Absence of persistent magnetic oscillations in type-II superconductors

M. R. Norman

Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439

A. H. MacDonald

Department of Physics, Indiana University, Bloomington, Indiana 47405

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We report on a numerical study intended to examine the possibility that magnetic oscillations persist in type-II superconductors beyond the point where the pairing self-energy exceeds the normal state Landau level separation. Our work is based on the self-consistent numerical solution for model superconductors of the Bogoliubov–de Gennes equations for the vortex lattice state. In the regime where the pairing self-energy is smaller than the cyclotron energy, magnetic oscillations resulting from Landau level quantization are suppressed by the broadening of quasiparticle Landau levels due to the nonuniform order parameter of the vortex lattice state and by splittings of the quasiparticle bands. Plausible arguments that the latter effect can lead to a sign change of the fundamental harmonic of the magnetic oscillations when the pairing self-energy is comparable to the cyclotron energy are shown to be flawed. Our calculations indicate that magnetic oscillations are strongly suppressed once the pairing self-energy exceeds the Landau level separation. [S0163-1829(96)07230-X]

I. INTRODUCTION

de Haas–van Alphen (dHvA) oscillations in the mixed state of type-II superconductors, discovered[1] in $NbSe_2$ some time ago, have recently[2–4] been observed in several additional materials. The oscillations are damped relative to those in the normal state and become unobservable at sufficiently weak external magnetic fields. These findings have led to a number of theoretical studies of the modification of normal state Landau level structure in the mixed state.[5–12] Conclusions from these studies are not always completely consistent and no widely accepted picture which covers all regimes of a magnetic field has emerged from this work. Recently we reported on a thorough numerical study of the quasiparticle band structure obtained by solving the Bogoliubov–de Gennes (BdG) mean-field equations in the vortex lattice state of a simple model two-dimensional (2D) superconductor.[13] We found that at fields near H_{c2}, magnetic oscillations were clearly present, but that these were rapidly damped as the superconducting self-energy strengthened at weaker fields. We argued and found partial numerical support for the assertion that the effect of superconductivity was similar to the effect of a disorder broadening of the normal state Landau levels, proportional to the pairing self-energy times $n_\mu^{-1/4}$ where n_μ is the Landau level index at the Fermi level. We also found that once the pairing self-energy became comparable to the Landau level separation, the quasiparticle electronic structure in the vortex lattice state entered a complicated crossover regime which simplified with increasing pairing self-energy only when unambiguous vortex cores with associated bound states emerged. While magnetic oscillations were essentially absent once the vortex cores became distinct, we were unable to draw any clear conclusions concerning magnetic oscillations in the crossover regime. These calculations did indicate the possibility of a phase shift of π for magnetic oscillations in the crossover regime, but the

origin of this phase shift was not understood. Such a phase shift was also found in earlier work by Maniv *et al.*[14] based on an expansion of the free energy to fourth order in the order parameter. Recently, Maniv *et al.*[15] have attributed this phase shift to a splitting of Landau levels in the vortex lattice state which they associate with the two vortices per electron magnetic flux quanta in the vortex lattice state. This suggestion has motivated us to examine the crossover regime in greater detail.

Our study is based on numerical solution of the BdG equations[16] for a model superconductor with a BCS pairing interaction, i.e., a δ-function attractive interaction modified by an energy cutoff. We solve the BdG equations in a Landau level basis so that the band energy quantization which is the source of magnetic oscillations is incorporated in an exact way. The formalism necessary to carry out these calculations in a convenient way is fully described in our earlier work[17,13] and briefly summarized below. This approach necessitates a number of practical limitations on the scope of our study: (i) Numerical problems which arise because of oscillations in high Landau index quantum wave functions make it convenient to restrict our attention to single-particle states with Landau level indices smaller than ≈ 60. (ii) We consider only two-dimensional electron systems; adding a third dimension creates no formal difficulty but does add to an already considerable computational burden. (iii) We approximate the magnetic field by its spatial average. The most serious of these limitations is the restriction to moderately large Landau level indices. Two-dimensional models will, if anything, overestimate the importance of magnetic oscillations and are even appropriate for some systems of current interest. The screening corrections to the uniform external magnetic field are small close to H_{c2} and are approximately uniform themselves, except for external fields close to H_{c1}.

In the present study Zeeman splitting is ignored since its effects are well understood. Most of the results we discuss

use the grand canonical ensemble rather than the canonical ensemble appropriate to experimental systems, since this eliminates the problem of determining the chemical potential self-consistently. In the normal state, there is little difference between magnetic oscillations in canonical and grand canonical ensembles for Landau level indices larger than about 6.[18] In the mixed state, however, canonical and grand canonical ensemble results may differ. We have therefore, in some cases, executed the Legendre transform from the grand canonical to the canonical ensemble numerically in order to quantify the importance of magnetic oscillations in the chemical potential.

In Sec. II of this paper we summarize the BdG formalism which is the basis of our numerical calculations. In Sec. III we discuss the devolution of the Landau level structure in the quasiparticle spectrum as the superconducting order strengthens. It is this devolution which underlies the damping of dHvA oscillations in the mixed state. We find that a picture in which the normal state Landau levels simply broaden captures little of the process, hence the substantial difficulty in developing a simple analytic theory for the influence of superconductivity on dHvA oscillations analogous to the simple and successful theory for the influence of disorder. In this section we discuss a plausible approximation which suggests that magnetic oscillations in the vortex lattice state in the crossover field regime will differ by a sign from those in the normal state. The possible sign change is associated with a splitting in the density of quasiparticle states associated with each Landau level at fields below H_{c2}. We explain the origin of this splitting and comment on the failure of the commonly used diagonal approximation for the quasiparticle spectrum. In Sec. IV we carefully examine magnetic oscillations in this regime and find that the sign change does not survive a more thorough analysis. Instead, the fundamental harmonic of the magnetization is strongly damped. The magnetization in this regime has substantial variation with field but the indication from our numerical calculations is that the field dependence is aperiodic. We conclude in Sec. V with a brief summary.

II. BOGOLIUBOV−de GENNES FORMALISM

In zero-field BCS theory, the pairing self-energy couples only single-particle states at wave vectors $\vec{k}$ and $-\vec{k}$. The property that the coupled states have the same band energy is favorable for the formation of a condensate of electron pairs. In a magnetic field the loss of time-reversal invariance makes it impossible to achieve this situation. The center-of-mass momentum of a pair of electrons, which is zero for condensate pairs at zero magnetic field, has quantum fluctuations in a magnetic field $\sim \hbar/\ell$ where $\ell = (\hbar c/eB)^{1/2}$ is the quantum magnetic length. Associated quantum fluctuations in the momenta of the individual electrons contributing to the pair lead[19] to pairing between electrons in different Landau levels and therefore with different single-particle energies. It is this qualitative difference which is responsible, from a microscopic point of view, for the decrease of T_c in a magnetic field. The well-known dependence of H_{c2} on field, obtained from semiclassical theory or (near T_{c0}) from Ginzburg-Landau theory, reflects in the microscopic theory primarily contributions from pairing between electrons in different or-

bital Landau levels.[20] It is not possible to understand the modification of Landau levels by superconductivity, even in the regime near H_{c2}, unless one includes these ''off-diagonal'' terms.[13]

The BdG mean-field equations for a superconductor in a constant magnetic field[17,20] replace the 2×2 secular matrix of BCS theory at zero magnetic field by a secular matrix of order $2N$ (where N is the number of Landau levels within a pairing cutoff energy) for each wave vector $\vec{k}$ in the Brillouin zone of the vortex lattice. The diagonal (normal) electron and hole blocks of the secular matrix are diagonal in the Landau level basis with elements given by ξ_n and $-\xi_n$, respectively, where $\xi_n \equiv (n+1/2)\hbar\omega_c - \mu$. Here $\omega_c = eB/mc$ is the cyclotron frequency and μ is the chemical potential. It is the simplicity of the diagonal block which makes such a basis convenient. The off-diagonal (pairing) blocks have matrix elements[17]

$$F_{\vec{k}NM} = \frac{-\lambda\hbar\omega_c}{2} \sum_j \chi_{M+N-j}(\vec{k}) D_j^{MN} \Delta_j, \qquad (1)$$

with

$$\chi_j(\vec{k}) = \sum_t e^{i2k_x a_x t} e^{-i\pi t^2/2} \chi_j(2k_y l^2 + 2ta_x), \qquad (2)$$

$$\chi_j(Y) = \left(\frac{1}{2^j j! \sqrt{2\pi}} \right)^{1/2} e^{-Y^2/4l^2} H_j\left(\frac{Y}{\sqrt{2}l} \right) \qquad (3)$$

(H_j a Hermite polynomial), and

$$D_j^{NM} = \left(\frac{j!(N+M-j)!N!M!}{2^{N+M}} \right)^{1/2}$$

$$\times \sum_{m=0}^{j} \frac{(-1)^{N-m}}{(j-m)!(N+m-j)!(M-m)!m!}. \qquad (4)$$

In these equations λ is the BCS coupling constant so that $\lambda\hbar\omega_c = V/(2\pi l^2)$ where V is the strength of the attractive interaction. The vortex lattice primitive vectors are $(0,a_y)$ and $(a_x, -a_y/2)$ with $a_x a_y = \pi l^2$ ($a_x = \sqrt{3}a_y/2$ for a triangular lattice). The sum over j in Eq. (1) is over the possible partitionings of the total quantized kinetic energy of the pair, $\hbar\omega_c(N+M+1)$, into contributions from the pair center-of-mass motion, $\hbar\omega_c(j+1/2)$, and the pair relative motion, $\hbar\omega_c(N+M-j+1/2)$, with $(D_j^{NM})^2$ the probability that a pair of electrons in Landau levels N and M will have center-of-mass kinetic energy $\hbar\omega_c(j+1/2)$.

In this formalism the order parameter in the vortex lattice state is paramatrized by a small set of numbers, Δ_j, which should be determined by solving the BdG equations self-consistently:[17]

$$\Delta_j = -\sum_{NM} D_j^{MN} \sum_{\vec{k}} \frac{2a_x}{N_k l} \chi^*_{M+N-j}(\vec{k}) \sum_{\mu} (1-2f^{\mu}_{\vec{k}}) u^{\mu}_{N\vec{k}} v^{*\mu}_{M\vec{k}}, \qquad (5)$$

where $E^{\mu}_{\vec{k}}$ is the μth positive eigenvalue of the secular matrix, $(u^{\mu}_{N\vec{k}}, v^{\mu}_{M\vec{k}})$ is the corresponding eigenvector, and $f^{\mu}_{\vec{k}}$ is the Fermi function. $N_k = L_x L_y/(2\pi l^2)$ is the number of k points ($L_x L_y$ is the area of the system). The Abrikosov

solution[21] for the order parameter near H_{c2} corresponds to a solution with only $\Delta_0 \neq 0$ and it is easy to verify that this solution is recovered in the appropriate limit. For a triangular flux lattice, the lowest-energy solution has Δ_j real and non-zero only for $j = 6m$ where m is an integer.

We determine the magnetization by numerically differentiating the appropriate thermodynamic potential with respect to magnetic field. The grand potential may be expressed in the following form which we use for our numerical calculations:[17]

$$\Omega = \sum_N \xi_N N_N + E_P - TS, \tag{6}$$

where the pairing self-energy is

$$E_P = -\lambda \hbar \omega_c \frac{l N_k}{4 a_x} \sum_j |\Delta_j|^2. \tag{7}$$

Here N_N is the occupation number of Landau level N,

$$N_N = \frac{2}{N_k} \sum_{\mu \vec{k}} f_{\vec{k}}^\mu |u_{N\vec{k}}^\mu|^2 + (1 - f_{\vec{k}}^\mu)|v_{N\vec{k}}^\mu|^2, \tag{8}$$

and S is the entropy:

$$S = -\frac{2 k_B}{N_k} \sum_{\mu \vec{k}} (1 - f_{\vec{k}}^\mu) \ln(1 - f_{\vec{k}}^\mu) + f_{\vec{k}}^\mu \ln f_{\vec{k}}^\mu. \tag{9}$$

For canonical ensemble calculations we calculate the free energy F over a range of electron densities from the grand potential calculated over a range of chemical potentials by using

$$F = \Omega + \mu \sum_N N_N, \tag{10}$$

where both F and the density depend parametrically on μ. The canonical ensemble magnetization is determined by numerically differentiating F with respect to field at fixed density. A portion of the discussion of our results is motivated by an equivalent alternate expression for Ω in terms of quasiparticle energies:[22,7,15]

$$\Omega = -\frac{2 k_B T}{N_k} \sum_{\mu \vec{k}} \ln \left[2 \cosh \left(\frac{E_{\vec{k}}^\mu}{2 k_B T} \right) \right] + \sum_n \xi_n + E_P. \tag{11}$$

The last term here is a double-counting correction for the pair interaction energy.

The magnetization is determined by numerically differentiation: $M(B) = -\partial \Omega / \partial B$. In practice, we generate results as a function of $n_\mu \equiv \mu / \hbar \omega_c - 1/2$ and calculate energies per state in the Landau level in units of $\hbar \omega_c$. Therefore, the derivative for the magnetization has two terms, the first coming from differentiating an explicit dependence on B (that is, $\Omega = \Omega_0 B^2$, with one power of B coming from $\hbar \omega_c$ and the other from the Landau level degeneracy factor), the second from the dependence of Ω_0 on n_μ which is determined numerically. Note that we do not need to perform separate calculations to determine the density dependence of Ω mentioned above and the field dependence of Ω required for the magnetization. Similarly, in the canonical ensemble the mag-
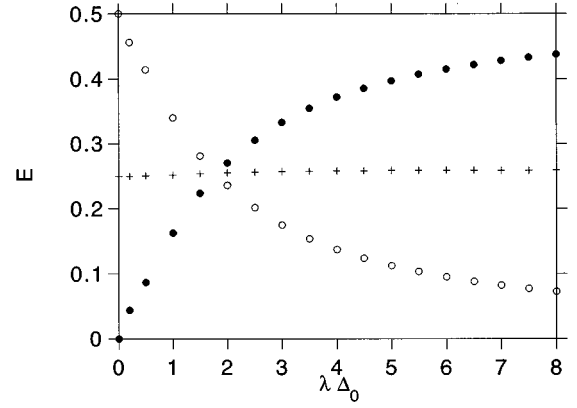

FIG. 1. Lowest eigenvalue ($\hbar \omega_c$ units) vs $\lambda \Delta_0$ in an approximation where all pairing matrix elements are the same constant $[-1/2(n_\mu \pi)^{-1/4} \Delta_0 \hbar \omega_c]$ for $n_\mu = 20$ (solid points), $n_\mu = 20.25$ (pluses), and $n_\mu = 20.5$ (open points).

netization can be expressed in terms of the derivative of the corresponding dimensionless free energy with respect to $N = \Sigma_N N_N$.

III. LANDAU LEVEL DEVOLUTION IN THE MIXED STATE

We first analyze the secular matrix in the limit of small Δ_0. Our objective here is to understand the behavior of the mixed state quasiparticle bands over one period of the normal state magnetic oscillations. Consider the case where $n_\mu = n$ (n an integer). For this case, we note that for each electron energy in the upper diagonal block, there will be a hole energy of the same value in the lower diagonal block. For the Landau level at μ, these two have the same index (n); otherwise, their indices are different ($n+m$ and $n-m$). When the order parameter is small, the strongest mixing of a particle in the Landau level $n+m$ will be with a hole in Landau level $n-m$. The degeneracy of the particle and hole levels will be lifted by the matrix elements in the pairing block which are, in general, off diagonal in Landau level index. At a given $\vec{k}$ the two levels will be split by $2|F_{n+m,n-m}|$ for all Landau levels within the pairing cutoff. In particular, one of the quasiparticle energy levels at zero in the normal state will be shifted up by $|F_{nn}|$ while one of the quasiparticle levels at $\hbar \omega_c$ in the normal state will be shifted down by $|F_{n+1,n-1}|$. Obviously, this splitting cannot continue to grow indefinitely since these two levels will eventually approach each other, leading to an avoided crossing. A similar degeneracy occurs when $n_\mu = n + 1/2$ with the electron level at Landau level index $n+1+m$ and the hole level at Landau level index $n-m$ being degenerate, resulting in a similar splitting of each Landau level. In this case, one of the two quasiparticle levels which has energy $1/2 \hbar \omega_c$ in the normal state will be shifted down by $|F_{n+1,n}|$. For $n_\mu = n + 1/4$ (or $n + 3/4$), the level repulsion effect is weak; that is, the Landau level splitting is most pronounced when degeneracies occur in the normal state, partially invalidating the analogy to Zeeman splitting suggested by Maniv et al.[15]

We illustrate these points in an approximation where all matrix elements in the pairing blocks are taken to be the

same constant, $-1/2(n_\mu\pi)^{-1/4}\Delta_0\hbar\omega_c$, which is the large-$N$ limit of the matrix element at the chemical potential if $\lambda=1$ and χ is set to unity.[20] For the cases considered in this paper, we take the cutoff ω_D, to be $1/2\mu$ (thus, for $n_\mu=20$, Landau levels 10–30 are involved in the pairing). The resulting eigenvalues for the above three cases are plotted as a function of Δ_0 in Fig. 1. In the large-Δ_0 limit, oscillations in the low-energy quasiparticle eigenvalue spectrum are about the same magnitude and are shifted by half a period relative to the oscillations in the normal state. The expression for the grand potential in terms of quasiparticle energies suggests that this might lead to a π phase shift in the Fourier transform of the magnetization relative to the normal state case, i.e., to a change in sign of the oscillatory contribution to the grand potential. We say ''might'' since it is not obvious, even from Eq. (11), that a phase shift in the oscillations of low-energy quasiparticle energies will necessarily show up as a phase shift of the magnetization. [Equation (11) involves three terms and each contributes strongly to the oscillatory dependence of $M(B)$.]

To examine this idea in more detail we have solved the BdG self-consistently at several different λ values for $n_\mu \in (20,21)$ and calculated coefficients of the Fourier expansions of quantities of interest within this interval. In the normal state the Fourier expansion coefficients vary slowly with the Landau level index associated with the interval over which the Fourier transform is performed, since the dominant variation with n_μ is periodic. In the mixed state we will have to check for this periodicity by verifying that the Fourier expansions in successive intervals are similar. We focus on the coefficient of the leading sine term in the Fourier expansion which is the dominant term in the normal state and refer to the Fourier expansion coefficients as harmonics of the magnetic oscillation; the terminology anticipates a repetition of the same pattern in successive intervals which does not always occur as we discuss in further detail below. For the interval $n_\mu \in (20,21)$ we find that the zero of the fundamental sine harmonic of the Fourier transform of $M(B)$ in this interval does indeed closely correspond to the point where the three curves in Fig. 1 cross. To test the degree of correspondence between the total oscillatory contribution to the grand potential and the contribution from the lowest band of quasiparticles, we have also verified that the quantity

$$\widetilde{E}_1 = -\frac{k_BT}{N_k}\sum_{\vec{k}} \ln\left[2\cosh\left(\frac{E_{\vec{k}}^1}{2k_BT}\right)\right] \quad (12)$$

(with 1 denoting the lowest quasiparticle band) has a magnetization whose fundamental sine harmonic agrees quite closely with that of the total magnetization. Note that $\widetilde{E}_1$ is essentially $-1/2$ the mean of the energies of the lowest quasiparticle band, suggesting that there is some validity in associating magnetization oscillations with oscillations in the low-lying quasiparticle bands. (This similarity of leading harmonics occurs even though the shapes of the two ''magnetizations'' with respect to n_μ are quite different; the correspondence does not hold for higher harmonics.) Finally, we again note the qualitative difference between Fig. 1 and what would be expected if the splittings were simply proportional to Δ as in the Zeeman-splitting analogy proposed by
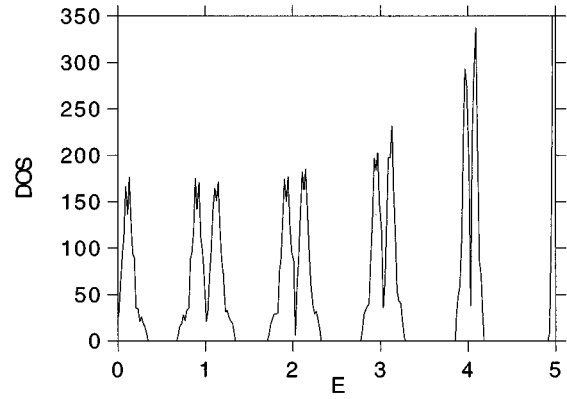

FIG. 2. Density of states versus energy ($\hbar\omega_c$ units) for $\lambda\Delta_0=1$ and $n_\mu=20$.

Maniv et al.[15] In this case, avoided crossing effects at larger Δ do not occur and additional zeros would occur in the harmonics at larger Δ.

The behavior seen in Fig. 1 should be contrasted with the commonly used diagonal approximation, where the only elements retained in the pairing blocks are diagonal in the Landau level index. In this case, the eigenvalues are simply shifted from ξ_N to $\sqrt{\xi_N^2+|F_{NN}|^2}$. In this approximation the level-splitting effect occurs only when $\xi_N=0$; otherwise, all quasiparticle Landau levels are shifted away from the Fermi level. Because of this qualitative failure, we do not feel that the diagonal approximation is useful for understanding the electronic structure of the vortex lattice state except for the Landau level closest to the Fermi level and then only when $n=n_\mu$.

To examine how Fig. 1 is changed when the constant matrix element approximation is abandoned and details of pairing in the vortex lattice state are properly accounted for, we have solved Eqs. (1)–(4) as a function of Δ_0. As discussed in our earlier work,[13] the use of a sharp cutoff when solving the secular matrix leads to spurious effects in $M(B)$ associated with the ratio of the cutoff energy to the cyclotron energy.[23] To eliminate this, we elect to use a smooth cutoff with the pairing interaction between Landau levels N and M scaled by $\sqrt{W_NW_M}$ where
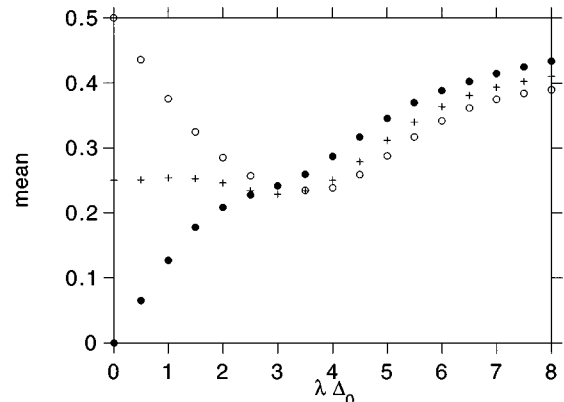

FIG. 3. Mean eigenvalue ($\hbar\omega_c$ units) of the lowest band vs $\lambda\Delta_0$ for the flux lattice. Same notation as Fig. 1.
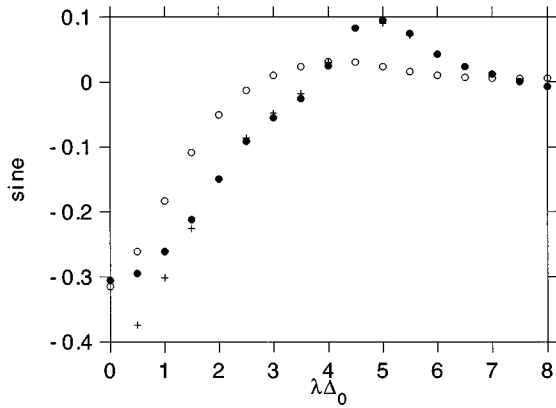
FIG. 4. Fundamental sine harmonic of $M(B)$ (solid points) and of the magnetization of $-1/2$ the mean of the first quasiparticle band (open points) vs $\lambda\Delta_0$ (grand canonical) from the period $n_\mu \in (20,21)$. The pluses are results for $M(B)$ generated in the canonical ensemble.

FIG. 6. Mean eigenvalue ($\hbar\omega_c$ units) of the lowest band vs $\lambda\Delta_0$ for $n_\mu=24$ (solid points), $n_\mu=24.25$ (pluses), and $n_\mu=24.5$ (open points).

$$W_N = 1.55 e^{-(\xi_N/0.5\omega_D)^4}. \tag{13}$$

In Fig. 2 we show a plot of the density of states for $n_\mu=20$ and $\lambda\Delta_0=1$. Each quasiparticle Landau level, not only the Landau level closest to the Fermi energy, is split into two roughly symmetric subbands. This splitting is due to particle-hole mixing. We have been unable to uncover a detailed connection between this splitting and the fact, emphasized by Maniv *et al.*, that two superconducting flux quanta pass through each area of the vortex lattice state enclosing one electronic flux quantum.

In Fig. 3, we show results for the vortex lattice quasiparticle bands which are analogous to those of Fig. 1 obtained using the constant matrix element approximation. The plotted eigenvalue in this case is the mean eigenvalue of the lowest band using a 66-$\vec{k}$-point grid in the irreducible triangle (1/12) of the vortex lattice magnetic Brillouin zone. The results look very similar to Fig. 1 up to the point where the curves cross. This crossing point is close to the point
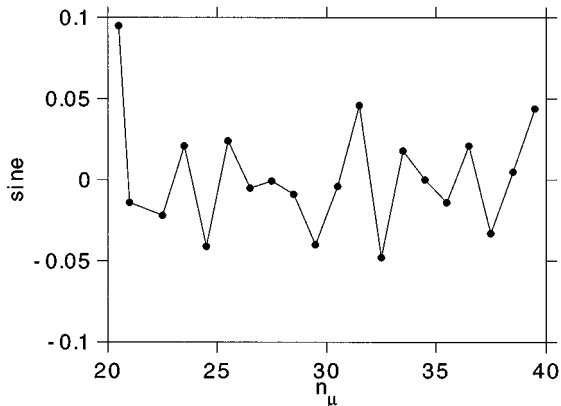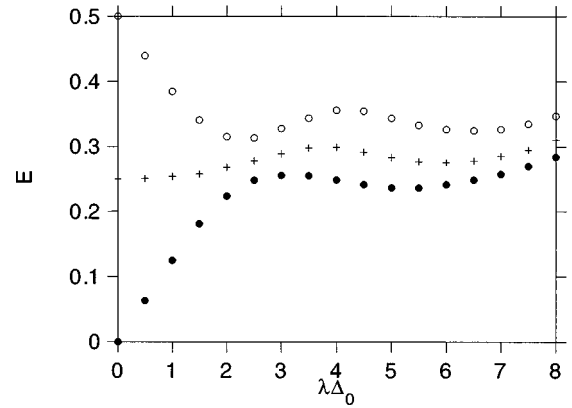
where the spatially averaged pairing self-energy in a coordinate representation ($F_0 \simeq 0.44\lambda\Delta_0\hbar\omega_c$) is equal to $\hbar\omega_c$ (F_0 being the vortex lattice analogue of the BCS gap). For smaller values of $\lambda\Delta_0$, we are in the quantum regime where we expect strong magnetic oscillations. As $\lambda\Delta_0$ increases the dependence of the eigenvalues on n_μ weakens and magnetic oscillations are correspondingly damped. The oscillations are further damped in this regime by the nonzero width of the Landau levels which reflects the nonuniformity of the order parameter. The width is linear in Δ_0 for small Δ_0 and should lead to an exponential suppression of magnetic oscillations with an effective scattering rate linear in Δ_0.[13] At higher values of $\lambda\Delta_0$ we initially enter into the crossover regime and then into the regime where well-defined vortex cores emerge. The fact that the mean eigenvalues increase in this regime reflects the crossover of the lowest-energy quasiparticle states to vortex-core bound states.[13] The eigenvalues clearly still have a substantial dependence on n_μ within the interval (20,21), at least in the crossover regime, although

FIG. 5. Fundamental sine harmonic of $M(B)$ vs n_μ for $\lambda\Delta_0 =4.75$. Each point represents a calculation over a single period.
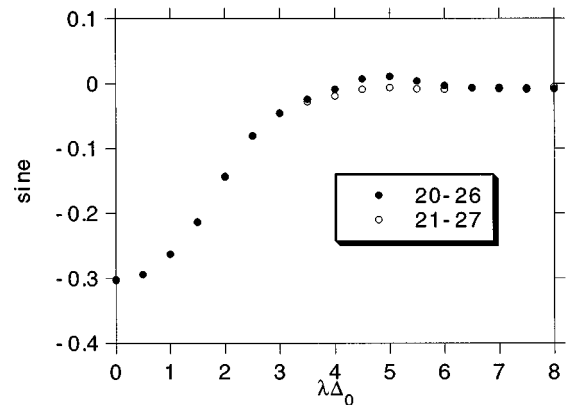

FIG. 7. Fundamental sine harmonic of $M(B)$ vs $\lambda\Delta_0$ for the periods $n_\mu \in (20,26)$ (solid points) and $n_\mu \in (21,27)$ (open points).

the dependence is much weaker than in the constant matrix element approximation.

Up to this point we have been performing calculations at fixed $\lambda \Delta_0$, i.e., at fixed pairing self-energy. To compute the magnetization we should in principle determine Δ_0 self-consistently at each value of n_μ and keep λ fixed. To facilitate comparisons with the preceding results for the quasiparticle bands we have chosen instead to allow λ to vary with n_μ so that self-consistency is achieved at a fixed value of $\lambda \Delta_0$. This self-consistent value of λ at a fixed $\lambda \Delta_0$ ($\tilde{\lambda}$) is easily determined[13] by using Eq. (5) to calculate the output value (Δ_0^{out}) at $\lambda = 1$: $\tilde{\lambda} \equiv \lambda \Delta_0^{\text{in}} / \Delta_0^{\text{out}}$. Results are shown in Fig. 4 for the fundamental sine harmonic of the Fourier transforms of $M(B)$ and $\tilde{E}_1$ versus $\lambda \Delta_0$. A zero in the harmonic of $M(B)$ occurs for $F_0 \sim 1.6 \hbar \omega_c$ (similar results are found for self-consistent calculations at fixed λ). The zero of the harmonic of $\tilde{E}_1$ is close to the zero for $M(B)$ as claimed earlier. We note that in the small-Δ_0 regime, the dependence of the harmonic on Δ_0 contains both linear and quadratic terms. The calculations are consistent with a crossover from quadratic to linear behavior when the quantity $F_0/n_\mu^{1/4}$ exceeds $2\pi k_B T$. We also present in Fig. 4 canonical ensemble results generated from the grand canonical calculations by a Legendre transform. Deviations from the grand canonical ensemble results occur at small Δ_0.[24] The important point, though, is that the zeros of the harmonics in the two schemes agree.

IV. ABSENCE OF PERSISTENT MAGNETIC OSCILLATIONS

The calculations in the previous section discussed the variation of different properties of the vortex lattice state within one particular period $[n_\mu \in (20,21)]$ of the normal state magnetic oscillations. In order for the magnetic oscillations to persist in the vortex lattice state, the same pattern of variation must occur over many periods of the normal state magnetic oscillations. To investigate whether or not this is the case we have studied the dependence of superconducting properties on field through a number of periods of the normal state oscillations. The small-Δ_0 behavior always involves the quantity $\Delta_0/n_\mu^{1/4}$ and retains the normal state magnetic oscillations with reduced amplitude. The zero and subsequent sign reversal of the fundamental harmonic with increasing Δ_0, however, does not occur in every normal state oscillation period. In Fig. 5, we plot the sine of the fundamental harmonic versus n_μ for a value of $\lambda \Delta_0$ equal to 4.75 (where the weak maximum in Fig. 4 occurs). These results show that no clear component of the magnetization with the normal state period survives in the crossover regime. The harmonic of the Fourier tranform of the magnetization in the finite interval from n_μ to $n_\mu + 1$ in this regime varies in sign and magnitude with no pattern we have been able to discern, consistent with results presented in our earlier work.[13] Corresponding variations occur in the lowest band quasiparticle energies. In Fig. 6 we plot the eigenvalue means as in Fig. 3 but for the case $n_\mu \in (24,25)$. One sees that the three curves converge together as in Fig. 3 but this time do not cross when the

crossover regime is entered. That is, the crossing effect of Fig. 3 may or may not occur depending on Landau level index. To emphasize this, we plot in Fig. 7 the fundamental harmonic averaged over two different six-period intervals. We see that the phase shift effect of Fig. 4 has been completely washed out and the fundamental harmonic is smoothly damped to zero as F_0 increases beyond $\hbar \omega_c$.

V. CONCLUDING REMARKS

Experimental evaluations of dHvA oscillation amplitudes are based on Fourier transforms over many periods of oscillations. The results in the preceding section indicate that no measurable oscillation with the normal state period or with any other period we have been able to recognize occurs once the typical value of the pairing self-energy becomes comparable to the Landau level separation. Because we work with relatively small Landau level indices compared with the typical experimental situation, we are not able to completely eliminate the possibility that oscillations in this regime are periodic with a different periodicity or with a periodicity in B rather[25] than in B^{-1}, although we have looked for such patterns without success and are reasonably confident that they do not exist. It seems clear that in the 3D case where many Landau levels contribute, even for a fixed field, magnetic oscillations will be even more strongly suppressed. Disorder broadening, which we have neglected, will damp the oscillatory signal beyond that calculated here.

In conclusion, we have done a detailed analysis of the nature of the quasiparticle states in the field regime near the upper critical field of a 2D type-II superconductor. We find that for small Δ, *all* Landau levels and not just the Landau level at the Fermi energy, are split. This property is associated with the absence of time-reversal symmetry in the presence of a magnetic field. The splitting would be naively expected to lead to a sign change in the fundamental harmonic of the Fourier transform of the magnetization for a value of the pairing self-energy of order the cyclotron energy, analogous to the sign changes which can occur due to the spin splitting of Landau levels. However, our numerical calculations show that once the pairing self-energy is comparable to the normal state Landau level separation, although the spectrum of quasiparticle excitations and the magnetization have sizable variations on the magnetic field scale of the normal state dHvA oscillations, the variations are aperiodic. Accordingly, we find that dHvA oscillations are strongly suppressed once this regime is reached.

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[24] Since the canonical ensemble and grand canonical ensemble results agree in the normal state, Fig. 4 implies a discontinuity in the canonical harmonic as Δ_0 approaches zero. We have verified that the harmonic rapidly crosses over to the normal state result for small Δ_0, with the crossover becoming sharper as the temperature is lowered. This effect was not seen for fixed λ, where we found that the two schemes always agreed quite closely.

[25] For two-dimensional electron systems in a periodic potential, oscillations *do* occur which are periodic in B and are associated with commensurability between the cyclotron orbit diameter and the potential period. See, for example, Dieter Weiss, in *Festkorperprobleme Advances in Solid State Physics*, edited by U. Rössler (Vieweg, Braunschweig, 1991), Vol. 31, p. 341.