Tricritical Lifshitz point in uniaxial ferroelectrics

Afaf Abdel-Hady and R. Folk
Institute for Theoretical Physics, University of Linz, Linz, Austria
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The critical behavior at a Lifshitz tricritical point in systems with a short-range and uniaxial dipolar interaction is calculated using renormalized group theory. We consider the case in which we separated the wave-vector space into the components $\mathbf{k}=(p_x,p_y,q)$, with the dimensions $\widetilde{m}$, m, and $d-m-\widetilde{m}$ for p_x, p_y, and q, respectively. The upper critical dimension d_c was found to be $d_c=2+[(2m+\widetilde{m})/3]$. The critical exponents and the logarithmic corrections have been calculated and compared with the available experimental and theoretical data. [S0163-1829(96)02230-8]

I. INTRODUCTION

As several second-order phase transition lines meet at a specific point, multicritical behavior may arise. Among these special transition points, much interest has been given to Lifshitz points (LP's)[1,2] (for a review, see Ref. 3), where three phases, a paraphase, a ferrophase, and a spatially modulated phase, coexist, and to tricritical points[4] (for a review, see Ref. 5), where a second-order phase transition line becomes first order and where in fact (in an enlarged space of the thermodynamic fields) three second-order lines meet. One may have even lines of such multicritical points. These lines are defined by specific conditions in the thermodynamic field space, and it may happen that these lines cross each other, leading to a Lifshitz tricritical point (LTP).[6–8] The critical behavior at this point has been studied within a model described by the usual Landau-Ginzburg-Wilson Hamiltonian, taking into account also the wave vector dependence of higher-order interactions.[7,8] Recently, a LTP within a variant of the axial next-nearest-neighbor Ising (ANNNI) model has been found.[9]

Physical realizations of such a high-order critical point have been suggested for the antiferrodistortive transition in $RbCaF_3$ (Ref. 10) and liquid crystals.[11] Promising systems for a realization of a LTP are the proper uniaxial ferroelectric $Sn_2P_2(Se_xS_{1-x})_6$ (Ref. 12) and solid mixtures of $(Pb_ySn_{1-y})_2P_2(Se_xS_{1-x})_6$.[13] These systems are of special interest because of the uniaxial dipolar interaction present, and in consequence a new kind of LP, namely, uniaxial dipolar Lifshitz points (ULP's) and also uniaxial dipolar Lifshitz tricritical points (ULTP's), may be observed.[14] The critical behavior at ULP's has already been calculated in one-loop order.[15] Here we study the ULTP's completing the table of exponents presented in Ref. 15.

The strength of critical fluctuations at this high-order multicritical compared with the usual second-order phase transition is the result of several competitive effects. On the one hand, at a Lifshitz point the fluctuations in $d=3$ are enhanced; on the other hand, tricriticality reduces the fluctuations and in the case considered here this reduction is even larger because of the uniaxial dipolar forces. The overall consequence is that, at an ULTP with a special modulation in one direction (this seems to be the physical case), the divergences of susceptibility and the specific heat in $d=3$ are described by mean-field theory with logarithmic corrections. The same is true at the usual second-order phase transition for systems with strong uniaxial dipolar interaction. However, the exponent x, characterizing the logarithmic correction in the susceptibility, is different from the ULTP; one has $x=1/3$ (Refs. 16, 17) instead of $x=1/10$. Without uniaxial dipolar forces one would find nonclassical power-law behavior.[7,8]

The paper is arranged as follows: In Sec. II we introduce the theoretical model (effective Hamiltonian) used in the renormalization group procedure which is presented in Sec. III. There we carry out a field-theoretical renormalization-group analysis[18] and give the general solutions for the renormalized vertex functions related to the susceptibility and the specific heat. In Sec. IV, we relate the critical exponents to the renormalizing functions, and in Sec. V we calculate the renormalization constants and the flow of the coupling constants for the isotropic and anisotropic cases in one-loop order. Finally, in Sec. VI, we summarize our findings and discuss their implications. In Appendix A we list the Feynman's diagrams and the corresponding analytical forms for the vertex functions in one-loop order. Appendixes B and C will include the evaluated integrals in the dimensional regularization scheme.

II. THEORETICAL MODEL

The Ginzburg-Landau-Wilson Hamiltonian suitable to describe LTP-multicritical behavior in systems with short-range interaction[7,8] can be written as

$$\begin{aligned}H=&\frac{1}{2}\int d^dk[r_0+c_0p^2+q^2+d_0p^4]\Phi_{0k}\Phi_{0-k}\\&+\frac{u_0}{4!}\int d^dk_1d^dk_2d^dk_3\Phi_{0k_1}\Phi_{0k_2}\Phi_{0k_3}\Phi_{0-k_1,-k_2,-k_3}\\&+\frac{w_0}{6!}\int d^dk_1d^dk_2d^dk_3d^dk_4d^dk_5\\&\times\Phi_{0k_1}\Phi_{0k_2}\Phi_{0k_3}\Phi_{0k_4}\Phi_{0k_5}\Phi_{0-k_1,-k_2,-k_3,-k_4,-k_5}\\&+\frac{v_0}{3!}\int d^dk_1d^dk_2d^dk_3p_1^2\Phi_{0k_1}\Phi_{0k_2}\Phi_{0k_3}\Phi_{0-k_1,-k_2,-k_3}.\end{aligned} \quad (1)$$

Here $\Phi_0(\mathbf{k})$ represents the scalar order parameter, e.g., magnetization, polarization, etc.. The d-dimensional wave vector

k is decomposed into **p** and **q** components of dimension m and $(d-m)$, respectively, for a m-fold LTP. $r_0=(T-T_c)/T_c$ is the bare reduced relative temperature distance to the phase transition temperature T_c, c_0 and d_0 are the parameters of the dispersion, and the coefficient of q^2 has been chosen to be 1. The coupling constants u_0, v_0, and w_0 may depend on temperature and/or other physical parameters (e.g., pressure, concentration, etc.). At the second-order critical point, only r_0 goes to zero; all other coefficients stay finite, and the terms d_0p^4, w_0, and v_0 turn out to be irrelevant and can be neglected. At a Lifshitz point, both r_0 and c_0 go to zero; then, the term d_0p^4 has to be taken into account. In the region, where $c_0<0$ and $d_0>0$, a second-order phase transition into an incommensurate phase, characterized by a nonzero m-dimensional wave vector, takes place. The ferroelectric phase and the incommensurable phase are separated by a line of first-order phase transitions. All three lines meet at the Lifshitz point. At a tricritical point, both r_0 and the fourth-order coupling u_0 vanish. Thus, we have to take into account the sixth-order coupling v_0. At a LTP, r_0, c_0, and u_0 vanish and all terms shown in (1) have to be taken into account.

So far, the argumentation was based on short-range interactions of the order parameter only. It is known that sufficiently long-range interactions lead to a different critical behavior. Uniaxial dipolar forces, present in ferroelectrics, are such long-range forces, which at a second-order phase transition lead to logarithmic corrections to the mean-field power laws.[16,17] The effect of uniaxial dipolar forces is to suppress fluctuation in wave vector space in the direction of the uniaxiality. For the anisotropic systems with a Lifshitz point, it is important in which direction the uniaxiality of the dipolar forces is directed.[15] Let us first consider the case where it is perpendicular to the m-dimensional subspace. We are at $d=3$, $m=2$, and the appropriate term to be included in the dispersion of (1) is $g_0^2q^2/k^2$.[19] Because of the presence of this term, the q^2 term becomes irrelevant and can be neglected. We also may neglect q^2 in the denominator of the dipolar term. So, finally, the effective Ginzburg-Landau-Wilson Hamiltonian for the ULTP with the m directions perpendicular to the uniaxiality reads

$$H=\frac{1}{2}\int d^dk\left[r_0+d_0p^4+g_0^2\frac{q^2}{p^2}\right]\Phi_{0k}\Phi_{0-k}+\frac{w_0}{6!}\int d^dk_1d^dk_2d^dk_3d^dk_4d^dk_5\Phi_{0k_1}\Phi_{0k_2}\Phi_{0k_3}\Phi_{0k_4}\Phi_{0k_5}\Phi_{0-k_1,-k_2,-k_3,-k_4,-k_5}$$
$$+\frac{v_0}{3!}\int d^dk_1d^dk_2d^dk_3p_1^2\Phi_{0k_1}\Phi_{0k_2}\Phi_{0k_3}\Phi_{0-k_1,-k_2,-k_3}. \quad (2)$$

The physical relevant case is $m=2$ so that q becomes $d-2$ dimensional and $d^dk=d^2pd^{d-2}q$.

The second case to be considered, and which may be realized in the above-mentioned ferroelectrics, is the case where the uniaxial direction lies within the m-dimensional subspace. However, the wave-vector-dependent terms in that direction are irrelevant and the Lifshitz character is changed from an m-fold to an $(m-1)$-fold Lifshitz point.[15] We therefore generalize the model Hamiltonian, Eq. (2), to include three subspaces: an m-dimensional (the subspace of the modulated behavior), an $\tilde{m}$-dimensional, and a $(d-m-\tilde{m})$-dimensional (the subspace of the ''uniaxial'' dipolar behavior) subspace. The physical relevant case is $m=1$, $\tilde{m}=1$. Then the Ginzburg-Landau-Wilson Hamiltonian reads (note that only the p_y^2 term survives in the denominator of the dipolar term)

$$H=\frac{1}{2}d^dk\left[r_0+c_0p_x^2+d_0p_y^4+g_0^2\frac{q^2}{p_y^2}\right]\Phi_{0k}\Phi_{0-k}$$
$$+\frac{w_0}{6!}\int d^dk_1d^dk_2d^dk_3d^dk_4d^dk_5\Phi_{0k_1}\Phi_{0k_2}\Phi_{0k_3}\Phi_{0k_4}\Phi_{0k_5}\Phi_{0-k_1,-k_2,-k_3,-k_4,-k_5}$$
$$+\frac{v_0}{3!}\int d^dk_1d^dk_2d^dk_3p_{y_1}^2\Phi_{0k_1}\Phi_{0k_2}\Phi_{0k_3}\Phi_{0-k_1,-k_2,-k_3}. \quad (3)$$

The Hamiltonians (2) and (3) constitute the two cases we study in this paper. Some aspects will be treated for the general case of arbitrary dimensions m and $\tilde{m}$ in the following.

III. RENORMALIZATION

A. Unscaled Hamiltonian

We study the critical behavior within the field-theoretical renormalization-group theory using the minimal subtraction scheme.[18]

If we choose c_0 as a dimensionless quantity, then the dimensions of the coefficients in Eq. (3) are

$$[r_0]=\mu^2,\quad [c_0]=\mu^0,\quad [d_0]=\mu^{-2},\quad [g_0]=\mu,\quad [\Phi_0]=\mu^{-(d+2)/2},\quad [v_0]=\mu^{2-d},\quad [w_0]=\mu^{6-2d},$$

and μ an arbitrary reference wave number. Therefore the following renormalizations of the original parameters are needed to compensate these poles in the loop expansion of the vertex functions:

$$r_0=Z_\Phi^{-1}Z_r r,\quad d_0=\mu^{-2}Z_\Phi^{-1}Z_d d,\quad g_0=\mu Z_\Phi^{-1/2}g,\quad v_0=\mu^{2-d}Z_\Phi^{-2}Z_v v A_d^{-1},\quad w_0=\mu^{6-2d}Z_\Phi^{-3}Z_w w B_d^{-1},\quad \Phi_0=Z_\Phi^{1/2}\Phi. \tag{4}$$

A_d is an appropriate dimension-dependent factor and $A_d^2=B_d$. There are no pole terms in the q^2/p^2 parts and (in one-loop order) none in the p_x^2, p_y^4 parts of the inverse propagator; therefore, there is no independent Z factor in g, and in one-loop expansion we have $Z_d=Z_\phi=1$.[15] Regarding the phase transition phenomena, the accessible quantities are, for instance, the susceptibility or specific heat. In order to obtain these quantities using the field renormalization-group procedure, we have to calculate the renormalized vertex functions $\Gamma_R^{(2,0)}$ and $\Gamma_R^{(0,2)}$. They are related to the physical unrenormalized vertex functions $\Gamma_0^{(2,0)}$ and $\Gamma_0^{(0,2)}$ by

$$\Gamma_0^{(2,0)}(p,q,r_0,d_0,g_0,v_0,w_0)=Z_\Phi^{-1}\Gamma_R^{(2,0)}(p,q,r,d,g,v,w,\mu),\quad \Gamma_0^{(0,2)}(p,q,r_0,d_0,g_0,v_0,w_0)=Z_r^2\Gamma_R^{(0,2)}(p,q,r,d,g,v,w,\mu).$$

This leads to the following renormalization-group equations for the renormalized vertex functions, representing the specific heat, and susceptibility (we have set $d=3$) stating the μ independence of the bare vertex functions:

$$\left(\mu\frac{\partial}{\partial\mu}+B_w\frac{\partial}{\partial w}+\beta_v\frac{\partial}{\partial v}-[\tfrac{1}{2}\zeta_\Phi+1]g\frac{\partial}{\partial g}-[-2+\zeta_\Phi-\zeta_d]d\frac{\partial}{\partial d}+\zeta_r\left\{\begin{bmatrix}2\\0\end{bmatrix}+r\frac{\partial}{\partial r}\right\}+\begin{bmatrix}0\\1\end{bmatrix}\zeta_\Phi\right)\begin{bmatrix}\Gamma_R^{(0,2)}\\ \Gamma_R^{(2,0)}\end{bmatrix}(p,q,r,d,g,v,w,\mu)$$
$$=\begin{bmatrix}-\hat{B}_r(p,q,d,g,v,w)\\0\end{bmatrix}. \tag{5}$$

As usual, we have defined $\beta_v=\mu\ \partial v/\partial\mu$, $\beta_w=\mu\ \partial w/\partial\mu$, and $\zeta_i=-\mu\,\partial\ln Z_i/\partial\mu$, $i=\Phi,r,g,d$. The inhomogeneity of Eq. (5), which is related to the additive renormalization of the specific heat, has the form

$$\hat{B}_r(d,g)=Z_r^2\mu\frac{d}{d\mu}Z_r^{-2}[Z_r^2\Gamma_0^{(0,2)}(p=0)]_{\rm sing}. \tag{6}$$

Equation (5) could be solved using the method of characteristics,[20] by means of $\mu(l)=\mu l$ and $\zeta_i(l)=\zeta_i(\Omega(l)),\Omega(l)=(g(l),d(l),v(l),w(l))$, and $\beta_j=\beta_j(\Omega(l))$, $j=v,w$, which in the present case leads to the set of the equations

$$l\frac{d}{dl}\mu(l)\equiv\mu(l),\quad l\frac{d}{dl}r(l)\equiv r(l)\zeta_r(l),\quad l\frac{d}{dl}g(l)\equiv g(l)\zeta_g(l),\quad l\frac{d}{dl}d(l)\equiv d(l)\zeta_d(l), \tag{7}$$

$$l\frac{dw(l)}{dl}\equiv\beta_w(l),\quad l\frac{dv(l)}{dl}\equiv\beta_v(l), \tag{8}$$

with the initial conditions $\mu(1)=\mu$, $r(1)=r$, $d(1)=d$, $g(1)=g$, $w(1)=w$, and $v(1)=v$. The flow, Eqs. (7), is solved by

$$\mu(l)=\mu l, r(l)=r\ \exp\left(\int_1^l\frac{d\rho}{\rho}\zeta_r(\rho)\right),\quad g(l)=g\ \exp\left(\int_1^l\frac{d\rho}{\rho}\zeta_g(\rho)\right),\quad d(l)=d\ \exp\left(\int_1^l\frac{d\rho}{\rho}\zeta_d(\rho)\right). \tag{9}$$

Via Eq. (9), a connection between the flow parameter l and the temperature distance $T-T_c$ is made by the condition $r(l)/\mu^2l^2=1$, which assures finiteness of the amplitude functions. The general solutions of the renormalization-group equation reads

$$\Gamma_R^{(N,L)}(r,d,g,v,w,\mu)=(\mu l)^{d+(N/2)[2-d]-2L}\exp\left(L\int_1^l\frac{d\rho}{\rho}\zeta_r\right)\exp\left(\frac{N}{2}\int_1^l\frac{d\rho}{\rho}\zeta\Phi\right)\hat{\Gamma}_R^{(N,L)}\left[\frac{r(l)}{\mu^2l^2},d(l),g(l),v(l),w(l)\right]. \tag{10}$$

Note that the higher-order interactions w_0 and v_0 have different dimensions. A more suitable form of the theory can be obtained by rescaling the variables in (3) such a way that the relevant interactions have the same μ dimension. This is performed in the following section.

B. Scaled Hamiltonian

The specific form of the Hamiltonians (3), where already irrelevant terms have been neglected, allows us to scale away the parameters d_0 and g_0. Indeed, by rescaling the wavelengths

$$\overline{q}=g_0d_0^{1/4}q,\quad \overline{p_x}=p_x,\quad \overline{p_y}=d_0^{1/4}p_y,\quad \overline{r_0}=r_0,\quad \overline{\Phi}_0=\Phi_0d_0^{-m/8}(g_0^2d_0^{1/2})^{-(d-m-\tilde{m})/4},$$

with dimensions $[\overline{p_y}]=\mu^{1/2}$, $[\overline{q}]=\mu^{3/2}$, and $[\overline{\Phi}_0]=\mu^{-(3d-2m-\tilde{m}+4)/4}$, effective interaction coefficients, instead of v_0 and w_0, are introduced (in fact, they are suggested by perturbation theory):

$$\overline{v_0}=v_0 d_0^{-m/4}(g_0^2 d_0^{1/2})^{-(1/2)(d-m-\widetilde{m})} d_0^{-1/2}, \quad \overline{w_0}=w_0 d_0^{-m/2}(g_0^2 d_0^{1/2})^{-(d-m-\widetilde{m})}, \tag{11}$$

with dimensions $[\overline{v_0}]=\mu^{3+m+\widetilde{m}/2-3d/2}$ and $[\overline{w_0}]=\mu^{6+2m+\widetilde{m}-3d}$. Then one may write the rescaled Hamiltonian in the form

$$H=\frac{1}{2}\int d^d\overline{k}\left[\overline{r_0}+\overline{p}_x^2+\overline{p}_y^4+\frac{\overline{q}^2}{\overline{p_y}^2}\right]\overline{\Phi}_{0k}\overline{\Phi}_{-0k}+\frac{\overline{w_0}}{6!}\int d^d\overline{k_1}\cdots d^d\overline{k_5}\overline{\Phi}_{0k_1}\cdots\overline{\Phi}_{0k_5}\overline{\Phi}_{0-k_1,\ldots,-k_5}$$
$$+\frac{\overline{v_0}}{3!}\int d^d\overline{k_1}d^d\overline{k_2}d^d\overline{k_3}\overline{p}_{y_1}^2\overline{\Phi}_{0k_1}\overline{\Phi}_{0k_2}\overline{\Phi}_{0k_3}\overline{\Phi}_{0-k_1,-k_2,-k_3}. \tag{12}$$

The upper critical dimensions for an ULTP, above which fluctuations may be neglected, is found from the condition that the effective couplings are marginal; thus, we have

$$d_c=2+\left(\frac{2m+\widetilde{m}}{3}\right). \tag{13}$$

For the isotropic case, $m=2$ and $\widetilde{m}=0$, the upper critical dimension turns out to be $d_c=3\ 1/3$, and in the anisotropic case, $m=\widetilde{m}=1$, the upper critical dimension is $d_c=3$. This may be compared with the case $g_0=0$, where the effective interaction coefficients are $\overline{v_0}=v_0 d_0^{-m/4} d_0^{-1/2}$ and $\overline{w_0}=w_0 d_0^{-m/2}$ with dimensions $[\overline{v_0}]=\mu^{3+m/2-d}$ and $[\overline{w_0}]=\mu^{6+m-2d}$ and $d_c=3+m/2$.[7,8] The relation between the scaled and unscaled bare vertex functions is given by

$$\overline{\Gamma}_0^{(N,L)}(\overline{p,q,r_0},\overline{v_0},\overline{w_0})=d_0^{-(m/4)(N/2-1)}(g_0^2 d_0^{1/2})^{-(1/2)(N/2-1)(d-m-\widetilde{m})}\Gamma_0^{(N,L)}(p,q,r_0,d_0,g_0,v_0,w_0). \tag{14}$$

Performing the bare loop expansion, pole terms of type $1/\epsilon$ where $\epsilon=d_c(m,\widetilde{m})-d$ appear (using the renormalization group theory with dimensional regularization and minimal subtraction[18]).

The renormalization constants for the rescaled theory are

$$\overline{r_0}=Z_{\overline{\Phi}}^{-1}Z_{\overline{r}}\overline{r}, \quad \overline{p_{y0}}=Z_{\overline{\Phi}}^{-1/4}Z_{\overline{d}}^{1/4}\overline{p_y}, \quad \overline{q_0}=Z_{\overline{\Phi}}^{-3/4}Z_{\overline{d}}^{1/4}\overline{q}, \quad \overline{\Phi}_0=Z_{\overline{\Phi}}^{(1/8)(3d-2m-3\widetilde{m})}Z_{\overline{d}}^{-(1/8)(d-\widetilde{m})}\overline{\Phi},$$

$$\overline{v_0}=\mu^{3+\mathrm{m}+\widetilde{m}/2-3d/2}Z_{\overline{\Phi}}^{(1/4)(3d-2m-3\widetilde{m})+1/2}Z_{\overline{d}}^{-(1/4)(d-\widetilde{m})-1/2}Z_{\overline{v}}\overline{v}\overline{A}_d^{-1},$$

$$\overline{w_0}=\mu^{6+2m+\widetilde{m}-3d}Z_{\overline{\Phi}}^{(1/2)(3d-2m-3\widetilde{m})}Z_{\overline{d}}^{-(1/4)(d-\widetilde{m})}Z_{\overline{w}}\overline{w}\overline{B}_d^{-1}. \tag{15}$$

From the definition of the renormalization factors of the scaled theory, it follows that they depend on the effective interactions only and we have the relation

$$Z_{\overline{i}}(\overline{v,w})=Z_i(d=1,g=1,v=\overline{v},w=\overline{w}). \tag{16}$$

Then for the renormalized vertex functions

$$\overline{\Gamma}_0^{(N,L)}(\overline{p,q,r_0},\overline{v_0},\overline{w_0})=Z_{\overline{\Phi}}^{-(1/2)\{N-(1/2)(N/2-1)[m+3(d-m-\widetilde{m})]\}}Z_{\overline{r}}^{L}Z_d^{-(1/4)(N/2-1)(d-\widetilde{m})}\overline{\Gamma}_R^{(N,L)}(\overline{r,w,v},\mu), \tag{17}$$

the renormalization group equation reads

$$\left\{\mu\frac{\partial}{\partial\mu}+\beta_{\overline{w}}\frac{\partial}{\partial\overline{w}}+\beta_{\overline{v}}\frac{\partial}{\partial\overline{v}}+\frac{1}{2}\left[N-\frac{1}{2}\left(\frac{N}{2}-1\right)[m+3(d-m-\widetilde{m})]\zeta_{\overline{\Phi}}+\left[L+\overline{r}\frac{\partial}{\partial\overline{r}}\right]\zeta_{\overline{r}}+\frac{1}{4}\left(\frac{N}{2}-1\right)(d-\widetilde{m})\zeta_d\right\}\overline{\Gamma}^{(N,L)}$$
$$\times(r,\overline{w,v},\mu)=B(\overline{v,w})\delta_{0,N}\delta_{2,L}, \tag{18}$$

with the general solution

$$\Gamma_R^{(N,L)}(\overline{r},\overline{d},\overline{g},\overline{v},\overline{w},\mu)=(\mu g)^{(N/2-1)(d-m-\widetilde{m})}(\mu l)^{d+(N/2)[2-d]-2L-(1/2)(N/2-1)(d-\widetilde{m}-2m)}(\mu^{-2}d)^{(1/4)(N/2-1)(d-\widetilde{m})}$$

$$\times\exp\left\{\frac{1}{2}\left[N-\left(\frac{N}{2}-1\right)(d-\overline{m}-2m)\right]\int_1^l-\frac{d\rho}{\rho}\zeta_{\overline{\Phi}}\right\}\exp\left(L\int_1^l\frac{d\rho}{\rho}\zeta_{\overline{r}}\right)$$

$$\times\exp\left[-\frac{1}{4}\left(\frac{N}{2}-1\right)(d-\overline{m})\int_1^l\frac{d\rho}{\rho}(\zeta_{\overline{\Phi}}-\zeta_{\overline{d}})\right]\hat{\overline{\Gamma}}_R^{(N,L)}\left[\frac{\overline{r}(l)}{\mu^2l^2},\overline{w}(l),\overline{v}(l)\right].\tag{19}$$

The ζ functions are defined correspondingly, $\zeta_{\overline{i}}=-\mu\,\partial\ln Z_{\overline{i}}/\partial\mu$, and depend on the effective interactions $\overline{v}$ and $\overline{w}$ only,

$$\zeta_{\overline{i}}(l)=\zeta_{\overline{i}}(\overline{\Omega}(l))=\zeta_i(\Omega(l)),$$

with $\overline{\Omega}(l)=(\overline{v}(l),\overline{w}(l))$, $\Omega(l)=(g(l)\equiv1,\ d(l)\equiv1,\ v(l)\equiv\overline{v}(l),\ w(l)\equiv\overline{w}(l))$, and $\overline{i}=\overline{\Phi},\overline{r},\overline{q},\overline{p_x}$. Note that now the two wave vector components $\overline{q}$ and $\overline{p_x}$ have to be renormalized. The β functions $\beta_{\overline{v}}=\mu\,\partial\overline{v}/\partial\mu$ and $\beta_{\overline{w}}=\mu\,\partial\overline{w}/\partial\mu$ also depend on $\overline{v}$ and $\overline{w}$ only, but there are additional terms, one connected with the shift in dimension and the others appearing in two-loop order:

$$\beta_{\overline{v}}(\overline{\Omega}(l))=(g^2)^{-(1/2)(d-m-\widetilde{m})}d^{-1/2-(1/4)(d-\widetilde{m})}(\beta_v(\Omega(l))+\{(d-m-\widetilde{m})(\tfrac{1}{2}\zeta_\Phi+1)-\tfrac{1}{2}[1+\tfrac{1}{2}(d-\widetilde{m})](2-\zeta_\Phi+\zeta_d)\}v),$$

$$\beta_{\overline{w}}(\overline{\Omega}(l))=(g^2)^{-(d-m-\widetilde{m})}d^{-(1/2)(d-\widetilde{m})}\{\beta_w(\Omega(l))+[2(d-m-\widetilde{m})(\tfrac{1}{2}\zeta_\Phi+1)-\tfrac{1}{2}(d-\widetilde{m})(2-\zeta_\Phi+\zeta_d)]w\}.$$

IV. IDENTIFICATION OF THE EXPONENTS

Let us consider the order parameter susceptibility $\chi^{-1}\sim\Gamma_R^{(2,0)}(\overline{r},\overline{d},\overline{g},\overline{v},\overline{w},\mu)$, where

$$\Gamma_R^{(2,0)}(\overline{r},\overline{d},\overline{g},\overline{v},\overline{w},\mu)=(\mu l)^2\exp\left(\int_1^l\frac{d\rho}{\rho}\,\zeta_{\overline{\Phi}}\right)\hat{\overline{\Gamma}}_R^{(2,0)}\left[\frac{\overline{r}(l)}{\mu^2l^2},\overline{w}(l),\overline{v}(l)\right].\tag{20}$$

Using the condition $r(l)/\mu^2l^2=1$, with its solution Eq. (9) for eliminating l, then asymptotically, for $l\to0$, Eq. (20) gives

$$\Gamma_R^{(2,0)}\approx\overline{r}^{(2+\zeta^*_{\overline{\Phi}})/(2-\zeta^*_{\overline{r}})},\tag{21}$$

with ζ_i^* is the ζ_i function taken at the fixed point $\overline{v^*},\overline{w^*}$. Since $\overline{r}\sim t=(T-T_c)/T_c$, this is to compare with

$$\chi^{-1}\approx t^\gamma;\tag{22}$$

thus, we identify the susceptibility exponent γ as

$$\gamma=\frac{2+\zeta^*_{\overline{\Phi}}}{2-\zeta^*_{\overline{r}}}.\tag{23}$$

Considering the wave-vector-dependent susceptibility, we can now define three correlation lengths ξ_x, ξ_y, and $\xi_\parallel$ diverging differently. They can be found from rescaling the wave-vector-dependent vertex function $\chi^{-1}(p_x,\overline{p_y},\overline{q},t)\sim\Gamma_R^{(2,0)}(p_x,\overline{p_y},\overline{q,r,v,w})$,

$$\Gamma_R^{(2,0)}(p_x,\overline{p_y},\overline{q,r,v,w})\approx\overline{r}^{\,\gamma}\Gamma_R^{(2,0)}(p_x\overline{r}^{-1/(2-\zeta^*_{\overline{r}})},\overline{p_yr}^{-(2+\zeta^*_{\overline{\Phi}}-\zeta^*_{\overline{d}})/4(2-\zeta^*_{\overline{r}})},\overline{q\ r}^{-(6+3\zeta^*_{\overline{\Phi}}-\zeta^*_{\overline{d}})/4(2-\zeta^*_{\overline{r}})},\overline{v^*},\overline{w^*}).\tag{24}$$

By comparing with

$$\chi^{-1}(p_x,\overline{p_y},\overline{q},t)\approx t^\gamma\chi^{-1}(p_x\xi_x,\overline{p_y}\xi_y,\overline{q}\overline{\xi_\parallel}),\tag{25}$$

it leads to

$$\xi_x=t^{-\nu_x},\quad \xi_y=t^{-\nu_y},\quad \xi_\parallel=t^{-\nu_\parallel},\tag{26}$$

where

$$\nu_x=\frac{1}{2-\zeta^*_{\overline{r}}},\quad \nu_y=\frac{2+\zeta^*_{\overline{\Phi}}-\zeta^*_{\overline{d}}}{4(2-\zeta^*_{\overline{r}})},\quad \nu_\parallel=\frac{6+3\zeta^*_{\overline{\Phi}}-\zeta^*_{\overline{d}}}{4(2-\zeta^*_{\overline{r}})}.\tag{27}$$

At T_c ($l\to0$), the susceptibility is given by

$$\chi(\bar{q},\bar{p}_x,\bar{p}_y)\sim(\mu l)^{-2}l^{-\zeta^*_{\bar{\Phi}}}\hat{\bar{\chi}}\left(\frac{\bar{q}(l)}{(\mu l)^{3/2}},\frac{\bar{p}_x(l)}{(\mu l)},\frac{\bar{p}_y(l)}{(\mu l)^{1/2}}\right). \tag{28}$$

The correlations decay as a power law for $\bar{q}=0$ with two different exponents η_x and η_y:

$$\chi(\bar{q}=0,\bar{p}_x,\bar{p}_y=0)\approx\bar{p}_x^{-2+\eta_x},\quad \chi(\bar{q}=0,\bar{p}_x=0,\bar{p}_y)\approx\bar{p}_y^{-4+\eta_y}. \tag{29}$$

Using the matching conditions $\bar{p}_x/(\mu l)=1$ and $\bar{p}_y/(\mu l)^{1/2}=1$ in Eq. (28), respectively, we identify

$$\eta_x=-\zeta^*_{\bar{\Phi}},\quad \eta_y=-\frac{4\zeta^*_{\bar{d}}}{2+\zeta^*_{\bar{\Phi}}-\zeta^*_{\bar{d}}}. \tag{30}$$

For the specific heat, $C\sim-\Gamma_R^{(0,2)}(\bar{r},\bar{d},\bar{g},\bar{v},\bar{w},\mu)$, one has

$$\begin{aligned}\Gamma_R^{(0,2)}(\bar{r},\bar{d},\bar{g},\bar{v},\bar{w},\mu)=&(\mu g)^{-(d-m-\tilde{m})}(\mu^{-2}d)^{-(1/4)(d-\tilde{m})}(\mu l)^{d-4+(d-\tilde{m}-m)-(1/2)(d-\tilde{m})}\\&\times\left\{\exp\int_1^l\left[\left(\frac{d-m-\tilde{m}}{2}\right)\zeta_{\bar{\Phi}}+\left(\frac{d-\tilde{m}}{4}\right)(\zeta_{\bar{\Phi}}-\zeta_{\bar{d}})+2\zeta_{\bar{r}}\right]\frac{d\rho}{\rho}\right\}\left(\hat{\bar{\Gamma}}_R^{(0,2)}\left(\frac{\bar{r}(l)}{\mu^2l^2},\bar{w}(l),\bar{v}(l)\right)\right.\\&\left.+\int_1^l\frac{d\rho}{\rho}\exp\left\{\int_l^\rho\left[\left(\frac{d-m-\tilde{m}}{2}\right)(2+\zeta_{\bar{\Phi}})+\frac{d-\tilde{m}}{4}(\zeta_{\bar{\Phi}}-\zeta_{\bar{d}}-2)+2\zeta_{\bar{r}}-4+d\right]\frac{d\rho'}{\rho'}\right\}\bar{B}_{\bar{r}}(\bar{w}(\rho),\bar{v}(\rho))\right).\end{aligned} \tag{31}$$

Asymptotically, with the condition $\bar{r}(l)/\mu^2l^2=1$, Eq. (31) gives

$$\Gamma_R^{(0,2)}(\bar{r},\bar{d},\bar{g},\bar{v},\bar{w},\mu)\approx\bar{r}^{\,\{[(d-m-\tilde{m})/2](2+\zeta^*_{\bar{\Phi}})+[(d-\tilde{m})/4](\zeta^*_{\bar{\Phi}}-\zeta^*_{\bar{d}}-2)+2\zeta^*_{\bar{r}}-4+d\}/(2-\zeta^*_{\bar{r}})}, \tag{32}$$

but

$$C\approx t^{-\alpha}; \tag{33}$$

thus, we identify the exponent α as

$$\alpha=-\tilde{m}\left[\frac{1}{(2-\zeta^*_{\bar{r}})}\right]-m\left[\frac{2+\zeta^*_{\bar{\Phi}}-\zeta^*_{\bar{d}}}{4(2-\zeta^*_{\bar{r}})}\right]-(d-m-\tilde{m})\times\left[\frac{6+3\zeta^*_{\bar{\Phi}}-\zeta^*_{\bar{d}}}{4(2-\zeta^*_{\bar{r}})}\right]+2. \tag{34}$$

It is easy now to see that

$$\nu_x=\nu_y\frac{4-\eta_y}{2-\eta_x}, \tag{35}$$

$$\nu_\parallel=\nu_y\left[3-\frac{\eta_y}{2}\right], \tag{36}$$

$$\gamma=\nu_x(2-\eta_x)=\nu_y(4-\eta_y), \tag{37}$$

$$2-\alpha=\tilde{m}\nu_x+m\nu_y+(d-m-\tilde{m})\nu_\parallel. \tag{38}$$

In order to get the values of the exponents in one-loop order, we have to calculate $\zeta_{\bar{r}},\beta_{\bar{v}}$, and $\beta_{\bar{w}}$ only. This will be done for the two cases $m=\tilde{m}=1$ and $m=2$, $\tilde{m}=0$ separately, and we will use the scaled Hamiltonian (12).

V. FLOW EQUATIONS FOR THE EFFECTIVE COUPLING

A. Anisotropic case ($m=\tilde{m}=1$)

Performing the usual loop expansion of the bare vertex functions in one-loop order (for the diagrams, see Appendix A), we obtain [note that, in the scaled Hamiltonian (12), $c_0=1$, $d_0=1$, $g_0=1$]

$$\Gamma_0^{(2)}(\bar{r}_0,\bar{v}_0,\bar{\mathbf{k}})=\bar{r}_0+p_x^2+\bar{p}_y^4+\frac{\bar{q}^2}{\bar{p}_y^2}+\frac{\bar{v}_0}{2}I_1^{0,4}, \tag{39}$$

$$\begin{aligned}\Gamma_0^{(4)}(\bar{r}_0,\bar{v}_0,\bar{\mathbf{k}})=&\bar{p}_y^2\bar{v}_0-\bar{p}_y^2\bar{v}_0^2[48I_2^{0,6}-44I_3^{2,8}+16I_4^{4,10}]\\&+96I_4^{2,14}-180I_3^{0,12}+144I_4^{0,18}],\end{aligned} \tag{40}$$

$$\Gamma_0^{(6)}(\bar{r}_0,\bar{v}_0,\bar{w}_0)=\bar{w}_0-30\bar{w}_0\bar{v}_0I_2^{0,6}+240\bar{v}_0^3I_3^{0,12}, \tag{41}$$

where the integrals $I_l^{i,j}$ are defined in Appendix B. The vertex functions, Eqs. (39)–(41), have poles in $\epsilon_1=3-d$.

The pole terms are absorbed into the renormalization factors, so that the renormalized vertex functions $\Gamma^{(2)}(\bar{r},\bar{\mathbf{k}}=0)$, $\partial\Gamma^{(4)}/\partial\bar{p}_y^2|_{\mathbf{k}=0}$, and $\Gamma^6(\bar{r},\bar{\mathbf{k}}=0)$ are finite. This leads to

$$Z_{\bar{r}}=1+\frac{1}{24\epsilon_1}\bar{v},\quad Z_{\bar{v}}=1+\frac{5}{12\epsilon_1}\bar{v},$$

$$Z_{\bar{w}}=1+\frac{5}{2\epsilon_1}\bar{v}-\frac{10}{\epsilon_1}\frac{\bar{v}^3}{\bar{w}}, \tag{42}$$

and the nonzero ζ and β functions

$$\zeta_{\bar{r}}=\tfrac{1}{8}\bar{v}, \tag{43}$$

$$\beta_{\bar{v}} = -3\epsilon_1 \overline{v} + \tfrac{5}{4}\overline{v}^2, \quad \beta_{\bar{w}} = -6\epsilon_1 \overline{w} + \tfrac{15}{2}\overline{v}\,\overline{w} - 30\overline{v}^3. \tag{44}$$

By means of $\beta_{\bar{v}} = \beta_{\bar{w}} = 0$, the following fixed points have been found:

$$\text{I:}\ \overline{v^*} = 0, \quad \overline{w^*} = 0; \quad \text{II:}\ \overline{v^*} = \tfrac{12}{5}\epsilon_1, \quad \overline{w^*} = \tfrac{864}{25}\epsilon_1^2. \tag{45}$$

Their stability is found from expanding around the fixed point and looking at the corresponding eigenvalues. They are $\lambda_1 = -3\epsilon_1 + \frac{5}{2}\overline{v}$ and $\lambda_2 = -6\epsilon_1 + \frac{15}{2}\overline{v}$. Thus for $\epsilon_1 > 0$ fixed point II is stable, whereas for $\epsilon_1 < 0$, the nontrivial fixed point I is stable.

The values of critical exponents, calculated for fixed point II read as $\eta = 0$, $\nu = \frac{1}{2} + \frac{3}{40}\epsilon_1$, $\alpha = \frac{1}{2} + \frac{11}{40}\epsilon_1$, $\beta = \frac{1}{4} - \frac{17}{80}\epsilon_1$, and $\gamma = 1 + \frac{3}{20}\epsilon_1$, for the correlation function, correlation length, specific heat, order parameter, and susceptibility, respectively.

Note that $\epsilon_1 = 0$ in the physically relevant dimension $d = 3$. In this case the power laws of mean-field theory are modified by powers of logarithms of t. They may be calculated from the solutions of the flow equation solved at borderline dimension.

1. Logarithmic correction

Solving the flow equation (44), with $\epsilon_1 = 0$,

$$\theta_{\bar{v}} = l\,\frac{d\overline{v}}{dl} = \frac{5}{4}\,\overline{v}^2, \tag{46}$$

we get

$$\overline{v} = -\tfrac{4}{5}\ \ln^{-1} l. \tag{47}$$

Since $\zeta_{\bar{r}} = \frac{1}{8}\overline{v}$ and $l\, d\overline{r}/dl = \overline{r}\zeta_{\bar{r}}$, we have $d\overline{r}/\overline{r} = -x \ln^{-1} l\, dl/l$, with $x = \frac{1}{10}$. Again, by integration one finds

$$\overline{r(l)} = \overline{r(1)}\,|\ln l|^{-x}. \tag{48}$$

Recall the matching condition $\overline{r(l)}/l^2 = 1$ (we take $\mu = 1$ for simplicity); the relation between the flow parameter l and the relative temperature distance reads $l^2 = t|\ln l|^{-x}$ and iteration gives

$$l = t^{1/2}|\ln l|^{-x/2} \approx \frac{1}{2^{-x/2}}\, t^{1/2}\, \frac{1}{2}\, |\ln t|^{-x/2}. \tag{49}$$

It is this relation between the flow parameter and the relative temperature distance which introduces logarithmic powers in the mean-field power laws. E.g., the correlation length reads

$$\zeta \sim t^{-1/2}|\ln l|^{x/2}, \tag{50}$$

the susceptibility reads

$$\chi \sim t^{-1}|\ln l|^{x}, \tag{51}$$

and the specific heat reads

$$C \sim t^{-1/2}|\ln l|^{x/2}. \tag{52}$$

B. Isotropic case ($m = 2$)

In the case of renormalization of the coupling constants and the flow diagram in one-loop order, the bare vertex functions for the case $m = 2$ are given by (see the Appendix C)

$$\Gamma_0^{(2)}(\overline{r_0}, \overline{v_0}, \overline{\mathbf{k}}) = \overline{r_0} + \overline{p}^4 + \frac{\tilde{q}^2}{\overline{p}^2} + \frac{8}{6}\,\overline{v_0} J_1^{5,0}, \tag{53}$$

$$\Gamma_0^{(4)}(\overline{r_0}, \overline{v_0}, \overline{\mathbf{k}}) = \overline{\mathbf{p}}^2 \overline{v_0} - 2\overline{\mathbf{p}}^2 \overline{v}_0^2 [\tfrac{160}{3} J_2^{7,0} - 184 J_3^{13,0} + 144 J_4^{19,0}], \tag{54}$$

$$\Gamma_0^{(6)}(\overline{r_0}, \overline{v_0}, \overline{w_0}) = \overline{w_0} - 80\overline{w_0}\,\overline{v_0} J_2^{7,0} + 640 \overline{v}_0^3 J_3^{13,0}, \tag{55}$$

where the integrals $J_l^{i,0}(\overline{r_0})$ are defined in Appendix C. Now, Eqs. (53)–(55) have the pole $\epsilon_2 = \frac{10}{3} - d$.

Keeping in mind the normalization conditions, the coupling renormalization constants and the corresponding β and ζ functions are

$$Z_{\bar{r}} = 1 + \frac{2}{9\epsilon_2}\,\overline{v}, \quad Z_{\bar{v}} = 1 + \frac{16}{9\epsilon_2}\,\overline{v},$$

$$Z_{\bar{w}} = 1 + \frac{40}{3\epsilon_2}\,\overline{v} - \frac{640}{9\epsilon_2}\,\frac{\overline{v}^3}{\overline{w}}, \tag{56}$$

$$\zeta_{\bar{r}} = \tfrac{2}{3}\overline{v}, \tag{57}$$

$$\beta_{\bar{v}} = -3\epsilon_2 \overline{v} + \tfrac{16}{3}\overline{v}^2, \quad \beta_{\bar{w}} = -6\epsilon_2 \overline{w} + 40\overline{v}\,\overline{w} - \tfrac{640}{3}\overline{v}^3. \tag{58}$$

Because of $Z_{\bar{\Phi}} = Z_{\bar{g}} = 1$, then $\zeta_\Phi = \zeta_g = 0$.

By means of $\beta_{\bar{v}} = \beta_{\bar{w}} = 0$, the following fixed points have been found:

$$\text{I:}\ \overline{v^*} = 0, \quad \overline{w^*} = 0; \quad \text{II:}\ \overline{v^*} = \tfrac{9}{16}\epsilon_2, \quad \overline{w^*} = \tfrac{23}{10}\epsilon_2^2. \tag{59}$$

The relevant eigenvalues are $\lambda_1 = -3\epsilon_2 + \frac{32}{3}\overline{v}$ and $\lambda_2 = -6\epsilon_2 + 40\overline{v}$. For $\epsilon_2 < 0$, then the fixed point I is stable. For $\epsilon_2 > 0$, then the fixed point II is stable. The calculated values of critical exponents can read as $\eta = 1$, $\nu = \frac{1}{2} + \frac{3}{32}\epsilon_2$, $\alpha = \frac{1}{3} + \frac{3}{16}\epsilon_2$, $\beta = \frac{1}{3} - \frac{3}{16}\epsilon_2$, and $\gamma = 1 + \frac{3}{16}\epsilon_2$, for the correlation function, correlation length, specific heat, order parameter, and susceptibility, respectively. Note that $\epsilon_2 = \frac{1}{3}$ at $d = 3$. Finally, with the help of the Eq. (52), the logarithmic correction, which is present at d_c, was calculated and found to be $x = \frac{1}{8}$.

VI. CONCLUSION

We have calculated the asymptotic static critical indices of tricritical Lifshitz behavior in a system with strong uniaxial dipolar interaction in one-loop order. Our results are collected in Table I. The values of the exponents are to be compared with the corresponding values when the uniaxial dipolar interaction is absent and/or one is not at a Lifshitz and/or a tricritical point. From this comparison one might guess the critical exponents to be found experimentally for the different phase transitions realized in the mixtures $Sn_2P_2(Se_xS_{1-x})_6$ (Ref. 12) or $(Pb_ySn_{1-y})2P_2(Se_xS_{1-x})_6$.[13] In fact, one would expect to see a crossover between the different values of asymptotic exponents corresponding to the specific fixed points. However, it is difficult to calculate the complicated crossover behavior because of the strong anisotropy of the complete dispersion. Only some limiting cases have been considered so far, namely, the crossover between uniaxial dipolar and isotropic short-range interac-

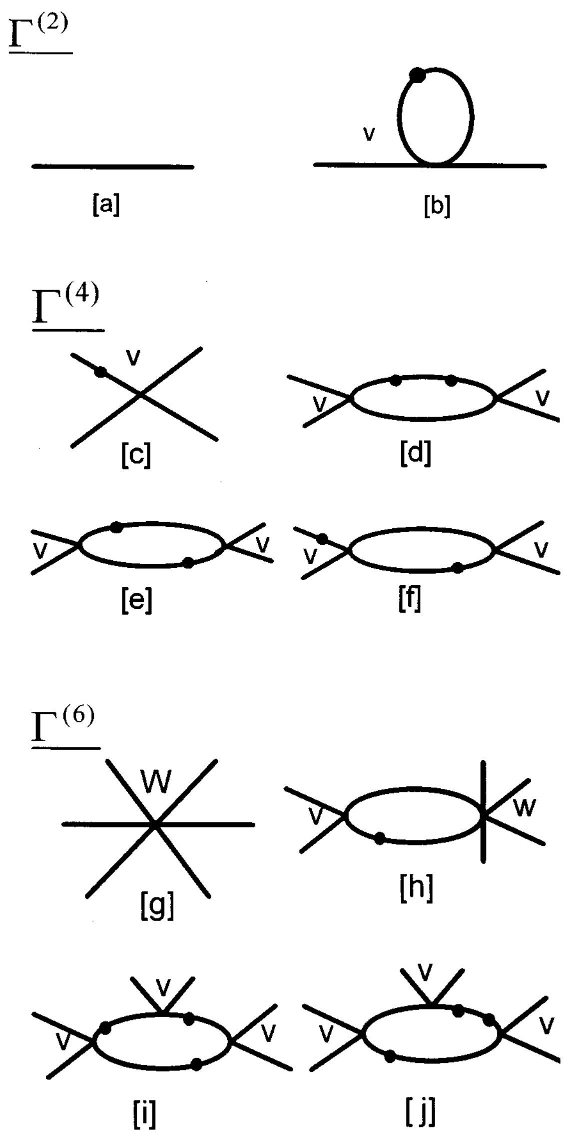

FIG. 1. One-loop order for the vertex functions $\Gamma_0^{(N)}$, with $N=2$, 4, and 6. The dot symbolizes the factor $F_i(\mathbf{p})$ from the $F_i(\mathbf{p})\phi^4$ interaction.

tion for the normal second-order transition[21] and the crossover between the normal second-order behavior and Lifshitz-type behavior.[22] One also has to bear in mind the fact that in solid-state systems other disturbances like defects or elastic coupling to the strain may be of relevance. Taking all this into account, it seems to be promising that indeed near the Lifshitz point in the phase diagram of $Sn_2P_2S_2$ logarithmic corrections to mean-field theory have been found.[23,24]

APPENDIX A: FEYNMAN'S DIAGRAMS AND CORRELATIONS FUNCTIONS

In this appendix, we list the explicit results for perturbation theory in one loop-order using Feynman's diagrams, as shown in Fig. 1, and the corresponding analytical expressions for both the isotropic and the anisotropic cases give the final forms for the vertex functions $\Gamma_0^{(N)}$. The corresponding analytical expressions, according to the graphs in Fig. 1, are (notice that we skipped the bar at the wave vectors; $\mathbf{k}$ denotes the whole wave vector space and $\mathbf{p}$ denotes the subspace perpendicular to the uniaxial dipolar space denoted by $\mathbf{q}$)

$$[a]\equiv G_i^{-1}(\mathbf{k}), \tag{A1a}$$

$$[b]\equiv \frac{\overline{v_0}}{2}\int F_i(\mathbf{p})G_i(\mathbf{k}), \tag{A1b}$$

$$[c]\equiv F_i(\mathbf{p})\overline{v_0}, \tag{A1c}$$

$$[d]\equiv -\tfrac{3}{2}\overline{v_0^2}\int F_i^2(\mathbf{p}_0-\mathbf{p})G_i(\mathbf{k})G_i(\mathbf{k}_0-\mathbf{k}), \tag{A1d}$$

$$[e]\equiv -\tfrac{3}{2}\overline{v_0^2}\int F_i(\mathbf{p})F_i(\mathbf{p}_0-\mathbf{p})G_i(\mathbf{k})G_i(\mathbf{k}_0-\mathbf{k}), \tag{A1e}$$

$$[f]\equiv -6\overline{v_0^2}\int F_i(\mathbf{p})G_i^2(\mathbf{k}), \tag{A1f}$$

$$[g]\equiv \overline{w_0}, \tag{A1g}$$

$$[h]\equiv -30\overline{w_0 v_0}F_i(\mathbf{p})\int F_i(\mathbf{p})G_i^2(\mathbf{k}), \tag{A1h}$$

$$[i]\equiv 60\overline{v_0^3}F_i(\mathbf{p})\int F_i^3(\mathbf{p})G_i^3(\mathbf{k}), \tag{A1i}$$

$$[j]\equiv 180\overline{v_0^3}F_i(\mathbf{p})\int F_i^3(\mathbf{p})G_i^3(\mathbf{k}). \tag{A1j}$$

In the anisotropic case $(m=\widetilde{m}=1)$ we have

$$\int = \frac{S_{d-3}}{(2\pi)^2}\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\pi dp_x dp_y d^{d-2}q\ \sin^{d-4}\theta\ d\theta,$$

$F_1(\mathbf{p})=p_y^2$, and $G_1^{-1}(\mathbf{k})=r_0+p_x^2+p_y^4+q^2/p_y^2$. In Eqs. (A1d) and (A1e), we expand and keep only the p_y^2 terms. The bare vertex functions in Eqs. (39), (40), and (41) could be calculated. In the isotropic case $(m=2,\ \widetilde{m}=0)$ we have

$$\int = \frac{S_{d-3}}{(2\pi)^2}\int_0^\infty\int_0^\infty\int_0^\pi d^2p d^{d-2}q\ \sin^{d-4}\theta\ d\theta,$$

$F_2(\mathbf{p})=p^2$, and $G_2^{-1}(\mathbf{k})=r_0+p^4+q^2/p^2$; then, one has

$$\Gamma_0^{(2)}(r_0,\overline{v_0},\mathbf{k})=G_{02}^{-1}(\mathbf{k})+\frac{8\overline{v_0}}{6}J_1^{5,0}, \tag{A2}$$

$$\Gamma_0^{(4)}(\overline{v_0},\mathbf{k})=\mathbf{p}^2\overline{v_0}-\mathbf{p}^2\overline{v_0^2}[16J_2^{7,0}+32J_2^{7,2}-12J_3^{13,0}-168J_3^{13,2}+144J_4^{19,2}], \tag{A3}$$

and finally

$$\Gamma_0^{(6)}(r_0,\overline{v_0},\overline{w_0})\equiv\overline{w_0}-80\overline{w_0v_0}J_2^{7,0}+640\overline{v_0^3}J_3^{13,0}. \tag{A4}$$

Using Appendix C, it is proved that $J_l^{i,0}=\frac{4}{3}J_l^{i,2}$. By collecting the terms in Eqs. (A2), (A3), and (A4), one can calculate Eqs. (53), (54), and (55), respectively.

TABLE I. Compilation of the critical dimension d_c and critical exponents for the system with one component order parameter, with and without uniaxial dipolar forces, in first order of ϵ at $d=3$. The asterisk indicates that one has logarithmic corrections to the power law (U≡uniaxial dipolar, L≡Lifshitz, T≡tricritical, and m is the dimension of Lifshitz subspace).

System	d_c	α	β	γ	x	Ref.
LT, $m=1$	$3\frac{1}{2}$	$\frac{9}{14}$	$\frac{1}{7}$	$1\frac{1}{28}$		7, 8
LT, $m=2$	4	$\frac{31}{40}$	$\frac{3}{80}$	$1\frac{3}{40}$		7, 8
ULT, $m=1$	3	$\frac{1}{2}$*	$\frac{1}{4}$*	1*	$\frac{1}{10}$	
ULT, $m=2$	$3\frac{1}{3}$	$\frac{19}{48}$	$\frac{13}{48}$	$1\frac{1}{16}$	$\frac{1}{8}$	

APPENDIX B: INTEGRALS OF $I_L^{I,J}(R)$

The integrals required for the calculation of $\Gamma^{(N)}$ in the anisotropic case [see Eqs. (39)–(41)] are

$$I_l^{i,j}(r) = f(d)\int_0^\infty\int_0^\infty\int_0^\infty \frac{p_x^i p_y^j}{[(r+p_x^2)p_y^2+p_y^6+q^2]^l}\, dp_x dp_y d^{d-2}q. \tag{B1}$$

Using the identities

$$\int_0^\infty dx\; x^{a-1}(p+qx^b)^{-c} = \frac{1}{b}\, p^{-c}\left(\frac{p}{q}\right)^{a/b}\Gamma\left(\frac{a}{b}\right) \times\Gamma\left(c-\frac{a}{\nu}\right)\Big/\Gamma(c), \tag{B2a}$$

$$f(d) = \frac{S_{d-3}}{(2\pi)^2}\int_0^\pi \sin^{d-4}\theta d\theta = (d-2)S_d = (3-\epsilon_1-2)S_d = S_d, \tag{B2b}$$

where $S_d^{-1} = 2^{d-1}\pi^{d/2}\Gamma(d/2)$, and consequently perform the integrations with respect to q, p_x, and p_y, it is straightforward to get

$$I_l^{i,j}(r) = \frac{M_l^{i,j}}{16}\, r^{(j+2i+3d-3-6l)/4}S_d, \tag{B3}$$

with

$$M_l^{i,j} = \Gamma\left(\frac{d-2}{2}\right)\Gamma\left(\frac{i+1}{2}\right)\Gamma\left(\frac{j+d-1-2l}{4}\right) \times\Gamma\left(\frac{6l+3-3d-2i-j}{4}\right)\Big/\Gamma(l). \tag{B4}$$

The pole in Eq. (B4) is $\epsilon_1 = 3-d$, and the diverging part of the integrals in our calculations correspond to the values (i,j,l)≡(0,4,1), (0,6,2), (0,12,3), (0,18,4), (2,8,3), (2,14,4), and (4,10,4).

For example, we will show in detail the divergent of the integral $I_1^{0,4}(r)$. Using Eq. (B4) for the values $(i,j,l) = (0,4,1)$, we get

$$I_1^{0,4}(r) = \frac{\pi^{1/2}}{16}\, S_d r^{(-5+3d)/4}\Gamma\left(\frac{d-2}{2}\right)\Gamma\left(\frac{5-3d}{4}\right)\Gamma\left(\frac{d+1}{4}\right), \tag{B5}$$

$$I_1^{0,4}(r) = \frac{\pi^{1/2}}{16}\, S_d r^{(-5+3d)/4}\Gamma\left(\frac{d-2}{2}\right)\Gamma\left(\frac{5-3d}{4}\right)\Gamma\left(\frac{d+1}{4}\right). \tag{B6}$$

By applying ϵ expansion with minimal subtraction we get

$$\Gamma\left(\frac{5-3d}{4}\right) = \Gamma\left(\frac{5-9+3\epsilon_1}{4}\right) = -\frac{4}{3\epsilon_1}\,\Gamma\left(\frac{3\epsilon_1+4}{4}\right) \tag{B7}$$

and

$$r^{(-5+3d)/4} = r^{-(-4+3\epsilon_1)/4} = r\;\exp\left(-\frac{3\epsilon_1}{4}\ln r\right) = r\left(1-\frac{3\epsilon_1}{4}\ln r+\cdots\right) \simeq r. \tag{B8}$$

This implies

$$I_1^{0,4}(r) = -\frac{1}{12\epsilon_1}\, rA_d, \tag{B9}$$

where

$$A_d = \pi^{1/2}\Gamma\left(\frac{d-2}{2}\right)\Gamma\left(\frac{3\epsilon_1+4}{4}\right)\Gamma\left(\frac{d+1}{4}\right)S_d. \tag{B10}$$

APPENDIX C: INTEGRALS OF $J_L^{I,A}(R)$

The integrals required for the calculation of $\Gamma^{(N)}$ in the isotropic case [see Eqs. (53)–(55)] are

$$J_l^{i,a}(r) = \int_0^\infty\int_0^\infty \frac{p^i q^{d-3}}{[rp^2+p^6+q^2]^l}\, X^a dp\; dq, \tag{C1}$$

with

$$X^a = \frac{S_{d-3}}{(2\pi)^2}\int_0^\pi \sin^{d-4}\theta\, \cos^a\theta\; d\theta \quad \text{with}\;\; a=0 \;\;\text{or}\;\; 2. \tag{C2}$$

Using the identity Eq. (B3a) and Eq. (B3c) with

$$X^0 = \frac{S_{d-3}}{(2\pi)^2}\int_0^\pi \sin^{d-4}\theta\; d\theta = (d-2)S_d = (\tfrac{10}{3}-\epsilon_2-2)S_d = \tfrac{4}{3}S_d, \tag{C3a}$$

$$X^2 = \frac{S_{d-3}}{(2\pi)^2}\int_0^\pi \sin^{d-4}\theta\, \cos^2\theta\; d\theta = S_d, \tag{C3b}$$

then performing the q and p integrations, it is straightforward to have

$$J_l^{i,2}(r)=\frac{M_l^i}{8}\,r^{(3d-5-6l+i)/4}S_d \quad \text{(C4a)}$$

and

$$J_l^{i,0}(r)=\tfrac{4}{3}J_l^{i,2}, \quad \text{(C4b)}$$

with

$$M_l^i=\Gamma\left(\frac{d-2}{2}\right)\Gamma\left(\frac{i+d-1-2l}{4}\right)\Gamma\left(\frac{6l-3d+4-i}{4}\right)\Bigg/\Gamma(l) \quad \text{(C5)}$$

containing the pole terms with $\epsilon_2=\frac{10}{3}-d$. The diverging parts of the integrals correspond to the values $(i,l)\equiv$(7,2), (13,3), (19,4), and (5,1).

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