Photon-assisted parity change and Andreev tunneling

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(Received 24 January 1996)

A microscopic theory has been constructed to investigate the tunneling current in a normal-superconductor-normal single-electron tunneling transistor with an oscillating potential coupled to the grain. The oscillating potential produces a photon-assisted Andreev tunneling and causes a photon-assisted parity change of the grain. [S0163-1829(96)11827-0]

When an electron tunnels through a potential barrier in the presence of an oscillating potential $\widetilde{V}\cos(\omega t)$, it may emit or absorb n photons with energy $n\hbar\omega$. When $\hbar\omega \gg k_B T$, such processes can be detected experimentally as steps in the current-voltage (I-V) characteristics or as peaks in the conductance-voltage curve. The pioneering theory of Tien and Gordon[1] explains qualitatively the photon-assisted (PA) electron tunneling observed in superconducting diodes.[2] Following the recent advancement of technology to fabricate samples of nanometer size, PA tunneling in semiconducting or metallic nanostructures of different geometries has been studied both theoretically and experimentally.[3–12] In a quantum dot where single-electron tunneling events are correlated due to Coulomb blockade, the observed PA tunneling process[13,14] has been explained with a combination of the Tien-Gordon theory[1] and the orthodox theory[15] of single-electron tunneling (SET).

The normal-superconductor-normal (NSN) SET transistor, the equivalent circuit of which is shown in Fig. 1, has a superconducting grain connected to two normal-metal leads through two tunnel junctions, which are characterized by the capacitance and tunneling conductance (C_s, G_s) and (C_d, G_d), respectively. The Coulomb blockade on the grain can be controlled by the gate voltage V_g and the gate capacitance C_g. The transport properties of the NSN SET are very sensitive to whether the number of excess electrons on the grain is even or odd. In the absence of an ac potential, when the gate charge $Q \equiv C_g V_g / e$ is an odd integer, at low dc bias voltage the tunneling current is due to the Andreev process where, effectively, two electrons tunnel coherently through the barrier between the superconducting grain and a normal-metal lead. Hence, the number of excess electrons on the grain is even. As the dc bias increases to the threshold value $V_{\rm th}$ for quasiparticle tunneling, the parity changes from even to odd, and the Andreev current is drastically suppressed. The I-V characteristics of an NSN SET transistor under a dc bias have been studied both experimentally[16,17] and theoretically.[18] When an additional oscillating potential $\widetilde{V}\cos(\omega t)$ is applied, besides the possible enhancement of Andreev current, the most interesting phenomenon is that under proper conditions, one expects the PA quasiparticle tunneling to occur before the dc bias reaches the threshold value $V_{\rm th}$. It then results in a PA parity change of the I-V characteristics of an NSN SET transistor.

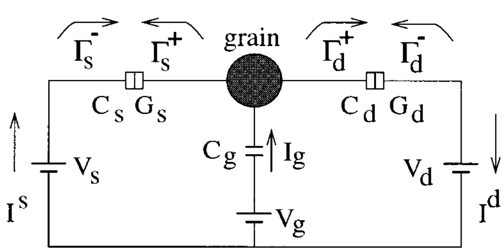

FIG. 1. Equivalent circuit of a SET transistor.

The purpose of this paper is to investigate these PA processes, which requires the calculation of not only the second-order PA quasiparticle tunneling, but also the fourth-order PA Andreev tunneling. To our knowledge, the present paper is the first attempt to study such higher-order effect. Other higher-order elastic and inelastic cotunneling[19] will be neglected because their contributions to the current are unimportant. In general, the total current consists of both tunneling current and displacement current. Since we will consider only the time-averaged current, the contribution of displacement current is zero.[10] We will show that under the high-frequency condition $\Delta > \omega \gg \Gamma$, where Δ is the superconducting gap and Γ the tunneling rate, the PA parity change can be observed.

The system we consider is illustrated in Fig. 1, where each voltage consists of a dc term and an ac term: $V_l(t) = V_l^0 + \widetilde{V}_l \cos\omega t$ with $l = s,\ d,$ and g. The Hamiltonians H_s and H_d for the normal leads can be expressed as

$$H_l = \sum_{p\sigma} [\xi_{l,p} + eV_l(t)] a^\dagger_{l,p\sigma} a_{l,p\sigma}; \quad l = s,d, \tag{1}$$

where the single-electron energy $\xi_{l,p}$ is measured relative to the chemical potential. The Hamiltonian for the superconducting grain,

$$H_g = \sum_{q\sigma} \epsilon_q \gamma^\dagger_{q\sigma} \gamma_{q\sigma} + E_n(t), \tag{2}$$

contains the electrostatic energy

$$E_n(t)=\frac{n^2e^2}{2C}+\frac{ne}{C}[C_sV_s(t)+C_dV_d(t)+C_gV_g(t)], \quad (3)$$

where $C\equiv C_s+C_d+C_g$ and n is the number of excess electrons on the grain. The quasiparticle operators $(\gamma_{q\sigma},\gamma^\dagger_{q\sigma})$ are connected to the single-electron operators via the Bogoliubov transformation, with the corresponding quasiparticle energy $\epsilon_q=\sqrt{\xi_q^2+\Delta^2}$. The tunneling between the grain and the lth lead is represented by the tunneling Hamiltonian

$$H_l^T=\sum_{pq\sigma}T_{l,pq}a^\dagger_{l,p\sigma}a_{q\sigma}+\text{H.c.}; \quad l=s,d. \quad (4)$$

In our notation the momenta in the leads are labeled by p, and the momenta in the grain are labeled by q.

To derive the I-V characteristics, we will first calculate the transition rates of all relevant tunneling processes, and then use the master equation approach to obtain the time-averaged current. In the absence of the additional ac bias, the detailed derivation has been given earlier.[20] To calculate the tunneling rate through the lth junction, we define $N_l(t)=\Sigma_{l,p\sigma}a^\dagger_{l,p\sigma}a_{l,p\sigma}$, $A_l(t)=\Sigma_{pq\sigma}T_{l,pq}a^\dagger_{l,p\sigma}a_{l,q\sigma}$, and $H(t)=H_s+H_d+H_s^T+H_d^T+H_g$, and start from the standard formula

$$\left\langle\frac{dN_l(t)}{dt}\right\rangle=\frac{i}{\hbar}\langle[H(t),N_l(t)]\rangle=\frac{2}{\hbar}\text{Im}\{\langle A_l(t)\rangle\}. \quad (5)$$

For the quasiparticle tunneling we only need to solve this equation to the second order in the tunneling Hamiltonian. The result will be the Tien-Gordon formula[1] as expected.

However, for the Andreev tunneling, it is necessary to include up to the fourth-order terms. The derivation is lengthy and complicated, and in this paper we will only outline the mathematical procedure and then present the final result. We will first calculate $\langle A_l(t)\rangle$. In the *interaction representation* $\hat{A}_l(t_i)$, we need to calculate 12 similar terms, each of which is a product of four operators of the typical form

$$\int_{-\infty}^{t}dt_1\int_{-\infty}^{t_1}dt_2\int_{-\infty}^{t_2}dt_3\langle\hat{A}_l(t)\hat{A}_l(t_1)\hat{A}_l^\dagger(t_2)\hat{A}_l^\dagger(t_3)\rangle. \quad (6)$$

When applying Wick's theorem to evaluate this quantity, we have neglected the terms proportional to $|T_{l,pq}|^2|T_{l,p'q'}|^2$ and $T_{l,pq}T_{l,p'q'}T^*_{l,p'q}T^*_{l,pq'}$, because they will only renormalize the quasiparticle tunneling rate. To deal with the time integrals in (6), we have expanded $\exp(iz\cos\phi)$ in terms of the Bessel functions $J_k(z)$. Since experiments measure the time-averaged current, we will first average over the time t in (6) and then perform the triple time integrations. After completing the calculation of these twelve terms, the so-obtained $\langle A_l(t)\rangle$ is inserted in (5), from which the Andreev tunneling rates are readily derived.

Let ν_l and ν_g be the density of states at Fermi energy for the lth lead and the grain, respectively. The number of quantum channels through the lth junction is specified as $\mathcal{N}_l$. We define $\alpha_l=e/c\,[C_s\tilde{V}_s+C_d\tilde{V}_d+C_g\tilde{V}_g]$-$e\tilde{V}_l$ and $E_l^\pm(n,m)=meV_l^0\pm(E_n^0$-$E_{n\pm m}^0)$, where E_n^0 is the electrostatic energy of (3) in the absence of ac potentials. Then, in terms of

$$\mathcal{F}^\pm_{l,pq}(n,k)=[E_l^+(n,1)+\xi_{l,p}\pm\epsilon_q+k\hbar\omega]^{-1},$$

$$\mathcal{G}^\pm_{l,pq}(n,k)=[E_l^-(n,1)+\xi_{l,p}\pm\epsilon_q+k\hbar\omega]^{-1}, \quad (7)$$

the Andreev tunneling rates have the compact analytical expressions

$$\begin{aligned}\Gamma_l^\mp(n)=&\pm\frac{2\pi G_l\Delta^2}{4\pi e^2\nu_l\nu_g\mathcal{N}_l}\sum_{kk'm}J_k\left(\frac{\alpha_l}{\hbar\omega}\right)J_{k'}\left(\frac{\alpha_l}{\hbar\omega}\right)J_m\left(\frac{\alpha_l}{\hbar\omega}\right)\\&\times J_{k+k'-m}\left(\frac{\alpha_l}{\hbar\omega}\right)\sum_{pqp'q'}\left(\frac{1}{2}\mp\frac{1}{2}-n_p\right)\\&\times\left(\frac{1}{2}\mp\frac{1}{2}-n_{p'}\right)\Big/(\epsilon_q\epsilon_{q'})\,\delta[E_l^\pm(n,2)+\xi_p+\xi_{p'}\\&+(k+k')\hbar\omega]\mathcal{Y}^\mp_{l,pqp'q'}(n,kk'm),\end{aligned} \quad (8)$$

$$\begin{aligned}\mathcal{Y}^-_{l,pqp'q'}(n,kk'm)=&[2(1-n_q)n_{q'}\mathcal{F}^-_{l,pq}(n,k)\\&-n_qn_{q'}\mathcal{F}^+_{l,pq}(n,k)]\\&\times[\mathcal{F}^+_{l,pq'}(n,k+k'-m)\\&+\mathcal{F}^+_{l,p'q'}(n,k+k'-m)]\\&-(1-n_q)(1-n_{q'})\mathcal{F}^-_{l,pq}(n,k)\\&\times[\mathcal{F}^-_{l,pq'}(n,k+k'-m)\\&+\mathcal{F}^-_{l,p'q'}(n,k+k'-m)],\end{aligned} \quad (9)$$

$$\begin{aligned}\mathcal{Y}^+_{l,pqp'q'}(n,kk'm)=&[2(1-n_q)n_{q'}\mathcal{G}^-_{l,pq'}(n,k)\\&-(1-n_q)(1-n_{q'})\\&\times\mathcal{G}^+_{l,pq'}(n,k)][\mathcal{G}^+_{l,pq}(n,k+k'-m)\\&+\mathcal{G}^+_{l,p'q}(n,k+k'-m)]\\&-n_qn_{q'}\mathcal{G}^-_{l,pq'}(n,k)[\mathcal{G}^-_{l,pq}(n,k+k'-m)\\&+\mathcal{G}^-_{l,p'q}(n,k+k'-m)],\end{aligned} \quad (10)$$

where n_q is the quasiparticle distribution on the grain. Here, $\Gamma_l^+(n)$ is the rate for tunneling out of the grain, and $\Gamma_l^-(n)$ is the rate for tunneling into the grain. It is clear from (7)–(10) that the effect of the ac potential is to shift the threshold voltage of Andreev tunneling by an integer multiple of $\hbar\omega$. If the ac potential is removed, in the limit $n_q\ll 1$ our results reduce to those in Ref. 18.

Knowing the tunneling rates, following the procedure in Ref. 20, the tunneling current can be readily computed. The formulas (7)–(10) are derived under the general situation that an ac potential is applied to each terminal. Since the tunneling current is effectively controlled by the gate, in our numerical calculation only one ac potential is applied to the gate ($\tilde{V}_s=\tilde{V}_d=0$) of a symmetric NSN SET transistor with $C_s=C_d=0.25C$, $G_s=G_d=0.05e^2/\hbar$, and $V_s=-V_d=V/2$. The values of other relevant physical quantities are set within the range of usual sample parameters and experimental conditions: $k_BT=0.005\Delta$, $E_c=0.4\Delta$, $C_g=0.5C$, and $\mathcal{N}_l=200$. So far we have not specified the exact form of the quasiparticle distribution n_q on the grain, which remains a controver-

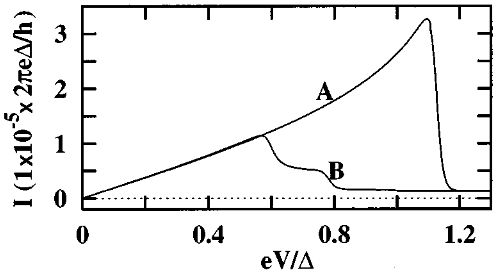

FIG. 2. I-V characteristics for a symmetric SET transistor with $Q \equiv C_g V_g^0/e = 1$. Curve A is for a pure dc gate voltage, and curve B is for a dc-ac gate voltage with $e\tilde{V}_g = 0.2\Delta$ and $\omega = 0.1\Delta/\hbar$. For Al with $\Delta = 245$ μeV, one unit of I is 596.4 fA.

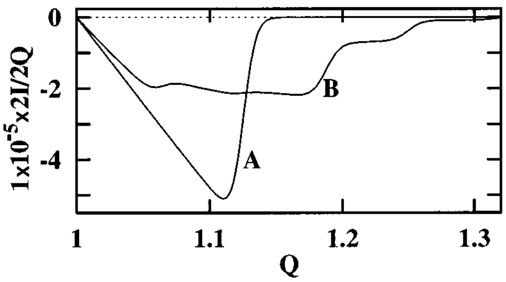

FIG. 4. Transconductances corresponding to the curves A and B in Fig. 3.

sial point in this field of research. Nevertheless, at such low temperature and in the experimental range of the bias voltage V, $n_q = 0$ is a well-justified approximation regardless of the functional form of n_q.

To demonstrate the PA parity change, we first tune the dc gate voltage to $Q \equiv C_g V_g^0/e = 1$ such that there is no Coulomb blockade of the Andreev tunneling at zero bias, $V = 0$. In the absence of an ac potential, the I-V characteristics are shown in Fig. 2 as curve A. The sharp drop of the Andreev current around the threshold voltage $eV_{\mathrm{th}}/\Delta \simeq 1.1$ is due to the onset of quasiparticle tunneling, indicating the change of number parity of the grain from even to odd.[18] The behavior of curve A has been discussed in detail in our previous work,[20] where its reasonable agreement with experiments has been demonstrated. When we apply an ac bias to the gate with an amplitude $e\tilde{V}_g = 0.2\Delta$ and a frequency $\omega = 0.1\Delta/\hbar$, the I-V curve is altered drastically from curve A to curve B. The ac potential causes a slight initial increase of Andreev current until the bias reaches $eV_{\mathrm{th}}/\Delta \simeq 0.6$, where the PA quasiparticle tunneling occurs. Consequently, the Andreev current is suppressed at a much smaller threshold voltage V_{th}, marking the PA parity change. In the PA quasiparticle tunneling region, the I-V curve has a negative differential resistance appearing in the form of several steps. For the symmetric SET transistor considered here, the width of the step is $2\hbar\omega C/(C_s + C_d + C_g) = 2\hbar\omega = 0.2\Delta$. The effect of electromagnetic environment on the I-V curve has been studied earlier using a broadband radiation model[21,22] with a probability for the *environment* to absorb photons of various frequencies. This model is different from the physical process of PA tunneling as described here in this paper, and therefore cannot produce the step structure on the I-V curve.

The initial increase of the current in Fig. 2 is due to the PA Andreev tunneling. To analyze this effect, we set the bias at $eV/\Delta = 0.2$ and calculate the current as a function of the gate charge Q. Since the I-Q curve is symmetric with respect to $Q = 1$, the result is shown in Fig. 3 for $Q \geqslant 1$. The PA Andreev tunneling not only increases the current from curve A to curve B, but also produces interesting fine structure, which is clearly seen in Fig. 4, where the corresponding transconductance $\partial I/\partial Q$ is plotted as a function of Q. By comparing curves A and B, we see that the onset of PA Andreev tunneling generates a sequence of steps, where the width of each step is $\hbar\omega/4E_c = 1/16$.

For $\tilde{V}_g = 0$, there is a Coulomb blockade of Andreev current if $Q \neq 1$. In this case, since $\tilde{V}_g$ enters the Bessel function in (8), a pronounced effect of $\tilde{V}_g$ on the PA Andreev current is expected. We see this in Fig. 5, where one curve (A) is plotted for $Q = 0.85$ and $\omega = 0$, and a series of I-V curves are plotted for $Q = 0.85$ and $\omega = 0.1\Delta/\hbar$. The values of $e\tilde{V}_g/\Delta$ for the six curves are 0 (A), 0.05 (B), 0.07 (C), 0.09 (D), 0.12 (E), and 0.16 (F). Around $eV/\Delta = 0.25$, the Coulomb blockade is destroyed by the PA Andreev tunneling.

Although our analytical formulas (8)–(10) are general for an asymmetric NSN SET transistor with an ac potential applied to each terminal, the numerical results are calculated for a symmetric NSN SET transistor with only one ac potential attached to the gate. All predictions of our theory lead to

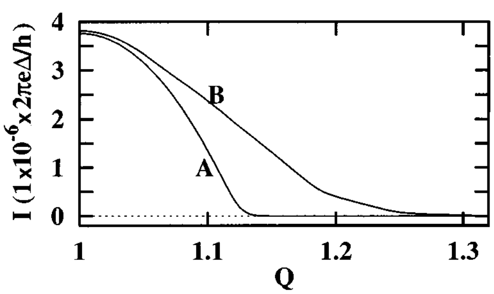

FIG. 3. I-Q curves for a symmetric SET transistor with $eV/\Delta = 0.2$. Curve A is for a pure dc gate voltage, and curve B is for a dc-ac gate voltage with $e\tilde{V}_g = 0.2\Delta$ and $\omega = 0.1\Delta/\hbar$.

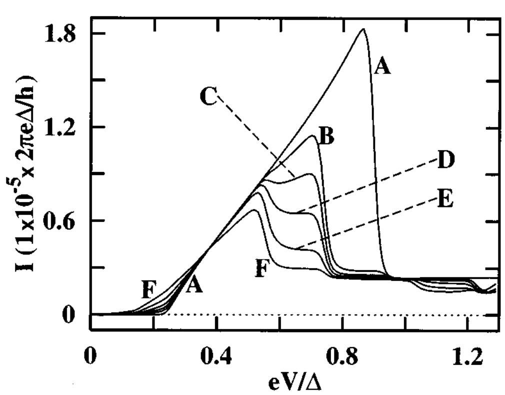

FIG. 5. I-V characteristics for a symmetric SET transistor with $Q = 0.85$, and with an ac potential of frequency $\omega = 0.1\Delta/\hbar$ attached to the gate. The ac amplitude is $e\tilde{V}_g/\Delta = 0$ (A), 0.05 (B), 0.07 (C), 0.09 (D), 0.12 (E), and 0.16 (F).

observably large changes of the tunneling current, and remain to be confirmed experimentally under the condition $\Delta > \hbar\omega \gg k_B T$, which is within the limit of present sample fabrication technology and experimental technique.

We would like to thank R. I. Shekhter for initiating this work, which was supported by the Norwegian Research Council Grant No. 100849/432 and the Swedish Research Council for Engineering Science (TFR).

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