Theory of optical-phonon limited hot-electron transport in quantum wires

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We present a kinetic theory of a nonequilibrium electron gas in a one-dimensional circular quantum wire interacting with acoustic and polar optical phonons. Besides these scattering mechanisms we also include an elastic interaction with interface roughness for the electron momentum relaxation. We have solved the Boltzmann kinetic equation analytically and obtained different distribution functions for a one-dimensional electron gas. A detailed kinetic analysis of the limiting case of the electron gas interacting solely with optical phonons is undertaken and the distribution function is found when this system can be described in a self-consistent way. Our analytical results are in good agreement with previous numerical studies of a similar system using Monte Carlo techniques. As an application of the developed theory we have calculated the electric-field dependences of electron mobility and average energy for different parameters of the quantum wire. It is shown that at high lattice temperature the electron mobility is a nonmonotonous function of the applied electric field and has its maximum value at intermediate electric fields when the transition from acoustic-phonon-limited to optical-phonon-limited transport takes place. [S0163-1829(96)01747-X]

I. INTRODUCTION

The quantization of the electron motion in systems of reduced dimensionality, such as quasi-one-dimensional (1D) quantum wires (QWI's), brings about different features in the electron kinetics compared with the usual three-dimensional (3D) electron systems with the same material parameters. The decrease of the electron density of states with increasing energy in a 1D QWI causes a reduction in the number of possible final states in momentum space for any electron transition at high energies. One consequence is that many of the interactions of a one-dimensional gas with the thermal bath become ineffective and this results in an enhanced electron mobility.[1]

If an external electric field is applied the electrons penetrate into the high-energy region. The main mechanisms which are responsible for electron energy relaxation are the interactions with acoustic and optical phonons, whose relative contributions depend on the lattice temperature T and the strength of the applied electric field F. Here we will deal with the situation when the lattice temperature T is small compared with the characteristic optical-phonon energy $\hbar\omega_L$, i.e., $T_0 \ll \hbar\omega_L$ (for brevity we write T_0 for $k_B T$ where k_B is the Boltzmann constant). In GaAs the LO phonon energy is 36.25 meV. The acoustic phonon scattering of a 1D electron gas has peculiarities[2–5] due to the quantization of the electron motion and this manifests itself in the electron kinetics through the appearance of a characteristic energy $E_c = \sqrt{\chi_0 m^* v_s^2 W_0}$, where W_0 is the quantum energy of the ground state, m^* is the electron effective mass, v_s is the sound velocity in the material, and χ_0 is a constant which depends on the shape of the quantizing potential. The behavior of the nonequilibrium electron system is quite different[5] at high ($T_0 > E_c$) and low ($T_0 < E_c$) lattice temperatures with respect to this characteristic energy E_c. For instance, in a GaAs QWI with transverse size $L_\perp \approx 100$ Å, $E_c = 3$ meV $\cong 35$ K, i.e., the value of the characteristic energy corresponds to the normal range of lattice temperature and electron energies for kinetic phenomena.

At high lattice temperatures the electron-acoustic-phonon interaction has a quasielastic character for the majority of the electrons at any strength of the electric field. The scattering rate in this case always decreases when the electron energy increases, following the energy dependence of the electron density of states. This means that the electron free path increases with increasing electron energy. As a result, in the presence of an external electric field, the total energy of the 1D electron gas gained from the electric field increases more rapidly than its loss due to the quasielastic interaction with acoustic phonons. The average energy balance between the 1D electron system and the thermal bath is then broken; the electrons increase their energy indefinitely and no steady state is established. This is the electron runaway effect.[6] In order to stabilize the electron system it is necessary to take into account an effective mechanism for energy relaxation other than the acoustic-phonon interaction. One possibility is to use the strongly inelastic interaction with optical phonons. Another consists of taking into account the transition of the electrons into the continuum (nonquantized) classical region of energy.[7] Both of the possibilities mentioned above for the stabilization of a 1D electron system are important under different conditions.

In an infinitely deep well with a transverse size $L_\perp \approx 100$ Å, we have for the energy of the ground state $W_0 = 110$ meV. The intersubband difference is $\Delta W \sim 3/2 W_0 = 165$ meV, which is 4–5 times that of the optical-phonon energy. This means that the interaction of a 1D electron with optical phonons is more important than the transition of electrons into the classical range of energy due to the effect of the electric field. Moreover, in the classical range of energy the electron energy is large compared with the optical-phonon energy and so this interaction becomes quasielastic, like the interaction with acoustic modes. However, due to the large value of the electron-optical-phonon coupling constant compared to that for acoustic modes, the interaction with optical

phonons will be more effective in controlling the energy relaxation of the electrons. As a result, at very high electric fields, when the electrons penetrate into the classical energy region, we can ignore the interaction with acoustic phonons. This regime corresponds to the optical-phonon-limited electron kinetics at high electric fields.

To realize the acoustic-phonon-limited electron kinetics it is necessary to use a thick QWI where, within the electron energy interval which corresponds to the energy $\hbar\omega_L$, there are several electron subbands. Then, under the influence of the electric field, the electrons will populate the upper subbands before interacting with optical phonons, and we can ignore this latter contribution to the electron energy relaxation. Estimations show us that the relevant width of a QWI for this case is within the range $L_\perp \approx 350$–450 Å.

The situation is as follows. At high lattice temperatures, $W_0 > T_0 > E_c$, when 1D electrons populate only the first subband at equilibrium, their kinetic properties in a strong electric field will be determined by the energy relaxation due to the interaction with acoustic and optical phonons for a thin QWI with $L_\perp \leqslant 200$ Å and only by the interaction with acoustic phonons for a thick QWI with $L_\perp \geqslant 400$ Å. The case which corresponds to the thick QWI was investigated in Ref. 7. In this paper we will develop a kinetic theory of hot electrons in a thin QWI at high lattice temperatures and will show that the interaction with optical phonons will suppress the runaway effect in a strong electric field. Our aim here is to derive analytic expressions for the distribution functions which arise and to calculate the electric-field dependence of different kinetic coefficients for the 1D electron gas in a QWI under the above conditions.

We now turn to the case of low lattice temperatures. In this regime the electron-acoustic phonon interaction has a strong inelastic character within a wide range of electric fields.[5] The scattering rate now increases very rapidly with increasing electron energy and the behavior of the nonequilibrium 1D electron gas is quite different compared with that in the high lattice temperature case. In particular, the electron-acoustic-phonon interaction at low lattice temperatures turns out to be strong enough to establish a steady state for the electron system at arbitrarily strong electric fields; no runaway effect occurs. This case was investigated in detail in Ref. 5 where only the first subband approximation was used. It was shown that at high electric fields the electron-acoustic-phonon interaction changes character again and becomes quasielastic. The mean electron energy increases with increasing electric field, and ultimately the electrons penetrate into the high-energy region where the interaction with optical phonons becomes important, just as it did for high lattice temperatures. In the present paper we will investigate the kinetic properties of the 1D electrons in the case where the interaction is with both acoustic and optical phonons at low lattice temperatures.

The paper is organized as follows. In the next section we describe the electron states and the phonon coupling terms for a cylindrical QWI. In Sec. III the relevant collision operators for the Boltzmann equation are found and in Sec. IV the limiting case of purely optical phonons is analyzed in detail. In Sec. V the distribution function is found for the total action of acoustic and optical phonons with the effects of interface roughness taken into account. Finally, in Sec. VI the mobility and average electron energy are presented with a summary in Sec. VII.

II. ELECTRON PROPERTIES AND SCATTERING MECHANISMS

In this paper we consider a wire of GaAs of circular cross section with radius R and length L_z, which is embedded in AlAs. The assumption of a cylindrical quantum wire is not only convenient from a mathematical point of view, but also has practical relevance as cylindrical wires have been fabricated with radii as small as 160 Å.[8]

The carriers are assumed to be confined by an infinite potential which gives a total wave function, in cylindrical coordinates (r,ϕ,z), of the form

$$\Psi_{mnk_z}(r,\phi,z)=\frac{1}{\sqrt{V_0}}\,e^{im\phi}e^{ik_z z}\psi_{mn}(r) \quad r<R, \quad (2.1)$$

where $V_0=\pi R^2 L_z$ is the volume of the QWI, $\Psi_{mn}(r)$ is the radial wave function, m is the azimuthal quantum number ($m=0,\pm1,\pm2..$), n is the radial quantum number ($n=1,2,3...$), and k_z is the axial electron wave vector. The radial wave function for a circular cross section is

$$\psi_{mn}(r)=\frac{1}{J_{m+1}(\beta_{mn})}\,J_m\left(\beta_{mn}\,\frac{r}{R}\right). \quad (2.2)$$

In (2.2) β_{mn} is the nth zero of the Bessel function of the first kind $J_m(\beta)$. The electron energy spectrum is given by $E_{mn}(k_z)=E(k_z)+\hbar^2/2m^*(\beta_{mn}/R)^2$, where $E(k_z)$ is the electron kinetic energy.

The transition probability from the initial electron state (k_z,ν) to the final state (k_z',ν') due to an interaction with a phonon with wave vector $\mathbf{q}$ is given by Fermi's golden rule,

$$W_{\nu\nu'}(k_z,k_z',\mathbf{q})=\frac{2\pi}{\hbar}\,|C_{\mathbf{q}}|^2 G^2_{\nu\nu'}(q_r)\,\delta_{k_z',k_z\mp q_z}\times\left[n_q+\frac{1}{2}\pm\frac{1}{2}\right]\times\delta[E_{\nu'}(k_z')-E_\nu(k_z)\pm\hbar\omega_q], \quad (2.3)$$

where the upper (lower) sign corresponds to emission (absorption) of the phonon, $n_q=[\exp(\hbar\omega_q/T_0)-1]^{-1}$ is the phonon occupation number and $\nu\equiv(m,n)$. Only bulk LA and polar optical modes are treated here where $\omega_q=v_s q$ is the long-wavelength approximation for the acoustic-mode dispersion and $\omega_q=\omega_L$ is the dispersionless optical-phonon frequency. For acoustic and optical phonons, the coupling constants are $C_{\mathbf{q}}^{\rm ac}=i\Xi_a\sqrt{\hbar q/2\rho v_s V_0}$ and $C_{\mathbf{q}}^{\rm op}=-i/q\sqrt{e^2\hbar\omega_L/2\varepsilon_0 V_0(1/\varepsilon_\infty-1/\varepsilon_s)}$, respectively. Here Ξ_a is the deformation acoustic potential, ρ is the density of the material, ε_∞ and ε_s are the high- and low-frequency dielectric constants, and ε_0 is the permitivity of free space. Strictly we should consider both GaAs hybrid optical modes and AlAs interface modes. In 2D systems it is known that the difference between rates calculated with these modes and calculated assuming a bulklike spectrum is not tremendously large. We will assume this to be the case for a QWI.

$G_{\nu\nu'}(q_r)$ is the form factor which takes into account the electron confinement in the 1D QWI. For the cylindrical case

$$G_{\nu\nu'}(q_r)=\frac{2}{R^2}\int_0^R J_{|m-m'|}(q_rR)\psi^*_{m',n'}(r)\psi_{mn}(r)r\,dr. \tag{2.4}$$

Unfortunately the radial wave function in (2.2) does not yield an analytical result for the form-factor in (2.4). Since we are only interested in the first subband approximation, we may employ the approximate ground-state radial wave function proposed in Ref. 9, viz. $\Psi_{01}(r)\approx\sqrt{3}(1-r^2/R^2)$, which gives the corresponding ground-state energy $W_0=6\hbar^2/2m^*R^2$ and the approximation to (2.4), $G_{0101}(q_r)=48J_3(q_rR)/(q_rR)^3$. These expressions give us the electron energy spectra and the electron interaction with acoustic and optical phonons which we shall use in our analysis.

III. THE BOLTZMANN EQUATION

The stationary-state electron distribution function $f(k_z)$ is defined by the Boltzmann kinetic equation

$$-\frac{e}{\hbar}F_z\frac{df(k_z)}{dk_z}=\hat{I}f(k_z), \tag{3.1}$$

with collision operator $\hat{I}f(k_z)$ which, for a nondegenerate electron gas, can be represented by the general form

$$\hat{I}f(k_z)=\sum_{k_z,\mathbf{q}}[W(k_z',k_z,\mathbf{q})f(k_z')-W(k_z,k_z',\mathbf{q})f(k_z)]. \tag{3.2}$$

We have omitted here and everywhere below the subband indices for all quantities since we assume the first subband approximation.

Let use specify the assumptions and approximations which we shall use in our kinetic analysis. First of all we assume that the electron energy relaxation is controlled by the interaction with both acoustic and optical phonons. For the electron momentum relaxation, as well as including the acoustic phonons and the interaction with optical phonons, we take into account the elastic interaction with the interface roughness of the QWI; the contribution of this form of scattering becomes important for small radii. The general collision operator in (3.2) has the same form for this type of scattering if we put $\mathbf{q}=\mathbf{0}$.

Another assumption concerns the character of the electron phonon interaction. In this paper we will assume that the electrons interact quasielastically with acoustic phonons and strongly inelastically with optical phonons. It was shown in Ref. 5 that for this to be realized

$$\sqrt{3m^*v_s^2W_0}<E(k_z),\quad T_0<\hbar\omega_L, \tag{3.3}$$

for an arbitrary electric field F_z.

The low lattice temperature case which can be defined as

$$T_0<\sqrt{3m^*v_s^2W_0}, \tag{3.4}$$

is more complicated compared with that for high lattice temperatures. Within a wide range of electric fields the 1D electrons predominantly interact with only the acoustic phonons, which is now a strongly inelastic interaction; the electron distribution function has a sharp anisotropic shape. The comprehensive analytical analysis for this case was carried out in Ref. 5.

The electron distribution function, $f(k_z)$, can be split into symmetric, $f_s(E)$, and antisymmetric, $f_a(E)$ parts given by

$$f_{s,a}(k_z)=\frac{1}{2}[f(k_z)\pm f(-k_z)], \tag{3.5}$$

and the Boltzmann equation then has the form,

$$-\frac{e}{\hbar}F_z\frac{df_{s,a}(k_z)}{dk_z}=\hat{I}f_{a,s}(k_z). \tag{3.6}$$

In accordance with the approximations discussed above, we can write the following expressions for the collision operators in (3.6) as

$$\hat{I}f_a(k_z)=-f_a(k_z)\frac{1}{\tau(E)}+\hat{I}_{\rm op}f_a(k_z), \tag{3.7}$$

$$\hat{I}f_s(E)=\hat{I}_{\rm ac}f_s(E)+\hat{I}_{\rm op}f_s(E), \tag{3.8}$$

where $1/\tau(E)$ is the electron momentum relaxation rate, which includes both the elastic acoustic phonon and the interface roughness contributions, i.e., $\tau^{-1}(E)=\tau_{\rm ac}^{-1}(E)+\tau_{ir}^{-1}(E)$. For interface roughness we obtain

$$\frac{1}{\tau_{\rm ir}(E)}=\frac{4\pi^{3/2}\hbar^3\beta_{01}^4\Delta^2\Lambda}{\varepsilon_s^2m^{*2}R^6}N(E)e^{-2m^*E/\hbar^2\Lambda^2}, \tag{3.9}$$

where $N(E)$ is the one dimensional electron density of states for a single spin, given by $N(E)=\sqrt{m^*/2\pi^2\hbar^2E}$. The parameters Δ and Λ for the interface roughness scattering are the average radial depth and lateral length of the unevenness along the QWI, respectively.

For acoustic-phonon scattering the momentum relaxation time in the elastic approximation is given by[5]

$$\frac{1}{\tau_{\rm ac}(E(k_z))}=\frac{\pi\Xi_a^2}{\rho v_sV_0}\sum_{\mathbf{q}}G_{11}^2(\mathbf{q}_r)(2n_q+1)\times\left[1-\left(1+\frac{q_z}{k_z}\right)\right]\delta[E(k_z+q_z)-E(k_z)]. \tag{3.10}$$

For the cylindrical quantum wire we obtain

$$\frac{1}{\tau_{\rm ac}(E)}=\frac{18\Xi_a^2T_0}{5\rho v_s^2\hbar R^2}N(E), \tag{3.11}$$

if the equipartition approximation for acoustic phonons is applied, viz., $n_q\approx T_0/\hbar v_sq$. At low temperatures, when the zero-point lattice approximation can be used, i.e., $n_q\approx0$, we obtain for the acoustic-phonon-scattering rate

$$\frac{1}{\tau_{\rm ac}(E)}=\frac{\Xi_a^2}{2\rho v_s}N(E)\int_0^\infty G^2(q_r)\left(q_r^2+\frac{8m^*E}{\hbar^2}\right)^{1/2}q_rdq_r. \tag{3.12}$$

To calculate the symmetric collision operator for the electron acoustic-phonon interaction in the quasielastic approximation, we average the total symmetric operator with

$$\hat{I}_{\rm ac}f_s(E)=\frac{\int\hat{I}_{\rm ac}f_s(E(k_z))\delta[E(k_z)-E]dk_z}{\int\delta[E(k_z)-E]dk_z}, \tag{3.13}$$

and expand the full δ function

$$\delta[E(k_z+q_z)-E\pm\hbar v_s q]\approx\left[1\mp\hbar v_s q\,\frac{d}{dE}+\frac{1}{2}(\hbar v_s q)^2\,\frac{d^2}{dE^2}\right]\times\delta[E(k_z)-E]. \quad (3.14)$$

After some algebra similar to Ref. 5, we obtain the final form for the symmetric operator as

$$\hat{I}_{\rm ac}f_s(E)=\frac{1}{N(E)}\,\frac{d}{dE}\left[N^2(E)\left(A(E)f_s(E)+B(E)\,\frac{df_s(E)}{dE}\right)\right]. \quad (3.15)$$

The coefficients $A(E)$ and $B(E)$ are given by

$$A(E)=\frac{m^*\Xi_a^2}{\rho\hbar}\int_0^\infty G^2(q_r)\left[\frac{\hbar^2q_r^2}{2m^*}+2E\right]q_r dq_r, \quad (3.16)$$

$$B(E)=\frac{m^*\Xi_a^2}{\rho\hbar R^2}\,\frac{\sum_{k_z,\mathbf{q}}G^2(q_r)\,\frac{\hbar^2q^2}{2m^*}\,\hbar v_s q(2n_q+1)\delta[E(k_z)-E]\delta[E(k_z+q_z)-E]}{\sum_{k_z,q_z}\delta[E(k_z)-E]\delta[E(k_z+q_z)-E]}. \quad (3.17)$$

By making use of the approximate form of the form factor in (2.4), we obtain for $A(E)$,

$$A(E)=\frac{12\hbar\Xi_a^2}{\rho R^4}\left(\frac{9E}{5W_0}+1\right). \quad (3.18)$$

$B(E)$ can be found for the high lattice temperatures in (3.3), when we can use the equipartition approximation, and for the low lattice temperatures in (3.4), when we can use the zero-point approximation. We obtain for high lattice temperature,

$$B(E)=T_0A(E), \quad (3.19)$$

and for low lattice temperature,

$$B(E)=\frac{\hbar^2v_s\Xi_a^2}{8\rho}\int_0^\infty G^2(q_r)[q_r^3+(q_r^2+4k_z^2)^{3/2}]q_r dq_r. \quad (3.20)$$

We can obtain expressions for the symmetric and antisymmetric part of the collision operator $\hat{I}_{\rm op}f(k_z)$,

$$\hat{I}_{\rm op}f_s(E)=\Gamma_0[N(E+\hbar\omega_L)[f_s(E+\hbar\omega_L)(n_L+1)-f_s(E)n_L]g_a^+(E)+N(E-\hbar\omega_L)[f_s(E-\hbar\omega_L)n_L-f_s(E)(n_L+1)]g_e^-(E)], \quad (3.21)$$

$$\hat{I}_{\rm op}f_a(k_z)=\Gamma_0[N(E+\hbar\omega_L)[f_a(k_0^+)(n_L+1)g_a^-(E)-f_a(k_z)n_Lg_a^+(E)]+N(E-\hbar\omega_L)[f_a(k_0^-)n_Lg_e^-(E)-f_a(k_z)(n_L+1)g_e^+(E)]], \quad (3.22)$$

where $\Gamma_0=e^2/4\varepsilon_0(1/\varepsilon_\infty-1/\varepsilon_s)$, $g_{a,e}^{\pm}(E)=g(q_{a,e}^{+})\pm g_a^{+}\times(q_{a,e}^{-})$, and

$$g(q)=\int_0^\infty\frac{G^2(q_r)}{q_r^2+q^2}\,q_r dq_r \approx\left[\frac{48}{(qR)^3}\right]^2\left[\frac{1}{6}-\frac{(qR)^2}{96}+\frac{(qR)^4}{640}-I_3(qR)K_3(qR)\right]. \quad (3.23)$$

Here $I_n(z)$ and $K_n(z)$ are modified Bessel functions and $q_a^{\pm}=-\sqrt{2m^*E/\hbar^2}\pm k_0^+$, $q_e^{\pm}=\sqrt{2m^*E/\hbar^2}\mp k_0^-$, $k_0^{\pm}=\sqrt{2m^*(E\pm\hbar\omega_L)/\hbar^2}$.

IV. KINETIC ANALYSIS FOR PURELY OPTICAL-PHONON SCATTERING

Including optical-phonons scattering as a possible relaxation mechanism for the electron system requires some additional justification. Due to the interaction with dispersionless optical phonons, electrons change their energy by the constant amount $\hbar\omega_L$. For an arbitrary initial electron distribution which is different from zero within the energy range ΔE, where $\Delta E<\hbar\omega_L$, the electron population will only be nonzero within the energy intervals $\Delta E+j\hbar\omega_L$, where $j=0$, 1, 2, 3..., if the only interaction is with optical phonons. As a result no electron thermalization takes place and even in the absence of external fields the steady-state solution (if it exists) is far from the Maxwellian distribution. [This is true for any relation between ΔE and $\hbar\omega_L$ (we used the above for

clarity).] In other words, the system is nonergodic. What is the mathematical evidence of this nonegodicity? For the equilibrium steady state, the distribution is governed by the equations $\hat{I}_{\rm op} f_s(E) = \hat{I}_{\rm op} f_a(k_z) = 0$. These are linear difference equations of the first order[10] which have the form

$$f_s(E+\hbar\omega_L)(n_L+1) - f_s(E) n_L = 0, \tag{4.1}$$

$$f_a(k_z^+)(n_L+1) g_a^-(E) - f_a(k_z) n_L g_a^+(E) = 0. \tag{4.2}$$

By using the methods in Ref. 10, we obtain the obvious solution $f_a(k_z) = 0$ and

$$f_s(E+j\hbar\omega_L) = \varphi_s(E) e^{-j(\hbar\omega_L/T_0)}, \quad 0 \leqslant E \leqslant \hbar\omega_L,$$

$$j = 0,1,2\ldots . \tag{4.3}$$

Here, $\varphi_s(E)$ is *an arbitrary function.*

Note that, in general, the solution in (4.3) undergoes a discontinuity at the points $E = 0, \hbar\omega_L$. In fact, we obtain

$$f_s(\hbar\omega_L) = \varphi_s(\hbar\omega_L), \quad j = 0, \quad E = \hbar\omega_L,$$

$$f_s(\hbar\omega_L) = \varphi_s(0) e^{-\hbar\omega_L/T_0}, \quad j = 1, \quad E = 0. \tag{4.4}$$

Only in the particular case when

$$\varphi_s(\hbar\omega_L) = \varphi_s(0) e^{-\hbar\omega_L/T_0}, \tag{4.5}$$

is the distribution function in (4.3) continuous everywhere. The Maxwellian distribution function is just a particular solution because it is in agreement with (4.1) and possesses the property in (4.5). This is a very important point. In any real physical system there are other mechanisms besides optical phonons and these modes will be dispersive. For the steady state these other mechanisms will establish a Maxwellian distribution, so that if we switch on the optical-phonon interaction in the system, the Maxwellian distribution will not be destroyed. Thus to define the solution it is necessary to know the function $\varphi_s(E)$. The electron-optical-phonon system does not produce this function in a self-consistent way and this is the evidence of nonergodicity of the system. It means that different initial electron distributions give different final results.

Furthermore, let us prepare some initial electron distribution at the moment of time $t = 0$, which is defined as $f_s(E, t=0) = f_s^0(E)$. The equation which describes the time evolution of this initial distribution in the absence of an external field is

$$\frac{\partial f_s(E,t)}{\partial t} = \hat{I}_{\rm op} f_s(E,t). \tag{4.6}$$

This is a first-order differential difference equation.[11] In accordance with the general theory of this type of equation, in order to find a solution it is necessary to know the function $f_s(E,t)$, within the elementary interval, at *all times* of the evolution of the process. In other words, for the solution of (4.6), it is not enough to know the initial condition. It follows from (4.6) that the function $f_s(E,t)$ within the next elementary interval $[\hbar\omega_L, 2\hbar\omega_L]$ is given by

$$f_s(E+\hbar\omega_L, t) = \frac{1}{n_L+1}\left[\frac{1}{\Gamma_0 N(E+\hbar\omega_L) g_a^+(E)} \frac{\partial f_s(E,t)}{\partial t} + n_L f_s(E,t)\right]. \tag{4.7}$$

As one can see the function $f_s(E+\hbar\omega_L, t)$ for the above interval can be found only if we know the function $f_s(E,t)$ within the elementary interval $[0, \hbar\omega_L]$ for all time t. Physically, this means that the electron-optical-phonon system does not produce its evolution in time self-consistently and the question about the final steady state remains open.

In the presence of external fields the situation is more complicated. Instead of one differential difference equation we will have a system of two coupled first-order differential difference equations for the functions $f_s(E,t)$ and $f_a(E,t)$, but the problem of time evolution of the initial electron distribution will be retained.

Carrying out the above analysis shows us that in the general case no steady-state electron distribution exists for the interaction solely with optical phonons. Moreover, this is true for any dimension, 3D, 2D, or 1D.

The 1D electron optical phonon system was studied in Ref. 12 using the Boltzmann equation where steady-state nonequilibrium and equilibrium electron distributions were found. This is clearly in disagreement with our conclusion above. The reason for the disagreement is as follows. The corresponding kinetic equations for the distribution function for $0 \leqslant E \leqslant \hbar\omega_L$ in this case are given by (3.6) with (3.21) and (3.22),

$$-\frac{e}{\hbar} F_z \frac{df_s(E)}{dk_z} = \Gamma_0 N(E+\hbar\omega_L)[f_a(k_0^+)(n_L+1) g_a^-(E) - f_a(k_z) n_L g_a^+(E)], \tag{4.8}$$

$$-\frac{e}{\hbar} F_z \frac{df_a(k_z)}{dk_z} = \Gamma_0 N(E+\hbar\omega_L)[f_s(E+\hbar\omega_L)(n_L+1) - f_s(E) n_L] g_a^+(E). \tag{4.9}$$

These equations coincide with (31a) and (31b) in Ref. 12. We can find from these equations the functions $f_s(E+\hbar\omega_L)$ and $f_a(E+\hbar\omega_L)$ if, and only if, we know the functions $f_s(E) \equiv \varphi_s(E)$ and $f_a(E) \equiv \varphi_a(E)$ within the first interval. After this we can find $f_s(E)$ and $f_s(E)$ within the intervals $[j\hbar\omega_L, (j+1)\hbar\omega_L]$. But there is nothing to tell us what the original two functions are. Hence, the initial system of Eqs. (4.8) and (4.9), in *principle*, does not allow us to obtain a unique solution.

To avoid this fundamental problem it was suggested in Ref. 12 to use, besides the system of Eqs. (4.8) and (4.9), the detailed balance conditions given by (4.1) and (4.2) which is (33) in Ref. 12. Note that for the antisymmetric function $f_a(E)$ this equation has to include additional multipliers as it is in (4.2). This suggestion produces a basic change in the solution. The point is that the detailed balance conditions are not only a simple correlation between different elementary intervals but that these are *additional equations* in the problem.

From (4.1) we obtain $\hat{I}_{\rm op} f_s(E)=0$ and (4.9) gives $f_a(k_z)=$const as obtained in Ref. 12 in (34). Substituting $f_a(E)$ into the right-hand side of (4.8) gives the first-order differential equation

$$-\frac{e}{\hbar}F_z\frac{df_s(E)}{dk_z}=\hat{I}_{\rm op}[f_a(k_z)=\text{const}]. \tag{4.10}$$

We can substitute into (4.10) the solution of the balance equation (4.1) given by (4.3). As a result it gives a differential equation for the function $\varphi_s(E)$ which can be easily solved. This scheme was essentially that used in Ref. 12, but, as we have pointed out, this cannot give a unique solution of the system of Eqs. (4.8), (4.9). A simple way to check this is to analyze the equilibrium solution, Eq. (40) obtained in Ref. 12, with the nonequilibrium solution, Eq. (37), by putting $F\rightarrow 0$. If we put $F=0$ in the initial Equations (4.8) and (4.9), we obtain the solutions $f_a(k_z)=0$ and (4.3). If we follow the method of Ref. 12 and apply this limiting condition to (4.10), we obtain the so-called singular disturbed differential equation[13] because the main derivative in the equation contains a small parameter. In the limit (4.10) is $\hat{I}_{\rm op}[f_a(k_z)=\text{const}]=0$, i.e., the equation undergoes a radical change because the function $f_s(E)$ disappears from the equation. In accordance with the general theory of singular perturbed differential equations,[13] this means that the limiting transition $F\rightarrow 0$ solution of (4.10) will translate into the equilibrium solution of the initial equations, (4.8) and (4.9) at $F=0$, only in some very particular cases. Since the equilibrium solution within the region $0\leqslant E\leqslant\hbar\omega_L$ given by (40) in Ref. 12 can only be extended in the region $E\geqslant\hbar\omega_L$ by using the detailed balance condition, which is an *additional* equation, this is not one of these particular cases.

There is only one particular situation when it is possible to use a self-consistent model resulting in a unique solution of (4.8) and (4.9). This is the case of low lattice temperature, $T_0\ll\hbar\omega_L$, and strong electric field. Here we only deal with two elementary energy intervals, where the field has to be in the range

$$\frac{\sqrt{2m^*\hbar\omega_L}}{e\tau_{\rm el}}<|F_z|<\frac{\sqrt{2m^*\hbar\omega_L}}{e\tau_{\rm op}}. \tag{4.11}$$

Here $\tau_{\rm el}$ is a scattering time with respect to any elastic (or quasielastic) interaction and $\tau_{\rm op}$ is the optical-phonon emission time.

Due to the low temperature we can put in (3.21) and (3.22) $n_L=0$ and due to (4.11) we can put

$$f(k_z)=0,\quad \text{if}\ \begin{matrix}E(k_z)\geqslant 2\hbar\omega_L, & k_z>0,\\ E(k_z)\geqslant\hbar\omega_L, & k_z<0.\end{matrix} \tag{4.12}$$

We assume here that $F_z=-\text{sign}(k_z)F$, i.e., the electric field pushes electrons along the positive direction of the k_z axis.

The low-temperature condition breaks the connection between the jth and $(j+1)$th elementary intervals of energy through the optical-phonon absorption process, but there remains the connection in the opposite direction, between the $(j+1)$th and jth intervals, through the spontaneous emission process of optical phonons. By means of the last process, kinetic balance is established for the nonequilibrium electrons within j elementary intervals of energy. The condition from (4.12) eliminates the infinite chain of the differential difference equations.

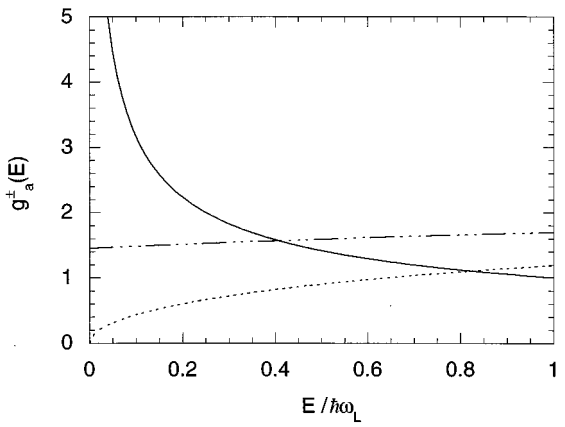

FIG. 1. The functions $g_a^+(E)$ (chained line) and $g_a^-(E)$ (dotted line) for a wire of circular cross section as a function of energy. The solid line is $1/E^{-1/2}$ which is proportional to the density of states in one dimension.

For only two elementary intervals we obtain from (3.6) with (3.21) and (3.22),

$$-\frac{e}{\hbar}F_z\frac{df_s(E)}{dk_z}=\Gamma_0 N(E+\hbar\omega_L)g_a^-(E)f_a(k_0^+), \tag{4.13}$$

$$-\frac{e}{\hbar}F_z\frac{df_a(k_z)}{dk_z}=\Gamma_0 N(E+\hbar\omega_L)g_a^+(E)f_s(E+\hbar\omega_L), \tag{4.14}$$

for $0\leqslant E(k_z)\leqslant\hbar\omega_L$, and

$$-\frac{e}{\hbar}F_z\frac{df_s(E)}{dk_z}=-\Gamma_0 N(E-\hbar\omega_L)g_e^+(E)f_a(k_z), \tag{4.15}$$

$$-\frac{e}{\hbar}F_z\frac{df_a(k_z)}{dk_z}=-\Gamma_0 N(E-\hbar\omega_L)g_e^+(E)f_s(E), \tag{4.16}$$

for $\hbar\omega_L\leqslant E(k_z)\leqslant 2\hbar\omega_L$. The solution of these equations is described in the Appendix where (A3)–(A9) give a full solution of the problem for the discussed case.

A similar situation was investigated by Magnusson[14] for electrons in a quantizing magnetic field. Our solution does not agree with Magnusson's result because of the different boundary conditions used. Another reason for this difference is the expressions for the functions $g_{a,e}^{\pm}(E)$. In Ref. 14 it was assumed that $g_{a,e}^-(E)=0$ and $g_{a,e}^+(E)=$const. In our case, as follows from Fig. 1, the functions $g_{a,e}^-(E)$ and $g_{a,e}^+(E)$ have approximately the same order of magnitude. Also within the actual energy range we can put these functions equal to a constant as their dependence on energy is much slower compared with the energy dependence of the density of states. With these approximations the integration in (A3)–(A7) can be carried out analytically, and we obtain the solutions

$$f_s(E)=f_s(\hbar\omega_L)\left[1+\frac{\alpha_F^-}{\alpha_F^+}(1+\sqrt{2})^{-\alpha_F^+}-\frac{\alpha_F^-}{\alpha_F^+}\left(\frac{|k_z|+\sqrt{k_z^2+k_0^2}}{k_0}\right)^{-\alpha_F^+}\right], \tag{4.17}$$

$$f_a(k_z)=\mathrm{sign}(k_z)f_s(\hbar\omega_L)\left[1+(1+\sqrt{2})^{-\alpha_F^+}-\left(\frac{|k_z|+\sqrt{k_z^2+k_0^2}}{k_0}\right)^{-\alpha_F^+}\right], \tag{4.18}$$

for $0\leqslant E(k_z)\leqslant\hbar\omega_L$, and

$$f_s(E)=\mathrm{sign}(k_z)f_a(k_z)=f_s(\hbar\omega_L)\left[\frac{|k_z|+\sqrt{k_z^2-k_0^2}}{k_0}\right]^{-\alpha_F^+}, \tag{4.19}$$

for $\hbar\omega_L\leqslant E(k_z)\leqslant 2\hbar\omega_L$. Here we have introduced the notation $\alpha_F^{\pm}=1/\pi(m^*\Gamma_0/e\hbar F)\overline{g_a^{\pm}}$, where $\overline{g_a^{\pm}}$ are averaged values. For the circular QWI numerical calculation gives $\overline{g_a^+}=1.57$ and $\overline{g_a^-}=0.59$. For the energy range $E\geqslant\hbar\omega_L$, we have used the relation $g_e^{\pm}(E)=g_a^{\pm}(E-\hbar\omega_L)$. These solutions are shown in Fig. 2 for a low and high electric field.

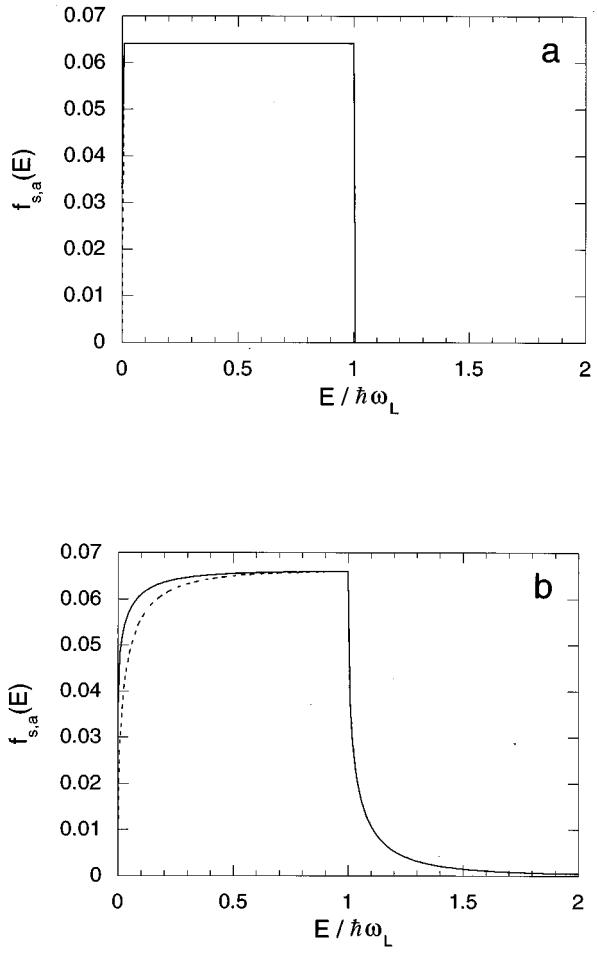

FIG. 2. The distribution function $f_s(E)$ (solid line) and $f_a(E)$ (dotted line) for purely optical-phonon scattering at low temperature as a function of energy for a wire of circular cross section with radius 100 Å and linear electron density of 10^5 cm^{-1} at (a) a very low field, $F\to 0$, and (b) a high field, $F=1000$ V cm^{-1}.

V. COMBINATION OF ACOUSTIC- AND OPTICAL-PHONON SCATTERING

In accordance with the model discussed in detail in Sec. III, the Boltzmann kinetic equation for the distribution function within the energy range defined in (3.3) has the form

$$-\frac{e}{\hbar}F_z\frac{df_s(E)}{dk_z}=-\frac{f_a(k_z)}{\tau(E)}+\Gamma_0N(E+\hbar\omega_L)\overline{g_a^-}f_a(k_0^+), \tag{5.1}$$

$$-\frac{e}{\hbar}F_z\frac{df_a(k_z)}{dk_z}=\frac{1}{N(E)}\frac{d}{dE}\left[N^2(E)\left(A(E)f_s(E)+B(E)\frac{df_s(E)}{dE}\right)\right]+\Gamma_0N(E+\hbar\omega_L)\overline{g_a^+}f_s(E+\hbar\omega_L). \tag{5.2}$$

We take the distribution function within the energy region $E(k_z)\geqslant\hbar\omega_L$ to be defined by (4.15) and (4.16). This means that in that region the optical-phonon scattering is much stronger than for the acoustic phonons or the interface roughness scattering, which is true if the electrons do not penetrate deeply into the region just above the threshold energy at $\hbar\omega_L$. For this case the distribution function within the energy region $\hbar\omega_L\leqslant E(k_z)\leqslant 2\hbar\omega_L$ is given in (4.19). Note that the inclusion of acoustic-phonon scattering in the Boltzmann equation removes the problem of the nonergodicity for the electron system discussed in Sec. IV.

Equation (5.1) gives us the solution for the antisymmetric distribution function,

$$f_a(k_z)=\frac{e\tau(E)}{\hbar}F_z\frac{df_s(E)}{dk_z}+\Gamma_0\tau(E)N(E+\hbar\omega_L)\overline{g_a^-}f_a(k_0^+). \tag{5.3}$$

By substituting $f_a(k_z)$ from (5.3) into (5.2), we obtain a second-order differential equation for $f_s(E)$,

$$-\frac{1}{N(E)}\frac{d}{dE}\left\{\left[B(E)N^2(E)+\left(\frac{eF_z}{\pi\hbar}\right)^2\frac{\tau(E)}{N(E)}\right]\frac{df_s(E)}{dE}+A(E)N^2(E)f_s(E)\right\}=\Gamma_0N(E+\hbar\omega_L)\overline{g_a^+}f_s(E+\hbar\omega_L)+\frac{eF_z}{\pi\hbar}\frac{\Gamma_0}{N(E)}\overline{g_a^-}\frac{d}{dE}[\tau(E)N(E+\hbar\omega_L)f_a(k_0^+)]. \tag{5.4}$$

The inhomogeneous part on the right-hand side of this equation describes the effect of optical-phonon scattering on the

symmetric distribution function. With the distribution functions from (4.19), the last equation can be directly integrated to give

$$\left[B(E)N^2(E)+\left(\frac{eF_z}{\pi\hbar}\right)^2\frac{\tau(E)}{N(E)}\right]\frac{df_s(E)}{dE}+A(E)N^2(E)f_s(E)=G(E), \tag{5.5}$$

where

$$G(E)=-\frac{eF_z}{\pi\hbar}[f_s(E+\hbar\omega_L)+\Gamma_0\overline{g_a}\,\tau(E)N(E+\hbar\omega_L)f_a(k_0^+)]+G_0, \tag{5.6}$$

and G_0 is the first integration constant.

To solve (5.5) it is necessary to find the solution of the homogeneous equation $[G(E)=0]$ to give the function $\tilde{f}_s(E)$,

$$\tilde{f}_s(E)=\exp\left\{-\int_0^E A(E')\left[B(E')+\left(\frac{eF_z}{\pi\hbar}\right)^2\frac{\tau(E')}{N^3(E')}\right]^{-1}dE'\right\}. \tag{5.7}$$

The function $\tilde{f}_s(E)$ is in fact the solution of the Boltzmann equation in the absence of optical-phonon scattering (to within an arbitrary multiplying constant). In the general case the relaxation time $\tau(E)$ in (5.7) can include any arbitrary elastic-scattering mechanisms.

The final solution of (5.5) is

$$f_s(E)=\tilde{f}_s(E)\left\{G_1-\int_E^{\hbar\omega_L}\frac{G(E')}{\tilde{f}_s(E')}\left[B(E')N^2(E')+\left(\frac{eF_z}{\pi\hbar}\right)^2\frac{\tau(E')}{N(E')}\right]^{-1}dE'\right\} \tag{5.8}$$

where G_1 is the second integration constant.

As one can see the solution in (5.8) includes three integration constants which are G_0, G_1, and $f_s(\hbar\omega_L)$. To find these constants we will use the matching conditions for both symmetric [from (4.19) and (5.8)] and antisymmetric [from (4.19) and (5.3)] distribution functions at the optical-phonon energy $E(k_z)=\hbar\omega_L$. The third constant can be found from the normalization condition,

$$2\int_0^\infty N(E)f_s(E)dE=n_0, \tag{5.9}$$

where n_0 is the linear density of electrons. We obtain the following expressions for the constants G_0 and G_1,

$$G_1=\frac{f_s(\hbar\omega_L)}{\tilde{f}_s(\hbar\omega_L)}, \tag{5.10}$$

$$G_0=f_s(\hbar\omega_L)\left[\begin{array}{c}A(\hbar\omega_L)N^2(\hbar\omega_L)+\dfrac{eF_z}{\pi\hbar}\left(1+\dfrac{\Gamma_0}{\sqrt{2}}N(\hbar\omega_L)\tau(\hbar\omega_L)\overline{g_a}\right)(1+\sqrt{2})^{-\alpha_F^+}\\ -\dfrac{\pi\hbar}{eF_z}A(\hbar\omega_L)N^2(\hbar\omega_L)\dfrac{N(\hbar\omega_L)}{\tau(\hbar\omega_L)}\left(\dfrac{dLn\tilde{f}_s}{dE}\right)^{-1}_{E=\hbar\omega_L}\\ \times\left(1-\dfrac{\Gamma_0}{\sqrt{2}}N(\hbar\omega_L)\tau(\hbar\omega_L)\overline{g_a}(1+\sqrt{2})^{-\alpha_F^+}\right)\end{array}\right]. \tag{5.11}$$

The constant $f_s(\hbar\omega_L)$ is found from (5.9).

The expressions in (5.3), (5.8), (5.10), and (5.11) give a full description of the electron distribution in a 1D QWI under the total action of acoustic- and optical-phonon scattering. It is noteworthy that this electron distribution was obtained without any simplifying assumptions concerning the boson occupation number for the acoustic phonons, which is included in the coefficients $A(E)$ and $B(E)$ in the above expressions.

The resultant distribution function is shown in Fig. 3 for four electric fields. For higher electric fields, which correspond to the range defined in (4.11), the electron distribution is controlled solely by optical-phonon scattering. The distribution function for this case is shown in Fig. 3(d). It can be seen from Fig. 3(a) that a Maxwellian distribution is found as expected at very low electric fields which *turns over* as the field is increased to ultimately retrieve the optical-phonon results. Our analytical results are in qualitative agreement with the results in Ref. 15 obtained by Monte Carlo simulation, but we did not obtain a steep slope on the energy dependence of the distribution function at low energies as described there [see Fig. 3(b) for the same electric field $F=200$ V cm^{-1}]. The physical explanation for this difference consists in the different energy dependences of the scattering rates at low energies used in Ref. 15 and in our paper. Our expressions in (3.9) and (3.12) for the momentum relaxation rates due to interaction with acoustic phonons are obtained within the energy range defined in (3.3). Using these dependences at low energies, where $\tau^{-1}(E)\approx N(E)\approx E^{-1/2}$, gives much higher values for the momentum relaxation rate com-

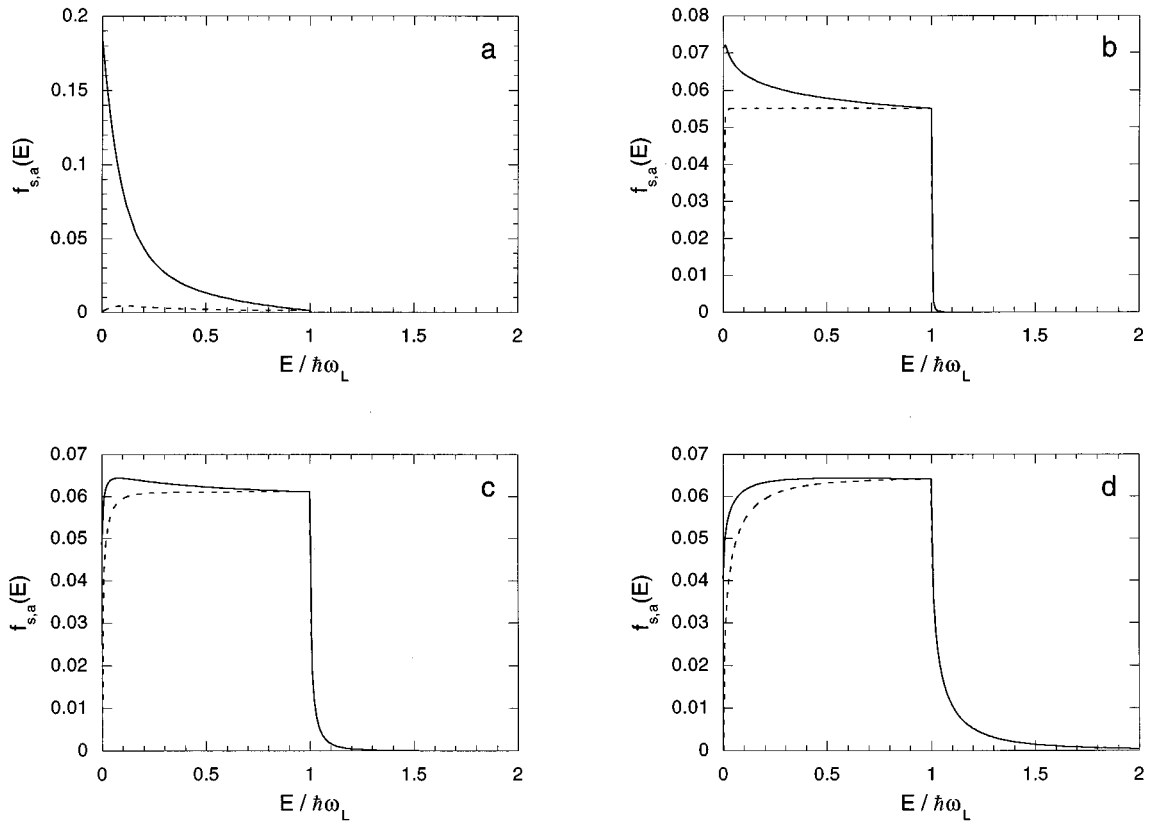

FIG. 3. The distribution function $f_s(E)$ (solid line) and $f_a(E)$ (dotted line) for the total action of elastic interface roughness scattering, acoustic scattering, and optical-phonon scattering at a temperature of 50 K for the same parameters as Fig. 2 at (a) a very low field, $F \to 0$, (b) $F = 200$ V cm^{-1}, (c) $F = 500$ V cm^{-1}, and (d) a high field, $F = 1000$ V cm^{-1}. The parameters for the interface roughness scattering are $\Lambda = 20$ Å and $\Delta = 3$ Å.

pared with the true dependence, which gives $\tau^{-1}(E) \approx$const at low energies (see Refs. 5 and 15). As a result, the electrons scattered into the low-energy region by means of emission of an optical phonon will undergo more intensive scattering from the acoustic phonons resulting in a slow diffusive motion in the momentum space near the point $k_z = 0$. This gives some accumulation of electrons within the low-energy region, i.e., an increased value of the distribution function. However, these distinctions of the distribution functions within the small energy region are not important for the calculation of the average characteristics of the electron gas. It follows from Fig. 3 that the majority of the electrons are distributed within the energy region defined in (3.3).

At higher electric fields, when penetration of electrons above the optical-phonon threshold becomes a factor, the gradient of the distribution function at low energy changes its sign from negative to positive, Figs. 3(c), 3(d). This is because the electric field is strong enough to push an electron deeply into the second region $[E(k_z) > \hbar\omega_L]$ before it emits an optical phonon. As a result the maximum flow of electrons in energy space due to the spontaneous emission of optical phonons is shifted from zero energy to some finite value. Since the electron penetration is not too deep (this is a restriction of our model) this energy value is small. It corresponds to the maximum value of the distribution function in Fig. 3(c).

VI. HOT-ELECTRON MOBILITY AND AVERAGE ENERGY

As an application of the developed kinetic theory we have calculated some macroscopic characteristics of the hot-electron gas in a QWI.

The electron mobility is given by

$$\mu = \frac{1}{\pi\hbar F} \int_0^\infty f_a(|k_z|)dE \Big/ \int_0^\infty N(E) f_s(E) dE. \quad (6.1)$$

The average electron energy is equal to

$$\overline{E} = \int_0^\infty E^{1/2} f_s(E) dE \Big/ \int_0^\infty E^{-1/2} f_s(E) dE. \quad (6.2)$$

To calculate μ and $\overline{E}$ we carried out the numerical integration in (6.1) and (6.2). For that we used the distribution functions (5.3) and (5.8) below and (4.19) above the optical-phonon energy.

Figure 4 shows the mobility for two different radii of the QWI. The case of only optical-phonon scattering produces near constant drift velocities so the electric-field dependence of the mobility obeys the law $1/F$. Including interface roughness and acoustic scattering produces a nonmonotonous behavior of the mobility as a function of electric field. At low electric fields (but these fields are high enough to deviate the

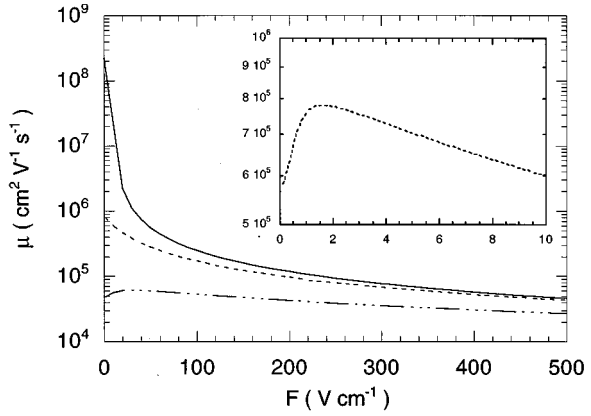

FIG. 4. The mobility as a function of electric field for electrons experiencing elastic interface roughness scattering, acoustic scattering, and optical-phonon scattering at a temperature of 50 K in a wire of circular cross section with radius $R=50$ Å (chained line) and $R=100$ Å (dotted line). The solid line is the mobility of electrons interacting with just optical phonons for both radii which is the high-field limit for the other two lines. The inset is the mobility at low fields for the wire of radius $R=100$ Å.

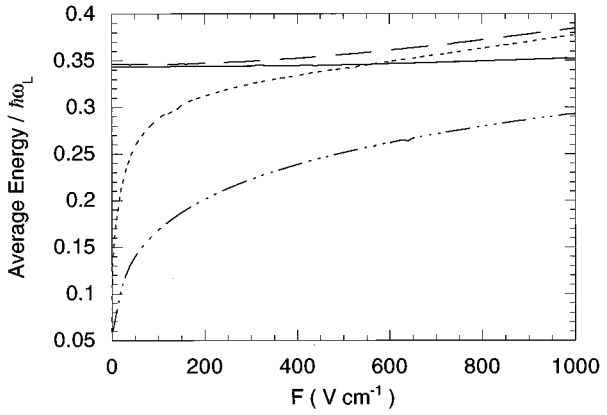

FIG. 5. The average energy as a function of field for electrons experiencing elastic interface roughness scattering, acoustic scattering, and optical-phonon scattering at a temperature of 50 K in a wire of circular cross-section with radius $R=50$ Å (chained line) and $R=100$ Å (dotted line). The high-field limits, which are electrons interacting with just optical phonons, are given by the solid line for $R=50$ Å and the dashed line for $R=100$ Å. The increase in the average energy with electric field for the last case is due to a small penetration of the electrons above the optical-phonon energy threshold.

electron system far from equilibrium) the electron mobility is controlled by the acoustic phonons and interface roughness scattering, the latter being more important as the radius is decreased. The electrons do not reach the optical-phonon energy so this interaction is ineffective. The momentum relaxation rate in this case decreases with increasing energy [see (3.9) and (3.11)]. When an electric field is applied it pushes the electrons into a higher-energy region resulting in an increase of the electron mobility. The current-voltage characteristics obey a superlinear behavior at these electric fields. A superlinear region for similar conditions was also obtained in Ref. 15 by a Monte Carlo simulation. For higher electric fields the electrons reach the optical-phonon energy threshold which, due to the intensive spontaneous emission of optical phonons, slows their further gain in energy. The electron mobility decreases with increasing electric field since the drift velocity here approaches saturation (this is a characteristic feature of the optical-phonon-scattering case).

At low lattice temperatures, defined in (3.4), the field dependence of the electron mobility is different. At low electric fields, when acoustic-phonon-scattering dominates, the electron mobility decreases with increasing electric field and the current-voltage characteristics have a sublinear behavior. This case is investigated in Ref. 5. At higher electric fields the electrons interact with both optical and acoustic phonons, however the latter become ineffective when the field increases. The electric-field dependence of the mobility for the range of fields allowed by (3.3) at zero temperature, $F>400$ V cm^{-1}, is close to that in Fig. 4.

The average energy as a function of electric field is presented in Fig. 5 for the high lattice temperature case. At low lattice temperature the situation is similar to that which takes place for the electron mobility: for electric fields $F>400$ V cm^{-1} the corresponding curves coincide with those for high lattice temperatures.

VII. SUMMARY

In this paper we presented a kinetic theory of a nonequilibrium electron gas in a circular QWI interacting with bulk acoustic and optical phonons. The lattice temperature is suggested to be small compared with the optical-phonon energy which is less than the energy separation between the electronic ground state and the first excited state. The theory can be easily generalized to include any other elastic-scattering mechanisms. As a demonstration of this we included the elastic electron interaction with interface roughness.

The Boltzmann kinetic equation was solved analytically and we obtained different distribution functions. With increasing electric field the efficiency of electron scattering by acoustic phonons decreases and the contribution of the optical-phonons scattering becomes critically important. The strong inelastic character of this interaction prevents the electrons penetrating into the high-energy region and stops the runaway effect for the one-dimensional electron gas. At high enough electric fields the electron distribution is controlled solely by the interaction with optical phonons.

We carried out detailed kinetic analysis of the particular case when the 1D electrons only interact with polar-optical phonons. For general conditions this system is nonergodic and cannot be described self-consistently. We have shown that only at low lattice temperatures and high enough electric fields can one use a model which gives a self-consistent solution and we obtained this solution analytically. This solution is the limit of the distribution function under the action of both acoustic and optical phonons for high electric fields.

As an application of the developed theory we calculated the mobility and average energy of the 1D electrons. We investigated their electric field dependences for different parameters of the QWI. It was found that the field dependence of the mobility is nonmonotonous at high lattice temperatures. At low electric field the mobility is defined by the interaction with acoustic phonons and interface roughness. Due to the decreasing efficiency of the scattering of both of these mechanisms with increasing energy the electron mobility increases with increasing electric field and reaches a

maximum. The maximum corresponds to the electric field when electrons begin to interact with optical phonons. After this the mobility decreases with increasing electric field. The field at which this maximum occurs depends on the size of the QWI since both acoustic-phonon and interface roughness scattering have a strong dependence on this. When the radius of the QWI decreases there is a decrease in mobility and an increase in the electric field corresponding to the maximum.

The average electron energy increases very rapidly at intermediate electric fields when the electrons interact mainly with acoustic phonons and the interface roughness, but changes slowly at high electric fields when the electrons interact predominantly with optical phonons. The effect of decreasing the radius of the QWI is similar to that for mobility. Since the electron mobility decreases for smaller radii, the efficiency of the electron heating drops as well and, as a result, the average electron energy decreases as the cross-sectional area becomes smaller for the same electric field.

ACKNOWLEDGMENTS

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APPENDIX

In this appendix we find the solutions of (4.13)–(4.16). We start by finding the equal solutions of (4.15) and (4.16), given by

$$f_{s,a}(k_z)=C_1\exp\left(-\frac{\hbar\Gamma_0}{eF}\int_{k_0}^{k_z}N[E(k_z')-\hbar\omega_L]\times g_e^+(E(k_z'))dk_z'\right)\pm C_2\times\exp\left(\frac{\hbar\Gamma_0}{eF}\int_{k_0}^{k_z}N[E(k_z')-\hbar\omega_L]\times g_e^+(E(k_z'))dk_z'\right), \tag{A1}$$

where the upper sign corresponds to the symmetric, the lower sign to the antisymmetric function, and $C_{1,2}$ are the integration constants.

Since due to (4.12) the *total* distribution function $f(k_z)=0$ within the region $E(k_z)>\hbar\omega_L$, $k_z<0$, this gives additional conditions for the functions in (A1) [see (3.5)], viz.,

$$f_s(E)=\begin{cases} f_a(k_z), & k_z>0 \\ -f_a(k_z), & k_z<0. \end{cases} \tag{A2}$$

Substituting the functions from (A1) into (A2) results in $C_2=0$ and

$$f_s(E)=f_a(k_z)=C_1\exp\left(-\frac{\hbar\Gamma_0}{eF}\int_{k_0}^{k_z}N[E(k_z')-\hbar\omega_L]\times g_e^+(E(k_z'))dk_z'\right), \tag{A3}$$

for $\hbar\omega_L\leqslant E(k_z)\leqslant 2\hbar\omega_L$, $k_z>0$. Here $k_0=\sqrt{2m^*\omega_L/\hbar}$. The function $f_a(k_z)$ with $k_z<0$ can be found by using the second condition in (A2).

With the functions from (A1) it is very easy to find solutions of (4.13) and (4.14). We obtain

$$f_s(E)=\frac{\hbar\Gamma_0}{eF}C_1\int_0^{|k_z|}N[E(k_z')+\hbar\omega_L]g_a^-(E(k_z'))\times\exp\left(-\frac{\hbar\Gamma_0}{eF}\int_{k_0}^{\sqrt{k_z'^2+k_0^2}}N[E(k_z'')-\hbar\omega_L]\times g_e^+(E(k_z''))dk_z''\right)dk_z'+C_3, \tag{A4}$$

$$f_a(k_z)=\mathrm{sign}(k_z)\frac{\hbar\Gamma_0}{eF}C_1\int_0^{|k_z|}N[E(k_z')+\hbar\omega_L]\times g_a^+(E(k_z'))\times\exp\left(-\frac{\hbar\Gamma_0}{eF}\int_{k_0}^{\sqrt{k_z'^2+k_0^2}}N[E(k_z'')-\hbar\omega_L]\times g_e^+(E(k_z''))dk_z''\right)dk_z'+C_4, \tag{A5}$$

for $0\leqslant E(k_z)\leqslant\hbar\omega_L$. Here $C_{3,4}$ are integration constants.

The expressions in (A3)–(A5) include three constants C_1, C_3, and C_4, which can be found from the matching conditions at the optical-phonon energy, $k_z=k_0$, for both the symmetric and antisymmetric functions, and from the normalization condition. This gives

$$C_3=f_s(\hbar\omega_L)\left[1-\frac{\hbar\Gamma_0}{eF}\int_0^{k_0}N[E(k_z')+\hbar\omega_L]g_a^-(E(k_z'))\exp\left(-\frac{\hbar\Gamma_0}{eF}\int_{k_0}^{\sqrt{k_z'^2+k_0^2}}N[E(k_z'')-\hbar\omega_L]g_e^+(E(k_z''))dk_z''\right)dk_z'\right], \tag{A6}$$

$$C_4=f_s(\hbar\omega_L)\left[1-\frac{\hbar\Gamma_0}{eF}\int_0^{k_0}N[E(k_z')+\hbar\omega_L]g_a^+(E(k_z'))\exp\left(-\frac{\hbar\Gamma_0}{eF}\int_{k_0}^{\sqrt{k_z'^2+k_0^2}}N[E(k_z'')-\hbar\omega_L]g_e^+(E(k_z''))dk_z''\right)dk_z'\right], \tag{A7}$$

where we have also put $C_1=f_s(\hbar\omega_L)$, which is the normalization constant. The expressions in (A3)–(A7) give a full solution of the problem for the discussed case.

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[1] H. Sakaki, Jpn. J. Appl. Phys. **19**, L735 (1980).
[2] R. Kubo, S. J. Miyake, and N. Hashitsume, *Quantum Theory of Galvanomagnetic Effect at Extremely Strong Magnetic Fields* (Academic, New York, 1965), pp. 299–317.
[3] V. Karpus, Fiz. Tekh. Poluprovodn. **20**, 12 (1986) [Sov. Phys. Semicond. **20**, 6 (1986)].
[4] R. Mickevicius and V. Mitin, Phys. Rev. B **48**, 17 194 (1993).
[5] B. K. Ridley and N. A. Zakhleniuk, J. Phys. Condens. Matter **8**, 8525 (1996); **8**, 8539 (1996); **8**, 8553 (1996).
[6] V. B. Levinson, Fiz. Tverd. Tela (Leningrad) **6**, 2113 (1964) [Sov. Phys. Solid State **6**, 1665 (1965)].
[7] S. D. Beneslavski and V. A. Korobov, Fiz. Tekh. Poluprovodn. **20**, 1030 (1986) [Sov. Phys. Semicond. **20**, 650 (1986)].
[8] R. J. Tonucci, B. L. Justus, A. J. Campillo, and C. E. Ford, Science **258**, 783 (1992).
[9] A. Gold and, A. Ghazali, Phys. Rev. B **41**, 7626 (1990).
[10] F. B. Hildebrand, *Finite-Difference Equations and Simulations* (Prentice Hall, New Jersey, 1968), p. 24.
[11] R. Bellman and K. L. Cooke, *Differential-Difference Equations* (Academic, New York, 1963), Chap. 3.
[12] J. P. Leburton, Phys. Rev. B **45**, 11 022 (1992).
[13] A. B. Vasilieva and V. F. Butuzov, *Asymtotic Expansions of Solutions of Singularly Perturbed Equations* (Nauka, Moscow, 1973).
[14] B. Magnusson, Phys. Status Solid B **52**, 316 (1972).
[15] R. Mickevicius, V. Mitin, V. K. Harithsa, D. Jovanovic, and J. P. Leburton, J. Appl. Phys. **75**, 973 (1994).