Electron diffraction by periodic arrays of quantum antidots

J.-P. Leburton and Yu. B. Lyanda-Geller
Beckman Institute for Advanced Science and Technology, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801
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Electron diffraction by a periodic array of repulsive δ barriers is an analytically solvable quantum-mechanical problem. In this geometry, bearing some analogy with single-barrier tunneling, incident electrons are perpendicular to the periodic barrier of antidots. In contrast to conventional quasi-one-dimensional tunneling, which conserves the component of the electron wave vector transverse to the current, electron diffraction occurs through multiple channels characterized by the transverse electron wave vectors differing by the reciprocal lattice vector of the periodic array. For a one-dimensional (1D) array of two-dimensional (2D) δ potentials we predict highly nonlinear characteristics in the vicinity of Fermi energies when a new channel for diffraction opens up. Two lines of 1D arrays reveal a rich resonant diffraction structure. [S0163-1829(96)04644-9]

I. INTRODUCTION

The possibility of realizing lateral superstructures by modulating the electric potential in a two-dimensional (2D) electron gas with the expectation of novel electronic properties has been anticipated by Sakaki[1] twenty years ago. In the meantime, with the continuous development of nanostructure technology, a wide class of superstructures has been proposed for the investigation of novel quantum transport effects and their applications in high functional devices.[2] Presently, many phenomena resulting from the periodic modulation of the electron gas have been observed at low temperature or in the mesoscopic regime, mainly because of the difficulty of confining or modulating the electron gas over short distances in more than one direction. Recent progresses in self-assembled microstructures with nanometer size features and the manipulation of single atoms by scanning tunneling microscopy have created new opportunities for realizing nanostructures with strong confinement of the order of the de Broglie wavelength at room temperature.[3–7] It becomes therefore possible to generate three-dimensional (3D) configurations of molecular scale structures with quantum-mechanical properties and transport phenomena not yet envisioned.

The simplest configurations of periodic nanostructures are short-period arrays of quantum antidots or quantum antiwires, which act as diffraction centers for incident electrons perpendicular to the plane of the arrays (see Fig. 1). From a physical point of view, this problem bears some analogy with the diffraction of light by a lattice of small apertures, but also with the von Laue diffraction of x rays by crystals. Aside from this analogy, the problem is also interesting from a transport viewpoint since the geometrical configuration is reminiscent of tunneling configuration across a (single or double) potential barrier. However, because of the periodicity in the plane (the direction perpendicular to the current), the transverse component of the electron wave vector is no longer conserved for coherent transport processes. Formally, the problem cannot be treated within a one-dimensional (1D) model by separation of variables as would be, for instance, the case in a tunneling problem across a periodic potential such as the Kroenig-Penney model, nor automatically treated with a perturbative technique using a scattering formalism, since the potential does not vanish at the infinity in the array plane.

In the present paper, we consider a class of 2D and 3D problems in diffraction geometry for which the periodicity of quantum antidots or antiwires permits exact analytical quantum-mechanical solutions, and provides the wave functions in the whole space. We find that in contrast to quasi-1D tunneling, which conserves the transverse component of the wave vector, electron diffraction occurs through multiple channels characterized by transverse electron wave vectors that differ from each other and from the wave vector of the incident electron by reciprocal lattice vectors of the periodic arrays as one can expect from the von Laue and the Wolf-Bragg formula for x-ray diffraction in crystals. As we will see by using the periodicity of the arrays it becomes possible to solve 2D or 3D Schrödinger equations analytically in cases when variables are inseparable and the problem cannot be reduced to 1D equations. Meanwhile, we will also show that the opening of diffraction channels results in highly nonlinear tunneling characteristics of two distinct conductance regimes separated by a sharp transition at the Fermi energy corresponding to the half of the reciprocal lattice vector of the periodic arrays.

We proceed as follows: In Sec. II we describe the diffraction geometry and the electron scattering model; Sec. III deals with solutions of the Schrödinger equation for several periodic barriers. Finally, in Sec. IV we calculate the tunneling current for these barriers.

II. DIFFRACTION GEOMETRIES AND MODEL

In this section we consider several configurations of quantum antidot and antiwire arrays for the 2D or 3D diffraction of electrons. In our search for analytical solutions of the Schrödinger equation, we model the repulsive potential of the quantum structures by a δ function. This approximation is justified if the geometrical dimensions of the diffraction center are relatively small, but its potential strength relatively important.

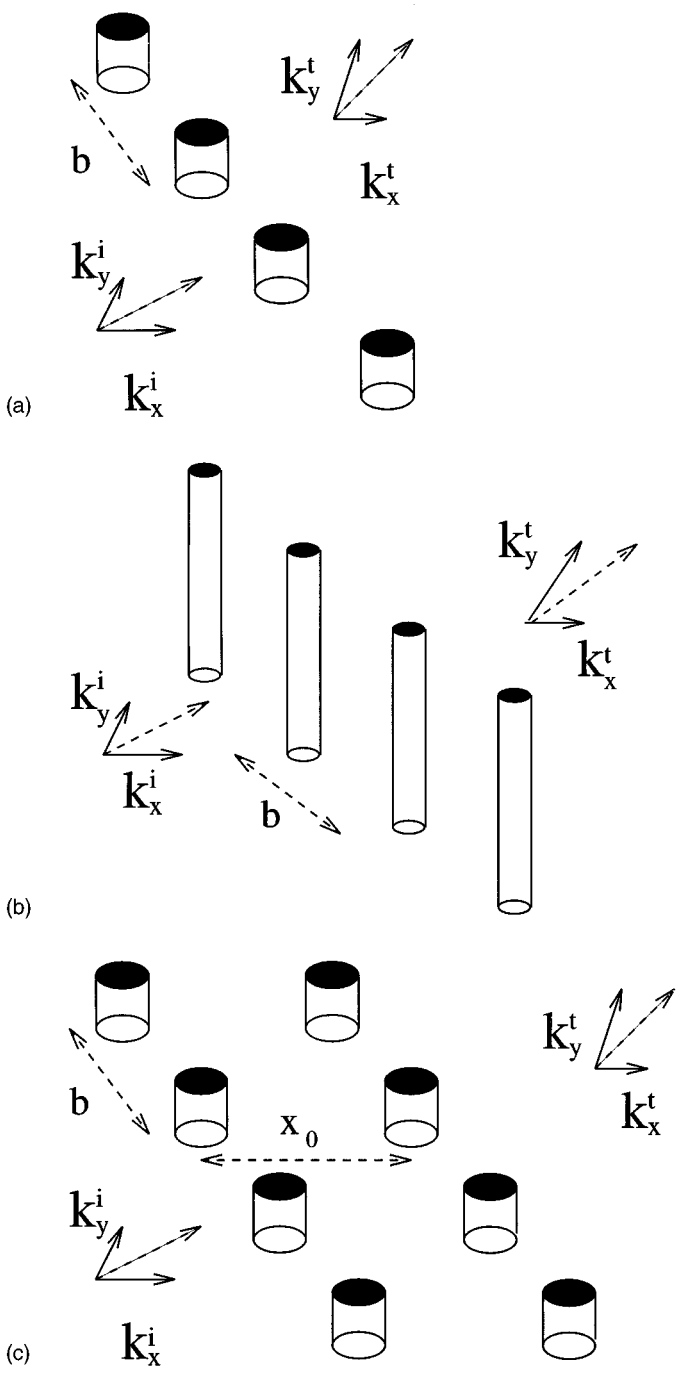

FIG. 1. Schematic representation of (a) diffraction across a line of antidots within the plane of a 2D electron gas. (b) 3D diffraction across a line of antiwires, (c) diffraction through two lines of antidots separated by a distance x_0 within the plane of a 2D electron gas. The period of arrays is b and $\mathbf{k}^i$ and $\mathbf{k}^t$ are the incident and transmitted wave vectors, respectively.

We study successively the following configurations.

(a) 2D diffraction by a line of periodic δ barriers. Such a problem corresponds to the 2D electron tunneling across the line of quantum antidots [Fig. 1(a)].

(b) 3D diffraction by periodically spaced set of quantum antiwire δ potentials [Fig. 1(b)].

(c) 2D diffraction by two lines of periodic δ barriers, which may be displaced from one another in the direction perpendicular to the current [Fig. 1(c)].

For simplicity, we will assume that the electron spectrum is described by an isotropic electron mass characteristic of III-V semiconductor compounds. We limit ourselves to the single-particle picture leaving the investigation of many-body effects for future work.

It is well known that exact solutions for the electron wave functions can easily be obtained for one-dimentional tunneling through a rectangular barrier or a set of barriers.[8,9] A typical case of an exactly solvable problem is tunneling through a 1D δ barrier or a 1D double δ barrier. As we discussed in the Introduction, for 2D and 3D quantum-mechanical configurations analytical solutions are known for problems in which the electron potential does not depend on one or two coordinates. In such a case one can separate the variables, writing the electron wave function as a product of plane waves propagating in one or two directions and a function depending on the remaining coordinate. Thus, the problem is effectively 1D and becomes easily solvable.

In the theory of electron diffraction[10–12] analysis is often given for the case of resonant scattering. The solution of diffraction equations is then obtained with the ''two-rod'' approximation in the theory of the reflection high-energy electron diffraction. This approximation retains only two components of the Fourier series expansion of the wave function, i.e., the component corresponding to zero reciprocal lattice vector (and elastic scattering) and the component corresponding to the reciprocal lattice vector satisfying the inequality related to resonant condition. This method is valid for an arbitrary form of the potential.

The case that we study is different since we do not consider resonant scattering nor the two-rod approximation. We solve the 2D Schrödinger equation for a 1D periodic array of antidots, and the 3D Schrödinger equation for a 2D periodic array of antidots by retaining all the coefficients of the Fourier series expansion, and not just two of them as in Refs. 10–12. For instance, we demonstrate that, if the potential barriers are δ functions, the analytical solution of the 2D and 3D Schrödinger equations for which the potential is unseparable, is found exactly.

The electron Hamiltonian for these systems is written as

$$H=\frac{p^2}{2m}+U(x,y,z), \tag{1}$$

where $U(x,y,z)$ is the potential periodic in one or two directions perpendicular to the x direction, which is the direction of the tunneling current m is the electron mass. We will discuss firstly the most simple case (a) and then will extend our consideration to cases (b) and (c).

A. Single chain of periodically spaced δ barriers: Two-dimensional case

We first consider tunneling across a barrier characterized by the following potential:

$$U(x,y)=W\left(\delta(x)\sum_n \delta(y-bn)\right), \tag{2}$$

where W is the strength of the potential barrier periodic in the y direction in units of (energy $\times$ length^2) and b is the lattice constant of the 1D array of the δ-potential dots in the

y direction. The electron wave function of the system described by the potential Eq. (2) satisfies the Lippman-Schwinger formula, which is equivalent to the Schrödinger equation:

$$\Psi(x,y)=\exp[i(k_x x+k_y y)] + \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' G(x,x',y,y') \times \Psi(x',y') U(x',y'), \tag{3}$$

where $G(x,x',y,y')$ is the electron Green function; $\mathbf{k}$ is the wave vector of an incident electron. In the basis of plane waves characterized by the 2D wave vector $\mathbf{p}$, the Green function reads

$$G(x,x',y,y')=\frac{2m}{\hbar^2}\int \frac{\exp[i(x-x')p_x+i(y-y')p_y]}{p^2-k^2} \times \frac{dp_x dp_y}{(2\pi)^2}. \tag{4}$$

We note that this expression for the Green function is easily generalized to a system of any dimensionality.

We solve now Eq. (3) using the symmetry properties of the problem. Since the potential is translationally invariant in the y direction, we write the wave function in the form of a Bloch function, i.e.,

$$\Psi(x,y)=\exp(ik_y y)u(x,y), \tag{5}$$

where

$$u(x,y)=u(x,y+nb) \tag{6}$$

is a periodic function. Integrating Eq. (3) over both coordinates x and y and using the δ-functional form of the potential barrier we obtain

$$\Psi(x,y)=\exp[i(k_x x+k_y y)]+\frac{2m}{\hbar^2}W\sum_n \Psi(0,bn) \times \int \frac{\exp[i(xp_x+(y-bn)p_y)]}{p^2-k^2}\frac{dp_x dp_y}{(2\pi)^2}. \tag{7}$$

By using the periodicity of the wave function we find

$$\Psi(x,y)=\exp[i(k_x x+k_y y)]+u(0,0)W\frac{2m}{\hbar^2}\sum_n \exp(ik_y bn) \times \int \frac{\exp[i(xp_x+(y-bn)p_y)]}{p^2-k^2}\frac{dp_x dp_y}{(2\pi)^2}, \tag{8}$$

where $u(0,0)=u(0,bn)$ is a constant that will be determined later. In our calculation, it is more convenient to retain the Green function in its most general form rather than to use its explicit form, which is the Hankel function in the 2D case (or the spherical wave in the 3D problem). Then, when the sum over the real space lattice vector is substituted by the sum over the space of the reciprocal lattice vector, the wave function reads

$$\Psi(x,y)=\exp[i(k_x x+k_y y)] + \frac{2m}{\hbar^2}u(0,0)W\int \left(\sum_l \delta_{k_y,p_y-2\pi l/b}\right) \times \frac{\exp[i(xp_x+yp_y)]}{p^2-k^2}\frac{dp_x dp_y}{(2\pi)^2}, \tag{9}$$

which allows easy summation over the components of the vector $\mathbf{p}$, yielding

$$\Psi(x,y)=\exp[i(k_x x+k_y y)]+\frac{2mu(0,0)W}{\hbar^2 b}\sum_l \frac{\exp[ik_x^{(l)}|x|]}{k_x^{(l)}} \times \exp\left[i\left(k_y-\frac{2\pi l}{b}\right)y\right], \tag{10}$$

where

$$k_x^{(l)}=\sqrt{k_x^2-\frac{4\pi l}{b}\left(k_y+\frac{\pi l}{b}\right)}=\sqrt{\frac{2mE}{\hbar^2}-\left(k_y+\frac{2\pi l}{b}\right)^2} \tag{11}$$

is the x component of the wave vector after diffraction by the periodic barrier. One can see that in contast to tunneling through a barrier that is uniform in the transverse direction, and that conserves the k_y component of the wave vector, diffraction through the periodic barrier allows the transverse wave vector to differ from its initial value by the reciprocal lattice vector of the periodic structure, i.e., $2\pi/b$. The magnitude of the longitudinal component of the wave vector of the diffracted electron is determined by energy conservation and its sign coincides with that of the incoming wave for the transmitted wave ($x>0$) and is opposite for the reflected wave ($x<0$). The constant $u(0,0)$ is determined by the linear equation obtained from Eq. (10) at $x=0$, $y=0$ and reads

$$u(0,0)=\frac{1}{1-it\sum_l (k_x^{(l)})^{-1}}, \tag{12}$$

where

$$t=\frac{mW}{\hbar^2 b}. \tag{13}$$

In general, summation in Eqs. (10) and (12) extends over all integer l, therefore including evanescent modes. However, since we are interested only in tunneling current through the barrier, our summation over l in Eqs. (10) and (12) effectively extends to numbers l, which correspond to real wave vectors of the diffracted electron $k_x^{(l)}$ characterizing the extended states. In particular, at zero temperature, the $k_x^{(l)}$ contributing to transmission are smaller than the Fermi wave vector $\sqrt{2mE_F}/\hbar$. We note that the second term in Eq. (10) contains an uncertainty, which in the limit of $k_x^{(l)}=0$ is resolved so that the wave function is finite.

B. Single chain of periodically spaced δ barriers: Three-dimensional case

In this subsection we consider a tunneling barrier characterized by the same potential as in Eq. (2), but in contrast to

the former situation, the wave vector of tunneling electrons has a k_z component in addition to the 2D k_x and k_y components. Physically, this situation corresponts to diffraction of 3D electrons by a plane containing periodic array of thin quantum antiwires made of a material with the bottom of conduction band being considerably higher than that in the rest of a device. Since $U(x,y,z)$ is z independent, the k_z component of the wave vector is conserved during tunneling. Therefore, the 3D problem at hand allows the trivial reduction to the 2D problem. The wave function corresponding to Eqs. (12) and (14) can be written as

$$\Phi(x,y,z)=\exp(ik_z z)\Psi(x,y), \tag{14}$$

where $\Psi(x,y)$ is given by the same Eq. (3) as in the 2D case (Sec. II A) with

$$E=\frac{\hbar^2}{2m}(k_x^2+k_y^2+k_z^2) \tag{15}$$

and

$$k_x^{(l)}=\sqrt{k_x^2-\frac{4\pi l}{b}\left(k_y+\frac{\pi l}{b}\right)}=\sqrt{\frac{2mE}{\hbar^2}-k_z^2-\left(k_y+\frac{2\pi l}{b}\right)^2}. \tag{16}$$

The wave function of the 3D system is given by Eqs. (12) and (14) with $k_x^{(l)}$ determined by Eq. (16).

C. Two chains of periodically spaced δ barriers

Electron diffraction through a single periodic chain of barriers bears some analogy with 1D tunneling through a single barrier. In quasi-1D configuration, one of the most intriguing phenomena is resonant tunneling, which occurs when the barrier contains a quantum well characterized by quasistationary states.[17] More general tunneling problems such as resonant transmission through a single trap contained in a quasi-1D potential barrier were discussed by Knauer, Richter, and Seidel[13] and Kalmeyer and Laughlin.[16] In that kind of problem a conventional scattering approach is usually utilized owing to the finite potential range of the trap. In this subsection we investigate a new kind of transmission: diffraction across two chains of periodically spaced δ barriers, which manifests certain features of resonant electron tunneling through quasistationary states located between the chains. The potential barrier is assumed to be given by

$$U(x,y)=W\left(\delta(x)\sum_n \delta(y-bn) + \delta(x-x_0)\sum_n \delta(y-y_0-bn)\right), \tag{17}$$

where x_0 is the distance between the two periodical chains, y_0 is the displacement in the y direction of δ barriers on the second chain relative to barriers of the first chain, W is the coefficient (the barrier strength) describing the potential periodical in the y direction. We assume that both chains are similar, i.e., described by the same coefficient W (see Sec. II A). In order to preserve the symmetry properties of the system, i.e., the translational invariance in the y direction, the lattice constant b is chosen to be equal for both chains. We note that the case $x_0=0$ and $y_0=0$ is reduced to model (i) with the barrier with the double strength. In the 2D configuration, we write the wave function in the Bloch form (see Sec. II A):

$$\Psi_2(x,y)=\exp(ik_y y)u_2(x,y), \tag{18}$$

where

$$u_2(x,y)=u_2(x,y+nb). \tag{19}$$

Following along the lines of Sec. II A, we obtain the following wave function:

$$\Psi_2(x,y)=\exp[i(k_x x+k_y y)] + \frac{2mu_2(0,0)W}{\hbar^2 b}\sum_l \frac{e^{ik_x^{(l)}|x|}}{k_x^{(l)}}e^{i(k_y-2\pi l/b)y} + \frac{2mu_2(x_0,y_0)e^{ik_y y_0}W}{\hbar^2 b}\sum_l \frac{e^{ik_x^{(l)}|x-x_0|}}{k_x^{(l)}} \times e^{i(k_y-2\pi l/b)(y-y_0)}. \tag{20}$$

The unknown amplitudes $u_2(0,0)$ and $u_2(x_0,y_0)$ are readily found from the system of two algebraic equations obtained by substituting $x=0$, $y=0$ and $x=x_0$, $y=y_0$ in Eq. (20). These amplitudes are given by

$$u_2(0,0)=\frac{1+\sum_n \frac{it}{k_x^{(n)}}-e^{ik_x x_0}\sum_n \frac{it}{k_x^{(n)}}e^{(ik_x^{(n)}x_0-2\pi i n y_0/b)}}{\left(1+\sum_n \frac{it}{k_x^{(n)}}\right)^2+t^2\sum_n \frac{1}{k_x^{(n)}}e^{i(ik_x^{(n)}x_0-2\pi i n y_0/b)}\sum_n \frac{1}{k_x^{(n)}}e^{(ik_x^{(n)}x_0+2\pi i n y_0/b)}}, \tag{21}$$

$$u_2(x_0,y_0)=\frac{\left(1+\sum_n \frac{it}{k_x^{(n)}}\right)e^{ik_x x_0}-\sum_n \frac{it}{k_x^{(n)}}e^{i(k_x^{(n)}x_0+2\pi i n y_0/b)}}{\left(1+\sum_n \frac{it}{k_x^{(n)}}\right)^2+t^2\sum_n \frac{1}{k_x^{(n)}}e^{(ik_x^{(n)}x_0-2\pi i n y_0/b)}\sum_n \frac{1}{k_x^{(n)}}e^{(ik_x^{(n)}x_0+2\pi i n y_0/b)}}. \tag{22}$$

III. THE ELECTRIC CURRENT AND CONDUCTANCE

A. General considerations

In order to calculate the electric current and conductance we use a formalism similar to the Landauer model.[14] We assume that the chemical potentials on each side of the active (barrier) region of the device, i.e., in the electrodes (electron reservoir), are uniform and constant but differ from one another by a small potential difference δV. The electron distribution functions in the electrodes are assumed to be the Fermi distribution functions $f(\epsilon)$. Calculating the difference of currents from the left to the right and from the right to the left we obtain

$$j = e\,\delta V \int_0^\infty dk_x \int_{\Omega_\perp} d\mathbf{k}_\perp j_{\mathbf{k}}^{(x)} \frac{\partial f(\epsilon)}{\partial \epsilon}, \tag{23}$$

where $\Omega_\perp$ is the phase space corresponding to the transverse wave-vector component. We note that the integral in Eq. (23) extends over positive values of k_x and over the whole $\Omega_\perp$ subspace. The matrix element of the current $j_{\mathbf{k}}^{(x)}$ in Eq. (23) is given by

$$j_{\mathbf{k}}^{(x)} = \int d^3\mathbf{r} \frac{ie\hbar}{2m}[(\nabla_x \Psi^*)\Psi - \Psi^*(\nabla_x \Psi)]. \tag{24}$$

The current matrix element can be calculated at any value of x (left or right to the barrier or in the barrier region) due to the current conservation. Equation (23) expressing the current via its matrix element is very convenient when the wave function in the whole space is known. In a 1D conductor Eq. (23) immediately yields the Landauer formula. We note that when the wave function cannot be easily calculated, but the Green function in the barrier region is known, the numerical approach of Kalmeyer and Laughlin[16] can be readily applied.

B. Current through a single chain of periodically spaced δ barriers

By using the general expression for the current matrix element Eq. (24) we obtain the current matrix element in the case of diffaction through a single chain of periodically spaced δ barriers:

$$j_{\mathbf{k}}^{(x)} = \frac{e\hbar}{m}\left(k_x - 2t\,\mathrm{Im}[u(0,0)] + t^2 \left| u(0,0) \right|^2 \sum_l (k_l)^{-1} \right), \tag{25}$$

where Im[ ] denotes the imaginary part, $k_l = k_x^{(l)}$. Substitution of Eq. (12) in Eq. (25) yields

$$j_{\mathbf{k}} = \frac{e\hbar}{m}\left(k_x - \frac{t^2\left(\sum_l (k_l)^{-1} \right)}{1 + t^2\left(\sum_l (k_l)^{-1} \right)^2} \right). \tag{26}$$

In the case of zero-temperature conductance $G = dj/d\,\delta V$ reads

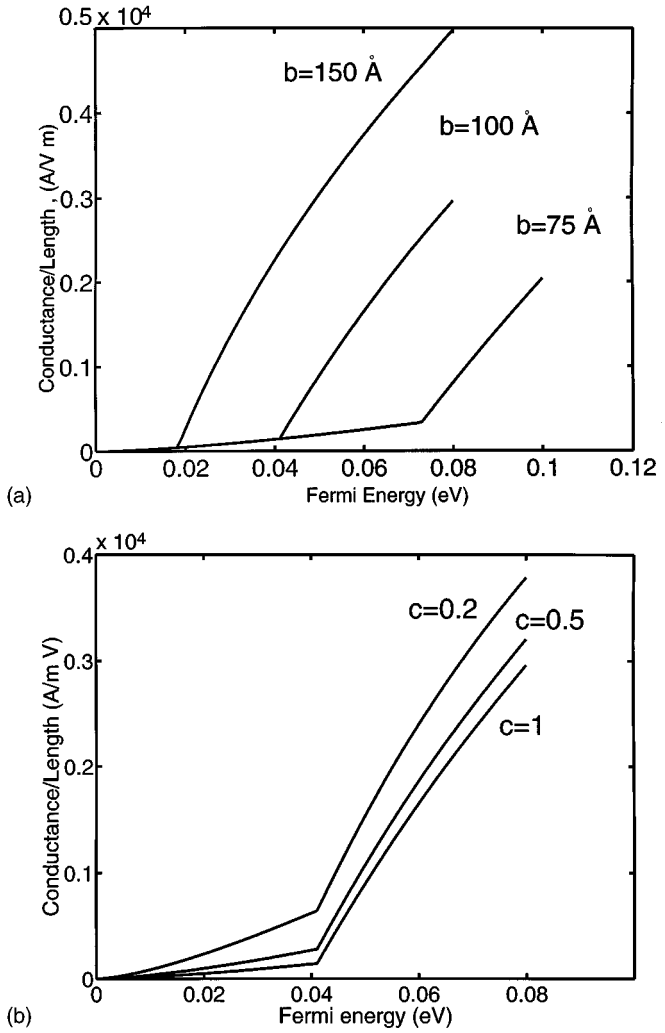

FIG. 2. (a) 2D conductance of a periodic 1D array of quantum antidots as a function of the Fermi energy for three different array periods at $T = 0$ K, c=1, (b) 2D conductance for different barrier strengths.

$$G = \frac{e^2}{\hbar}\int_{\Omega_\perp} d\mathbf{k}_\perp \left(1 - \frac{t^2\left(\sum_l (k_l k_0)^{-1} \right)}{\left\{ 1 + t^2\left[\sum_l (k_l)^{-1} \right]^2 \right\}} \right). \tag{27}$$

At zero temperature, channels for electron diffraction open up successively: hence for Fermi energy E_F smaller than the energy corresponding to half of the reciprocal lattice vector, i.e., $(\hbar\pi)^2/2mb^2$, the transverse component of the wave vector cannot be changed during the tunneling since the transmitted wave vector would be larger than the Fermi wave vector, which is forbidden for coherent processes. For $E_F > (\hbar\pi)^2/2mb^2$ the incident electron wave vector can be changed by the amount equal to the smallest reciprocal lattice vector, first. Then with the opening of new diffraction channels the wave vector can be changed by a larger and larger amount.

If only one diffraction channel $l = 0$ is open (i.e., when the electron wavelength is much larger than the distance between barriers so that tunneling electrons are effectively sen-

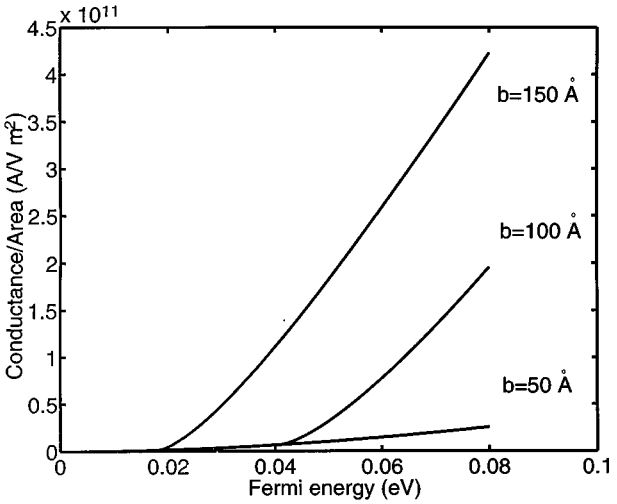

FIG. 3. 3D conductance of periodic arrays of antiwires for different periods, $c=1$.

sitive to a uniform potential) Eq. (26) is nothing but the expression for the current through the uniform 1D δ barrier obtained from the Landauer formula.[14] When, with the increase of the Fermi energy and the reduction of the electron wavelength, a new channel of diffraction opens up the electric current increases correspondingly.

The current increase with the variation of the Fermi energy may be abrupt and much stronger than in the case of 1D tunneling through a single δ barrier. In Fig. 2(a) we show the 2D conductance of a periodic 1D array of quantum antidots as a function of the Fermi energy of incident electrons for three different array periods at $T=0$ K and at high potential strength. To describe the potential strength, we use the dimensionless parameter c, rather than using parameter t given by Eq. (13):

$$c=t^2/Q^2(1\text{eV}), \tag{28}$$

where $Q(1\text{ eV})$ is the wave vector corresponding to the electron energy $E=\hbar^2Q^2/2m=1$ eV. By writing $W=Ua_xa_y$, where U is the barrier height, and a_x and a_y are the barrier thickness in the x direction and the width in the y *direction*, respectively, the value $c=1$ could represent a rectangular barrier of height $U=0.3$ eV, with $a_x=75$ Å, and with $a_y=b/2$. The height of the barrier corresponds to the difference between the bottom of conduction bands in GaAs and in the solid solution $Al_{0.35}Ga_{0.65}As$ located at the Γ point. The electron mass is taken to be $m=0.068m_0$, as in GaAs. Let us point out that for other materials, alternative combinations of barrier heights and thicknesses can be determined.

In Fig. 2(a) it is seen that at high potential strength the conductance exhibits low and high conductivity regimes separated by an abrupt transition when a new diffraction channel opens up. The transition occurs at the Fermi energy for which the wave vector of the incident electrons is exactly equal to half of the reciprocal lattice vector of the periodic array π/b. In the new diffraction channel the y *component* of the wave vector of electrons changes by the smallest reciprocal lattice vector, $2\pi/b$, which means that the values of the y component of the wave vectors of the incident and the transmitted electron are π/b and $-\pi/b$, i.e., have the same

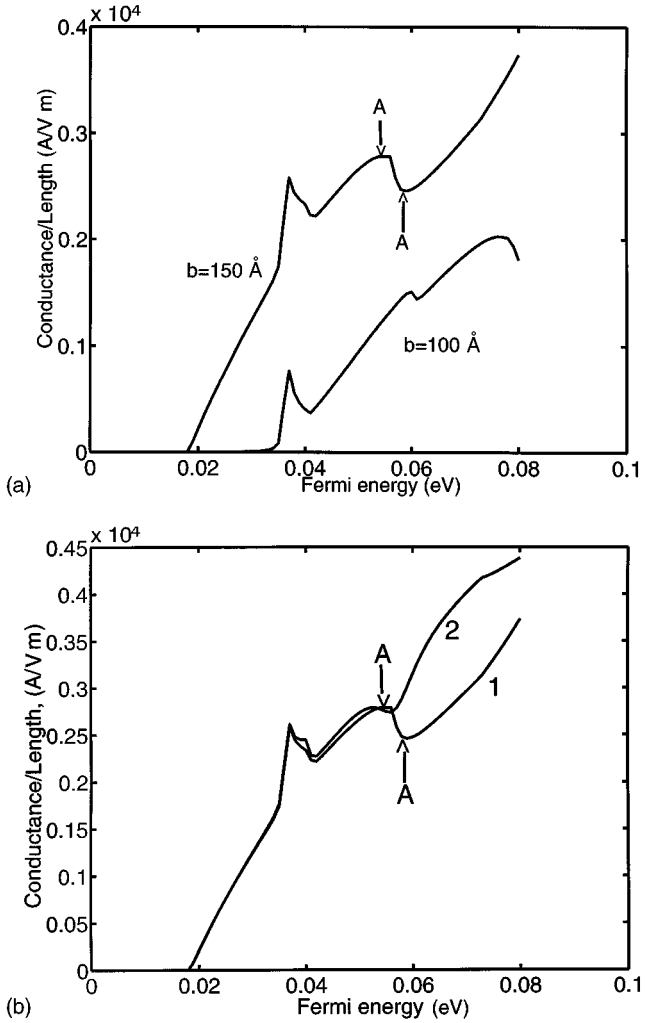

FIG. 4. Current-voltage characteristics for the case of tunneling across two parallel periodic arrays of antidots at $T=0$ K and $c=1$: (a) the two arrays separated by $x_0=100$ Å (no shift of arrays relative to each other) with two different periods of the antidot lattice. (b) Comparison of conductance resonant features between the two arrays with the period $b=150$ Å, (curve 1, no relative shift between arrays; curve 2, with a relative shift of $y_0=40$ Å).

magnitude but the opposite sign. For our choice of parameters, in the three cases, the abrupt transition for the opening of the first new channel is particularly well defined while transitions for higher-order channels are much weaker and barely noticeable, as follows from Eq. (27). Figure 2(b) shows the 2D conductance for different parameters c, which correspond to different potential barrier strength. The potential strength is regulated by the barrier height and the barrier sizes in the x and y directions. It is seen that the abrupt transition still occurs at the same Fermi energy, but the conductance for the low conductivity regime decreases as the strength of the barrier increases. Note also that the abruptness of the transition is enchanced with c. Let us mention that in the Landauer formalism, as a result of Eq. (23) the current-voltage characteristic, at low voltages, is obtained by the simple multiplication of the conductance by the voltage drop accross the arrays.

In Fig. 3, we plot the 3D conductance [case (b)], which shows a behavior similar to the 2D case, but with slightly

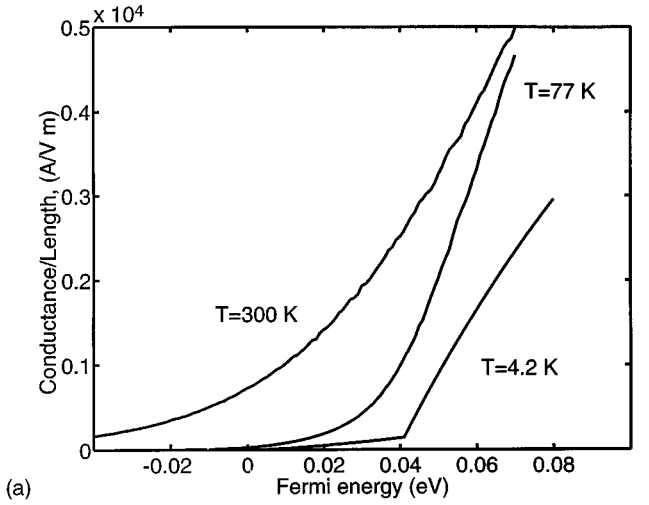

FIG. 5. (a) 2D conductance vs Fermi energy at finite temperatures. (b) 3D conductance vs Fermi energy at finite temperatures.

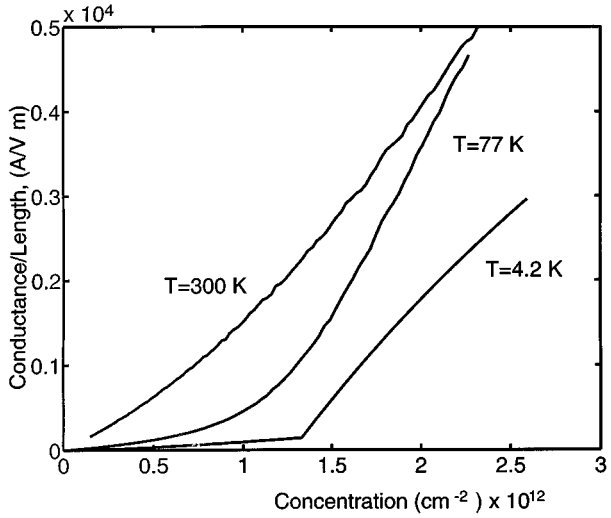

FIG. 6. 2D conductance vs electron concentration for three different temperatures.

smoother transitions between the low and high conductivity regimes. We note that experimental observation of such highly nonlinear characteristics could be realized by both current (conductance) measurements and, especially by the current derivative with respect to the Fermi energy (transconductance) which, for 2D electron gases, could be related to the variation of the gate voltage in a field effect transistor. Such high nonlinearities make possible the application of electron diffraction in field effect transistors.[15]

C. Current across two chains of periodically spaced δ barriers

The current matrix element for the electron transmission through two parallel lines of δ barriers reads

$$j_{\mathbf{k}}=\frac{e\hbar}{m}\Bigg\{k_x+2t\,\mathrm{Im}[u(0,0)+u(x_0,y_0)e^{-ik_x x_0}]+t^2[|u(0,0)|^2+|u(x_0,y_0)|^2]\sum_l (k_l)^{-1}+2t^2\,\mathrm{Re}\times\Bigg[u^*(0,0)u(x_0,y_0)\sum_l (k_l)^{-1}e^{-i(k_l x_0+2\pi l y_0/a)}\Bigg]\Bigg\}. \tag{29}$$

Here we use Eq. (20) for the expression of the wave function. Explicit expression for the current as a function of the potential strength and period is obtained by substituting the expressions of $u(0,0)$ and $u(x_0,y_0)$ given by Eqs. (21) and (22) in Eq. (28). This procedure is rather tedious and we will not present it here.

It is possible to show by solving transcendental equations that the denominators of $u_2(x_0,y_0)$ and $u_2(0,0)$ are minimum and equal to unity for specific magnitudes of the incident electron wave vector. This situation that is similar to 1D resonant tunneling through the δ barrier corresponds to 2D and 3D resonant tunneling in the present case. We note that the case of a single diffraction channel is described by exactly the same relations as 1D tunneling through the double δ barrier. Figure 4(a) shows that resonant tunneling in the diffraction picture can be characterized by resonance features in both the low and high impedance regimes depending on the period of the diffraction array. One can see that the periodic barrier exhibits not only resonances that characterize tunneling through the continuous double δ barrier, but also additional resonances indicated by the arrows A. Figure 4(b) shows that these additional resonant features can be tuned by varying the displacement y_0. The closer y_0 to a half of a period of the array, the larger the shadowing effect that eliminates the resonant feature A.

IV. TEMPERATURE DEPENDENCE OF THE CURRENT

The current and conductance characteristics at finite temperatures are derived from Eq. (23) by computing the derivative of the Fermi function.

Because of the spreading of the electron distribution at high energy, the simultaneous contribution of several diffraction channels occurs at finite temperatures even in the limit of long wavelength. In this case the contribution of different channels to the conductance is determined by the exponential

cutoff of the carrier distribution at large wave vectors after diffraction. From this consideration, it is seen that by choosing chains of smaller periodicity (and, correspondingly, larger reciprocal lattice vectors) it is possible to suppress the contribution of diffraction channels characterized by a significant change in the electron wave vector at small chemical potential, and, thereby, achieving more abrupt increase in the conductance with increasing Fermi energy. Conductance versus electrochemical potential for finite temperatures is presented on Fig. 5(a) for 2D diffraction by linear chains and Fig. 5(b) for 3D diffraction by linear chains. At liquid helium and nitrogen temperatures, we clearly distinguish the low and high impedance regimes. At room temperature, however, thermal smearing of the transition between low and high impedance states becomes substantial and the two regimes are undistinguishable. Figure 6 shows the conductance versus 2D electron concentration at different temperatures, which shows little difference from Fig. 5(a) except at high temperatures and negative Fermi energies. Since the concentration of the 2D electron gas could be tuned by the gate of a field effect transistor, it is therefore possible to control the transition from the low to the high impedance regime simply by varying the gate voltage.

V. CONCLUSION

We have considered a class of quantum problems of electron diffraction by periodic structures. In the particular geometry considered here, diffraction occurs through multiple channels with transverse electron wave vectors differing by the reciprocal lattice vector of the periodic array. We also showed that tunneling in this diffraction geometry is analytically solvable in two and three dimensions for 1D and 2D arrays of δ potentials. Highly non-linear conductance charateristics in the vicinity of the Fermi energy, corresponding to the opening of a new diffraction channel, have been found. Conductance characteristics show low and high impedance regimes as a function of the electrochemical potential. For diffraction by two chains of periodic δ-potential arrays, the conductance exhibits rich resonant structures determined by the array period and spacing between chains.

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[1] H. Sakaki, K. Wagatsuma, J. Hamasaki, and S. Saito, Thin Solid Films **36**, 497 (1976).
[2] D.A. Antoniadis, K. Ismail, and H.I. Smith, in *Science and Engineering of One and Zero-Dimensional Semiconductors*, Volume 214 of *NATO Advanced Studies Institute Series B: Physics*, edited by S. Beaumont and C. Sotomayor Torres (Plenum, New York, 1990), p. 179.
[3] D. Weiss, M. Roukes, A. Menschig, P. Grambow, K. von Klitzing, and G. Weimann, Phys. Rev. Lett. **66**, 2790 (1991).
[4] R. Schuster, K. Ensslin, J.P. Kotthaus, M. Holland, and S.P. Beaumont, Superlatt. Microstruct. **12**, 93 (1992).
[5] R. Schuster, K. Ensslin, J.P. Kotthaus, M. Holland, and C. Stanley, Phys. Rev. B **47**, 6843 (1993).
[6] D. Weiss, K. Richter, A. Menschig, H. Schweizer, K. von Klitzing, and G. Weimann, Phys. Rev. Lett. **70**, 4118 (1993).
[7] H. Drexler, D. Leonard, W. Hansen, J.P. Kotthaus, and P.M. Petroff, Phys. Rev. Lett. **73**, 2252 (1994).
[8] A. Messiah, *Quantum Mechanics* (North-Holland, Amsterdam, 1964), Vol. 1, p. 504.
[9] S. Flügge, *Practical Quantum Mechanics* (Springer-Verlag, New York, 1974).
[10] S.L. Dudarev and M.J. Whelan, Phys. Rev. Lett. **72**, 1032 (1994).
[11] E.G. McRae, Rev. Mod. Phys. **51**, 541 (1979).
[12] J.B. Pendry, *Low Energy Electron Diffraction* (Academic, New York, 1974).
[13] H. Knauer, J. Richter, and P. Seidel, Phys. Status Solidi A **44**, 303 (1977).
[14] R. Landauer, IBM J. Res. Dev. **1**, 223 (1957).
[15] Yu. Lyanda-Geller and J.P. Leburton (unpublished).
[16] V. Kalmeyer and R. Laughlin, Phys. Rev. B **35**, 9805 (1987).
[17] L. Esaki and R. Tsu, Appl. Phys. Lett. **24**, 593 (1974).