Role of anisotropic impurity scattering in anisotropic superconductors

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A theory of nonmagnetic impurities in an anisotropic superconductor including the effect of anisotropic (momentum-dependent) impurity scattering is given. It is shown that for a strongly anisotropic scattering the reduction of the pair-breaking effect of the impurities is large. For a significant overlap between the anisotropy functions of the scattering potential and that of the pair potential and for a large amount of anisotropic scattering rate in impurity potential the superconductivity becomes robust vis-à-vis impurity concentration. The implications of our result for YBCO high-temperature superconductor are discussed. The experimental data of electron irradiation-induced T_c suppression [Phys. Rev. B **50**, 15 967 (1994)] are understood quantitatively and a good qualitative agreement with the ion (Ne^+) damage and Pr substitution-induced T_c decrease data [Phys. Rev. B **50**, 3266 (1994)] is obtained. [S0163-1829(96)02145-5]

There now exists considerable experimental evidence supporting the d-wave superconductivity in the cuprates (for review see Refs. 1–4). Nevertheless, this scenario still faces some theoretical difficulties. One of these is the predicted extreme suppression of the critical temperature T_c by nonmagnetic impurities.[5–10] Experimentally, however, the observed suppression of T_c by impurities or radiation damage in YBCO is much more gradual.[11,12] This issue was critically examined by Radtke *et al.*[6] who considered isotropic impurity scattering within the second Born approximation by applying the Eliashberg formalism. Their predictions in both weak- and strong-coupling theory gave a T_c suppression which was close to the Abrikosov-Gorkov scaling function[6,13] with an effective impurity scattering rate. This led to an approximate universal dependence of T_c on the planar residual resistivity ρ_0, which did not depend on the details of the microscopic pairing. In order to verify the results of Radtke *et al.*[6] systematic electron irradiation experiments on YBCO were carried out by Giapintzakis *et al.*[12] The measured initial slope of impurity-induced T_c suppression was $dT_c/d\rho \sim -0.30$ K/$\mu\Omega$ cm,[12] whereas the predicted value was in the range from ~ -0.74 to -1.2 K/$\mu\Omega$ cm.[6] While discussing the experimental results in Ref. 12 the authors invoked the issue of the anisotropic impurity scattering. They understood their data within a model of Millis *et al.*[8] assuming a value of 0.5 for a dimensionless parameter g_I which describes the anisotropy of the scattering potential and modifies the bare isotropic impurity scattering rate $1/\tau$ according to $1/\tau^\star = (1-g_I)/\tau$, where $1/\tau^\star$ is the effective scattering rate. Thus the analysis by Giapintzakis *et al.*[12] brings out the significant role of the anisotropic scattering in understanding the impurity effect on d-wave superconductivity and calls for more detailed theoretical studies.

In this paper we consider in detail the problem of nonmagnetic impurities in an anisotropic superconductor for the case of anisotropic (momentum-dependent) impurity scattering by applying weak-coupling approximation. We find a remarkable change in the T_c suppression which becomes more gradual when the anisotropy function defining anisotropy of the impurity potential overlaps with the anisotropy function of the order parameter. Although our formalism is general and valid for any superconducting order parameter described by a one-dimensional (1D) irreducible representation of the crystal point group we discuss the results for a d-wave superconductor in the context of high-temperature superconductivity. In a certain limit, the effective scattering rate in our model is identical to that of Millis *et al.*[8] We compute T_c as a function of planar residual resistivity. Within a certain range of scattering potential parameters values we find a quantitative agreement of our results with the electron irradiation data.[12] Also for an appropriate choice of the impurity potential coefficients a good qualitative fit to the Pr substitution and Ne^+-irradiation data[11] is obtained. We take $\hbar = k_B = 1$ throughout the paper.

We consider randomly distributed nonmagnetic impurities in an anisotropic superconductor. Treating the electron-impurity scattering within second Born approximation and neglecting the impurity-impurity interaction,[13] the normal and anomalous temperature Green's functions averaged over the impurity positions read

$$G(\omega,\mathbf{k}) = -\frac{i\widetilde{\omega} + \xi_k}{\widetilde{\omega}^2 + \xi_k{}^2 + |\widetilde{\Delta}(\mathbf{k})|^2}, \tag{1}$$

$$F(\omega,\mathbf{k}) = \frac{\widetilde{\Delta}(\mathbf{k})}{\widetilde{\omega}^2 + \xi_k{}^2 + |\widetilde{\Delta}(\mathbf{k})|^2}, \tag{2}$$

where the renormalized Matsubara frequency $\widetilde{\omega}(\mathbf{k})$ and the renormalized order parameter $\widetilde{\Delta}(\mathbf{k})$ are given by

$$\widetilde{\omega}(\mathbf{k}) = \omega + i n_i \int |w(\mathbf{k}-\mathbf{k}')|^2 G(\omega,\mathbf{k}') \frac{d^3k'}{(2\pi)^3}, \tag{3}$$

$$\widetilde{\Delta}(\mathbf{k}) = \Delta(\mathbf{k}) + n_i \int |w(\mathbf{k}-\mathbf{k}')|^2 F(\omega,\mathbf{k}') \frac{d^3k'}{(2\pi)^3}. \tag{4}$$

In the above $\omega = \pi T(2n+1)$ (T is temperature and n is an integer number), ξ_k is the quasiparticle energy, n_i is impurity (defect) concentration, $w(\mathbf{k}-\mathbf{k}')$ is a momentum-dependent impurity potential, and $\Delta(\mathbf{k})$ is the orbital part of a singlet[14] superconducting order parameter defined as

$$\Delta(\mathbf{k})=\Delta e(\mathbf{k}), \tag{5}$$

where $e(\mathbf{k})$ is a real basis function of a 1D irreducible representation of an appropriate point group, which seems to be good approximation for high-T_c superconductors.[1] We normalize $e(\mathbf{k})$ by taking $\langle e^2\rangle=1$, where $\langle\cdots\rangle$ denotes the average value over the Fermi surface.

The impurity scattering potential is assumed to be separable and given by

$$|w(\mathbf{k}-\mathbf{k}')|^2=|w_0|^2+|w_1|^2 f(\mathbf{k})f(\mathbf{k}'), \tag{6}$$

where $|w_0|$ ($|w_1|$) is isotropic (anisotropic) scattering amplitude and $f(\mathbf{k})$ is the momentum-dependent anisotropy function. We assume that the overall scattering rate is determined by the isotropic component and impose the constraints

$$|w_1|^2\leqslant|w_0|^2, \quad \langle f\rangle=0, \quad \langle f^2\rangle=1. \tag{7}$$

Therefore the Fermi surface average of the scattering potential is $\langle|w(\mathbf{k}-\mathbf{k}')|^2\rangle=|w_0|^2$ and the momentum-dependent part in Eq. (6) represents the deviations from the isotropic scattering. It is clear that this kind of anisotropic scattering cannot affect the properties of the isotropic superconductor, but it can play a certain role in the case of a superconductor with an anisotropic order parameter. Although the structure of scattering potential is postulated in Eq. (6) this approach is rather general since no additional assumption about $f(\mathbf{k})$ is made in contrast to previous methods.[8,15] We note from Eq. (3) and from the form of impurity potential [Eq. (6)] that $\tilde{\omega}$ is $\mathbf{k}$ dependent. This means that the electron self-energy due to impurity scattering and consequently the quasiparticle lifetime are anisotropic and change over the Fermi surface. Further, it yields from Eqs. (4) and (6) that the impurity scattering may change the symmetry of the renormalized order parameter $\tilde{\Delta}(\mathbf{k})$ depending on the $f(\mathbf{k})$ symmetry. In this respect our approximation differs from that by Markowitz and Kadanoff[15] who assumed only a change of a degree of order-parameter anisotropy but not the anisotropy function itself. Moreover in Ref. 15, the anisotropy of the order parameter was introduced in a way appropriate for weak anisotropy only. In the more recent study of anisotropic scattering by Millis *et al.*[8] the authors also assumed that the anisotropic impurity potential does not change the symmetry of the electron anomalous self-energy. We may also mention that our approach is different than that by Brink and Zuckermann,[16] where the scattering potential was essentially isotropic but its amplitude varied with the superconducting channels.

To proceed further, we restrict the wave vectors of the electron self-energy and pairing potential to the Fermi surface and replace $\int d^3k/(2\pi)^3$ by $N_0\int_{\mathrm{FS}}dS_k n(\mathbf{k})\int d\xi_k$, where N_0 is the overall density of states at the Fermi surface (FS), $n(\mathbf{k})$ is the angle-resolved FS density of states normalized to unity, i.e., $\int_{\mathrm{FS}} dS_k n(\mathbf{k})=1$, and $\int_{\mathrm{FS}}dS_k$ denotes integration over the Fermi surface. Using Eqs. (1), (2), (5), and (6) in Eqs. (3) and (4) and performing the integration over ξ_k (particle-hole symmetry of quasiparticle spectrum is assumed) we write

$$\tilde{\omega}(\mathbf{k})=\omega[1+u(\omega,\mathbf{k})], \tag{8}$$

$$\tilde{\Delta}(\mathbf{k})=\Delta[e(\mathbf{k})+e(\omega,\mathbf{k})], \tag{9}$$

where $u(\omega,\mathbf{k})$ and $e(\omega,\mathbf{k})$ separate into the isotropic (subscript s) and anisotropic (subscript a) parts as follows:

$$u(\omega,\mathbf{k})=u_s(\omega)+u_a(\omega)f(\mathbf{k}), \tag{10}$$

$$e(\omega,\mathbf{k})=e_s(\omega)+e_a(\omega)f(\mathbf{k}), \tag{11}$$

which are determined by the self-consistent equations

$$u_s(\omega)=\Gamma_0\int_{\mathrm{FS}}dS_k n(\mathbf{k})\,\frac{1+u(\omega,\mathbf{k})}{[\tilde{\omega}^2+|\tilde{\Delta}(\mathbf{k})|^2]^{1/2}}, \tag{12}$$

$$u_a(\omega)=\Gamma_1\int_{\mathrm{FS}}dS_k n(\mathbf{k})f(\mathbf{k})\,\frac{1+u(\omega,\mathbf{k})}{[\tilde{\omega}^2+|\tilde{\Delta}(\mathbf{k})|^2]^{1/2}}, \tag{13}$$

$$e_s(\omega)=\Gamma_0\int_{\mathrm{FS}}dS_k n(\mathbf{k})\,\frac{e(\mathbf{k})+e(\omega,\mathbf{k})}{[\tilde{\omega}^2+|\tilde{\Delta}(\mathbf{k})|^2]^{1/2}}, \tag{14}$$

$$e_a(\omega)=\Gamma_1\int_{\mathrm{FS}}dS_k n(\mathbf{k})f(\mathbf{k})\,\frac{e(\mathbf{k})+e(\omega,\mathbf{k})}{[\tilde{\omega}^2+|\tilde{\Delta}(\mathbf{k})|^2]^{1/2}}. \tag{15}$$

In writing the above we have introduced the isotropic Γ_0 and anisotropic Γ_1 impurity scattering rates ($\Gamma_1\leqslant\Gamma_0$)

$$\Gamma_0=\pi N_0 n_i|w_0|^2, \quad \Gamma_1=\pi N_0 n_i|w_1|^2. \tag{16}$$

The gap function is given by the weak-coupling self-consistent equation

$$\Delta(\mathbf{k})=-T\sum_{\omega}\sum_{\mathbf{k}'} V(\mathbf{k},\mathbf{k}')\,\frac{\tilde{\Delta}(\mathbf{k}')}{\tilde{\omega}^2+\xi_{k'}^2+|\tilde{\Delta}(\mathbf{k}')|^2} \tag{17}$$

with the phenomenological separable pair potential $V(\mathbf{k},\mathbf{k}')$ taken as

$$V(\mathbf{k},\mathbf{k}')=-V_0 e(\mathbf{k})e(\mathbf{k}'). \tag{18}$$

Following standard procedure,[17] we obtain the equation for the critical temperature T_c as

$$\ln\frac{T_c}{T_{c_0}}=2\pi T_c\sum_{\omega>0}\left[[f(\omega)]_{\Delta=0}-\frac{1}{\omega}\right] \tag{19}$$

with

$$[f(\omega)]_{\Delta=0}=\int_{\mathrm{FS}}dS_k n(\mathbf{k})\,\frac{e(\mathbf{k})}{\tilde{\omega}_0(\mathbf{k})}\left[\frac{\tilde{\Delta}(\mathbf{k})}{\Delta}\right]_{\Delta=0}, \tag{20}$$

where T_{c_0} is the critical temperature in the absence of impurities and $\tilde{\omega}_0(\mathbf{k})=\tilde{\omega}(\mathbf{k})_{\Delta=0}$. Using Eqs. (8)–(15), we get for a $\Delta\rightarrow0$ limit

$$\tilde{\omega}_0(\mathbf{k})=\omega+\Gamma_0\mathrm{sgn}(\omega) \tag{21}$$

and

$$\left[\frac{\tilde{\Delta}(\mathbf{k})}{\Delta}\right]_{\Delta=0}=e(\mathbf{k})+\frac{|\Gamma_0|}{|\omega|}\langle e\rangle+\frac{\Gamma_1}{|\omega|+\Gamma_0-\Gamma_1}\langle ef\rangle f(\mathbf{k}). \tag{22}$$

The last two terms on the right-hand side of Eq. (22) represent the possible impurity-induced anisotropy of the renormalized order parameter, which is absent in the works of Markowitz and Kadanoff[15] as well as of Millis *et al.*,[8] where $\tilde{\Delta}(\mathbf{k})=\tilde{\Delta}e(\mathbf{k})$ is assumed. It may be mentioned that $\tilde{\Delta}(\mathbf{k})$ and $\Delta(\mathbf{k})$ have the same anisotropy given by $e(\mathbf{k})$ function if $f(\mathbf{k})=\pm e(\mathbf{k})(\langle e\rangle=0)$ only. Based on Eqs. (20)–(22) we get from Eq. (19)

$$\ln\frac{T_c}{T_{c_0}}=(\langle e\rangle^2-1)\left[\psi\left(\frac{1}{2}+\frac{\Gamma_0}{2\pi T_c}\right)-\psi\left(\frac{1}{2}\right)\right]+\langle ef\rangle^2\,\frac{\Gamma_1}{2\pi T_c}\sum_{n\geqslant 0}\frac{1}{(n+1/2+\Gamma_0/2\pi T_c)[n+1/2+(\Gamma_0-\Gamma_1)/2\pi T_c]}\quad(23)$$

where $\psi(z)$ is digamma function.[18] The first term on the right-hand side of Eq. (23) gives the T_c suppression due to the isotropic scattering. Since the second term which couples the anisotropy functions $e(\mathbf{k})$ and $f(\mathbf{k})$ is always non-negative, T_c does not decrease as fast as for the isotropic scattering only. In other words, an anisotropic potential of the form given by Eq. (6) diminishes the suppression of superconductivity if the scalar product $\langle ef\rangle$ value is nonzero, which may be the case in many cuprate superconducting compounds. For an isotropic superconductor $e(\mathbf{k})=1(\langle e\rangle=1,\langle ef\rangle=0)$ and it yields from Eq. (23) that the critical temperature does not depend on the impurity scattering which is in accordance with the Anderson's theorem.[19] Finally, Eq. (23) may be written in a more compact form as

$$\ln\frac{T_c}{T_{c_0}}=(\langle e\rangle^2+\langle ef\rangle^2-1)\left[\psi\left(\frac{1}{2}+\frac{\Gamma_0}{2\pi T_c}\right)-\psi\left(\frac{1}{2}\right)\right]+\langle ef\rangle^2\left\{\psi\left(\frac{1}{2}\right)-\psi\left[\frac{1}{2}+\frac{\Gamma_0}{2\pi T_c}\left(1-\frac{\Gamma_1}{\Gamma_0}\right)\right]\right\}.\quad(24)$$

Our model has two more dimensionless parameters than the isotropic scattering model. First is $\langle ef\rangle^2=[\int_{\mathrm{FS}}dS_k n(\mathbf{k})e(\mathbf{k})f(\mathbf{k})]^2$, which describes the interplay between the pair potential $V(\mathbf{k},\mathbf{k}')$ [Eq. (18)] and the anisotropic part of the scattering potential $|w(\mathbf{k}-\mathbf{k}')|^2$ [Eq. (6)]. This parameter is determined by the symmetry of the superconducting state $[e(\mathbf{k})]$ and that of the impurity scattering matrix element $[f(\mathbf{k})]$. According to the normalization of the order-parameter orbital function $\langle e^2\rangle=1$ [Eq. (5)] and that of the anisotropy function of the impurity potential $\langle f^2\rangle=1$ [Eq. (7)] the parameter $\langle ef\rangle^2$ takes values between 0 and 1. When $\langle ef\rangle^2=0$ then the $e(\mathbf{k})$ and $f(\mathbf{k})$ functions are orthogonal, which means that the pair potential $V(\mathbf{k},\mathbf{k}')$ and the impurity scattering potential $|w(\mathbf{k}-\mathbf{k}')|^2$ do not couple and the T_c decrease in Eq. (24) is due to isotropic scattering only. On the other hand, for $\langle ef\rangle^2=1$ we deal with $f(\mathbf{k})=\pm e(\mathbf{k})$ and the pair-breaking effect is minimized by the anisotropic part of the scattering potential which is proportional to the pair potential. In this case Eq. (7) yields $\langle e\rangle=0$ and Eq. (24) becomes[20]

$$\ln\frac{T_c}{T_{c_0}}=\psi\left(\frac{1}{2}\right)-\psi\left(\frac{1}{2}+(1-g_I)\,\frac{\Gamma_0}{2\pi T_c}\right)\quad(25)$$

with $g_I=\Gamma_1/\Gamma_0$ which leads to the pair-breaking parameter $(1-g_I)\Gamma_0/(2\pi T_c)$. On the other hand, if we calculate g_I coefficient defined in Ref. 8 with the impurity potential from Eqs. (6) and (7) we get $g_I=\langle e\rangle^2+\langle ef\rangle^2(\Gamma_1/\Gamma_0)$, which reduces to our value of g_I for $\langle ef\rangle^2=1$ and $\langle e\rangle=0$. Thus in this case our pair-breaking parameter is identical to the one obtained by Millis *et al.*[8] The second parameter in our model (Γ_1/Γ_0) represents the amount of anisotropic scattering rate in impurity potential normalized by the isotropic scattering rate [Eq. (16)], its value ranges also from 0 to 1. For $\Gamma_1/\Gamma_0=0$ we obtain

$$\ln\frac{T_c}{T_{c_0}}=(\langle e\rangle^2-1)\left[\psi\left(\frac{1}{2}+\frac{\Gamma_0}{2\pi T_c}\right)-\psi\left(\frac{1}{2}\right)\right]\quad(26)$$

which is the isotropic scattering case[5] and yields a considerable critical temperature suppression for $\langle e\rangle\neq 1$. When $\langle e\rangle=0$ then Eq. (26) gives a T_c suppression curve for a d-wave superconductor with isotropic scattering and is the weak-coupling version of the form used by Radtke *et al.*[6] In the case of strong anisotropic scattering $\Gamma_1/\Gamma_0=1$ and the T_c equation reads

$$\ln\frac{T_c}{T_{c_0}}=(\langle e\rangle^2+\langle ef\rangle^2-1)\left[\psi\left(\frac{1}{2}+\frac{\Gamma_0}{2\pi T_c}\right)-\psi\left(\frac{1}{2}\right)\right].\quad(27)$$

It is easy to see that the critical temperature suppression becomes more gradual now and may be even reversed into T_c increase for a significant overlap between $e(\mathbf{k})$ and $f(\mathbf{k})$ functions, that is when $\langle ef\rangle^2\sim 1$.

Our results for the dependence of T_c/T_{c_0} on the isotropic scattering rate $\Gamma_0/2\pi T_{c_0}$ are shown in Figs. 1(a)–1(d) for a selection of the model parameters $\langle ef\rangle^2=0.2$, 0.4, 0.8, and 0.95 and $\Gamma_1/\Gamma_0=0$, 0.5, 0.9, 0.95, and 1.0. We have assumed here $\langle e\rangle=0$.[20] Based on these we make the following remarks: (1) In all curves the depression of T_c in the limit of impurity concentration $n_i\rightarrow 0$ is given by the initial slope

$$\frac{d(T_c/T_{c_0})}{d(\Gamma_0/2\pi T_{c_0})}=-\frac{\pi^2}{2}\left[1-\langle ef\rangle^2\,\frac{\Gamma_1}{\Gamma_0}\right]\quad(28)$$

which decreases drastically as $\langle ef\rangle^2\Gamma_1/\Gamma_0$ approaches unity; (2) for a given value of Γ_1/Γ_0, the value of $\Gamma_0/2\pi T_{c_0}$ needed to suppress superconductivity increases as $\langle ef\rangle^2$ is increased; (3) when there is a significant overlap between the anisotropy functions $e(\mathbf{k})$ and $f(\mathbf{k})$, e.g., $\langle ef\rangle^2\sim 0.8$ [Fig. 1(c)] the value of $\Gamma_0/2\pi T_{c_0}$ needed to destroy superconductivity is increased considerably when Γ_1/Γ_0 becomes large.

In order to make contact with experiment, we estimate the planar residual resistivity ρ_0, which is a normal-state property and according to Eq. (21) depends on the isotropic scattering rate Γ_0 exclusively. It is worth mentioning here that in

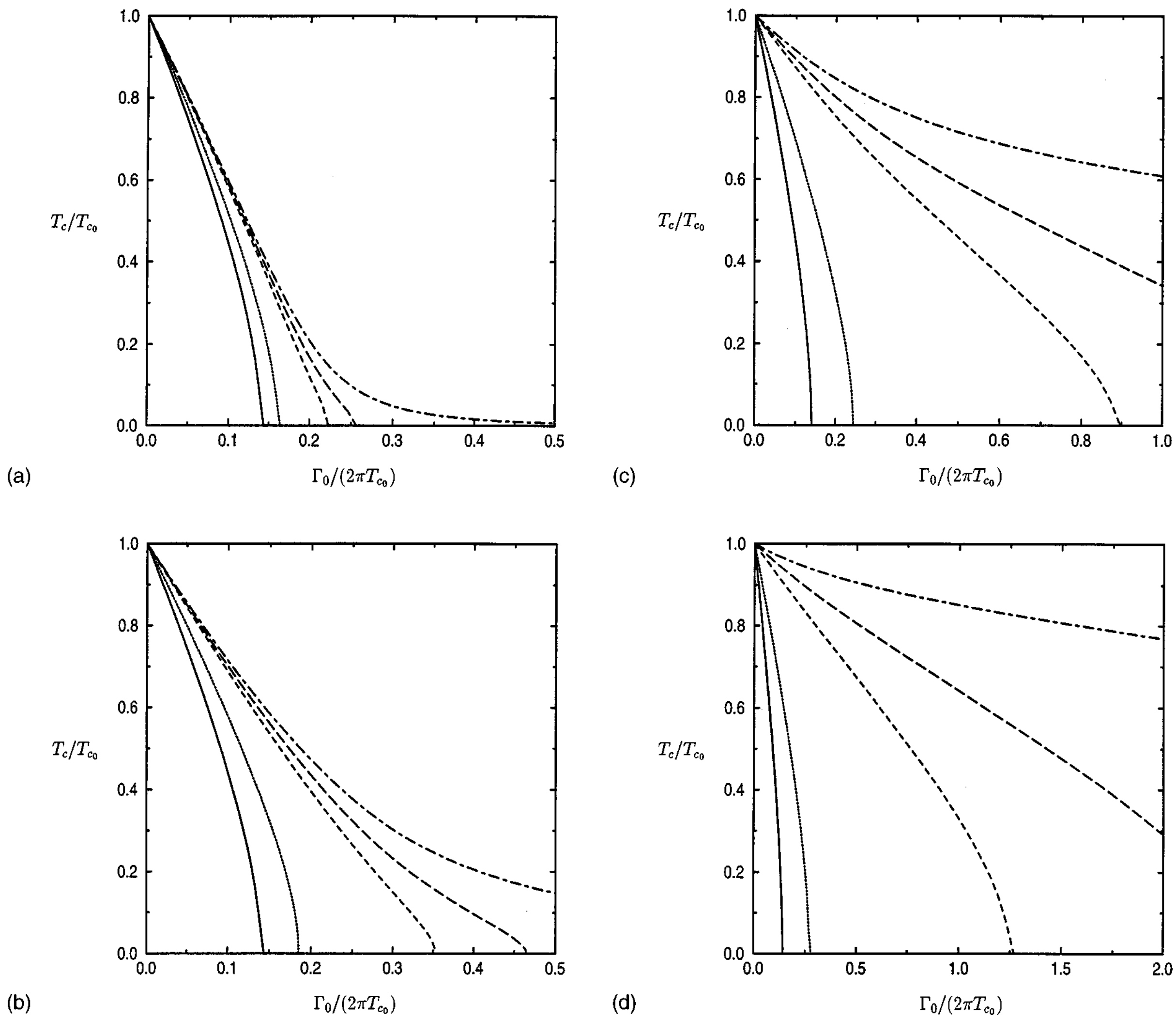

FIG. 1. Normalized critical temperature T_c/T_{c_0} as a function of the normalized isotropic scattering rate $\Gamma_0/2\pi T_{c_0}$ for different values of the normalized anisotropic scattering rate $\Gamma_1/\Gamma_0=0.5$ (dotted curve), 0.9 (short-dashed curve), 0.95 (long-dashed curve), 1.0 (dot-dashed curve). The solid curve represents the isotropic scattering pair-breaking effect ($\Gamma_1/\Gamma_0=0$). We have taken $\langle ef\rangle^2=0.2$ (a), 0.4 (b), 0.8 (c), 0.95 (d), and $\langle e\rangle=0$.

the normal state the influence of the impurity scattering on the electron self-energy is reflected by the frequency rescaling only [Eq. (21)], and hence the scattering process is characterized by Γ_0 parameter completely. Therefore neither of anisotropic scattering parameters enters the equations determining the normal state properties. Using a Drude form of the low-frequency residual electrical conductivity at zero frequency $\sigma=\omega_{\rm pl}^2\tau/4\pi$, where $\omega_{\rm pl}$ is the plasma frequency and $1/\tau=2\Gamma_0$, we represent[6] the planar residual resistivity in terms of the dimensionless pair-breaking parameter $\Gamma_0/2\pi T_{c_0}$

$$\rho_0\simeq 10.18\times 10^{-2}\,\frac{8\pi^2}{\omega_{\rm pl}^2}\,T_{c_0}\left(\frac{\Gamma_0}{2\pi T_{c_0}}\right)\;\;\mu\Omega\ \mathrm{cm} \tag{29}$$

with $\omega_{\rm pl}$ in eV and T_{c_0} in K. From Eqs. (28) and (29) we get the initial slope for a d-wave superconductor

$$\frac{dT_c}{d\rho_0}\simeq -0.615\times\omega_{\rm pl}^2\left(1-\langle ef\rangle^2\,\frac{\Gamma_1}{\Gamma_0}\right)\;\;\mathrm{K}/\mu\Omega\ \mathrm{cm}. \tag{30}$$

In the electron irradiation experiment in YBCO Giapintzakis *et al.*[12] obtained $dT_c/d\rho\sim-0.30\pm0.04$ K/$\mu\Omega$ cm (ρ is resistivity at 145 K and $dT_c/d\rho\simeq dT_c/d\rho_0$). Taking the plasma frequency $\omega_{\rm pl}$ ranging from 1.1 to 1.4 eV, which is the experimental estimate of $\omega_{\rm pl}$ for YBCO,[12] we find from Eq. (30) that the experimental data can be reproduced by the anisotropic scattering parameters with values given by a constraint $0.55\leqslant(\Gamma_1/\Gamma_0)\langle ef\rangle^2\leqslant 0.78$. The range of values of the scattering parameters stem from an uncertainty of the plasma frequency and the $dT_c/d\rho$ measurement accuracy.[12] Our calculation focused entirely on a single CuO_2 plane seems to be a good approximation here since the low-energy electron irradiation, used in this experiment, displaces the oxygen atoms only and an appropriate measurement method probes the contribution to T_c suppression due to these oxygen defects on the CuO_2 planes.[12] The two-dimensional approach is not so justified in the interpretation of the experimental data of Ref. 11 where Pr substitution and ion (Ne^+) damage were applied. The Pr substitutes onto the Y site and a similar defect is probably induced by ion irradiation since the T_c suppression induced by both methods is analogous. Consid-

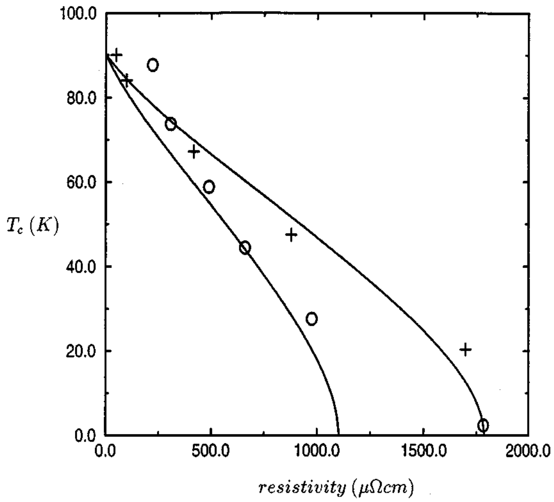

FIG. 2. Critical temperature T_c (area between solid curves) of a superconductor with the critical temperature in the absence of impurities $T_{c_0}=90$ K vs residual resistivity for $\langle ef\rangle^2=0.95$, $\Gamma_1/\Gamma_0=0.96$, $\langle e\rangle=0$ and plasma frequencies $\omega_{\rm pl}$ between 1.1 and 1.4 eV. The experimental data of Ref. 11 for YBCO are shown with circles (Pr substitution) and crosses (ion damage).

ering this caveat, our theoretical results and the experimental data of Ref. 11 are shown in Fig. 2 for an illustrative purpose mainly. The data were read from Fig. 4 of Sun *et al.*[11] and the region between the curves corresponds to the T_c computed from Eqs. (24) and (29) with $\langle e\rangle=0$, $\langle ef\rangle^2=0.95$, and $\Gamma_1/\Gamma_0=0.96$ for plasma frequencies $\omega_{\rm pl}$ ranging from 1.1 to 1.4 eV. We did not try to adjust the amount of anisotropic scattering present in our model so as to get a best fit to the data. Nevertheless, we note a good qualitative agreement of the theoretical results with the experimental data of Ref. 11. The experimental data show, however a long tail T_c suppression which is not reflected in the computed T_c. We think that this feature may be due to a slight orthorhombic anisotropy of the system,[21] which was neglected in our calculation by the assumption of $\langle e\rangle=0$.

Before concluding, we give some critical remarks concerning our approach. We have employed a weak-coupling approximation neglecting the strong-coupling corrections. We expect that as in Ref. 6, the strong-coupling effects would rescale the scattering rates. Further, while calculating T_c as a function of residual resistivity, we have neglected the interaction between the nearest CuO_2 planes, restricting our considerations to a single copper-oxide plane. This simplification may not be valid for the interpretation of the experimental data of Ref. 11, where the defects are not in the CuO_2 planes. Finally, we have assumed a model separable momentum-dependent impurity potential, which is obviously not the most general way of treating the problem, but is more general than the one applied in the previous studies.[8,15]

In summary, we have given a theory of anisotropic impurity scattering in anisotropic superconductors. The impurity potential is assumed to be separable according to Eq. (6). There are two parameters characterizing the scattering anisotropy in our approach. The first of them ($\langle ef\rangle^2$) represents the interplay between the symmetry of the superconducting order parameter and that of the impurity potential, the second (Γ_1/Γ_0) gives the amount of anisotropic scattering versus the isotropic one. We find that for a significant overlap between the pair potential and the impurity potential that is for large $\langle ef\rangle^2$ values, and for a large value of Γ_1/Γ_0, the anisotropic superconductivity becomes robust vis a vis the impurity concentration. The experimental data of the electron irradiation-induced T_c suppression in YBCO (Ref. 12) is understood quantitatively within our model. We also obtain a good qualitative agreement with the observed T_c decrease in YBCO due to ion (Ne^+) damage and Pr substitution.[11]

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