Edge and bulk transport in variably connected quantum Hall conductor

M. Rahman, J. H. Davies, I. A. Larkin, and M. C. Holland
Department of Electronics and Electrical Engineering, University of Glasgow, Glasgow G12 8QQ, United Kingdom

A. R. Long
Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom

J. G. Williamson
Department of Electronics and Electrical Engineering, University of Glasgow, Glasgow G12 8QQ, United Kingdom
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We have performed an experiment in the quantum Hall regime using a geometry that has connectivity between two and three depending on the applied gate voltage. The measurements may be interpreted in terms of either bulk or edge transport. However, the variable connectivity reveals fundamental differences in the current flow through the gated region in the two pictures. In the edge picture this current does not link the measurement contacts and is dissipationless; in the bulk picture it links the contacts and requires dissipation to flow. This extends previous results on the equivalence of the two pictures in simply connected geometries. [S0163-1829(96)08748-6]

In initial analyses[1,2] of transport in quantum Hall devices the two-dimensional electron gas (2DEG) was regarded as a conducting sheet with a completely antisymmetric conductivity tensor. The edge potential was the only property of the sample edges considered to be of importance. In this bulk picture current flow is not restricted *a priori* to any particular region of the sample. The structure of electronic states at the sample edges was first considered by Halperin[3] and led to the subsequent formulation of the edge-channel picture by Büttiker.[4] In contrast to the bulk picture, in the edge picture the current is constrained to flow only along the edges of the 2DEG.

Detailed calculations of the Hall conductance in the bulk picture require the evaluation of a Kubo formula.[1,2] Generally, this is difficult for samples with barriers. On the other hand, the edge-channel picture, as embodied in the Landauer-Büttiker formalism, provides a ready framework for understanding measurements on such samples.[5] Experiments suggest that the current density is indeed highest near the edges.[6–8] However, the maximum current density supportable by edge states and the screening of edge charges suggest that current flow cannot be limited just to infinitesimal filamentary paths. This points to transport in the quantum Hall regime being described by something intermediate to the above pictures.[9–11]

In Hall and other simply connected geometries the bulk and edge pictures lead to equivalent interpretations of experimental measurements.[9,12] Thus, it is immaterial which picture is used and calculations in both pictures yield identical results. In this paper we ask whether this is true also of more complex topologies. Specifically, we employ a structure having a connectivity that varies between two and three depending on the value of applied gate voltage. We find that the two pictures can both explain observed properties but that they make fundamentally different statements regarding current flow in the gated region. A structure with connectivity varying between one (simply connected) and two has been studied recently by Sachrajda *et al.*[13]

For the experiment we used a GaAs/$Al_xGa_{1-x}As$ heterostructure grown by molecular-beam epitaxy. The 2DEG was a little over 92 nm below the surface and had a carrier concentration $n_{2D}=3.0\times10^{11}$ cm^{-2} and Hall mobility (1.5–2)$\times10^6$ cm^2/V s at a temperature 1.3 K. Sample fabrication was by electron-beam lithography. A schematic diagram of the device geometry used (device A) is shown in the inset to Fig. 1. A second device with no gates (device B) acted as a ''control'' sample and a conventional Hall bar with gated

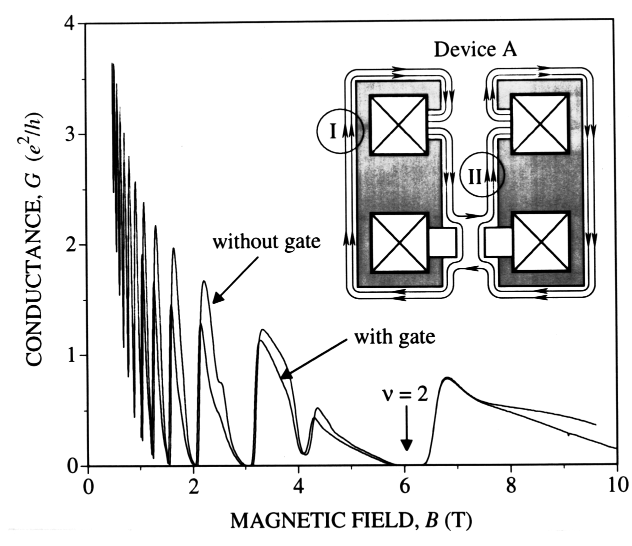

FIG. 1. Two-terminal magnetoconductance for devices with (device A) and without (device B) the bridging gate, at $T=1.3$ K. The gate voltage on device A was set to zero. The magnetic field points out of the page. Inset: Schematic diagram of device A, showing two ''Corbino islands'' with bridging gates. The shaded area represents wet-etched regions.

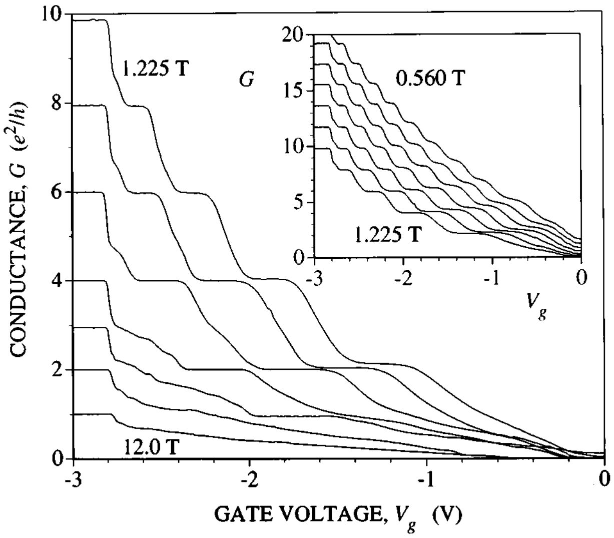

FIG. 2. Measured conductance curves with varying V_g at successive integer bulk filling factors $\nu \lesssim 10$. The fields used were 12.00, 5.90, 4.08, 3.05, 2.04, 1.53, and 1.225 T. Inset: Measured conductances for the integer bulk filling factors at fields 1.225, 1.025, 0.878, 0.768, 0.682, 0.615, and 0.560 T.

and ungated regions allowed comparative measurements.

Only two Ohmic contacts were patterned allowing just two-terminal conductance measurements: dissipation in the electron reservoirs associated with separate voltage contacts could be ignored. Regions surrounding the contacts were removed by wet etching, creating two ''islands'' (Corbino contacts) in the 2DEG with a separation of roughly 40 μm. Gates extended from each island to form a constriction of lithographic width 320 nm and length 120 nm.

Measurements were made at temperatures down to 1.3 K using low-frequency lock-in techniques. Figure 1 shows the two-terminal conductances of devices A and B as a function of the magnetic field at $T=1.3$ K, with the gate voltage on device A set to zero, $V_g=0$. Although the curves differ slightly in amplitude their minima coincide, occurring at integer bulk filling factors ν as determined from Shubnikov–de Haas and quantum Hall effect measurements on the regular Hall geometry.

These measurements show that at conductance minima for magnetic fields $B \gtrsim 1$ T the bulk conductance in device A is small, $G_b \lesssim e^2/h$. Specifically, at even filling factors for $B \gtrsim 2$ T we have $G_b \approx 0$. For the material and temperature used, spin splitting can be resolved only for $B \gtrsim 2$ T so states of odd filling factor are weak: an embryonic $\nu=5$ state is visible at $B=2.5$ T with suppression of G_b more evident at $\nu=3$. Thus for $V_g=0$ device A behaves much as if the gate was not present.

Figure 2 shows measurements on device A in which the two-terminal conductance G was measured while sweeping V_g, with B fixed to give the even filling factors $\nu \lesssim 10$ and the odd filling factors $\nu=1,3$. Apart from the $\nu=3$ and $\nu=10$ curves all have $G_b \approx 0$. At each filling factor $\nu<10$, G goes through a sequence of values quantized in units of e^2/h terminating at a conductance $G_p=\nu e^2/h$ when the constriction is pinched off (at $V_g=V_p \approx -2.85$ V). At $\nu=6,8$ the sequence is given by the formula $G_N=Ne^2/h$, $N=2,4,\ldots,\nu$. For $B \gtrsim 2.5$ T spin splitting appears and $G_N=Ne^2/h$, $N=1,2,\ldots,\nu$, although at $(\nu,N)=(3,3)$ the conductance does not flatten properly into a plateau.

The inset to Fig. 2 shows curves measured for filling factors $\nu \gtrsim 10$. For these curves the bulk conductance is finite, $G_b \gtrsim 0$. Again G goes through a sequence of quantized values, but this time not exactly in units of e^2/h. The sequences terminate at conductances $G_p \lesssim \nu e^2/h$ and both G_b and $\delta G_p=\nu e^2/h-G_p$ grow with increasing ν. The threshold voltage for the material, as measured on the gated portion (wide solid gate) of the regular Hall bar, was $V_t=-0.39$ V.

The curves bear a strong resemblance to measurements with point contacts on a standard Hall geometry. One immediate difference is that in a Hall geometry the conductance decreases with more negative gate voltage whereas in device A the conductance increases. A further important inequivalence regarding the application of the edge and bulk pictures to the structure is brought into focus below. All these factors relate to topological differences between the two geometries.

The experiment is readily interpreted in the edge-channel picture. We calculate the two-terminal conductance in the Landauer-Büttiker formalism.[4] The inset to Fig. 1 depicts the edge-current flow. The regions marked I and II contain adjacent edge channels at different chemical potentials. The resulting interedge channel scattering does not affect the analysis since it does not affect the net current flow into the contacts.

The constriction acts as a tunable barrier passing only certain edge channels. Consider N_R+1+N_T edge channels emerging from one of the contacts, of which N_R pass through the constriction and return to the contact (are ''reflected'') and N_T proceed to the other contact (are ''transmitted''). The remaining channel is partially reflected and partially transmitted with coefficients r and t. From current conservation $r+t=1$, whence the two-terminal conductance is

$$G=\frac{e^2}{h}(N_T+t),$$

as observed when $G_b=0$. Plateaus in G occur when $t=0$ or 1, with intermediate values describing transitions. Note that a net current appears to flow through the constriction, but that it does not link the measurement contacts.

For weaker magnetic fields the small bulk conductance $G_b \gtrsim 0$ corresponds to the case where states at the chemical potential in the bulk are not perfectly localized even though they lie between Landau levels. In this case the transport has a dissipative component. Near the contacts, the dissipative transport permits coupling between outgoing and incoming edge-current flows,[5] i.e., a fraction of the current injected into outgoing edge states is backscattered into incoming edge states. This causes the conductance steps in the inset to Fig. 2 to be less than the perfectly quantized value $2e^2/h$.

The two-terminal conductance for $\nu=10$ was also measured at higher temperature and excitation as shown in Fig. 3. The two sets of measurements are similar qualitatively, the conductance plateaus smearing with both increased temperature and increased excitation. This is readily attributed to heating effects with the heating being ''local'' (i.e., near the current paths) in the case of increased excitation and ''glo-

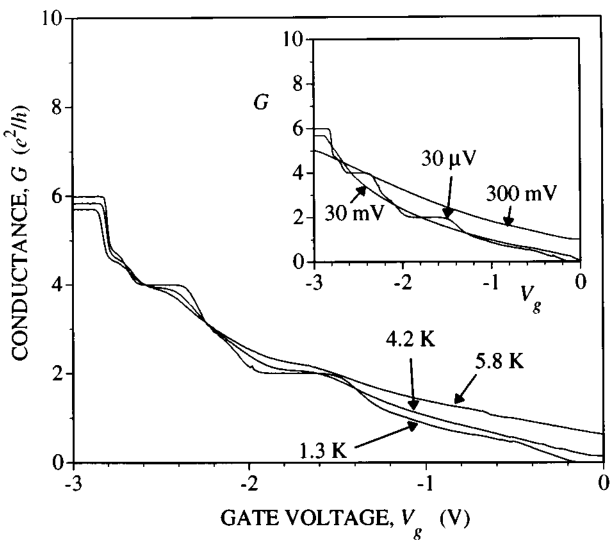

FIG. 3. Measured conductance curves with increasing temperature for $\nu=10$ at a voltage excitation of 30 μV. As the temperature is raised, the plateaus structure smears. Inset: Measured conductance curves with increasing excitation for $\nu=10$ at $T=1.3$ K.

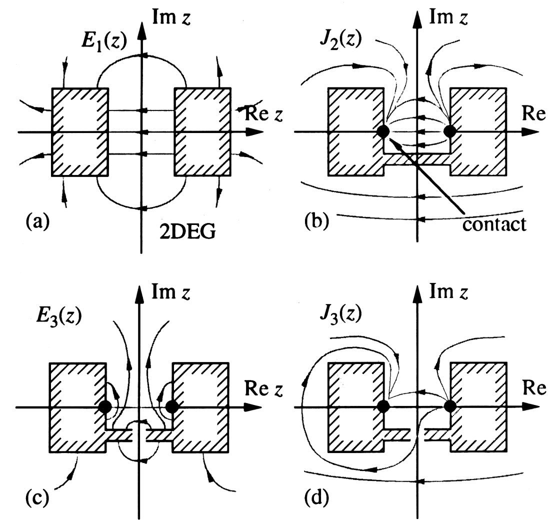

FIG. 4. Schematic bulk distributions for the variably connected geometry with $G_b=0$. The right-hand contact is positively biased with respect to the left-hand one and the magnetic field points out of the page. (a) $V_g=0$. The current lines are orthogonal to $E_1(z)$ and circulate the islands. (b) $V_g=V_p$. (c) Quadrupolic field distribution $E_3(z)$. (d) Current flow resulting from $E_3(z)$. When $V_g=V_p$ the constriction becomes closed and $J_3(z)\rightarrow J_2(z)$.

bal'' in the case of increased temperature, accounting for the difference in detail between the two sets of curves.

In a Hall geometry all closed-loop paths may be contracted to a point. The geometry of Fig. 1, however, has two macroscopic holes in the conducting plane, which merge into one by the application of a sufficiently large gate voltage. Thus, whereas the Hall bar is simply connected, the geometry of Fig. 1 has a connectivity that varies between two and three depending on V_g. In view of this nonstandard topology we now ask whether the experiment affords an alternative interpretation in a bulk picture without invoking edge states.

When the chemical potential lies within a Landau level the bulk conductivity is diagonal and we may show that the current lines have a distorted dipole form. We are more interested in the case of integer bulk filling factor when the chemical potential is midgap and the diagonal conductivity is zero, so $G_b=0$. The device is represented in the complex z plane as shown in Fig. 4(a). We ignore charge density variations in the 2DEG, so only the edge potential matters, and use conformal methods to find the current patterns.

For $V_g=0$ we assume that the edges of the islands charge to the two contact potentials, corresponding to the condition of zero current normal to the edges. From the conformal mapping of canonical domains[14] and since the electric field strength goes to zero at infinity, the conducting region may be mapped onto the w plane with two disks cut in it centered at $w=\pm a/2$. Solving for the field strength $E_1(w)$ in bipolar coordinates gives the current lines $J_1(z)=i\sigma_H E_1(z)$, with the bulk Hall conductivity $\sigma_H=\nu e^2/h$. These current lines are circulating and do not connect the contacts.

For $V_g=V_p$ the conducting region is doubly connected [Fig. 4(b)]. It may be mapped[14] onto the w plane with the contacts at $w=\pm b/2$ and a cut along the real axis between $\pm b/2$. Opposite sides of the cut charge to the two contact potentials. Using the solution for a dipole, we see that the current flow (not the electric field) $J_2(z)$ has a distorted dipole form. By contrast to $J_1(z)$, these current paths all link the two contacts. The distributions $J_1(z)$ and $J_2(z)$ give conductances of 0 and $\nu e^2/h$, respectively, in agreement with the edge picture.

The situation for intermediate gate voltage, $0<V_g<V_p$, is more complex. The edge potentials must interpolate in a continuous manner between the two extremes above. This requires four distinct edge potentials, the edges in question connecting the two contacts and the constriction. Such boundary conditions arise if there is charge transport through the constriction; they give $J_1(z)$ and $J_2(z)$ in the limits $V_g=0$ and $V_g=V_p$, respectively. Away from these limits, the field has a distorted quadrupole form,[15] which we expect from Laplace's equation with four specified potentials [Fig. 4(c)].

The quadrupole field firstly induces dipolar current flow [converging to $J_2(z)$ in the limit $V_g=V_p$] along equipotential lines linking the two contacts. It further induces a potential difference across the constriction, leading to current flow there. This current must have some associated dissipation. The net current pattern is then as shown in Fig. 4(d). Smooth variation in the gate voltage causes transitions in the filling factor at the saddle point of the constriction. This leads to transitions in the edge potentials and hence in the quadrupolic-field distribution, and would explain the observed quantization. Effects of heating in Fig. 3 may be attributed to additional dissipation in regions of integer filling factor.

The measurements may thus be explained in either a bulk or an edge picture despite the nontrivial topological character of the device. However, there are interesting and fundamental differences between the two pictures. Both pictures give a current flow through the constriction. For edge transport this current is dissipationless, but in the bulk picture it requires additional dissipation. Moreover, in the edge picture the current through the constriction is circulating; i.e., it does not connect the contacts. By contrast in the bulk picture it does. Thus, in the bulk picture the current through the gated region is a measurement current, whereas in the edge picture it is not. In standard Hall geometries the current through the gated region connects the measurement contacts irrespective of the picture used.[12]

The two pictures therefore represent two distinct microscopic transport processes in the gated region of the variably connected geometry. Although both explain experimental observations, it is not true to say that they are entirely equivalent. To distinguish between the pictures it is not sufficient to detect a maximum of current density near the sample edge, since this is found in the bulk picture too. The direct detection of edge channels has been reported in photoeffect experiments on Hall geometries,[7] though not under small-signal conditions. In general, the magnitude of bulk currents induced by edge charges is not clear, but must be known before transport processes in complex topologies may be categorized definitively as being edge like or bulk like.

We finally mention that topology also enters the study of quantum Hall systems in another way. In the topological approach to the quantum Hall effect Thouless *et al.*[16] showed that the magnetic wave functions of electrons in certain special geometries may be characterized by a topological invariant, the first Chern class, and that this invariant determines the Hall conductance. In the present experiment we find that the global conductance of the variably connected geometry goes through integer multiples of e^2/h as the connectivity is varied between two and three. Thus, whereas in a Hall geometry the conductance state is determined solely by the magnetic field (alternatively n_{2D}) and hence the first Chern class, here it is determined in addition by the nonintegral connectivity[17] of the structure.

In summary, we have studied the conductance of a geometry that has connectivity between two and three. Despite the variable connectivity the measured conductance plateaus may be interpreted in either bulk- or edge-transport pictures. This observation generalizes similar assertions for simply connected geometries. However, the current flow through the gated region in the present device is dependent fundamentally on the picture used: it is dissipationless and circulating in the edge picture, but dissipative and connects the contacts in the bulk picture. In terms of microscopic transport properties through the gated region of such structures it matters which picture is used to interpret the measurements.

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