Dynamic dielectric properties of a bounded solid-state plasma and a two-dimensional electron sheet: Inverse dielectric function and coupled collective modes

N. J. M. Horing, T. Jena, and H. L. Cui
Department of Physics and Engineering Physics, Stevens Institute of Technology, Hoboken, New Jersey 07030

J. D. Mancini
Department of Physics, Fordham University, Bronx, New York 10458
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We examine the dynamic dielectric response properties of a planar two-dimensional (2D) electron system embedded in and coupled to a semi-infinite, local dynamic dielectric medium whose bounding surface is parallel to the 2D sheet at a distance z_0 from it. For this system, we carry out an explicit position space inversion of the longitudinal dielectric function, and analyze the concomitant coupled collective plasmon dispersion relation. [S0163-1829(96)00828-4]

I. INTRODUCTION

The confined geometries of semiconductor nanostructures have yielded an impressive array of plasmons[1,2] for quantum wells, wires, dots, periodic superlattices of such structures, and other spatial configurations. Even within the simplest description of the random-phase approximation, it is clear that such longitudinal collective modes can couple and interact among themselves. In this paper we treat the dynamic dielectric properties of a planar two-dimensional (2D) electron sheet (in a quantum well or inversion layer) embedded in and coupled to a semi-infinite, local dynamic dielectric medium whose bounding surface is parallel to the 2D sheet at a distance z_0 from it. Our analysis involves the construction of the direct longitudinal dielectric function $\varepsilon(1,2)$ $(1=x_1,y_1,z_1,t_1=\mathbf{r}_1,t_1=\overline{r_1},z_1,t_1,$ etc.) for the combined system of this geometry, and its explicit inversion in position representation. The importance of the inverse dielectric function $K(1,2)$ stems from its significance as a propagator of longitudinal potential in the dynamic, inhomogeneous system at hand. Here, the effective potential $V(1)$ due to an impressed potential $U(2)$ is given by the linear functional relation $V(1)=\int d^4(2)K(1,2)U(2)$, or $K(1,2)=\delta V(1)/\delta U(2)$ in terms of variational differentiation. One could alternatively examine the electrostatic fields for the present geometrical configuration to treat the coupled mode dispersion relation in a straightforward manner. However, our explicit determination of the inverse dielectric function of the combined system provides not only the coupled mode frequencies at the poles, but also their excitation amplitudes (oscillator strengths) as given by the residues at the poles. In this model calculation, the medium on the far side of the local semi-infinite bulk dielectric is taken to be vacuum, as shown in Fig. 1.

While there is spatial translational invariance of this system in the plane parallel to the 2D electron sheet and the semi-infinite plasma interface, so that Fourier transformation $\overline{r}_1-\overline{r}_2\rightarrow Q$ and $t_1-t_2\rightarrow\omega$ is useful, the essential spatial inhomogeneity of this confined system in the perpendicular z direction renders the Fourier-transform technique useless in this direction. We therefore proceed with the determination of the inverse dielectric function $K(1,2)$ explicitly by executing the inversion of the direct dielectric function $\varepsilon(1,2)$ in the z representation, employing the condition

$$\int d^4 3\, K(1,3)\varepsilon(3,2)=\delta^4(1-2)=\delta^3(\mathbf{r}_1-\mathbf{r}_2)\,\delta(t_1-t_2). \tag{1}$$

Or, using Fourier transformation in the parallel plane,

$$\int_{-\infty}^{\infty} dz_3 K(z_1,z_3;\overline{Q},\omega)\,\varepsilon(z_3,z_2;\overline{Q},\omega)=\delta(z_1-z_2). \tag{2a}$$

Suppressing $\overline{Q}$ and ω,

$$\int_{-\infty}^{\infty} dz_3 K(z_1,z_3)\,\varepsilon(z_3,z_2)=\delta(z_1-z_2). \tag{2b}$$

This inverse relation holds for the direct and inverse dielectric functions of any (planar) plasma geometry, including that of the 2D plasma sheet alone, that of the semi-infinite dielectric medium alone, and that of the combined system presently under consideration.

Our determination of $K(1,2)$ in direct position representation proceeds as follows: based on earlier experience[3] with

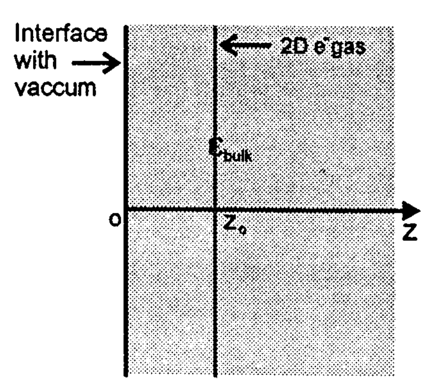

FIG. 1. Planar 2D electron sheet parallel to and at a distance z_0 from the interface of a semi-infinite plasma with vacuum.

semi-infinite (and slab) solid-state plasmas, we employ the known result for the semi-infinite dielectric in the local limit, $K_{\mathrm{semi}}(1,2)$, and invert it to obtain $\varepsilon_{\mathrm{semi}}(1,2)$ in position representation using Eq. (2). Considering next the 2D electron sheet at z_0, we employ the well-known 2D result[4,5] for $\varepsilon_{2\mathrm{D}}(1,2)$ and then combine polarizabilities with that of $\varepsilon_{\mathrm{semi}}(1,2)$ to obtain the direct dielectric function of the combined system $\varepsilon(1,2)$ in position representation as $\varepsilon(1,2)=\varepsilon_{\mathrm{semi}}(1,2)+\varepsilon_{2\mathrm{D}}(1,2)-\delta^4(1,2)$. Finally, we again employ the inversion condition, Eq. (2), to obtain the inverse dielectric function $K(1,2)$ for the combined system from $\varepsilon(1,2)$ in position representation.

The simple additivity of the polarizabilities employed here is an aspect of the random-phase approximation (RPA). To clarify this point, we consider the defining relation

$$K(1,2)=\frac{\delta V(1)}{\delta U(2)},$$

which describes the inverse dielectric function $K(1,2)$ as the linear connection between the effective potential $V(1)$ and an impressed potential $U(2)$, alternatively expressed as

$$V(1)=\int d^4 2\, K(1,2)U(2).$$

Considering the instantaneous electron-electron Coulomb interaction $v(1-2)=(e^2/|\mathbf{r}_1-\mathbf{r}_2|)\delta(t_1-t_2)$, the effective potential is given by

$$V(1)=U(1)+\int d^4 3\, v(1-3)\rho(3),$$

where $\rho(3)$ is the perturbed density in the presence of the fields. Taking the variational derivative $\delta/\delta U(2)$, this yields

$$K(1,2)=\frac{\delta V(1)}{\delta U(2)}=\delta^4(1-2)+\int d^4 3\, v(1-3)\,\frac{\delta\rho(3)}{\delta U(2)},$$

and using the chain rule for variational differentiation, we have the integral equation

$$K(1,2)=\delta^4(1-2)+\int d^4 3\int d^4 4\, v(1-3)\,\frac{\delta\rho(3)}{\delta V(4)}\,K(4,2).$$

Employing the inversion relation, Eq. (1), this may be solved exactly for the direct dielectric function

$$\varepsilon(1,2)=\delta^4(1-2)-\int d^4 3\, v(1-3)\,\frac{\delta\rho(3)}{\delta V(2)},$$

so that the polarizability may be identified as

$$\varepsilon(1,2)-\delta^4(1-2)=4\pi\alpha(1,2)=-\int d^4 3\, v(1-3)\,\frac{\delta\rho(3)}{\delta V(2)}.$$

The RPA is constituted of approximating $4\pi\alpha(1,2)$ by its free-electron form $4\pi\alpha_0$, in the absence of Coulomb interactions. With this, $\delta\rho(3)/\delta V(4)=R(3,4)$, the density perturbation response function, may be seen to be the lowest order ''ring'' diagram. [The sum of the infinite series of ring diagrams then places $R(3,4)$ in the kernel of the integral equation above, as one should expect.] For systems of the type considered here, which have several distinct contributing density distributions (e.g., the 2D electron sheet and the semi-infinite plasma in which it is embedded), $\rho=\Sigma_i\rho_i$, and since the RPA involves $R(3,4)=\delta\rho(3)/\delta V(4)=\Sigma_i\delta\rho_i(3)/\delta V(4)=\Sigma_i R_i(3,4)$ in the absence of Coulomb interactions, the determination of $R(3,4)$ involves only the sum of noninteracting density, or ring diagram, contributions from the various parts of the system. Correspondingly, in the RPA, the noninteracting polarizability

$$4\pi\alpha\to 4\pi\alpha_0=\sum_i 4\pi\alpha_{i0}$$

is composed of contributions from the various parts of the system in a simple additive manner.

II. INVERSION OF THE DIELECTRIC FUNCTION

Following the method outlined above, we determine the joint dielectric function $\varepsilon(z,z')$ of the combined semi-infinite dielectric medium plus the 2D sheet by adding the individual polarizabilities of the constituent parts in a position z-representation. To find $\varepsilon_{\mathrm{semi}}(z,z')$ of the semi-infinite medium, we start from the result of Ref. 3 for its inverse dielectric function $K_{\mathrm{semi}}(z,z')$, taken here in the local cold plasma limit,

$$\begin{aligned}K_{\mathrm{semi}}(z,z')=&\,\eta_+(-z)[\delta(z-z')+\delta(z')e^{Qz}\Gamma]\\&+\eta_+(z)[\delta(z-z')/\varepsilon-\delta(z')e^{-Qz}\Gamma/\varepsilon],\end{aligned}\tag{3}$$

where $\Gamma=(1-\varepsilon)/(1+\varepsilon)$, and $\eta_+(z)$ is the Heaviside unit step function. It is to be noted that this result properly incorporates the role of dynamic screening through the 3D bulk dielectric function

$$\varepsilon=1+4\pi\alpha_0^{3\mathrm{D}}\tag{4}$$

and image strength potential $(\varepsilon-1)/(\varepsilon+1)$. Here, $4\pi\alpha_0^{3\mathrm{D}}$ is the 3D bulk polarizability having the local cold plasma limit $\to-\omega_p^2/\omega^2$, where $\omega_p^2=4\pi\rho_{3\mathrm{D}}e^2/m$ is the electron plasma frequency squared, with m as effective band mass and $\rho_{3\mathrm{D}}$ is the 3D conduction-band electron density.

To determine the direct dielectric function $\varepsilon_{\mathrm{semi}}(z,z')$ of the semi-infinite plasma, we make an ansatz on the basis of our earlier experience [$\eta_+(z)$ is the Heaviside unit step function: $\eta_+(z)=1$ for $z>0$, 0 for $z<0$, $\frac{1}{2}$ for $z=0$]:

$$\begin{aligned}\varepsilon_{\mathrm{semi}}(z,z')=&\,\eta_+(-z)[\delta(z-z')f_1(z)+\delta(z')g_1(z)]\\&+\eta_+(z)[\delta(z-z')f_2(z)+\delta(z')g_2(z)].\end{aligned}\tag{5}$$

Here, $f_1(z)$, $f_2(z)$, $g_1(z)$ and $g_2(z)$ are continuous functions to be determined by the inversion condition, Eq. (2b), using $K_{\mathrm{semi}}(z,z')$ as given by Eq. (3), from which we obtain

$$\delta(z-z')=\eta_+(-z)[\delta(z-z')f_1(z)+\delta(z')g_1(z)+\eta_+(-z')\delta(z')f_2(z')\Gamma e^{Qz}+\eta_+(-0)\delta(z')g_1(0)\Gamma e^{Qz} + \eta_+(z')\delta(z')f_2(z')\Gamma e^{Qz}+\eta_+(0)\delta(z')g_2(0)\Gamma e^{Qz}]+\frac{\eta_+(z)}{\varepsilon}[\delta(z-z')f_2(z)+\delta(z')g_2(z) - \eta_+(-z')\delta(z')f_1(z')\Gamma e^{-Qz}-\eta_+(-0)\delta(z')g_1(0)\Gamma e^{-Qz}-\eta_+(z')\delta(z')f_2(z')\Gamma e^{-Qz} - \eta_+(0)\delta(z')g_2(0)\Gamma e^{-Qz}]. \tag{6}$$

Equating coefficients of $\delta(z-z')$, we have

$$1=\eta_+(-z)f_1(z)+\eta_+(z)f_2(z)/\varepsilon, \tag{7}$$

which implies that

$$\text{for } z>0 \rightarrow f_2(z)=\varepsilon, \tag{8a}$$

$$\text{for } z<0 \rightarrow f_1(z)=1. \tag{8b}$$

Furthermore, equating coefficients of $\delta(z')$, we obtain

$$0=\eta_+(-z)[g_1(z)+\eta_+(-z')f_1(z')\Gamma e^{Qz}+\eta_+(-0)g_1(0)\Gamma e^{Qz}+\eta_+(z')f_2(z')\Gamma e^{Qz}+\eta_+(0)g_2(0)\Gamma e^{Qz}] + \frac{\eta_+(z)}{\varepsilon}[g_2(z)-\eta_+(-z')f_1(z')\Gamma e^{-Qz}-\eta_+(-0)g_1(0)\Gamma e^{-Qz}-\eta_+(z)'f_2(z')\Gamma e^{-Qz}-\eta_+(0)g_2(0)\Gamma e^{-Qz}]. \tag{9}$$

At the interface $z'=0$, Eq. (9) yields

$$0=\eta_+(-z)\{g_1(z)+(\Gamma e^{Qz}/2)[f_1(0)+g_1(0)+f_2(0) +g_2(0)]\}+\frac{\eta_+(z)}{\varepsilon}\{g_2(z)-(\Gamma e^{-Qz}/2)[f_1(0)+g_1(0) +f_2(0)+g_2(0)]\}. \tag{10}$$

Considering first the semi-infinite plasma region $z>0$, Eq. (10) becomes

$$0=g_2(z)-(\Gamma e^{-Qz}/2)[f_1(0)+g_1(0)+f_2(0)+g_2(0)]. \tag{11}$$

Since $f_1(z)$ and $f_2(z)$ are continuous, $f_1(0)=1$ and $f_2(0)=\varepsilon$, we have

$$0=g_2(z)-(\Gamma e^{-Qz}/2)[1+\varepsilon+g_1(0)+g_2(0)]. \tag{12a}$$

In the limit $z\rightarrow 0$, $g_1(z)$ and $g_2(z)$ are also continuous, and we find

$$0=g_2(0)[1-\Gamma/2]-\Gamma g_1(0)/2-(1+\varepsilon)\Gamma/2. \tag{12b}$$

Considering next the vacuum region $z<0$, Eq. (10) yields

$$0=g_1(z)+(\Gamma e^{Qz}/2)[1+\varepsilon+g_1(0)+g_2(0)], \tag{13a}$$

and, for the limit $z\rightarrow 0$, we have

$$0=g_1(0)[1-\Gamma/2]+\Gamma g_2(0)/2+(1+\varepsilon)\Gamma/2. \tag{13b}$$

Solving Eqs. (12b) and (13b), we obtain

$$g_1(0)=-g_2(0)=(\varepsilon-1)/2. \tag{14}$$

Clearly, $g_1(0)+g_2(0)=0$, so that substitution in Eqs. (12a) and (13a) yields

$$g_1(z<0)=\frac{\varepsilon-1}{2}e^{Qz}, \tag{15a}$$

$$g_2(z>0)=\frac{1-\varepsilon}{2}e^{-Qz}. \tag{15b}$$

Hence, the direct dielectric function for the semi-infinite plasma is given by

$$\varepsilon_{\mathrm{semi}}(z',z'')=\eta_+(-z')[\delta(z'-z'')+\delta(z'')(\varepsilon-1)e^{Qz'}/2] +\eta_+(z')[\delta(z'-z'')\varepsilon+\delta(z'')(1-\varepsilon)e^{-Qz'}/2]. \tag{16}$$

Following the calculational program described above, we now add the polarizability of the semi-infinite dielectric medium to the polarizability of the 2D sheet of the electron plasma to obtain the direct dielectric function of the combined system in position z-representation as

$$\varepsilon(z,z')=\varepsilon_{\mathrm{semi}}(z,z')+\varepsilon_{\mathrm{2D}}(z,z')-\delta(z,z'). \tag{17}$$

The direct dielectric function of the 2D electron sheet at z_0 in three-dimensional (3D) space has been determined[4,5] in the position z representation as

$$\varepsilon_{\mathrm{2D}}(z,z')=\delta(z-z')+4\pi\alpha_0^{\mathrm{2D}}e^{-Q|z-z_0|}\delta(z'-z_0), \tag{18}$$

where $4\pi\alpha_0^{\mathrm{2D}}$ is the 2D polarizability of the electron sheet in transverse momentum/frequency (Q,ω) representation. This 2D polarizability is well known from semiconductor inversion layer and quantum-well studies.[1] In the local cold plasma limit, it takes the form $4\pi\alpha_0^{\mathrm{2D}}\rightarrow -2\pi\rho_{\mathrm{2D}}e^2Q/m\omega^2$, where ρ_{2D} is the 2D electron sheet density.

Forming the joint direct dielectric function $\varepsilon(z,z')$ of the combined system following Eqs. (16)–(18), we obtain

$$\varepsilon(z,z')=\delta(z-z')+4\pi\{\eta_+(-z)\alpha_0^{3\mathrm{D}}\delta(z')e^{Qz}/2 + \eta_+(z)\alpha_0^{3\mathrm{D}}[\delta(z-z')-\delta(z')e^{-Qz}/2] + \delta(z'-z_0)\alpha_0^{2\mathrm{D}}e^{-Q|z-z_0|}\}. \tag{19}$$

Again, our experience suggests that the inverse dielectric function $K(z,z')$ should take the ansatz form

$$K(z,z')=\eta_+(-z)[\delta(z-z')f_1(z)+\delta(z')g_1(z)] + \eta_+(z)[\delta(z-z')f_2(z)+\delta(z')g_2(z)] + \delta(z'-z_0)f(z), \tag{20}$$

with $f(z)$, $f_1(z)$, $f_2(z)$, $g_1(z)$, and $g_2(z)$ to be determined by the inversion condition Eq. (2) for the combined system. This yields

$$\begin{aligned}\delta(z-z')={}&\eta_+(-z)f_1(z)\delta(z-z')+\eta_+(-z)g_1(z)\delta(z')+\eta_+(z)f_2(z)\delta(z-z')+\eta_+(z)g_2(z)\delta(z')+f(z)\delta(z'-z_0)\\&+\eta_+(-z)\eta_+(-0)4\pi\alpha_0^{3\mathrm{D}}e^{Qz}[f_1(0)+f_2(0)+g_1(0)+g_2(0)]\delta(z')/2+\eta_+(-z)4\pi\alpha_0^{3\mathrm{D}}e^{Qz}f(0)\delta(z'-z_0)/2\\&+\eta_+(z)\eta_+(-z)4\pi\alpha_0^{3\mathrm{D}}f_1(z)\delta(z-z')+\eta_+(z)\eta_+(-z)4\pi\alpha_0^{3\mathrm{D}}g_1(z)\delta(z')+\eta_+(z)4\pi\alpha_0^{3\mathrm{D}}f_2(z)\delta(z-z')\\&+\eta_+(z)4\pi\alpha_0^{3\mathrm{D}}g_2(z)\delta(z')-\eta_+(z)\eta_+(-0)4\pi\alpha_0^{3\mathrm{D}}e^{-Qz}[f_1(0)+f_2(0)+g_1(0)+g_2(0)]\delta(z')/2\\&-\eta_+(z)4\pi\alpha_0^{3\mathrm{D}}e^{-Qz}f(0)\delta(z'-z_0)/2+\eta_+(z)4\pi\alpha_0^{3\mathrm{D}}f(z_0)\delta(z-z_0)/2+\eta_+(-z_0)f_1(z_0)\delta(z'-z_0)\\&\times4\pi\alpha_0^{2\mathrm{D}}e^{-Q|z-z_0|}+\eta_+(-z_0)g_1(z_0)\delta(z')4\pi\alpha_0^{2\mathrm{D}}e^{-Q|z-z_0|}+\eta_+(z_0)f_2(z_0)\delta(z'-z_0)4\pi\alpha_0^{2\mathrm{D}}e^{-Q|z-z_0|}\\&+\eta_+(z_0)g_2(z_0)\delta(z')4\pi\alpha_0^{2\mathrm{D}}e^{-Q|z-z_0|}+f(z_0)\delta(z'-z_0)4\pi\alpha_0^{2\mathrm{D}}e^{-Q|z-z_0|}.\end{aligned} \tag{21}$$

Again, we equate coefficients of the various δ functions. Equating the coefficients of $\delta(z-z')$, we find

$$1=\eta_+(-z)f_1(z)+\eta_+(z)f_2(z)+\eta_+(z)f_2(z)4\pi\alpha_0^{3\mathrm{D}}, \tag{22}$$

which yields, for $z>0$,

$$f_2(z>0)=1/\varepsilon, \tag{23}$$

and, for $z<0$,

$$f_1(z<0)=1. \tag{24}$$

Furthermore, we equate coefficients of $\delta(z'-z_0)$ to obtain

$$\begin{aligned}0={}&f(z)+\eta_+(-z)f(0)4\pi\alpha_0^{3\mathrm{D}}e^{Qz}/2+\eta_+(z)f(z)4\pi\alpha_0^{3\mathrm{D}}\\&-\eta_+(z)f(0)4\pi\alpha_0^{3\mathrm{D}}e^{-Qz}/2\\&+\eta_+(-z_0)f_1(z_0)4\pi\alpha_0^{2\mathrm{D}}e^{-Q|z-z_0|}\\&+\eta_+(z_0)f_2(z_0)4\pi\alpha_0^{2\mathrm{D}}e^{-Q|z-z_0|}\\&+f(z_0)4\pi\alpha_0^{2\mathrm{D}}e^{-Q|z-z_0|}.\end{aligned} \tag{25}$$

To determine $f(0)$ and $f(z_0)$, we set $z=0$ and $z=z_0$ in Eq. (25) in succession, obtaining two simultaneous equations for $f(0)$ and $f(z_0)$. Taking $z=0$ in Eq. (25) first, we have

$$0=f(0)(1+4\pi\alpha_0^{3\mathrm{D}}/2)+[\eta_+(-z_0)f_1(z_0)+\eta_+(z_0)f_2(z_0)+f(z_0)]4\pi\alpha_0^{2\mathrm{D}}e^{-Qz_0}. \tag{26}$$

Since the 2D electron sheet is in the material medium (as opposed to the vacuum region), $z_0>0$, and from Eq. (23) we have $f_2(z_0)=1/\varepsilon$. Thus Eq. (26) yields

$$f(z_0)=-f(0)\,\frac{\varepsilon+1}{8\pi\alpha_0^{2\mathrm{D}}e^{-Qz_0}}-\frac{1}{\varepsilon}. \tag{27}$$

Now, taking $z\to z_0>0$ in Eq. (25), we find

$$f(z_0)\varepsilon=f(0)\{e^{-Qz_0}4\pi\alpha_0^{3\mathrm{D}}/2+[(1+\varepsilon)/(2e^{-Qz_0})]\}. \tag{28}$$

Solving Eqs. (27) and (28) simultaneously for $f(0)$, we find

$$f(0)=-\frac{1}{\varepsilon}\left[\frac{4\pi\alpha_0^{3\mathrm{D}}e^{-Qz_0}}{2\varepsilon}+\frac{1+\varepsilon}{2\varepsilon e^{-Qz_0}}+\frac{1+\varepsilon}{8\pi\alpha_0^{2\mathrm{D}}e^{-Qz_0}}\right]^{-1}. \tag{29}$$

Turning now to the solution of Eq. (25), we first consider the case $z>0$, obtaining

$$f(z>0)\varepsilon=\tfrac{1}{2}f(0)[4\pi\alpha_0^{3\mathrm{D}}e^{-Qz}+(\varepsilon+1)e^{(Qz_0-Q|z-z_0|)}]. \tag{30}$$

On the other hand, for $z<0$, Eq. (25) yields

$$f(z<0)=\tfrac{1}{2}f(0)[-4\pi\alpha_0^{3\mathrm{D}}e^{Qz}+(\varepsilon+1)e^{(Qz_0-Q|z-z_0|)}]. \tag{31}$$

$f(z)$ may now be formed as

$$f(z)=\eta_+(z)f(z>0)+\eta_+(-z)f(z<0). \tag{32}$$

At this point, we focus on the determination of $g_1(z)$ and $g_2(z)$ by equating coefficients of $\delta(z')$ in Eq. (21), from which

$$\begin{aligned}0=&\eta_+(-z)[g_1(z)+\eta_+(-0)f_1(0)4\pi\alpha_0^{3D}e^{Qz}/2+\eta_+(-0)g_1(0)4\pi\alpha_0^{3D}e^{Qz}/2+\eta_+(0)f_2(0)4\pi\alpha_0^{3D}e^{Qz}/2\\&+\eta_+(0)g_2(0)4\pi\alpha_0^{3D}e^{Qz}/2]+\eta_+(z)[g_2(z)(1+4\pi\alpha_0^{3D})-\eta_+(-0)f_1(0)4\pi\alpha_0^{3D}e^{-Qz}/2\\&-\eta_+(-0)g_1(0)4\pi\alpha_0^{3D}e^{-Qz}/2-\eta_+(0)f_2(0)4\pi\alpha_0^{3D}e^{-Qz}/2]+g_2(z_0)4\pi\alpha_0^{2D}e^{-Q|z-z_0|}.\end{aligned}\tag{33}$$

However, recall $f_1(0)=1$ and $f_2(0)=1/\varepsilon$, so that for $z<0$ we have

$$\begin{aligned}0=&g_1(z<0)+4\pi\alpha_0^{3D}e^{Qz}[1+g_1(0)+g_2(0)+1/\varepsilon]/4\\&+g_2(z_0)4\pi\alpha_0^{2D}e^{-Q|z-z_0|},\end{aligned}\tag{34}$$

and for $z>0$ we find

$$\begin{aligned}0=&\varepsilon g_2(z>0)-4\pi\alpha_0^{3D}e^{-Qz}[1+g_1(0)+g_2(0)+1/\varepsilon]/4\\&+g_2(z_0)4\pi\alpha_0^{2D}e^{-Q|z-z_0|}.\end{aligned}\tag{35}$$

In order to match the limits from $z\to0^+$ and $z\to0^-$, we first consider $z>0$ and let $z\to0$, with the result

$$\begin{aligned}0=&\varepsilon g_2(0)-(\varepsilon-1)[1+g_1(0)+g_2(0)+1/\varepsilon]/4\\&+g_2(z_0)4\pi\alpha_0^{2D}e^{-Qz_0}.\end{aligned}\tag{36}$$

Next, we consider $z<0$ and let $z\to0$, finding

$$\begin{aligned}0=&g_1(0)+(\varepsilon-1)[1+g_1(0)+g_2(0)+1/\varepsilon]/4\\&+g_2(z_0)4\pi\alpha_0^{2D}e^{-Qz_0}.\end{aligned}\tag{37}$$

Finally, we consider Eq. (35) in the limit $z\to z_0>0$, obtaining

$$\begin{aligned}0=&\varepsilon g_2(z_0)-(\varepsilon-1)e^{-Qz_0}[1+g_1(0)+g_2(0)+1/\varepsilon]/4\\&+g_2(z_0)4\pi\alpha_0^{2D}.\end{aligned}\tag{38}$$

We can now solve for $g_1(0)$, $g_2(0)$, and $g_2(z_0)$ from Eqs. (36)–(38) as follows:

$$g_1(0)=\left(1+\frac{4\pi\alpha_0^{2D}e^{-2Qz_0}}{\varepsilon+4\pi\alpha_0^{2D}}\right)\left(\frac{1}{\Gamma}-\frac{4\pi\alpha_0^{2D}e^{-2Qz_0}}{\varepsilon+4\pi\alpha_0^{2D}}\right)^{-1},\tag{39}$$

and substituting in Eq. (34) for g_1 $(z<0)$, we find

$$\begin{aligned}g_1(z)=&\frac{1-\varepsilon}{2}[1+g_1(0)]\\&\times\left[e^{Qz}+\frac{4\pi\alpha_0^{2D}}{\varepsilon+4\pi\alpha_0^{2D}}e^{-Qz_0}e^{-Q|z-z_0|}\right],\end{aligned}\tag{40}$$

and similarly for g_2 $(z>0)$, we use Eq. (35) to obtain

$$\begin{aligned}g_2(z)=&\frac{\varepsilon-1}{2\varepsilon}[1+g_1(0)]\\&\times\left[e^{-Qz}-\frac{4\pi\alpha_0^{2D}}{\varepsilon+4\pi\alpha_0^{2D}}e^{-Qz_0}e^{-Q|z-z_0|}\right].\end{aligned}\tag{41}$$

Finally, we note that

$$g_2(z_0)=\frac{\varepsilon-1}{2}\frac{1+g_1(0)}{\varepsilon+4\pi\alpha_0^{2D}}e^{-Qz_0}.\tag{42}$$

Returning to the ansatz of Eq. (20), we see that it does indeed satisfy the inversion condition with $f(z)$, $f_1(z)$, $f_2(z)$, $g_1(z)$, and $g_2(z)$ as given above, yielding the result

$$\begin{aligned}K(z,z')=&\eta_+(-z)[\delta(z-z')+g_1(z)\delta(z')]+\eta_+(z)[\delta(z-z')/\varepsilon+g_2(z)\delta(z')]+(\eta_+(z)\{f(0)[4\pi\alpha_0^{3D}e^{-Qz}/2\\&+(1+\varepsilon)e^{-Q|z-z_0|}/(2e^{-Qz_0})]/\varepsilon\}+\eta_+(-z)\{-f(0)[4\pi\alpha_0^{3D}e^{Qz}/2-(1+\varepsilon)e^{-Q|z-z_0|}/(2e^{-Qz_0})]\})\delta(z'-z_0).\end{aligned}\tag{43}$$

III. COUPLED COLLECTIVE PLASMA OSCILLATIONS OF THE COMBINED SEMI-INFINITE PLASMA AND 2D ELECTRON SHEET

The coupled plasmons of the confined semi-infinite plasma and 2D electron sheet are given by the frequency poles of $K(z,z')$. These poles can be identified by inspection of the ways that the right-hand side of Eq. (43) can diverge. There are several categories of such coupled plasmon frequency poles.

(a) $\varepsilon=0$, the usual bulk plasma oscillation:

$$\omega^2=\omega_p^2=4\pi e^2\rho_{3D}/m.\tag{44}$$

(b) $\varepsilon+4\pi\alpha_0^{2D}=0$, a hybridization of the bulk and 2D plasmons:

$$\omega^2=\omega_p^2+\omega_{2D}^2.\tag{45}$$

(c) $g_1(0)\to\infty$ and $f(0)\to\infty$:

$$\frac{1}{\Gamma}-\frac{4\pi\alpha_0^{2D}e^{-2Qz_0}}{\varepsilon+4\pi\alpha_0^{2D}}=0,\tag{46}$$

so that

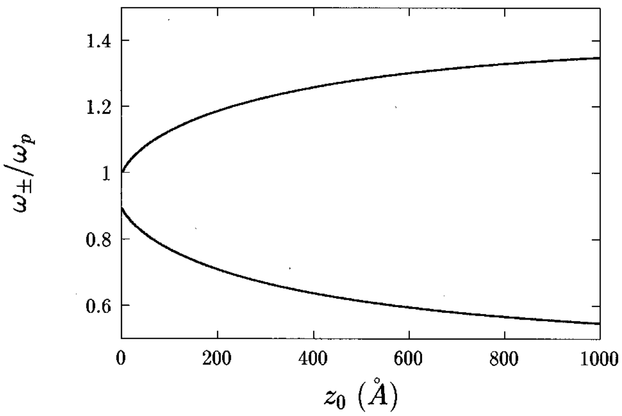

FIG. 2. $\omega_\pm/\omega_p$ as a function of z_0 for a Si-SiO$_2$ inversion layer, with $\rho_{2D}=10^{11}$ cm^{-2}, $\rho_{3D}=10^{16}$ cm^{-3}, and $Q=0.1Q_F$ (Q_F is the 2D Fermi wave number).

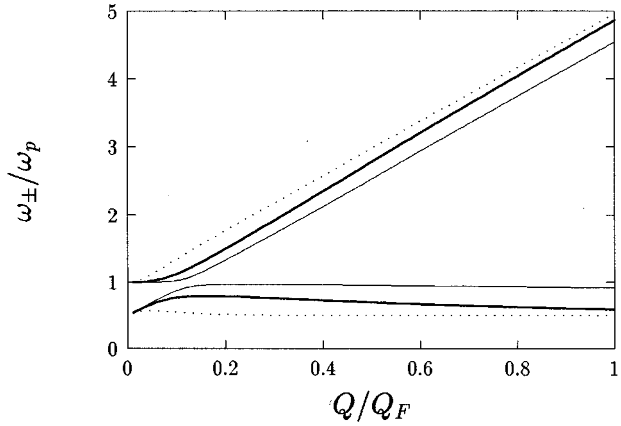

FIG. 3. $\omega_\pm/\omega_p$ as a function of Q/Q_F for various z_0 values. Thin solid curves: $z_0=10$ Å; solid curves: $z_0=100$ Å; dotted curves: $z_0=1000$ Å. Other parameters are the same as in Fig. 2.

$$4\omega_\pm^2=3\omega_p^2+2\omega_{2D}^2\pm\{(3\omega_p^2+2\omega_{2D}^2)^2 -8(\omega_p^4+\omega_p^2\omega_{2D}^2[1+e^{-2Qz_0}])\}^{1/2}. \tag{47}$$

We examine two limits of interest in case (c):

(i) As $z_0\to\infty$, we have $\omega_+^2=\omega_p^2+\omega_{2D}^2$ as in case (b), and $\omega_-^2=\omega_p^2/2$, the surface plasmon.

(ii) As $z_0\to 0$, we have $\omega_+^2=\omega_p^2$, and a hybridization of the surface plasmon and the 2D plasmon $\omega_-^2=\omega_p^2/2+\omega_{2D}^2$.

For finite nonvanishing z_0, these modes are admixed and coupled as indicated in Eq. (47). In Fig. 2 we plot $\omega_\pm/\omega_p$ as a function of z_0 for a Si-SiO$_2$ inversion layer, with $\rho_{2D}=10^{11}$ cm^{-2}, $\rho_{3D}=10^{16}$ cm^{-3}, and $Q=0.1Q_F$ (Q_F is the 2D Fermi wave number). Furthermore, in Fig. 3 we plot the dispersion curves $\omega_\pm/\omega_p$ as functions of Q/Q_F for $z_0=10$, 100, and 1000 Å.

The plasmon roots arising from the frequency poles of Eq. (43) are intuitively reasonable for the system at hand. The bulk plasmon $\omega^2=\omega_p^2$ will always exist deep inside the semi-infinite bulk, and its hybridization with the 2D plasmon $\omega^2=\omega_p^2+\omega_{2D}^2$ is to be expected with installation of the 2D plasma sheet, no matter how far the 2D sheet is from the surface, including the limit $z_0\to\infty$ deep in the bulk. Of course, the surface plasmon emerges, $\omega^2=\omega_p^2/2$, and its hybridization with the 2D plasmon, $\omega^2=\omega_p^2/2+\omega_{2D}^2$, is most robust when the 2D sheet overlays the surface, $z_0\to 0$. For finite values of z_0, the modes depend on z_0, with the detailed results exhibited in Fig. 2. Their relative excitation amplitudes may be determined from the residues at the frequency poles of $K(z,z')$, using Eq. (43), which provides the full description of the dynamic linear response of the joint system (of the 2D plasma coupled to a semi-infinite plasma) to a longitudinal potential field, for any separation (z_0) between the 2D sheet and the surface terminating the bulk plasma. The direct position space inversion of the dielectric function achieved here can also be extended to more complex nanostructure systems to provide analytic, closed-form expressions for their dynamic linear-response functions. This result promises to facilitate a variety of calculations pertaining to nanostructure potential interactions involving external perturbation as well as self-interaction.

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