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# Tunneling in two-channel Kondo superconducting junctions

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Charge transport in the short weak links ($K$) between two superconductors ($S$) or between a normal metal ($N$) and a superconductor is considered. The electron scattering in the weak link is accomplished by two-level systems acting as two-channel Kondo impurities. The impact of this interaction on conductance in the $NKS$ tunnel structure as well as on the Josephson current in $SKS$ junctions is analyzed. It is shown that the conductance of $NKS$ systems as a function of temperature $T$ has a dip at low temperatures, an effect similar to that in $NKN$ junctions. The critical Josephson current of $SKS$ junctions shows a temperature dependence which deviates from such a dependence of ballistic superconducting contacts. [S0163-1829(96)09029-7]

In recent experiments on Cu-point contacts,[1,2] an anomaly consisting in a dip in the differential conductance around zero bias was observed. This zero-bias anomaly in the temperature dependence of the conductance was attributed to the interaction of the electrons with two-level systems acting as two-channel Kondo (2CK) impurities.[1,2] Here the role of the spin state is played by some orbital degree of freedom, while the two states of the physical spin play the role of the channel indexes.[3] The 2CK model shows non-Fermi-liquid behavior and was proposed for the explanation of the thermodynamic properties and resistivity of some heavy fermion compounds.[4] Non-Fermi-liquid effects of this model take place because channels are unable to lift completely the degeneracy of the impurities energy level, which tend to overscreen the impurity. A renormalization group study[5] reveals that the effective exchange flows towards an intermediate coupling fixed point. Using simple Abelian bosonization Emery and Kivelson[6] mapped a 2CK system on a resonance level model and recovered information on the dynamic properties as well as exact results known from a Bethe ansatz solution of the thermodynamic properties. [7]

A conformal-field-theory approach which was developed in Ref. 8 for the overscreened multichannel Kondo problem happened to be very fruitful for obtaining the single-particle Green's functions (GF's) and dynamic characteristics of this system in the region of low temperatures. In our calculations of the tunneling current we will use these results. The universal zero-temperature resistivity and the temperature-dependent correction to the resistivity which originates from a leading irrelevant ''dangerous'' operator was found.[8] Unlike the case of a simple Kondo impurity,[9,10] the dip in the conductance around $T=0$ that appears in Cu point junctions was explained[1] by theory[8] using the 2CK model. An alternative explanation of this effect based on the dip in the density of states in disordered metals has been recently suggested by Wingreen, Altshuler, and Meir.[11] Therefore it seems useful to study other tunneling systems like a short constriction between two superconductors or between the superconductor and normal metal that consists of a 2CK-type weak link.

In this paper we calculate the tunneling current in superconductor-2CK-metal-superconductor ($SKS$) junctions. We also consider the limits when one ($NKS$) or both ($NKN$) banks are normal metals. The method we used,[12–14] generally, can be applied to the nonequilibrium as well as to the equilibrium (for example, to obtain the Josephson current) problem. The method deals with two types of GF's: an exact one which describes the entire junction and GF's of every separate component of the junction. Below we denote the separate layer functions by small letters $g$: $g_{L,R}$ describes the banks and $g_b$ stands for a weak link. The first derivatives of $g$ on the coordinates normal to the junction's plane vanish at the interfaces. The theory[12,13] is a convenient tool to investigate nonequilibrium and nonstationary effects in superconducting tunnel structures. However, there is a limitation which comes from neglect of the renormalization of the exact quasiparticle self-energy. For two cases this approximation is justified: for insulating barriers and for the restricted short junctions (point contacts). For such a type of junction the entire voltage drop occurs across the weak link.

We investigate the tunneling current in point junctions which are the objects of experimental interest.[1,2] Some results of the conformal field theory for the 2CK model will be used. The scattering matrix of electrons interacting with 2CK centers is of the main interest. Although this matrix was obtained for the bulk system,[8] we, nevertheless, can use it for the point contacts if the size of the weak link is of the order of Kondo coherence length $\xi_K \cong v_F/T_K$ ($T_K$ is the Kondo temperature). However, the tunneling characteristics are qualitatively correct even for smaller junctions.[1] Let us consider low-temperature superconductors with a critical temperature for the superconducting transition, $T_c < T_K$. In this case for short junctions which have the length of the weak link $2d$ that satisfy inequality $\xi_K < 2d < \xi_c$ the theory[8] can be applied.

The tunneling current is given by the second derivative of the nonequilibrium Green's function $G^<$ of the three-layer $SKS$ system. This derivative is taken at the interfaces between the weak link and superconductors. The method relates the exact GF's of the entire system to the GF's for three isolated layers. Moreover, as usual for the nonequilibrium case, the less than functions $G^<$ are expressed in terms of retarded and advanced GF's and distribution function. Let $M^<$ represent this second derivative of $G^<$ at the left interface $L$,

$$M^< = -(\hbar^2/(2m))\tau_3(\hat{x}\nabla_{\vec{r}})(\hat{x}'\nabla_{\vec{r}'})G^<(\vec{r},\vec{r}')_{\vec{r}=\vec{r}'=L}\tau_3; \tag{1}$$

 

then the current density in the $\hat{x}$ direction normal to the planes of the junction's interfaces assumes the form

$$j(t)=e/(\hbar 2m)\sum_{\vec{p}} \langle \vec{p}t|\mathrm{Tr}P_+(M^<g_L^a-g_L^rM^< \\ +M^rg_L^<-g_L^<M^a)|\vec{p}t\rangle, \quad (2)$$

where $P_+=0.5(1+\tau_3)$; $\tau_3$ denotes the Pauli matrix which acts in Nambu space created by two channels. The superscripts $a$ and $r$ stand for advanced and retarded GF's, respectively. Here $g_L=g_L(L,L)$ is the layer GF of the left electrode at the points of left $SK$ interface; $\vec{p}$ is the component of total momentum parallel to the $SK$ interfaces and Dirac notation for the matrix elements has been used.

Unlike Refs. 13,14, we consider the metallic weak link with the Hamiltonian that includes some constant diagonal potential $U$.[15] Such a potential can be produced for example by the mismatches in Fermi velocities of superconductors and normal metal.[16] If $U>\mu_F$ ($\mu_F$ is the Fermi level), then we have the tunnel junction; when $U<\mu_F$ or $U=0$, then the weak link is the normal metal. At the points of $SK$ boundaries the function $M^<$ in the same way as $G^<$ is related to $M^r$ and $M^a$ (Refs. 12–14) as follows:

$$-\frac{2m}{\hbar^2}M^<=M^r(g_L^<+g_{bL}^<)M^a-M^rg_{bLR}^<M^a(RL) \\ -M^r(LR)g_{bRL}^<M^a+M^r(LR) \\ \times(g_{bR}^<+g_R^<)M^a(RL) \quad (3)$$

Here the GF's of the weak link are involved: $g_{bL}\equiv g_b(LL)$, $g_{bLR}\equiv g_b(LR)$, $g_{bLR}\equiv g_b(LR)$, $g_{bRL}\equiv g_b(RL)$, and $g_R=g_R(RR)$ describes the right superconductor.

The functions $M^a$ and $M^r$ obey the matrix equation which directly follows from the boundary conditions on exact ($G$) and layer ($g$) GF's,

$$\begin{pmatrix} M & M(LR) \\ M(RL) & M(RR)\end{pmatrix}=\frac{2m}{\hbar^2}\begin{pmatrix} g_L+g_{bL} & -g_{bLR} \\ -g_{bRL} & g_R+g_{bR}\end{pmatrix}^{-1}. \quad (4)$$

Equations (3) and (4) are a complete set that permits one to determine the nonequilibrium functions $M_<$. What we need now are GF's for every separate layer $g$ which have vanishing derivatives at $SK$ interfaces. In the barrier region these GF's can be obtained using the solutions found by Affleck and Ludwig[8] and which are based on conformal field theory arguments. To simplify the expression for the current and at the same time not damage the physical results we consider an approximation that takes the positions of two-channel Kondo centers in the middle plane of the weak link; i.e., their $x$ coordinates are equal to zero. In this case we have

$$g_{bRL}=g_{bLR}=-(2m/k_x)a\tau_3\delta_{\vec{p},\vec{p}'}+b(\vec{p})b(\vec{p}')\tau_3\hat{T}(E), \quad (5)$$

where $k_x=\sqrt{p_F^2-p^2}$, $\kappa=i\sqrt{2m(\mu_F-U)/\hbar^2-p^2}$, $p_F$ stands for Fermi momentum, $a=k_x[\kappa\sinh(2\kappa d)]^{-1}$, and $b(p)=m[\kappa\sinh(\kappa d)]^{-1}$. In this equation and also below we drop the index $r$ for the retarded functions. The corresponding advanced forms are obtained by taking Hermitian conjugations of the retarded ones. The interaction of 2CK centers with electrons is given by the scattering matrix $\hat{T}(E)$.[8] In Nambu space it assumes the form

$$\tau_3\hat{T}(E)=(2i\pi\nu)^{-1}\left[1+\frac{24\lambda}{\sqrt{2\pi}}[1-i\epsilon(E)]\sqrt{|E|}\right] \\ \equiv(2i\pi\nu)^{-1}z^{-1}; \quad (6)$$

here $\epsilon(E)$ is the step function, $\nu$ is the electronic density of states, per spin and per channel, and $\lambda$ stands for the coupling constant with 2CK center. $\lambda$ is negative at low temperature[8] and $|\lambda|=\alpha/\sqrt{T_K}$. In Ref. 8 the value of $\alpha$ was assumed to be unity. However, here we consider it as a free parameter which can be determined from experiment.

The two additional GF's that are involved in Eq. (4), $g_{bL}=g_{bR}$, can be obtained from Eq. (5) by simply replacing $a$ in $w=-a\cosh(2\kappa d)$. An $SKS$ contact under constant bias voltage shows a time-dependent current. We calculate the zero-frequency component of this current ($I$) that reflects the $I$-$V$ characteristic of the junction. Next, we insert expressions (5) and (6) into the basic equations (3) and (4) to get the advanced, retarded, and less functions $M^a$, $M^r$, and $M^<$. At this stage, the analysis of the different terms in formula (2) is completed. Next, we calculate the current directly. The energy representation is the most useful for this purpose. Therefore we write Eq. (3) in $E$ representation and after some algebra (see Ref. 14) arrive at our principal result $I=I_{\mathrm{dir}}+I_K$, where

$$I_{\mathrm{dir}}=\frac{2e}{\pi}\mathrm{Re}\left\langle\int_{-\infty}^{\infty}dED\,\mathrm{Tr}P_+[\Gamma^r(g_L^<+Dg_R^<) \\ \times\Gamma^ag_R^a-\Gamma^rg_R^<]\right\rangle, \quad (7)$$

$$I_K=\frac{e}{\pi\hbar}N_i\int_{-\infty}^{\infty}dE\,\mathrm{Tr}[(W^rA+\mathrm{H.c.})-W^rFW^aB]. \quad (8)$$

Here $\langle\ \rangle$ denote a two-dimensional sum over $p$, $N_i$ stands for the number of two-channel Kondo centers in the weak link with constriction area $S$, $D$ is the variable parameter that characterizes the transparency of the barrier, $D=a^2(1+w^2)^{-1}$, and the abbreviation H.c. stands for Hermitian conjugation of the first term. $I_{\mathrm{dir}}$ describes the direct tunneling contribution to the total current, and coincides with the result obtained by Arnold.[13] $\Gamma$ is the resolvent matrix for direct tunneling and has the form

$$\Gamma^{-1}=g_L+Dg_R+w(1-D)\tau_3. \quad (9)$$

The Kondo component $I_K$ of the current depends on the complete resolvent propagator $W$. This propagator incorporates the interaction of electrons with 2CK levels. For its retarded form we can write $(W^r)^{-1}=(\tau_3T^r)^{-1}-\gamma^r$ where $\gamma^r$ is given by

$$\gamma^r=(2m)^{-1}\langle k_xb^2[(g_{1R}^r)^{-1}+Q^r\tau_3\Gamma^r\tau_3Q^r]\rangle, \quad (10)$$

where $Q=\tau_3+ag_{1R}^{-1}$, $g_{1R}=g_R+w\tau_3$, and $b\equiv b(\vec{p})$. The values $A$ and $F$ are related to the nonequilibrium Green's function $G^<$ and therefore they depend on the distribution function

$$2mA=\langle k_x b^2[\hat{Q}^r g_L^< \Gamma^a(P_+(\Gamma^r)^{-1}+D(g_R^a P_+ - P_+ g_R^r)) - D(\tau_3/a+\hat{Q}^r)g_R^<\Gamma^a(g_L^a P_+ - P_+ g_L^r)]\hat{Q}^r\rangle, \quad (11)$$

$$2mF=\langle k_x b^2[\hat{Q}^r g_L^< \hat{Q}^a-(1+a\hat{Q}^r\tau_3)(g_R^{-1})^<(1+a\tau_3\hat{Q}^a)]\rangle + 2mF_K, \quad (12)$$

where $F_K=f(\tau_3 T^a)^{-1}-(\tau_3 T^r)^{-1}f$, $\hat{Q}^a=\Gamma^a\tau_3 Q^a$, $\hat{Q}^r=(\hat{Q}^a)^+$, and $g_{R,L}^<=fg_{R,L}^a-g_{R,L}^r f$; in the $E$ representation $f$ is the Fermi distribution function.

In Eq. (8) the matrix $B$ is a combination of retarded and advanced Green's functions and thus does not depend on $f$,

$$2mB=\langle k_x b^2[\hat{Q}^r(g_L^a P_+ - P_+ g_L^r)\hat{Q}^a]\rangle. \quad (13)$$

So far, we have considered the general case. Equation (4) represents a complicated operator equation in energy space. For the nonequilibrium state, it contains nondiagonal matrix elements. Therefore, the entire problem requires numerical calculations. Below, we concentrate on the cases when an analytical solution is possible. The approximation which will be used corresponds to the metallic barrier with no elastic reflection at the boundaries ($U=0$, $D=1$). For $NKS$ junctions we have $g_L^r=i$, $(E|g_L^<|E')=-2i[f(E+eV)P_+ + f(E-eV)P_-]\delta_{E,E'}$, and $g_R^r=i(\epsilon_1+\eta\tau_1)$ with

$$\epsilon_1=\frac{|E|\theta(|E|-\Delta)}{\sqrt{E^2-\Delta^2}}+\frac{E\theta(\Delta-|E|)}{i\sqrt{\Delta^2-E^2}}, \quad \eta=\frac{\epsilon_1\Delta}{E}. \quad (14)$$

Here $\theta(x)=1$ if $x>0$, and $\theta(x)=0$ for $x<0$.

If the right electrode is also normal metal ($NKN$ junction), then $g_R^r=i$. This case was discussed in Refs. 1,2. Inserting these expressions for GF's into formulas (10)–(13) we get $A=-if(E)P_+\pi\nu$, $F=-i\pi\nu\{f(E+eV)+f(E)[1+2(z^r+z^a)]\}$, $B=-iP_+\pi\nu$, $\gamma=-i\pi\nu$, and

$$(W^r)^{-1}=i\pi\nu[1+2z^r]. \quad (15)$$

Equations (7) and (8) are simplified and we arrive at the conductance $\sigma_{nn}$ which has a dip as the function of temperature around $T=0$ and the $\sqrt{T}$ dependence,

$$\sigma_{nn}=R_S^{-1}\left(1-\frac{2e^2N_iR_S}{9h}+\frac{16\alpha e^2N_iR_S}{9h}\sqrt{\pi T/T_K}\right). \quad (16)$$

The zero-temperature contact resistance $R_0$ is related to the Sharvin resistance $R_S$ as $R_0=R_S(1+2e^2N_iR_S/9h)$. Sharvin conductance $R_S^{-1}=Sp_F^2/(4\pi^2)$ defines the total number of tunneling channels $N_t$; thus $R_S^{-1}=e^2N_t/h$. In experiments[1,2] the number of 2CK centers $N_i\ll N_t$. Therefore for the small number of 2CK channels $R_0=R_S[1+2N_i/(9N_t)]$ and we can rewrite Eq. (16) in the form

$$\sigma_{nn}=R_0^{-1}\left(1+\frac{16\alpha N_i}{9N_t}\sqrt{\pi T/T_K}\right). \quad (17)$$

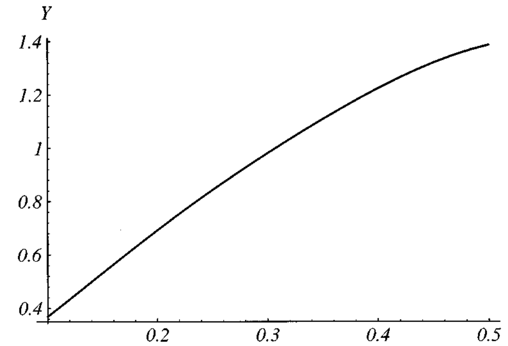


FIG. 1. The temperature dependence of the 2CK part of the conductance. Plot $Y$ [see Eq. (3)] as a function of $t=\sqrt{T/\Delta}$. The region of low temperatures $T\ll T_c$ has been considered.

The $NKS$ junction contains one electrode which is a superconductor. Therefore Andreev reflection takes place, an effect which is especially pronounced at low temperatures $T\ll\Delta$. With the help of Eq. (14) we can calculate all the values $A$, $B$, $F$, and $W$ and find the conductance $\sigma_{sn}$ of $NKS$ junctions. Let us first consider the case which has direct contact with Blonder-Tinkham-Klapwijik[17] (BTK) and Beenakker[18] theories. In the limit of a pure ballistic junction ($z\to\infty$) at zero temperature, $\sigma_{sn}=2\sigma_{nn}$, the well-known BTK result. This relation also holds if we have a resonant level[18] in the junction ($\mathrm{Re}z\to 0$). The full resolvet of this level includes only tunneling rates [see $W^{-1}$ for $NKN$ in Eq. (15) ($z=0$)]. A similar situation takes place in the insulating or disordered semiconducting barriers with resonant levels (see Refs. 14, 18, 19 and recent work[20]). The resonant tunneling through these levels defines the current. Here the conductance for the single channel is determined by the tunneling widths which deviate from unity. Therefore, Andreev reflection does not exactly double the $\sigma_{nn}$ conductance in $NKS$ junctions.[18] This conclusion is valid for a fixed energy and position of the resonant level. After averaging over the position and the energy of resonance impurity is performed[14,19,20] the real conductance deviates from theory.[17,18] The scattering from a 2CK center or from a one-channel Kondo impurity strongly interferes with the tunneling, which is a cause for more complicated relations following from Eq. (8) and Eq. (6).

Two models (2CK and one-channel Kondo) differ in the unitary limit ($\lambda=0$) where we have [see Eq. (6)] $\hat{T}_{\mathrm{Kondo}}=2\hat{T}_{\mathrm{2CK}}$. For our purpose the difference between the energy dependent terms in Eq. (6) is more important, because they contribute to the temperature dependence of conductance. For the one-channel Kondo case we have $E^2$ behavior[8] which results in a small $T^2$ correction to the zero-temperature resistivity. In the case of the 2CK system the difference between the zero-voltage conductance of $NKS$ junctions and the zero-bias anomaly in conductance of $NKN$ point contacts as the function of temperature becomes

$$[\sigma_{ns}(T)-\sigma_{nn}(T=0)]_{V\to 0}=R_1^{-1}\left(1+\frac{2\alpha N_i}{N_t}Y\sqrt{\frac{\Delta}{T_K}}\right), \quad (18)$$

where $R_1=R_S(1+1.54N_i/N_t)$. The function $Y$ is plotted in Fig. 1 and shows near $\sqrt{T}$ dependence as in the normal

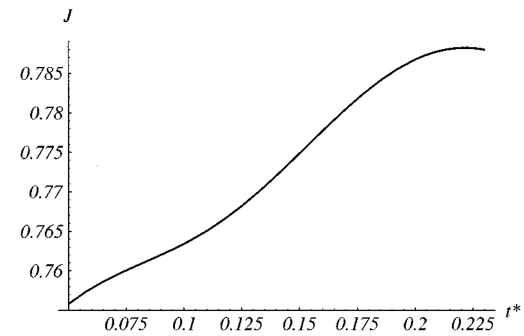


FIG. 2. The 2CK component of the Josephson current $J$ as a function of reduced temperature $t^* = T/\Delta$ for particular values of the Josephson phase, $\phi = \pi/2$ and $\alpha = 0.07$.

$NKN$ structures. The unity in brackets of Eq. (18) is the consequence of Andreev reflection at the $NS$ boundary.[15]

A nonzero constant voltage bias applied to a $SKS$ Josephson junction causes the strong time dependence of the left-hand GF via its phase difference: $g_L(t,t') = \exp[-\phi(t)\tau_3/2]g(t-t')\exp[\phi(t')\tau_3/2]$, where the Fourier transform of $g(t-t')$ is given by Eq. (14). When $V=0$, only the constant Josephson phase $\phi$ remains and $g_L$ takes the form $g_L = i[\epsilon_1 + \eta\tau_1\exp(i\phi\tau_3)]$. Substituting this definition for $g^r$ and a similar expression for the advanced GF in Eqs. (7) and (8), we obtain the Josephson current. However, the best way to perform the calculation of Josephson critical current is to start directly from the thermodynamic (Matsubura) representation of Eqs. (7) and (8) and GF's. The dc Josephson effect in clean point contacts corresponds to the ballistic motion of carriers and was studied by Kulik and Omel'ynchuk (KO).[21] At low temperatures $T \ll T_c$ the current phase relation deviates from $\sin\phi$, resulting in a higher Josephson critical current than that of tunnel junctions. Interaction with 2CK levels brings an additional contribution to the current. Thus the total Josephson current can be expressed as the sum of two terms $I = \pi\Delta(J_{\mathrm{KO}} + JN_i/N_t)/(e\tilde{R}_S)$ where $\tilde{R}_S = R_S(1+N_i/N_t)$,

$$J_{\mathrm{KO}} = T\Delta\sin\phi \sum_{n=-\infty}^{\infty} [\omega_n^2 + \Delta^2\cos^2(\phi/2)]^{-1}$$
$$= \sin(\phi/2)\tanh\left(\frac{\Delta\cos(\phi/2)}{2T}\right), \quad |\phi| < \pi, \tag{19}$$

$$J = 4T\Delta\sin\phi \sum_{n=-\infty}^{\infty} \{\Omega_n^2 S^2 + 4[\omega_n^2 + \Delta^2\cos^2(\phi/2)]$$
$$+ 4\omega_n S\Omega_n\}^{-1}. \tag{20}$$

Here $\Omega_n = \sqrt{\omega_n^2 + \Delta^2}$ where $\omega_n = \pi(2n+1)$ are odd Matsubara frequencies; $S = 1 + 24\lambda\sqrt{(|\omega_n|)/(2\pi)}$ is related to the scattering matrix [see Eq. (6)].

For the particular values of the phase difference $\phi = \pi/2$, the ratio $\Delta/T_K = 0.1$, and $\alpha = 0.07$, we plot (see Fig. 2) $J$ as a function of the reduced temperature $t = T/\Delta$. This figure shows that the Kondo component of the critical current increases with $T$, while the $J_{\mathrm{KO}}$ term decreases as a function of temperature. However, the number of Kondo centers $N_i$ has to be not too small to make this effect observable.

To compare this effect with related effects, we note that the critical Josephson current also has been calculated in the case of resonant tunneling in disordered tunnel junctions[14,22,23] and for a one-channel Kondo impurity in a tunnel barrier.[24] (There is a difference for the Josephson current at $T \ll T_c$ in Refs. 14 and 23. This deviation, probably, occurred because a phase-dependent contribution to the spectrum in Ref. 23 was missed.) Aslamazov and Fistil'[22] considered the resonant tunneling in superconductor-semiconductor-superconductor junctions through periodically arranged impurity atoms, where the important parameter is the width of the impurity band. For a not too thick weak link only one impurity tunneling is relevant. Let us put this impurity in the middle of the barrier and fix the energy of the resonant level at resonance. Then the Josephson current from a such level is proportional to the (KO) contribution [Eq. (19)] per one channel [see the equation after Eq. (58) in Ref. 14]. The same is valid for the Kondo impurity [Eq. (5) of Ref. 24 with equal tunneling rates]. In this case at $T=0$ the impurity spin is screened and the situation is similar to the fixed resonance level problem. In the ballistic junctions $(2d > \xi_K)$ which we consider here, the 2CK centers depress the Josephson current ($\tilde{R}_S$ grows) in the unitary limit for the Kondo scattering $(\lambda = 0)$. They decrease the conductance $R_0^{-1}$ [Eq. (17)] in the $NKN$ junction. However, at low temperatures $T \ll T_K$ the 2CK model contains an interaction which vanishes at zero temperature; i.e., it is irrelevant in the renormalization group sense. This interaction, represented by the term which is proportional to $\lambda$ in Eq. (6), causes an inelastic transport which can provide additional $J_{\mathrm{KO}}$ supercurrent with strong temperature dependence. This effect is like the formation of a dip in the conductance of $NKN$ and $NKS$ junctions.

An alternative explanation for experimentally observed dip in the conductance of $NKN$ junctions[11] is related to the disordered point contacts. This explanation is based on the known dip in the density of states caused by the interactions between electrons. For the Josephson junction we can use the expression for the critical Josephson current in a dirty point contact[25,26] to estimate the effect of an electron-electron interaction. If $T \ll T_c$, then we have[26]

$$I_d = \frac{\pi\Delta}{e} G\cos(\phi/2)\operatorname{arctan}h[\sin(\phi/2)]. \tag{21}$$

Here $G$ is the average conductance which has a dip as a function of temperature, $G = G(T=0)[1+\Gamma_d(T)]$. For a three-dimensional (3D) system,[27]

$$\Gamma_d(T) = \frac{\sqrt{T}}{\nu(\hbar D_0)^{3/2}}. \tag{22}$$

Here $D_0$ denotes the diffusion coefficient. Thus there is a noticeable difference between the Josephson current of dirty point contacts [Eq. (21)] and that of a ballistic junction [Eqs. (19) and 20)]. The current-phase relationship at $(T \ll T_c)$ of a ballistic point contact in the regime of the direct tunneling [the KO part, Eq. (19)] and a disordered point junction shows large deviation in the vicinity of $|\phi| \approx \pi/2$ (see Fig. 2 of Ref. 26). In the disordered limit also the coefficient of the temperature-dependent term $\sim \sqrt{T}$ is a strong function of the electron mean path. We suggest that these characteristic features, the current phase relation, make the investigation of the Josephson junctions promising for fixing the real mechanism of current transport in the considered systems.

In conclusion, we have calculated the conductance of $NKS$ junctions and the Josephson current in $SKS$ point junctions. In both cases, a new temperature-dependent contribution appears—an effect which reflects the importance of the interaction between the carriers and the 2CK centers. Future experimental investigation of these junctions will be helpful for an analysis of effects which are related to the existence of 2CK centers.

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