiodic layered superconductor may be now directly obtained in the $\gamma_1=\gamma_2$ and $\gamma_1 \gg \gamma_2$ limits by just using in Eq. (6.6) the new A_σ and B_σ given by

$$A_\sigma = 2A_{\mathrm{AL}}, \quad B_\sigma = 4B_{\mathrm{LD}} \quad (\text{if } N=2 \text{ and } \gamma_1=\gamma_2), \tag{6.10a}$$

$$A_\sigma = A_{\mathrm{AL}}, \quad B_\sigma = B_{\mathrm{LD}} \quad (\text{if } N=2 \text{ and } \gamma_1 \gg \gamma_2). \tag{6.10b}$$

In the case of the in-plane fluctuation-induced magnetoconductivity, these contributions are given, for $N=1$, by[17]

$$\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{MTR}}(\epsilon,H) = \frac{h^2 \kappa_{\mathrm{MTR}} A_\sigma}{3\epsilon^2} \frac{2+\frac{B_\sigma}{\epsilon}}{\left(1+\frac{B_\sigma}{\epsilon}\right)^{3/2}}, \tag{6.11a}$$

$$\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{DOS}}(\epsilon,H) = \frac{h^2 \kappa_{\mathrm{DOS}} A_\sigma}{3\epsilon^2} \frac{2+\frac{B_\sigma}{\epsilon}}{\left(1+\frac{B_\sigma}{\epsilon}\right)^{3/2}}. \tag{6.11b}$$

Again, for $N=2$ in the $\gamma_1=\gamma_2$ and $\gamma_1 \gg \gamma_2$ limits, the same expressions are applicable if we use for A_σ and B_σ the values given by Eq. (6.10) instead of the $N=1$ values given by Eq. (6.7).

VII. AN EXAMPLE OF APPLICATION: ANALYSIS OF THE IN-PLANE EXPERIMENTAL DATA OBTAINED IN UNTWINNED $YBa_2Cu_3O_{7-\delta}$ CRYSTALS

As an example of their interest, we are going to use here our theoretical results on the in-plane paraconductivity and fluctuation-induced magnetoconductivity in biperiodic layered superconductors to analyze the data obtained by Pomar and co-workers[10,24] in the *a* direction of two untwinned $YBa_2Cu_3O_{7-\delta}$ (Y-123) crystals. The high quality of these almost full oxygenated Y-123 crystals, with $T_c>92$ K and a resistive upper half width of less than 0.1 K, and also the reproducibility of the data from sample to sample well to within the 15% of the estimated experimental uncertainty, strongly suggests that these $\Delta\sigma_a(\epsilon)$ and $\Delta\tilde{\sigma}_a(\epsilon,H)$ data must be close to the intrinsic ones.[42,43] Let us stress here that in the *a* direction these fluctuation-induced effects are due solely to the CuO_2 superconducting layers, without any contribution from the CuO chains also present in this compound and which role on the transport properties in the *b* direction is not well settled up to now. As a crucial test of consistency, simultaneously we will briefly analyze also, in terms of our previous theoretical results for the fluctuation-induced diamagnetism in biperiodic layered superconductors,[20] the available experimental data on $\Delta\chi_{ab}(\epsilon)$ measured in the weak magnetic-field limit in the same crystals.[19] Moreover, as it was shown in Ref. 20, the OPF effects on $\chi_c(\epsilon)$, the magnetic suseceptibility for weak magnetic fields applied in the *ab* planes, may be neglected in this case. So, for the three different observables, the $H=0$ mean field critical temperature, T_{c0}, which is never directly accessible, was estimated from measurements of $\chi_c(\epsilon)$. These $\Delta\sigma_a(\epsilon)$ and $\Delta\tilde{\sigma}_a(\epsilon,H)$ experimental data were already compared with the AL contribution (i.e., neglecting any indirect contribution) for $N=2$ with $\gamma_1=\gamma_2$.[10,24] So, we will extend here these results to other γ_1/γ_2 values and, mainly, we will analyze in detail the possible relevance of the indirect contributions, included the DOS and MTR terms recently proposed by Dorin and co-workers,[17] on both the in-plane paraconductivity and the fluctuation-induced magnetoconductivity. Until now these last contributions were used to analyze only the experimental data obtained in the transversal (perpendicular to the ab plane) direction of different HTSC. So, the interest of our present analysis is enhanced by the fact that these indirect contributions are confronted with high quality in-plane experimental data.

A. The one single periodicity approach

We will first check if the conventional approach for layered superconductors with one single periodicity ($N=1$) and with the anomalous MT term as the only indirect contribution is compatible with the experimental $\Delta\sigma_a(\epsilon)$, $\Delta\widetilde{\sigma}_a(\epsilon,H)$, and $\Delta\chi_{ab}(\epsilon)$ data in Y-123 crystals. As noted before, this conventional approach, that corresponds to the limit $\gamma_1/\gamma_2\rightarrow\infty$ of the $N=2$ theory and which leads to $N_e=1$, was used by many of the authors, without other justification than its simplicity, in analyzing their data in biperiodic layered superconductors.[3–9,37,42] In this case, the AL and the MT contributions are given by, respectively, Eqs. (3.4) and (3.12). In Fig. 3(a), the average values of the paraconductivity measured in the a direction of two Y-123 crystals having a central part without twins is compared with the theoretical $\Delta\sigma_{ab}^{N=1}(\epsilon)$. The solid line in this figure corresponds to the best fit of Eq. (3.1), with $\Delta\sigma_{ab\mathrm{AL}}(\epsilon)$ and $\Delta\sigma_{ab\mathrm{MT}}(\epsilon)$ given by Eq. (3.4) and, respectively, Eq. (3.12). The fitting was done in the ϵ region bounded by the arrows, i.e, for $2\times10^{-2}<\epsilon<10^{-1}$, with τ_ϕ, τ, and $\xi_c(0)$ as free parameters but by imposing $\tau_\phi\geqslant\tau$, in accordance with the Matthiessen rule.[30] Note also that the lower limit of the ϵ region considered here coincides with the estimated Ginzburg temperature for Y-123 compounds,[10,19,24] whereas the upper limit has been chosen to avoid the possible influence on the paraconductivity of nonlocal or other high-temperature effects.[21,37,38] So, we will consider this ϵ region as the mean-field-like temperature region (MFR) above T_{c0} for the Y-123 compounds. Note here that τ_ϕ and τ are expected to be temperature dependent, but they may be approximated as constants (and equal to their respective value at $T=100$ K) in that MFR. The resulting values from this fit are $\tau_\phi(100\ \mathrm{K})\approx\tau(100\ \mathrm{K})=1.4\times10^{-14}$ s, and $\xi_c(0)=0.11$ nm. As it may be seen in this figure, the agreement between this conventional approach and the experimental data is quite good, the rms error being of the order of 3%. These values of the free parameters lead to a contribution of the anomalous MT term of about 15% of the total paraconductivity at $\epsilon=10^{-2}$ and of the order of 35% at $\epsilon=10^{-1}$.

The comparison between the theoretical in-plane fluctuation-induced magnetoconductivity, always in the limit $\gamma_1/\gamma_2\rightarrow\infty$, and the experimental data obtained in the weak magnetic-field limit in the same Y-123 crystals studied before,[10] is presented in Fig. 3(b). In this figure, the solid line corresponds to the best fit, in the same ϵ region as before for the paraconductivity, of Eq. (4.1), with the ALO term given by Eq. (4.5) and the MTO term given by Eq. (4.10) and by imposing the above found values of τ_ϕ, τ, and $\xi_c(0)$. The resulting value of the in-plane superconducting coherence length amplitude (at $T=0$ K), the only remaining free parameter, is $\xi_{ab}(0)=1.05$ nm, the rms error being of the order of 5%. We see, therefore, that this scenario, noted A in Table I, may reasonably explain simultaneously the experimental $\Delta\sigma_a(\epsilon)$ and $\Delta\widetilde{\sigma}_a(\epsilon,H)$ in the MFR in Y-123 crystals. However, a crucial test of consistency for this scenario is provided by the fluctuation-induced diamagnetism, $\Delta\chi_{ab}(\epsilon)$, because (as seen previously in Sec. III) this observable depends on the same parameters as $\Delta\sigma_a(\epsilon)$ and $\Delta\widetilde{\sigma}_a(\epsilon,H)$. In Figs. 3(c) and 3(d) we compare $\Delta\chi_{ab}(\epsilon)/T\Delta\sigma_a(\epsilon)$ and $\Delta\chi_{ab}(\epsilon)/T$ with the experimental results obtained in the

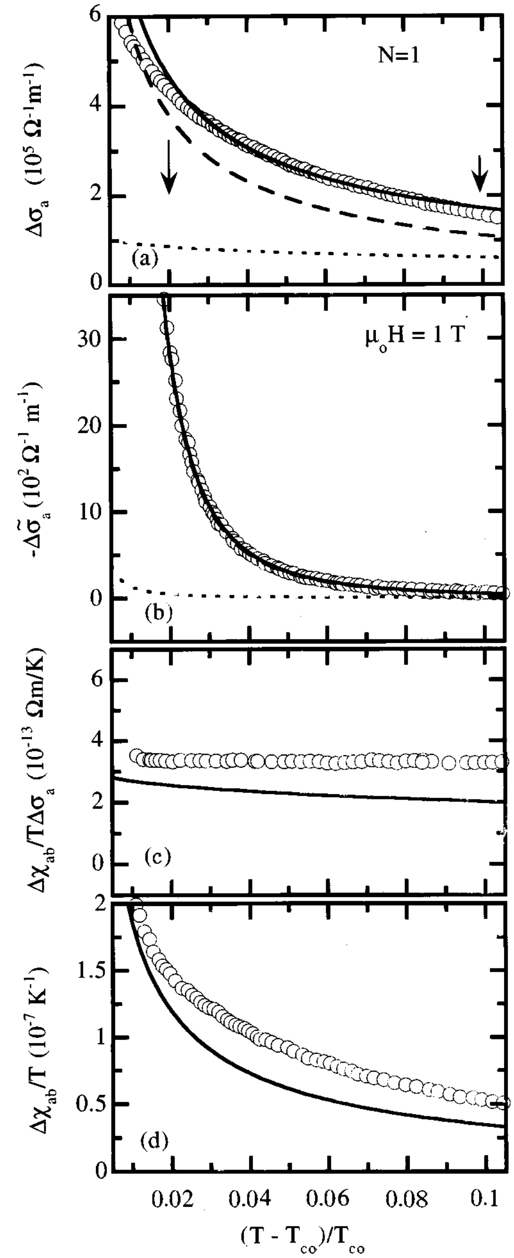

FIG. 3. Comparison between the theoretical results for layered superconductors with a single periodicity ($N=1$), but by taking into account only the direct Aslamazov-Larkin (dashed line) and the indirect anomalous Maki-Thompson (dotted line) contributions, and the experimental data of the paraconductivity along the a axis (a), the fluctuation-induced magnetoconductivity (b), the fluctuation-induced diamagnetism (d), and the relationship between $\Delta\sigma_a$ and $\Delta\chi_{ab}/T$ (c), measured in untwinned Y-123 crystals by Pomar and co-workers (Refs. 10 and 24) and by Torrón and co-workers (Refs. 19). This $N=1$ approximation is equivalent to $\gamma_1/\gamma_2\rightarrow\infty$ in the $N=2$ theory. The solid lines correspond to the best fits of the theory to the experimental data in the mean-field-like region bounded by the arrows. The resulting values of the various parameters arising in the theory are summarized in Table I (scenario A).

TABLE I. Values of the characteristic parameters arising in the different theoretical scenarios compared here with the experimental data of the in-plane paraconductivity, fluctuation induced magnetoconductivity, and fluctuation induced diamagnetism measured in untwinned $YBa_2Cu_3O_{7-\delta}$ crystals (Refs. 10, 19, and 24). The different symbols are defined in the main text. Scenarios A and B, with $\gamma_1/\gamma_2\to\infty$, correspond to the approaches with one single periodicity ($N=1$), whereas the scenarios C and D correspond to the biperiodic approach, in the limiting case $\gamma_1/\gamma_2\approx1$, i.e., by assuming a similar tunneling coupling strength between adjacent layers. In B and D the possible indirect DOS and MTR contributions to $\Delta\sigma_a(\epsilon,H)$ and $\Delta\widetilde{\sigma}_a(\epsilon,H)$ are taken into account, whereas these effects are neglected in A and C. The too low (unphysical) values of τ_ϕ and τ in scenario C, the most plausible one to explain the in-plane OPF effects in Y-123 crystals, just suggest the absence in this scenario of appreciable indirect OPF contributions to $\Delta\sigma_a(\epsilon,H)$ and $\Delta\widetilde{\sigma}_a(\epsilon,H)$ in all the mean-field-like region. Note also the excellent agreement between the experimental paraconductivity and the different theories in the four scenarios analyzed here, which is a clear illustration that the analysis of only one observable does not suffice to discriminate between very different theoretical approaches. However, the ambiguity disappears by simultaneously analyzing also $\Delta\widetilde{\sigma}_a(\epsilon,H)$ and $\Delta\chi_{ab}(\epsilon)$. The uncertainties of these different values, associated with the different error sources, remain below 15%.

Scenario	γ_1/γ_2	$\xi_c(0)$ (nm)	τ (s)	τ_ϕ (s)	$\xi_{ab}(0)$ (nm)	rms error $\Delta\sigma_a$	rms error $\Delta\widetilde{\sigma}_a$	rms error $\Delta\chi_{ab}/T\Delta\sigma_a$	rms error $\Delta\chi_{ab}/T$
A	$\to\infty$	0.11	1.4×10^{-14}	1.4×10^{-14}	1.05	3%	5%	35%	35%
B	$\to\infty$	0.17	1.5×10^{-14}	5.7×10^{-13}	0.90	3%	90%	>100%	>100%
C	1	0.12	2.2×10^{-16}	2.2×10^{-16}	1.10	3%	5%	1%	3%
D	1	0.13	1.3×10^{-14}	3.6×10^{-13}	0.95	3%	70%	>100%	>100%

same samples,[19,24] in both cases by using the values of the different parameters found before, i.e., without any fitting parameter. The disagreement between the theory for $N=1$ (solid lines) and the experimental data is evident, the rms error in the MFR being of the order of 35%. These results show that the conventional approach cannot explain quantitatively and simultaneously the experimental data on $\Delta\sigma_a(\epsilon)$, $\Delta\widetilde{\sigma}_a(\epsilon,H)$, and $\Delta\chi_{ab}(\epsilon)$ obtained in the MFR in high quality Y-123 crystals.

To check if the presence of the new indirect contributions recently proposed by Dorin and co-workers[17] could eliminate the difficulties found before for the one single periodicity approach, in Fig. 4 we compare again this approach with the same experimental data, but adding to the theoretical expressions of $\Delta\sigma_a(\epsilon)$ and of $\Delta\tilde{\tilde{\sigma}}_a(\epsilon,H)$ the corresponding new terms, given by Eqs. (6.6) and, respectively, (6.11). The solid line in Fig. 4(a) was obtained by fitting (6.6) to the experimental data in the same MFR as above, and with τ_ϕ, τ, and $\xi_c(0)$ as free parameters, but again by imposing $\tau_\phi\geqslant\tau$. As it may be seen, the agreement is again excellent, the resulting values for the free parameters being $\xi_c(0)=0.17$ nm, $\tau(100\ \mathrm{K})=1.5\times10^{-14}$ s, and $\tau_\phi(100\ \mathrm{K})=5.7\times10^{-13}$ s. In this case, the AL term represents about 60% of the total paraconductivity whereas the net (the anomalous MT term is positive, i.e., enhances the conductivity, whereas the DOS and the regular MT contributions are negative, i.e., they decrease the conductivity) contribution of the indirect terms is around 40%. However, such a good agreement of the one single periodicity approach for the in-plane paraconductivity is severely questioned by analyzing the in-plane fluctuation-induced magnetoconductivity. In Fig. 4(b), we present the best fit of the total (included the new indirect contributions) theoretical fluctuation-induced magnetoconductivity to the experimental data, by imposing the values of $\xi_c(0)$, τ_ϕ and τ found above, but with $\xi_{ab}(0)$ as the unique free parameter. As it may be seen in this figure, the agreement between the theory and the experimental data is very poor, the rms error of the fit being of the order of 90%. The resulting $\xi_{ab}(0)$ value is 0.9 nm. In addition, it is very easy to check that with such a low $\xi_{ab}(0)$, this scenario, noted B in Table I, cannot explain the $\Delta\chi_{ab}(\epsilon)$ data, the rms error being [always in the same MFR, i.e., the ϵ region bounded by the arrows in Fig. 4(a)] bigger than 100%.

We may, therefore, conclude already here that the conventional approaches with one single layered periodicity cannot explain simultaneously and consistently the experimental $\Delta\sigma_a(\epsilon)$, $\Delta\widetilde{\sigma}_a(\epsilon)$, and $\Delta\chi_{ab}(\epsilon)$ data obtained in the MFR in the same Y-123 crystals, even if the new indirect contributions proposed by Dorin and co-workers[17] are taken into account. In fact, the presence of appreciable DOS and regular MT contributions will still enhance the importance of the disagreement found between the $N=1$ theory and the experimental results in Y-123 crystals.

B. The biperiodic approach

We will now compare the same experimental data already used in the precedent subsection with our theoretical results for biperiodic layered (also called bilayered) superconductors. To avoid the introduction of new free parameters in the theoretical expressions, we are going to impose henceforth in this comparison the condition $\gamma_1/\gamma_2=1$. Let us stress already here that this γ_1/γ_2 value is close to those that are obtained if, as it may be expected, the strength of the tunneling coupling between adjacent superconducting layers is approximately proportional to the inverse of the distances between them, or even proportional to the square of these interlayer distances.[44] As it may be easily inferred from the plots shown in Figs. 2(a) and 2(b), in this $\gamma_1/\gamma_2=1$ limit the biperiodic approach leads to very appreciable differences with the one single periodicity approach ($\gamma_1/\gamma_2\to\infty$).

In comparing the biperiodic approach with the experimental data, we will follow the same procedure as used before for the $N=1$ theory. The solid line in Fig. 5(a) corresponds to

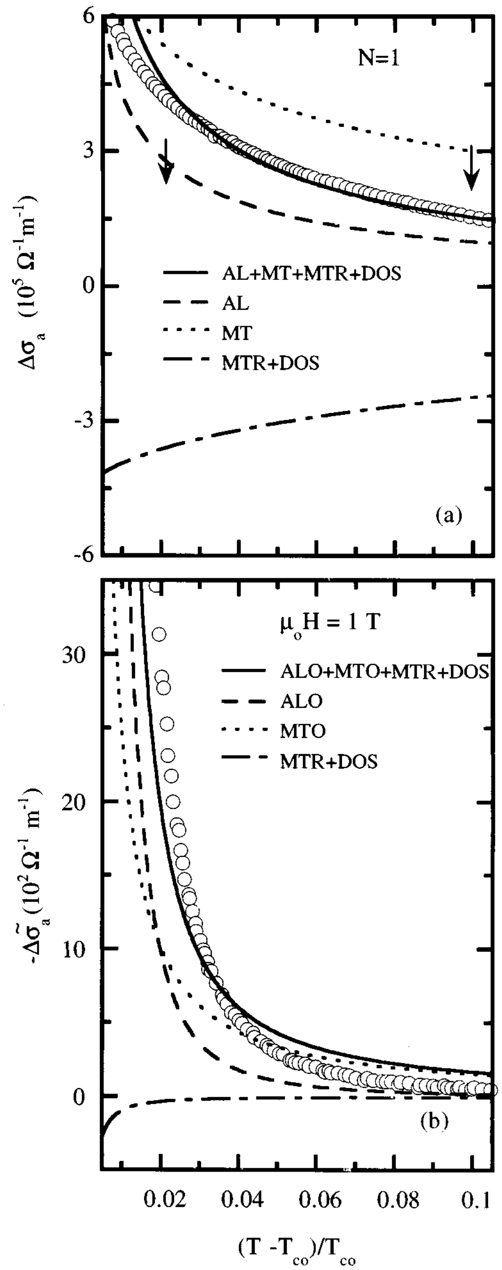

FIG. 4. Comparison between the theoretical expressions for one single periodicity layered superconductors, but now including the indirect DOS and MTR terms, with the same experimental results as in Fig. 3. The solid lines are the best fits of the theory to the data. The disagreement between this scenario, noted *B* in Table I, and the experimental results is evident.

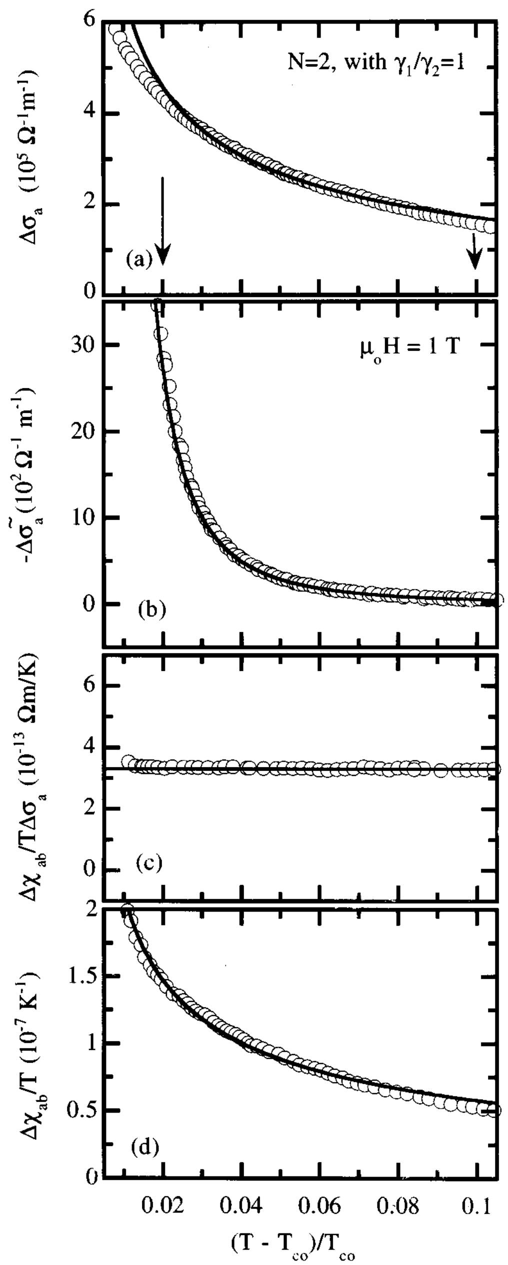

FIG. 5. Comparison between the same experimental results as in Fig. 3 with the theoretical results for biperiodic layered superconductors, in the limit $\gamma_1/\gamma_2=1$, and by neglecting the indirect DOS and MTR contributions. The corresponding scenario is noted *C* in Table I. The solid lines correspond to the best fits of the theory to the data. This scenario leads to the absence of appreciable anomalous MT contributions to the paraconductivity and to the fluctuation-induced magnetoconductivity.

the best fit, in the same MFR as before (the ϵ region between the arrows, in this figure), of the total paraconductivity given by Eq. (3.1) with the AL and the anomalous MT terms given by, respectively, Eqs. (3.6) and (3.13), to the experimental $\Delta\sigma_a(\epsilon)$ data, with again τ_ϕ, τ, and $\xi_c(0)$ as free parameters and $\tau \geq \tau$. The resulting best fit values are $\xi_c(0)=0.12$ nm and $\tau(100\ \text{K})=\tau_\phi(100\ \text{K})=2.2\times10^{-16}$ s. The agreement between the theory and the experimental data is excellent, the rms error being 3% in such a MFR. These values lead to an anomalous MT contribution of less than 10% of the total paraconductivity in all the MFR. In fact, as we will see below, this τ is much more smaller than the relaxation time that may be obtained from the analysis of the normal conductivity and, therefore, the physical meaning of these so low τ and τ_ϕ values may also suggest the nonapplicability to the HTSC of the BCS-based calculations of these indirect contributions to the in-plane paraconductivity (see below).

The comparison between the theoretical in-plane fluctuation-induced magnetoconductivity in biperiodic layered superconductors, always in the case $\gamma_1/\gamma_2=1$, and the experimental data is presented in Fig. 5(b). In this figure, the solid line correspond to the best fit, in the same MFR as before, of Eq. (4.1), with $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{ALO}}(\epsilon,H)$ and $\Delta\tilde{\tilde{\sigma}}_{ab\mathrm{MTO}}(\epsilon,H)$ given by Eqs. (4.6) and, respectively, (4.11), and by imposing the above found values of τ_ϕ, τ, and $\xi_c(0)$. The resulting value of the only remaining free parameter, the zero-temperature in-plane coherence length, is $\xi_{ab}(0)=1.1$ nm, and the rms error is 5%. This scenario, which leads to the absence also in the fluctuation-induced magnetoconductivity of appreciable indirect contributions, is noted *C* in Table I and it may also explain at a quantitative level the fluctuation-induced diamagnetism measured in the same samples. Such a comparison is shown in Figs. 5(c) and 5(d) for $\Delta\chi_{ab}(\epsilon)/T\Delta\sigma_a(\epsilon)$ and, respectively $\Delta\chi_{ab}(\epsilon)$. In agreement with the irrelevance of the anomalous MT contribution to the paraconductivity in all the MFR, we see in Fig. 5(c) that the theoretical $\Delta\chi_{ab}/T\Delta\sigma_a(\epsilon)$ is practically ϵ independent in all the MFR, in excellent agreement with the experimental data, the rms error being of the order of 1% in this ϵ region. Such a good agreement is also found for $\Delta\chi_{ab}(\epsilon)$, the rms error being less than 3%.

We will check now the possible relevance, in the case of this biperiodic layered description (always with $\gamma_1/\gamma_2=1$), of the new indirect terms (DOS and MTR) on the $\Delta\sigma_a(\epsilon)$ and $\Delta\tilde{\sigma}_a(\epsilon,H)$ measured in Y-123 crystals. The solid line in Fig. 6(a) correspond to the best fit of the total in-plane paraconductivity given by

$$\Delta\sigma_{ab}(\epsilon)=\Delta\sigma^{N=2}_{ab\mathrm{AL}}(\epsilon)+\Delta\sigma^{N=2}_{ab\mathrm{MT}}(\epsilon)+\Delta\sigma^{N=2}_{ab\mathrm{MTR}}(\epsilon)+\Delta\sigma^{N=2}_{ab\mathrm{DOS}}(\epsilon), \quad (7.1)$$

with these terms given by Eqs. (3.6), (3.13), and (6.6), with $\gamma_1/\gamma_2=1$ or, equivalently, by imposing the conditions given by Eq. (6.10a). The fit was done in the same MFR as before, and again with τ_ϕ, τ, and $\xi_c(0)$ as free parameters. The resulting values are $\xi_c(0)=0.13$ nm, $\tau(100\ \mathrm{K})=1.3\times10^{-14}$ s, and $\tau_\phi(100\ \mathrm{K})=3.6\times10^{-13}$ s, and the agreement between the theory and the experimental data is excellent, the rms error being again of the order of 3%. In fact, as it may be seen in Fig. 6(a), in this case the anomalous MT term from one side, and the DOS and the regular MT contributions from the other, are almost mutually compensated, so their net contribution to the paraconductivity is of the order of 15%. However, the comparison with the fluctuation-induced magnetoconductivity will allow us to further check the possible presence of these indirect contributions: The solid line in Fig. 6(a) corresponds to the best fit of the total in-plane fluctuation-induced magnetoconductivity, given by

$$\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)=\Delta\tilde{\tilde{\sigma}}^{N=2}_{ab\mathrm{ALO}}(\epsilon,H)+\Delta\tilde{\tilde{\sigma}}^{N=2}_{ab\mathrm{MTO}}(\epsilon,H)+\Delta\tilde{\tilde{\sigma}}^{N=2}_{ab\mathrm{MTR}}(\epsilon,H)+\Delta\tilde{\tilde{\sigma}}^{N=2}_{ab\mathrm{DOS}}(\epsilon,H), \quad (7.2)$$

where these terms are given by Eqs. (4.6), (4.11), and (6.11), with $\gamma_1=\gamma_2$, or equivalently, under the conditions given by Eq. (6.10a). In this fit, we have imposed the values of $\xi_c(0)$, τ_ϕ, and τ found before, but with $\xi_{ab}(0)$ as free parameter. From this fit we obtain $\xi_{ab}(0)=0.95$ nm and the disagreement between the theory and the experimental data is very important, the rms error being of the order of 70%. It will be very easy to check that this scenario, noted *D* in Table I, is also not at all able to account for the $\Delta\chi_{ab}(\epsilon)$ data.

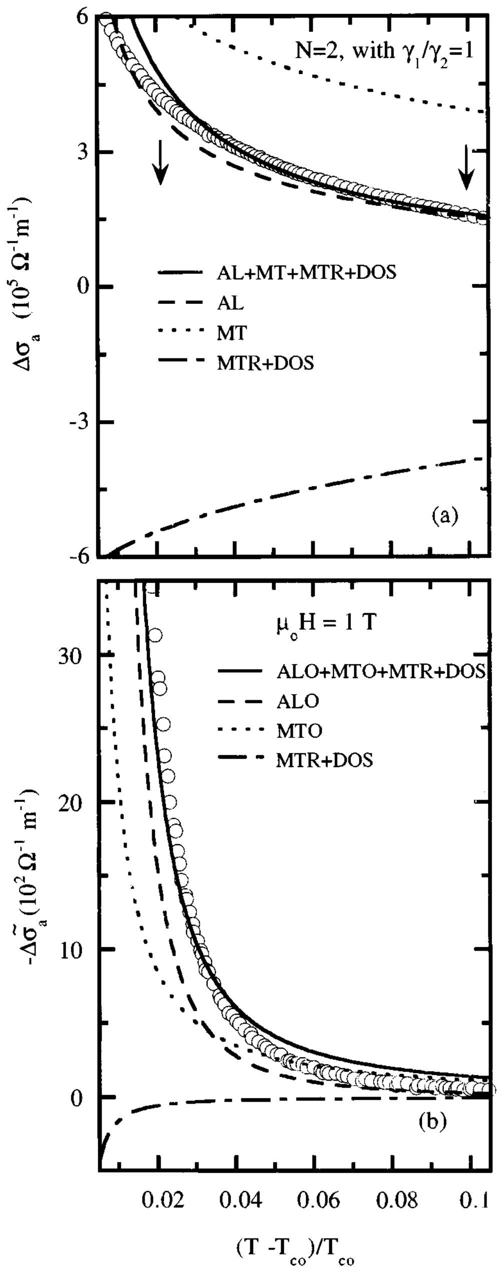

FIG. 6. Comparison of the theoretical expressions for biperiodic layered superconductors in the limit $\gamma_1/\gamma_2\approx1$, but now including the indirect DOS and MTR terms, with the same experimental data as in Fig. 3. The solid lines are the best fits of the theory to the data. The disagreement between this scenario, noted *D* in Table I, and the experimental results is evident.

We may conclude already here, therefore, that the only scenario which explains quantitative and simultaneously the intrinsic $\Delta\sigma_a(\epsilon)$, $\Delta\tilde{\sigma}_a(\epsilon,H)$, and $\Delta\chi_{ab}(\epsilon)$ data obtained in the MFR in Y-123 untwinned crystals is the scenario *C*, which corresponds to a biperiodic layered superconductor with $\gamma_1\approx\gamma_2$. In this scenario, $2\xi_c(0)\geqslant s=1.17$ nm for $\epsilon\lesssim0.04$, so the OPF have a 2D–3D crossover in the MFR, the effective critical exponent and the effective number of fluctuating planes varying to within $-0.75\lesssim x(\epsilon)\lesssim-0.5$ and $1.2\lesssim N_e(\epsilon)\lesssim1.6$ in the MFR ($0.02\lesssim\epsilon\lesssim0.1$). Another important aspect of this scenario is that the OPF effects on the three magnitudes analyzed here may be explained at a quantitative level in terms of the so-called direct effects: The AL contribution in the case of the in-plane paraconductivity and the ALO contribution in the case of the fluctuation-induced magnetoconductivity. These results confirm at a quantitative level our earlier proposal, based on the analysis of the paraconductivity and of the fluctuation-induced diamagnetism in different HTSC systems,[23,45] that the indirect contributions (now including also the DOS and regular-MT ones) are negligible in the MFR in Y-123 compounds and that, therefore, the wave pairing in the HTSC could be unconventional (extended or non-$^1s_0$) pair breaking.[46,47] In fact, by taking into account the in-plane normal electrical conductivity of our samples at $T=100$ K, and using a carrier density of about $n_H=4\times10^{27}$ m^3 we obtain for the mean free path of the normal carriers, $l\approx8$ nm. By combining this value with the value of τ in this scenario C, we obtain for the Fermi velocity, $v_F=6\times10^2$ m/s, a value in disagreement with those currently proposed for Y-123 compounds in the literature[48] ($v_F\approx10^5$ m/s). In addition, such a low τ will indicate that the Y-123 compounds will be in the so-called dirty limit, also in contrast with most of the existing proposals.[7,16,38] As τ only appears in the MT contributions and these terms were calculated on the grounds of the BCS-like theory, with a conventional $^1s_0$ wave pairing, these last results suggest again the nonapplicability to the copper oxide superconductors of such an approach. Complementary, another conclusion to be stressed here is that our present results also confirm, at a quantitative level, our earlier suggestion[45] that the use of the conventional LD-like approaches for layered superconductors with one single periodicity will introduce a considerable error in analyzing the biperiodic layered HTSC.

Finally, let us also stress here that the *simultaneous* analysis of high quality experimental data of in-plane paraconductivity and of the fluctuation-induced magnetoconductivity (together with the analysis of the in-plane fluctuation-induced diamagnetism measured in the same samples) is crucial to discriminate between the different scenarios of Table I. The results shown in this table clearly illustrate that the analysis of an unique observable measured in an unique HTSC family will *not* allow, even by using high quality data,[49] a discrimination between these different (or even other) possible scenarios for the OPF effects. To further illustrate this point, in Fig. 7 we present the relationship between $\Delta\sigma_{ab\mathrm{AL}}^{N=2}(\epsilon)$ with $\gamma_1/\gamma_2=1$ and $\Delta\sigma_{ab\mathrm{AL}}^{N=1}(\epsilon)+\Delta\sigma_{ab\mathrm{MT}}^{N=1}(\epsilon)$, for different relative strengths of the anomalous MT term, i.e., for different τ_ϕ values, but always with $\xi_c(0)=0.12$ nm and $\tau\sim10^{-15}$ s. As we may see in this figure, these two different theoretical approaches almost agree in all the MFR if $\tau_\phi\sim2\times10^{-15}$ s in the $N=1$ approach. So, the discrimination between these two very different scenarios is only possible by analyzing simultaneously the OPF effects on other magnitudes, as for instance $\Delta\chi_{ab}(\epsilon)$ and $\Delta\tilde{\sigma}_a(\epsilon,H)$, measured in the same samples.

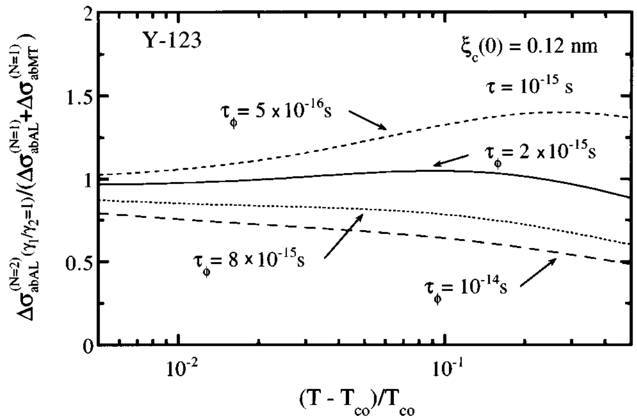

FIG. 7. Relationship between $\Delta\sigma_{ab\mathrm{AL}}^{N=2}(\epsilon)$, with $\gamma_1/\gamma_2=1$, and $\Delta\sigma_{ab\mathrm{AL}}^{N=1}(\epsilon)+\Delta\sigma_{ab\mathrm{MT}}^{N=1}(\epsilon)$, for different τ_ϕ values and $\tau=10^{-15}$ s. We see that for $\tau_\phi\approx2\times10^{-15}$ s, $\Delta\sigma_{ab\mathrm{AL}}^{N=2}(\epsilon)$ almost coincides, in all the MFR, with $\Delta\sigma_{ab\mathrm{AL}}^{N=1}(\epsilon)+\Delta\sigma_{ab\mathrm{MT}}^{N=1}(\epsilon)$.

VIII. CONCLUSIONS

In this work the different direct and indirect contributions, associated with the Cooper pairs created by thermal fluctuations above the superconducting transition, to the in-plane paraconductivity, $\Delta\sigma_{ab}(\epsilon)$, and to the fluctuation-induced magnetoconductivity, $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$, in a biperiodic layered superconductor, with two interlayer distances in the unit cell length and with two different tunneling coupling strengths between adjacent superconducting layers, were calculated. Our results show, in particular, that by introducing an effective number, N_e, of independent fluctuating layers per unit cell length, it is possible to express the Aslamazov-Larkin and the anomalous Maki-Thompson contributions to $\Delta\sigma_{ab}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$ in terms of the corresponding quantities in one single periodicity layered superconductors with the same unit cell length. Also, N_e is the same effective number of fluctuating planes that we have previously found in studying the fluctuation-induced diamagnetism, $\Delta\chi_{ab}(\epsilon)$, in biperiodic layered superconductors.[20] As an example of their interest, these theoretical results in biperiodic and in single layered superconductors have been used to analyze the experimental data on the paraconductivity and on the fluctuation-induced magnetoconductivity in the a direction (nonaffected by the presence of CuO chains) obtained in untwinned $YBa_2Cu_3O_{7-\delta}$ crystals.[10,24] As an important test of consistency, we have analyzed also the data of the in-plane fluctuation-induced diamagnetism measured in the same crystals.[19] In these analyses taken into account are all the different direct and indirect OPF contributions to $\Delta\sigma_{ab}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$, including the indirect contributions associated with the fluctuations of the normal quasiparticle density of states (DOS), recently proposed by Dorin and co-workers.[17] Some of the conclusions of these analysis are as follows. (i) The currently used Lawrence-Doniach (single layered) approaches for $\Delta\sigma_{ab}(\epsilon)$, $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$, and $\Delta\chi_{ab}(\epsilon)$, which consider the Y-123 compounds as stacks of superconducting layers with only one superconducting layer per unit cell length cannot explain simultaneous and consistently the experimental $\Delta\sigma_a(\epsilon)$, $\Delta\tilde{\sigma}_a(\epsilon,H)$, and $\Delta\chi_{ab}(\epsilon)$ in these compounds, and this with or without the consideration of indirect OPF contributions. (ii) The presence of appreciable DOS contributions is not compatible with the $\Delta\sigma_a(\epsilon)$, $\Delta\tilde{\sigma}_a(\epsilon,H)$, and $\Delta\chi_{ab}(\epsilon)$ data in the Y-123 crystals, and this in both the single or the biperiodic layered approaches. This conclusion directly affects the recent proposals[17,39–41] of the existence of important DOS effects as an explanation for the behavior near T_{c0} of the magnetoconductivity in the c direction in

HTSC. New theoretical and experimental work on the OPF effects in this transversal direction in HTSC will be therefore suitable. (iii) If the important modifications associated with the biperiodicity of the Y-123 compounds are taken into account, it is possible, then, to explain simultaneously, consistently and at a quantitative level the $\Delta\sigma_a(\epsilon)$, $\Delta\tilde{\sigma}_a(\epsilon,H)$, and $\Delta\chi_{ab}(\epsilon)$ experimental data obtained in untwinned Y-123 crystals in the reduced temperature region bounded by $2\times10^{-2}\leqslant\epsilon\leqslant10^{-1}$. This ϵ region corresponds quite well to the expected mean-field region in these compounds, where the mean-field OPF theories are expected to be applicable without the inclusion of nonlocal and high-temperature contributions[21,37,38] which may appear at higher ϵ values, and also without the modifications that appear in the so-called full critical region closer to T_{c0}.[10,24] In this biperiodic scenario for the Y-123, the tunneling coupling strengths between different adjacent superconducting CuO_2 layers are of the same order, just as it could be expected if such couplings are inversely proportional to the interlayer distances or even to the squares of these distances.[44] Let us comment here that quite recent measurements in Y-123 crystals by Ling and co-workers support also the existence in this compound of two tunneling junctions per crystallographic unit cell.[50] Moreover, another important aspect of this scenario is that the indirect (regular-MT and DOS included) contributions to $\Delta\sigma_a(\epsilon)$ and $\Delta\tilde{\sigma}_a(\epsilon,H)$ are negligible in all the MFR: the direct (i.e., Aslamazov-Larkin) OPF contributions to $\Delta\sigma_{ab}(\epsilon)$ and $\Delta\tilde{\tilde{\sigma}}_{ab}(\epsilon,H)$ suffice to explain at a quantitative level the corresponding experimental data. These last results confirm at a quantitative level our earlier proposals, based on the preliminary analysis of the in-plane paraconductivity and of the fluctuation-induced diamagnetism in different HTSC families, that the indirect OPF effects are negligible in HTSC, and they again suggest, therefore, the possibility of an unconventional (extended or non-$^1s_0$), pair breaking, wave pairing in these compounds.[23,45] Also, our analysis confirms at a quantitative level the presence of a dimensionality (2D–3D) crossover of the superconducting OPF in the MFR of the Y-123 crystals.

ACKNOWLEDGMENTS

This work was supported by the Comisión Interministerial de Ciencia y Tecnología (MAT 92-0841 and MAT 95-0279) and by the European Community Grant No. CHRX-CT93-0325. One of us, M.V.R., acknowledges financial support from the Xunta de Galicia.

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