Ground state of an antiferromagnetic Heisenberg spin system on a nontranslational lattice of dimension between one and two

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The ground-state properties of the antiferromagnetic, nearest-neighbor, frustrated Heisenberg spin-1/2 system on a nontranslational lattice of dimension $d=\ln 3/\ln 2$ are investigated. The argument is presented indicating that quantum fluctuations, frustration, lack of translational symmetry, and reduction of dimension of the system from $d=2$, but kept above $d=1$, taken together lead to a disordered ground state in quantum antiferromagnets. [S0163-1829(96)07425-5]

I. INTRODUCTION

In quantum spin systems such factors as high coordination number of the interacting moments, large spin S, and high dimensionality of the system suppressing quantum and thermal fluctuations, as well as translational order, favor magnetic order. By contrast, it is generally more difficult to find magnetic order in substances with low coordination and spin S (pronounced quantum fluctuations), low dimensionality, in the presence of frustration, thermal, positional (chemical), or topological disorder disturbing periodicity. A complex interplay of these factors is not rare in nature and leads to nontrivial physical situations. Some of them can be found among the low-dimensional antiferromagnets described by the quantum, spin-$\frac{1}{2}$ Heisenberg model investigated for a long time now.[1,2] Specific types of disorder can be related to noninteger or fractal dimensionality, e.g., that of a percolation cluster backbone in diluted substances near percolation.[3] The topological properties of the backbone of a percolating cluster have been modeled with the help of the Sierpiński gasket.[4] The fractal self-similar structure with geometrical inhomogeneities extended over many length scales is inherent also to many other random physical systems. It has recently been shown that fractal concepts can be helpful in describing the properties of granular superconductors near percolation[5] characterized by the combined effect of disorder and frustration. In this context note that the magnetism of low-dimensional antiferromagnets has been the subject of interest over the past few years mainly due to its relevance to the phenomenon of high-T_c superconductivity. It is widely accepted now that the latter is related to weakly magnetically coupled copper-oxygen layers. They allow a description with the help of the Hubbard model, which in the strong-coupling limit and for a half-filled band can be considered as equivalent to the Heisenberg model with antiferromagnetic coupling:

$$H=J\sum_{(i,j)}\vec{S}_i\cdot\vec{S}_j\,. \tag{1.1}$$

Among many questions regarding the Heisenberg model perhaps the most essential one concerns the existence of long-range magnetic order (LRO) in its ground state. Despite numerous efforts, beyond one dimension the exact ground state of the antiferromagnetic infinite system remains unknown. It is known from the exact Bethe solution that the ground state of a one-dimensional chain of antiferromagnetically coupled Heisenberg spins is disordered: It means that in one dimension quantum fluctuations completely destroy the long-range order conceived as the magnetization of the system. However, in two dimensions the quantum fluctuations sometimes seem not to be strong enough to destroy the long-range order in a Heisenberg antiferromagnet even if they act together with frustration (the case of the Heisenberg spin system on a triangular lattice).[6–10]

The question of magnetic order in the ground state is related to the role the quantum fluctuations represented by the expressions $S_i^+S_j^-+S_i^-S_j^+$ in the Hamiltonian play in destroying the long-range magnetic order in the ground state. As yet most of the results[11–22] concerning triangular and square two-dimensional (2D) lattices indicate the existence of a nonzero staggered magnetization in the ground state, i.e., LRO. This means that in the presence of nearest-neighbor interactions only, quantum fluctuations (in the square lattice case) or quantum fluctuations together with frustration (in the triangular lattice case) seem not to be able to destroy the long-range ordering in the ground state of the 2D antiferromagnetic spin-$\frac{1}{2}$ Heisenberg models on those lattices. More specifically, for an antiferromagnetic square lattice without frustration, i.e., with nearest-neighbor interactions only, there is a well-established opinion that LRO is present. There are also a few indications of a sublattice LRO for a Heisenberg antiferromagnet on a triangular

lattice[11,23–25] as well as the honeycomb lattice,[26] despite the low coordination number $z=3$ of the latter. While magnetic LRO rather persists in the ground states of square, triangular, and honeycomb lattices, perhaps the *kagomé* lattice could be an exception. Essentially, for a *kagomé* lattice with nearest-neighbor antiferromagnetic interactions the situation does not seem to be quite clear as well,[27] although indications exist for a quantum spin liquid state[28–31] for $S=\frac{1}{2}$. Note, however, that for *large S* a *kagomé* lattice attains a coplanar sublattice order.[32] In this respect notice its low coordination number $z=4$, which by the way concerns the fractal system considered here as well, in comparison with $z=6$ in the triangular lattice. Another common point represents the size of the hollow places surrounded by six spins in the *kagomé* lattice and the same perimeter of the least lacuna in the Sierpiński gasket. Hence similarities of the present nontranslational system and the *kagomé* system with translational symmetry noted in the following (Sec. III) should not be surprising.

One way to destroy LRO in the ground state would lead through introducing a disorder of interactions.[33,34] Note that the periodicity of the lattice is maintained in the latter approaches, while only the values of the interactions are randomly varied, except for the limiting case of random bond dilution.[33] Another approach, which can be considered as leading closer to real materials mentioned above, is to refrain from the periodicity of the system and, in addition, to look for the effect of dimensionality. This is what we do here, in particular by reducing the dimension of the system below two. A different but somewhat related problem of the strength of interchain coupling needed for the transition from a disordered phase in one dimension to the ordered one in the two-dimensional square lattice has also been discussed.[35]

Recently, there appeared a number of papers dealing with ferromagnetic properties of Ising spin systems on fractal lattices.[36–46] However, the complexity of a *quantum* spin system on a frustrated fractal lattice appeared prohibitive until very recently.[47,48] It is the latter kind of system which is investigated in the present paper.

II. METHOD

Let us recall the method of creating a Sierpiński gasket lattice. If one takes three equilateral triangles with sides a and attains mutual covering of three of their corners, one obtains a larger triangle of sides $2a$. Connecting three such triangles as before one gets a triangle of sides $4a$. Repeating such a process n times (Fig. 1) one gets a fractal Sierpiński lattice cluster of the order n and dimension $d=\ln 3/\ln 2$. The number N of sites in the cluster amounts to $N=(3^n+3)/2$. The sites of such lattices bear Heisenberg spins with antiferromagnetic interactions between nearest neighbors. So one has a frustrated quantum Heisenberg spin system. Our main aim is to answer the question: Do the quantum fluctuations, frustration, and dimension reduced below two (but kept above one) of the nontranslational antiferromagnetic quantum spin system on such a lattice taken together destroy the long-range magnetic order in the ground state?

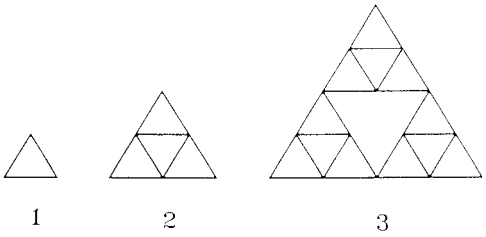

FIG. 1. First three steps of the construction of the Sierpiński gasket lattice having the fractal dimension $d_f=\ln 3/\ln 2$. Number of spins, N, contained in the cluster of the order n amounts to $N=\frac{1}{2}(3^n+3)$.

A. Numerical diagonalization of the Hamiltonian

To get the answer the exact ground state of the homogeneous, antiferromagnetic, frustrated, nearest-neighbor Heisenberg spin-$\frac{1}{2}$ system on the Sierpiński gasket fractal lattice of dimension $d=\ln 3/\ln 2\approx 1.58$ is calculated by standard direct diagonalization[49,50] of the Hamiltonian (1.1) on finite clusters consisting of $N=6$ and $N=15$ spins.

B. Variational Monte Carlo calculations

The same and larger clusters of N up to 1095 spins are examined by a variational Monte Carlo (MC) technique adapted from Huse and Elser.[11] The variational wave function is expanded, following the Huse-Elser[11] method, into the complete orthonormal set of Ising states $|\alpha\rangle$,

$$|\Psi\rangle=\sum_{\alpha}\exp(\tfrac{1}{2}\widetilde{H})|\alpha\rangle, \tag{2.1}$$

and the operator $\widetilde{H}$ containing the variational parameters is diagonal in that base:

$$\widetilde{H}|\alpha\rangle=\lambda_{\alpha}|\alpha\rangle. \tag{2.2}$$

The expectation value of an arbitrary operator $\hat{A}$,

$$\frac{\langle\Psi|\hat{A}|\Psi\rangle}{\langle\Psi|\Psi\rangle}=\frac{\Sigma_{\alpha\beta}\exp(\lambda_{\alpha}^{*}/2)A_{\alpha\beta}\exp(\lambda_{\beta}/2)}{\Sigma_{\alpha}\exp\,\mathrm{Re}\lambda_{\alpha}}$$

$$=\frac{\Sigma_{\alpha}\exp\,\mathrm{Re}\lambda_{\alpha}\Sigma_{\beta}A_{\alpha\beta}\exp\frac{1}{2}(\lambda_{\beta}-\lambda_{\alpha})}{\Sigma_{\alpha}\exp\,\mathrm{Re}\lambda_{\alpha}}, \tag{2.3}$$

where $\mathrm{Re}\lambda_{\alpha}$ stands for the real part of λ_{α}, is easily calculated by the MC method,[11,51] since $\exp(\mathrm{Re}\lambda_{\alpha})$ can formally be treated as a Boltzmann factor attributed to the quantity $\Sigma_{\beta}A_{\alpha\beta}\exp\frac{1}{2}(\lambda_{\beta}-\lambda_{\alpha})$ to be averaged. It is clear that the procedure described above can be applied to any finite system, whether translationally invariant or not.

Now, we shall discuss the way of defining the variational wave function on the Sierpiński gasket lattice. The lattice is divided into three sublattices and the sublattice spin-spin correlation can indicate the eventual magnetic long-range order. Let us label the three interpenetrating equivalent sublattices A, B, C so as to have a site (say, A) surrounded by two pairs of neighbors, each pair belonging to one of the remaining sublattices [two B and two C neighbours; see Fig. 2(a)]. To avoid any distinction among sites we assume boundary con-

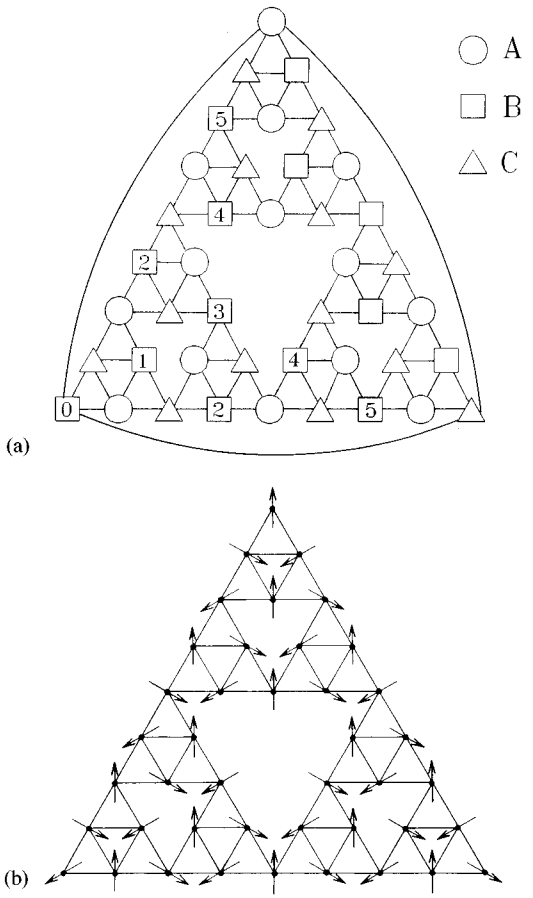

FIG. 2. (a) Partition of the 42-spin cluster taken from a Sierpiński lattice into three interpenetrating equivalent sublattices. Boundary conditions described in the text are marked. The correlation functions $\langle S_0^z S_j^z \rangle$ are calculated for pairs of sites: 0 and those marked $j=1, 2, \ldots$ belonging to the same sublattice B. (b) The 120° classical spin structure on the same cluster.

ditions which guarantee that the corner sites also have four neighbors, each fulfilling the above rule. More specifically, the spins positioned at the three corners of the lattice are allowed to interact mutually with each other with the same strength as common over the system. Note that every attempt at treating the gasket as a building block of a translational lattice would break the requirement of the same coordination number $z=4$ everywhere in the system.[5,36]

The starting point for defining the wave function represents its ''classical'' form (i.e., one with no quantum corrections) in which the spins form a 120° structure[22,11] [Fig. 2(b)] and the corresponding part of the operator $\frac{1}{2}\widetilde{H}$ present in the expansion (2.1) is written as

$$\frac{1}{2}\widetilde{H}_1=\frac{2}{3}\pi i\left(\sum_{i\in B} S_i^z-\sum_{i\in C} S_i^z\right), \tag{2.4}$$

where the sums run over spins on the B and C sublattices, respectively. This wave function provides the energy -0.125/bond, and the three classical sublattice magnetizations lie symmetrically in the xy plane.

Quantum corrections, essentially allowing the system to be free to build correlations among the S^z's, are introduced by adding to $\widetilde{H}_1$ yet another expression

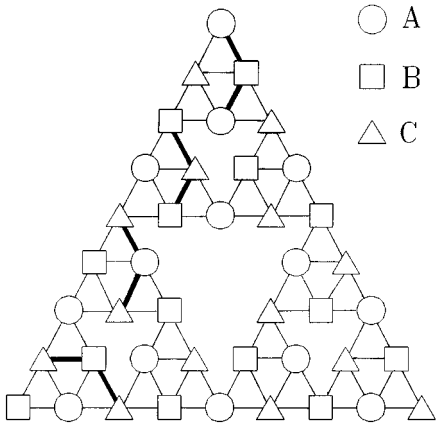

FIG. 3. Examples of the three-spin interactions present in the variational Hamiltonian $\widetilde{H}_3$ [Eq. (2.6)]. $\gamma_{ABA}=+1$, $\gamma_{BCB}=+1$, $\gamma_{CAC}=+1$, $\gamma_{CBC}=-1$. Cyclic permutations of A, B, C do not change the sign of γ_{ijk}.

$$\widetilde{H}_2=\sum_{i,j} kS_i^z S_j^z, \tag{2.5}$$

where $k=K/r^{\sigma}$. K and σ represent variational parameters somehow describing the strength of the correlation and its decay with distance, respectively. At this stage a problem arises of how to measure the distance r. A simplistic approach would suggest the Euclidean distance, as commonly used in translational lattices.[52] Another approach, apparently more suitable to nontranslational lattices with connected parts separated by large lacunas (hollow places), is to take the shortest path over bonds (Manhattan metric) as the physical measure of distance, and this is adopted here. In this way the lacunarity of the fractal lattice, which seems to belong to the essential differences between our system and a translationally invariant one, is taken into account. The term of lacunarity, which does not seem to be rigorously defined mathematically for the Sierpiński gasket (in contrast to the Sierpiński carpet), is used to mark the presence of large hollow places (with a possibility of drawing circles into them) of sizes comparable to the whole cluster at each stage of construction of the lattice. The preliminary calculations concerning the same system,[52] but using the Euclidean distances, have not led to very different conclusions concerning the correlations, but the energy value of the ground state was higher.

The third part of the operator $\frac{1}{2}\widetilde{H}$ present in the expansion (2.1),

$$\frac{1}{2}\widetilde{H}_3=\gamma_{ijk} iL\sum_{i,j,k} S_i^z S_j^z S_k^z, \tag{2.6}$$

contains yet another variational parameter L taking into account the effect of frustration on the wave function by a slight variation of its phase. The three-spin interactions, exemplified in Fig. 3, are chosen so that both neighbors of the spin S_i, i.e., S_j and S_k, belong to the same sublattice A, B, or C. Thus the ground-state energy is minimized with respect to the amplitude of a given ''Ising'' configuration $|\alpha\rangle$ of spins and its phase. The latter provides a factor $\cos[\mathrm{Im}(\lambda_1+\lambda_3)]$ in the energy corresponding to a given Ising configuration of spins, calculated by applying Eq. (2.3).

The choice of $\tilde{H}_3$ in the form given by Eq. (2.6) ensures the most favorable energetically distribution of frustrated bonds. The value $+1$ or -1 of γ_{ijk} is chosen if $i \in A$ and $j \in B$ or $i \in B$ and $j \in A$, respectively. The total variational operator $\tilde{H}$ depending on the parameters K, σ, and L can thus be written in the form

$$\tilde{H}(K,\sigma,L) = \tilde{H}_1 + \tilde{H}_2 + \tilde{H}_3 . \tag{2.7}$$

III. RESULTS

To judge the quality of such an approach to the problem, the ground-state energy E and sublattice spin-spin correlation function $\langle S^z(0)S^z(r)\rangle$ have been calculated by the variational method and compared with exact results obtained by direct diagonalization of the Hamiltonian (1.1) for systems consisting of 6 and 15 spins (second and third Sierpiński gasket construction steps). The usual extrapolation to the thermodynamic limit starts from quantities calculated for small clusters and is based on scaling formulas obtained from spin-wave theory. Such an approach makes no sense, since similarly as for the *kagomé* antiferromagnet[53] there is no good reason to believe that the ground state of an antiferromagnetic system on the Sierpiński gasket shows long-range magnetic order. Instead, the sublattice spin-spin correlation function $\langle S_i^z S_j^z\rangle$ is calculated for various $r = |i-j|$ with both spins belonging to the same sublattice. Thereafter the character of decay of the correlation functions with distance is analyzed to find if it does correspond to an ordered ground state of the system or not.

A. Results of the exact diagonalization of the Hamiltonian

We start by comparing the exact diagonalization result for the six-spin system and the variational calculation using *all* 2^6 Ising states $|\alpha\rangle$ of the complete base in the expansion (2.1) of the variational wave function $|\Psi\rangle$. It happens that for the six-spin system, the variational approach leads to the rigorous singlet wave function for values $K = -93.9$, $\sigma = 0.763$, and $L = 2.62$, corresponding to the energy of -0.25/bond.

For the cluster of 15 spins the exact ground-state energy $E = -0.2208$ and the sublattice spin-spin correlation function $\langle S^z(0)S^z(\sqrt{3})\rangle = 0.0554$. It is worth noting that the exact ground state is fourfold degenerated. Therefore, an average with equal weights over all ground states is taken as the value of the spin correlation function. The conditions imposed by the symmetry of the cluster are fulfilled in this way and the right symmetry properties of the correlation functions to be calculated are ensured.

The origin of the ground-state degeneracy can be explained as follows: The Heisenberg model on a triangle has two fourfold-degenerate energy levels, namely, two Kramers doublets in the ground state ($E = -\frac{3}{4}, S^z_{\rm tot} = \pm\frac{1}{2}$) and a quartet of excited states ($E = \frac{3}{4}, S^z_{\rm tot} = \pm\frac{3}{2}$). The Kramers degeneracy of the ground state is the consequence of the odd number (3) of spins in the system. An additional twofold degeneracy of the ground state comes from the chirality of the triangle.[54] The symmetry group of the 15-spin Sierpiński gasket cluster is C_{3v}, the same as the group of symmetry of three interacting spins on the triangle. The $N = 15$ cluster can be divided into three equivalent sublattices of five spins each. The spins belonging to each sublattice transform among themselves under the action of the elements of the C_{3v} group in a similar way as three interacting spins on a triangle do. In the ground state each sublattice of $S^z_{\rm tot} = \pm\frac{1}{2}$ can be treated as a $\frac{1}{2}$ spin. Thus the degeneracy of the ground state of the 15-spin Sierpiński gasket cluster should be fourfold. For the same reasons, a similar fourfold degeneracy should be present in any Sierpiński gasket cluster consisting of an odd number of spins. The multiplicity of degeneracy of the ground state of the 15-spin Sierpiński gasket cluster might suggest a breaking of rotational symmetry, similarly as it occurs in a single triangle of Heisenberg spins with antiferromagnetic interactions.[54]

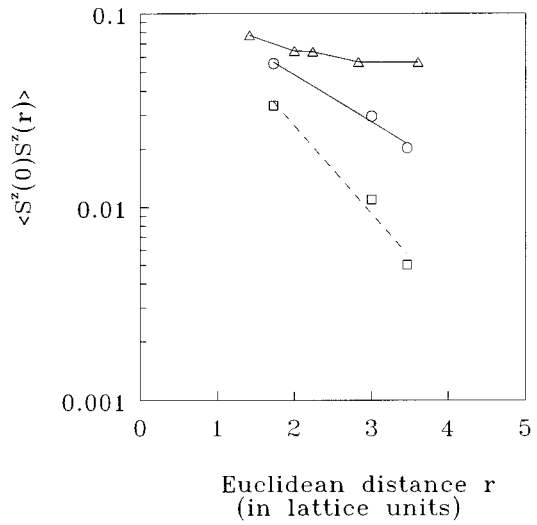

FIG. 4. Plot in logarithmic scale of the spin-spin correlation function $\langle S^z(0)S^z(r)\rangle$ as a function of Euclidean distance r for three pairs of spins situated on the 15-spin cluster taken from a Sierpiński lattice. Each spin of the pair belongs to the same sublattice. Circles, exact results; solid straight line, exponential fit. Squares, values obtained by applying the variational method with all 2^{15} wave functions taken into account. Dashed straight line, exponential fit to the squares. Triangles, values of the correlation functions of a cluster of 26 spins on a square lattice (exact results). Here the negative values have been multiplied by -1. Note a different character of decay of the latter data.

For the sake of comparison, note that on the *kagomé* lattice the cluster containing an odd number N of spins has a degenerate ground state as well.[55]

The values of exact correlation functions obtained by direct diagonalization of the Hamiltonian (1.1) for 15 spins are plotted as a function of r in Fig. 4 and compared with their values obtained by applying the variational MC method. The former values of $\langle S_i^z S_j^z\rangle$ for three different distances $r = |i-j|$ between spins belonging to the same sublattice are well fitted by a straight line in logarithmic scale. The latter set of data shows the same tendency. Hence, the plot reveals an exponential decay of the correlation function with distance r, suggesting a disordered ground state. To check this hint analogous calculations for larger clusters have been performed by applying the variational procedure described above (Sec. II B).

The specific tendency of decay with distance mentioned above becomes even more remarkable if contrasted with the exact results for a system of 26 spins on a square lattice (with periodic boundary conditions), which behaves in a quite different manner (Fig. 4). This is not surprising since the Heisenberg spin system on a square lattice is known to have an ordered ground state.

B. Results of the variational Monte Carlo procedure

The variational procedure applying again the complete set of 2^{15} states of the base to construct the variational wave functions leads in the present case to the approximate minimum ground-state energy value $E(15)=-0.2048$ and sublattice spin-spin correlation function $\langle S^z(0)S^z(\sqrt{3})\rangle = 0.0336$ for $K=-3.56$, $L=-0.427$, and $\sigma=1.38$. This represents a substantial overestimation of energy and an underestimation of the correlation function with respect to the exact value. The difference between the above exact and approximate values of energy (obtained by the variational method and the complete set of functions) amounts to 7%. At this point it should be noted that the approximate minimum ground-state energy value would amount to $E(15)=-0.1982$ if the Euclidean metric were applied [cf. Eq. (2.5)] instead of the Manhattan one, i.e., still 3% worse a value than in the latter case.

We have also applied the MC method as described above to get the ground-state energy value $E_{\mathrm{MC}}(15)=-0.2048$. The accuracy to the fourth digit is achieved at 20 000 MC steps and 1000 MC steps needed to reach equilibrium. Note that to the fourth digit at such a number of MC steps there is no difference between the latter MC result and the value obtained with the variational method using the complete base. Hence at this stage ($N=15$) the MC variational method does not worsen essentially the energy value with respect to that obtained by a variational procedure without further approximations such as choosing only a selection of wave functions. Also, we infer that for the 15-spin cluster of the Sierpiński gasket the ground-state energy value obtained by the variational MC method (similarly as that with a complete set of wave functions) differs by about 7% from the exact one. The MC values of the correlation functions $\langle S^z(0)S^z(r)\rangle$ (see Fig. 5) are calculated for pairs of spins $\langle 01\rangle$, $\langle 02\rangle$, ... $\langle 05\rangle$ [Fig. 2(a)]. Again, similarly as in the case of the above exact results, these approximate correlation functions show an exponential decay with distance r. This forms the basis for judging on how far the following MC calculations deserve confidence. Note that for the finite clusters investigated the variational approximate Monte Carlo results for the correlation functions, unlike the energy value, do not allow one to discern between the Manhattan metric and the Euclidean one within the given limits of accuracy. So we felt free to plot them as a function of the Euclidean distances, as usually done.

It has been checked that the energy minimum for the lattice of 42 spins is attained for values of the variational parameters K, L, and σ different but very slightly from those given above for 15 spins, and so we decided to keep them unchanged in the following calculations.[11] With the assumption of fixed values of the variational parameters made above, the ground-state energy $E=-0.2054$/bond for the largest cluster of 1095 spins. Further the correlation functions have been checked to find how they decay with distance. A least squares linear fit to $\ln\langle S^z(0)S^z(r)\rangle$ vs r has been performed with a high correlation coefficient 0.973. This represents evidence of a disordered ground state,[56] in a contrast to the square and triangular lattices, but probably rather like the *kagomé* lattice. The similarity to the latter appears even closer if one takes into account a very recent result by one of us (P.T.): The Heisenberg spin-$\frac{1}{2}$ antiferromagnetic system on a Sierpiński gasket[48] *and* such a system on a *kagomé* lattice[28,57–59] show a similar marked structure in the low-temperature part of their temperature dependence of the specific heat.

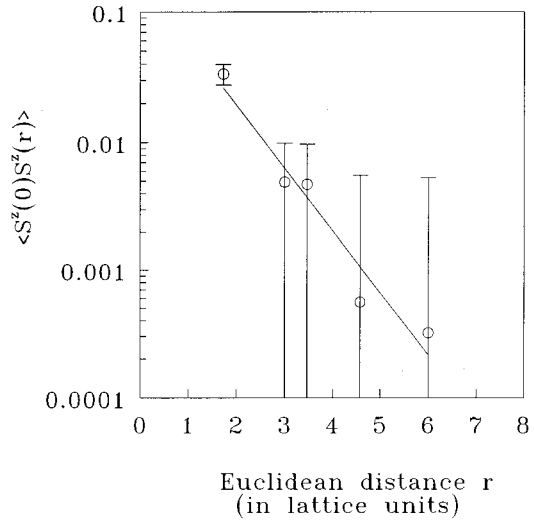

FIG. 5. Plot in logarithmic scale of the spin-spin correlation function $\langle S^z(0)S^z(r)\rangle$, with each spin of the pair on the same sublattice, as a function of Euclidean distance r on the cluster of $N=1095$ spins. Note the exponential decay of correlations with growing r. The error bars represent the statistical errors of the Monte Carlo data and amount to (from left to right) 0.006, 0.005, 0.005, 0.006, 0.005.

It deserves a comment why the conclusion should be based on the correlation $\langle S^z(0)S^z(r)\rangle$ and not on $\langle \vec{S}(0)\cdot\vec{S}(r)\rangle$. It is the consequence of the variational method applied. The point is that the quantum correction part $\widetilde{H}_2$ of the variational Hamiltonian essentially corresponds to enforcing the long-range correlations among the z components of the spins (and not the total quantum spin vector operators) with parametrized weights. However, it happens that the Huse-Elser scheme provides rather bad values of the correlation functions of S^x and S^y operators.[51] We have checked it also for a system of the present form consisting of 15 spins with the same conclusion. Let us recall that the exact correlation function $\langle S^z(0)S^z(r)\rangle$ decayed exponentially with the spin-spin distance r. A similar feature can be seen in Fig. 5, which shows the correlations vs distance for a Sierpiński gasket cluster of 1095 spins calculated by applying the variational Monte Carlo method. There have been performed 240 000 MC steps and 5000 MC steps needed to attain equilibrium. The deviations of particular data points from a straight line stem in our opinion from the fact that for a given r not all sites on the lattice are equivalent from the point of view of symmetry. In other words, for short distances the

correlations depend not only on the value of r, but also on its direction. For smaller clusters a similar linear dependence of $\ln\langle S^z(0)S^z(r)\rangle$ vs r has been observed, but the values of $\langle S^z(0)S^z(r)\rangle$ for larger r revealed relatively large statistical errors due to large fluctuations in small systems. For $N=1095$ the error bars are given in Fig. 5. Note that the logarithmic scale exaggerates the errors the more the larger the distance r. Actually the errors do not vary much with r: cf. the caption to Fig. 5. It appears [cf. Eq. (2.5) with $k=K/r^{\sigma}$] that despite the form of the input variational scheme ''suggesting'' to the system a possibility of an algebraic (nearly hyperbolic) decay of correlations with distance, the system prefers rather an exponential decay. This convinces us that the result is not an artifact of the method.

IV. CONCLUSIONS

We conclude by stating that on applying the exact diagonalization and a variational procedure due to Huse and Elser we have not found evidence for the three-sublattice long-range magnetic order in a Heisenberg spin-$\frac{1}{2}$ system on the Sierpiński fractal lattice. In this respect, as well as some others, it resembles the antiferromagnetic translational *kagomé* lattice. The present result can be considered as a rather strong hint that the ground state of the system considered may correspond to the notion of a spin liquid.

One could also ask a question what would happen to the same antiferromagnet on fractal lattices of different dimension? Examples of Ising systems on a series of Sierpiński-like fractal lattices of dimensions between that considered here and two have recently been examined[60] and the phase transition found first in the limiting dimension two. In our system the only mechanism preventing LRO in the ground state is provided by quantum fluctuations. In the Ising systems considered by Stošić *et al.*[60] a similar role is played by thermal fluctuations which destroy LRO for $d<2$ at any finite temperature. The analogy between the effect of both kinds of fluctuations on LRO enables one to speculate that Heisenberg antiferromagnets on other Sierpiński-like fractal lattices also may not order below dimension of two.

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