Finite-size effects in a metallic multichannel ring with Kondo impurity: Persistent currents and magnetoresistance

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We consider a spin-1/2 impurity interacting with conduction electrons in two different orbital channels via an isotropic spin exchange. The exchange is the same for both channels, but a crystalline field breaks the symmetry between the orbital channels. This corresponds to a splitting of the conduction electron Γ_8 into two doublets in the quadrupolar Kondo effect, or to the electron-assisted tunneling of an atom in a double-well potential in an external magnetic field. Another possible realization could be a quantum dot coupled to two equal rings of the same length subject to an electrostatic potential difference. We consider the Bethe ansatz equations for this model and derive the tower structure of the finite-size corrections to the ground-state energy. These results are used to discuss the Aharonov-Bohm-Casher interference pattern in the persistent charge and spin currents, and the magnetoresistivity due to the scattering of electrons off the impurity. [S0163-1829(96)09945-6]

I. INTRODUCTION

Probably the most exciting system of a magnetic impurity embedded in a metal is the n-channel Kondo problem,[1] where the impurity of spin S interacts via a contact exchange potential with the conduction electrons propagating through the lattice in n different ''orbital'' channels. The model is characterized by three parameters, namely, the spin S, the number of channels n, and an energy scale T_K, referred to as the Kondo temperature. An exhaustive analysis of the model is due to Nozières and Blandin[1] within a perturbative renormalization-group approach. The Hamiltonian was then later exactly diagonalized by means of Bethe's ansatz by Andrei and Destri[2] and Wiegmann and Tsvelick.[3–5] Other treatments of the model include the conformal field theory,[6] the numerical renormalization group,[7] the bosonization of the conduction electrons,[8] and a $1/n$ expansion.[9]

As a function of the number of channels n and the impurity spin S, we have to distinguish three qualitatively different situations.[1–12] (i) If $n=2S$ the spins of the conduction electrons exactly compensate the impurity spin into a singlet, giving rise to Fermi-liquid behavior at low T ($T \ll T_K$). This situation is realized for Fe and Cr impurities in Cu and Ag.[12,13] (ii) If $n<2S$ the impurity spin is undercompensated, leaving an effective spin degeneracy (in zero field) at low T of $(2S+1-n)$. This situation could correspond to impurities with two magnetic configurations, e.g., Tm (Refs. 12 and 14) or Tb (Ref. 15) embedded in a metal. (iii) If $n>2S$ the impurity is overcompensated and critical behavior is obtained as the temperature and the external field tend to zero.[2–5,11,12] Applications for this case are the quadrupolar Kondo effect[16,17] and electron-assisted tunneling of an atom in a double-well potential.[18–20] Experimentally it was observed in the differential resistance of metal point contacts containing structural disorder,[21] which scales with T with an exponent 1/2 as expected from conformal field theory for the two-channel Kondo problem. Another possible application could be a quantum dot coupled to two equal rings of the same length subject to an electrostatic potential difference.

Here we limit ourselves to $S=1/2$ and two channels. The non-Fermi-liquid behavior can be understood in terms of an essential singularity in the entropy at $H=T=0$. For $H=0$ the entropy is $\mathcal{S}(T=H=0)=(1/2)\ln(2)$,[5,11,12] while if $H\neq 0$ the ground state is a singlet and the entropy is zero. This singular behavior leads to logarithmic divergencies as a function of field,[2,4,11,12] and temperature[5,9–12]

$$M_{\rm imp}\propto (H/T_K)\ln(H/T_K), \quad \chi_{\rm imp}\propto \ln(H/T_K),$$
$$C_{\rm imp}\propto (T/T_K)\ln(T/T_K), \quad \chi_{\rm imp}\propto \ln(T/T_K). \tag{1.1}$$

The instability of the overcompensated fixed point to a magnetic field was confirmed both by numerical renormalization-group calculations[22] and conformal field theory.[23] The stability of the non-Fermi-liquid fixed point to perturbations is of great interest. Besides the magnetic field, other symmetry breaking fields have been investigated. (i) The exchange anisotropy, i.e., $J_\parallel \neq J_\perp$, is irrelevant at the $T=0$ fixed point.[6,22,23] (ii) The fixed point is unstable to a channel-symmetry breaking in the exchange coupling, i.e., the channels have different J.[22–25] (iii) The fixed point is also unstable to a crystalline field splitting of the orbital channels.[26,27]

Here we consider the two-channel Kondo problem with a crystalline field.[26,27] A crystalline field breaks the symmetry between the channels by changing their electron population. The exchange coupling is kept isotropic and the same for both orbital channels. The ground-state Bethe ansatz equations consist of two populated rapidity bands, two-strings and simple spin waves, while all other bands are empty. This

contrasts to the situation without crystalline field where only the two-strings contribute. The channels contribute differently to the screening of the impurity leading to a singlet (Fermi-liquid) ground-state for the impurity. Hence, the crystalline field leads to similar results as a weak channel asymmetry in the exchange coupling,[25] although the underlying mechanism breaking the symmetry between the channels is physically different.

In this paper we study (i) persistent charge and spin currents and (ii) the magnetoresistivity due to scattering off the impurity for a two-channel metallic ring with a spin-1/2 impurity. Persistent currents arise due to the phase shifts picked up by the wave functions as a consequence of the gauge invariance of the electromagnetic field. The phase shifts are proportional to the magnetic and electric fluxes through the ring. The quantization of the fluxes give rise to periodic oscillations of the currents with sawtoothlike shape, which are the consequence of interferences of the Aharonov-Bohm-Casher type.[28]

Persistent currents were studied theoretically with exact methods, e.g., the Bethe ansatz[29–34] and the bosonization technique[35] for a variety of models, and observed experimentally in mesoscopic rings.[36] The description of persistent currents and the magnetoresistance due to scattering of electrons off the impurity require an evaluation of the finite-size corrections to the ground-state energy. Finite-size corrections are used in conformal field theory[37] to calculate the critical exponents of the long distance asymptotic of correlation functions.[38] The fact that both the magnetic and the crystalline fields are relevant variables that quench the non-Fermi-liquid properties of the overscreened fixed point, manifests itself in the finite-size spectrum of the $Z(2)$ parafermion sector, which becomes massive (and does not contribute to mesoscopic order) if at least one of the fields is nonzero.

The rest of the paper is organized as follows. In Sec. II we briefly restate the ground-state Bethe ansatz solution of the two-channel Kondo model with crystalline field splitting.[26,27] In Sec. III we obtain the finite-size corrections to the ground-state energy, i.e., the tower structure of the excitations of the model, first for the case of nonzero fields and then we restate results for the singular zero-field situation (parafermionic sector). In Sec. IV the Aharonov-Bohm-Casher interference pattern as a function of the external fluxes and the magnetoresistance, consequence of the scattering of electrons off the impurity, are discussed. Concluding remarks follow in Sec. V.

II. BETHE ANSATZ EQUATIONS

We consider a spin-1/2 impurity interacting via an isotropic spin exchange with conduction electrons moving along a ring of length L. The conduction electrons can be in *two* different orbital channels. The exchange coupling is assumed to be isotropic and the same for both channels. The Hamiltonian for the two-channel Kondo problem is given by[1]

$$H_K=\sum_{m,\alpha,\sigma}\int dx c^{\dagger}_{m\alpha\sigma}(x)\left(-i\alpha\frac{\partial}{\partial x}+\vartheta_{\alpha\sigma}\right)c_{m\alpha\sigma}(x)+\frac{J}{2}\sum_{m,\alpha,\alpha',\sigma,\sigma'}\mathbf{S}\cdot\int dx\,\delta(x)c^{\dagger}_{m\alpha\sigma}(x)\boldsymbol{\sigma}_{\sigma\sigma'}c_{m\alpha'\sigma'}(x), \tag{2.1}$$

where $\mathbf{S}$ are the spin operators describing the magnetic impurity, J is the exchange coupling, $\boldsymbol{\sigma}$ are Pauli matrices, m labels the two orbital channels, and $\alpha=+,-$ represents the chiral index (forward or backward moving particles). The kinetic energy in Eq. (2.1) has been linearized in momentum space about the Fermi point, i.e., all electrons move with Fermi velocity, which is a necessary condition for the model to be integrable. The parameter $\vartheta_{\alpha\sigma}$ is the phase shift picked up by the electron due to the electric and/or magnetic-field fluxes through the ring. $\vartheta_{\alpha\sigma}$ is responsible for interference effects of the Aharonov-Bohm-Casher type[28] and gives rise to the persistent currents.

The crystalline field splitting is incorporated into Eq. (2.1) by adding the term[26,27]

$$H_{\rm cf}=\frac{\Delta}{2}\sum_{m,\alpha,\sigma}(-1)^m\int dx c^{\dagger}_{m\alpha\sigma}c_{m\alpha\sigma}, \tag{2.2}$$

which lowers the energy of the $m=1$ band by $\Delta/2$ and raises the energy of the $m=2$ band by the same amount. Since $H_{\rm cf}$ commutes with Hamiltonian Eq. (2.1), the crystalline field does not affect the integrability of the model. Applications for this model are the quadrupolar Kondo effect[16] (with a crystalline field splitting of the Γ_8 conduction states) and electron-assisted tunneling of an atom in a double-well potential,[18–20] which has its experimental realization in the differential resistance of metal point contacts containing structural disorder.[21] In the latter case an external magnetic field plays the role of Δ. Another possible realization of the model could be a quantum dot coupled to two equal rings of the same length L, labeled by the index m, and Δ represents the electrostatic potential difference between the two rings.

The phase shift $\vartheta_{\alpha\sigma}$ picked up by a charged particle is proportional to the magnetic-field flux enclosed by the ring, ϕ, and quantized by the elemental magnetic field flux, $\phi_0=hc/e$. If the particles carry a magnetic moment (spin) a radially directed electric field (generated by a string passing through the center of the ring with linear charge density τ) gives rise to an additional phase in the wave function proportional to $F=4\pi\tau$ and quantized in units of $F_0=hc/\mu$. This phase shift is the consequence of the gauge invariance of the electromagnetic field. Although F does not have the dimensions of a flux, we will call F a flux throughout the remainder of the article. The phase shift $\vartheta_{\alpha\sigma}$ is then given by ($\sigma=\pm1$)

$$\vartheta_{\alpha\sigma}=2\pi\alpha\left[\frac{\phi}{\phi_0}+\sigma\frac{F}{F_0}\right]. \tag{2.3}$$

In view of the α dependence of $\vartheta_{\alpha\sigma}$ it is convenient to rewrite the Hamiltonian in terms of states with even and odd parity with respect to the impurity site. The states completely decouple, i.e., the Hamiltonian takes the form $H=H_e+H_o$, where H_e (H_o) involves only states with even (odd) parity. Since H_e and H_o commute, they can be treated independently. Note that H_e couples to the impurity but not to the

phase shifts, while states of H_o are affected by the ϑ dependence but are not scattered by the impurity.

Although the Hamiltonian is diagonal in m the different channels are not independent of each other. On the contrary, the exact solution shows that the channels are strongly correlated close to the impurity and form an orbital singlet.[1–5] In this way, the spins of the bulk electrons are glued together to form a composite of total spin one, which overcompensates the impurity spin degrees of freedom. However, due to the crystalline field splitting, Eq. (2.2), the population of the two orbital channels is not equal, such that not all electrons can participate in the spin composites and a fraction remains orbitally unpaired.[26,27] Hence, for $\Delta\neq 0$ two bands will play a role in the ground state, while if $\Delta=0$ only the spin *one* composites determine the low-T properties. In the former case we obtain the usual Fermi-liquid properties, while $\Delta=0$ and zero magnetic field gives rise to critical behavior, i.e., the non-Fermi-liquid properties discussed in the Introduction.

Following the procedure developed by Andrei and Destri,[2] the Hamiltonian Eq. (2.1) for the even-parity states is diagonalized in terms of *three* sets of rapidities: one set of charge rapidities $\{k_j\}$, $j=1,\ldots,N_e$, where N_e is the total number of electrons in even-parity states, one set of spin rapidities $\{\chi_\gamma\}$, $\gamma=1,...,M$, where M is the number of electrons with down-spin in even-parity states, and one set for the orbital (flavor) degree of freedom $\{\omega_\alpha\}$, $\alpha=1,...,m$, where m is the number of electrons in the minority orbital of even parity. This solution corresponds to the sector $N_e-M\geqslant M$ and $N_e-m\geqslant m$. As already mentioned above the crystalline field splitting Eq. (2.2) leaves the scattering matrices, the wave functions, and the discrete Bethe ansatz equations (but not the energy) unchanged.

Andrei and Destri[2] have shown for $\Delta=0$ that in the thermodynamic limit $L\rightarrow\infty$ the orbital rapidities form bound states (two strings) with the charge rapidities of the form

$$k_\alpha^{\pm}/\Upsilon=\omega_\alpha\pm i(J/2),\quad \alpha=1,\ldots,N_e/2, \tag{2.4}$$

where we assumed that N_e is even and Υ is a cutoff parameter, which eventually tends to infinity. This string arrangement assumes equal population of all orbital bands and corresponds to orbital singlet states. The crystalline field changes the population of the bands and hence the string structure of the orbital rapidities. As shown in Ref. 26 for $\Delta\neq 0$ there are not sufficient ω_α to accommodate all charge rapidities in bound states, such that in addition to the bound states there are real k_j. The index α in Eq. (2.4) now runs from 1 to m_e, while $j=1,\ldots,m_g-m_e$. Here m_e and m_g denote the populations of the excited and ground orbital channels, respectively. m_g and m_e are related to the total number of electrons with even parity through $N_e=m_g+m_e$.

Inserting the above solutions into the discrete Bethe ansatz equations one obtains, as the cutoff Υ tends to infinity, the following ''fused'' Bethe ansatz equations for even parity:[26,27]

$$\frac{\Lambda_\gamma+1/J+i/2}{\Lambda_\gamma+1/J-i/2}\left(\frac{\Lambda_\gamma+i}{\Lambda_\gamma-i}\right)^{m_e}\left(\frac{\Lambda_\gamma+i/2}{\Lambda_\gamma-i/2}\right)^{m_g-m_e} =-e^{i\vartheta_P}\prod_{\alpha=1}^{M}\frac{\Lambda_\gamma-\Lambda_\alpha+i}{\Lambda_\gamma-\Lambda_\alpha-i}, \tag{2.5}$$

for $\gamma=1,...,M$. Here the spin rapidities χ_γ have been rescaled by J so that the electron gas factors do not depend on J, i.e., $\Lambda_\gamma=\chi_\gamma/J$. The fused equations are the consequence of the strong interaction among the orbital channels forming composite spin operators of effective spin *one*. For $m_e=m_g$ (two channels, no crystalline fields) or $m_e=0$ (one channel) equations (2.5) are identical to those of Andrei and Destri.[2]

The first factor on the left-hand side of Eq. (2.5) represents the impurity and the first factor on the right-hand side is the phase shift picked up by the spin of the electrons due to the radial electrostatic field. The remaining factors parametrize the noninteracting gas of electrons. Consider first states with even parity. Since particles with even parity do not interact with the field fluxes, the phase shift ϑ_P vanishes for even parity states. The energy of the system is given by the sum over all (real or complex conjugated) k rapidities and the magnetization is $S_z=N_e/2-M+1/2$, where the 1/2 refers to the impurity spin. Note that the energy of the system is not defined without a cutoff procedure.

Consider now the states with odd parity, which are parametrized in the same way as those of even parity, leading again to Bethe ansatz equations of the form Eq. (2.5). Since states with odd parity do not interact with the impurity, the impurity factor (first factor on the left-hand side) is to be suppressed for odd-parity states. On the other hand, $\vartheta_P=4\pi F/F_0$ for odd-parity states. This phase shift gives rise to the Aharonov-Casher interference pattern. It is straightforward to see that ϑ_P can be replaced by its value modulo 2π, giving rise to a periodicity in F.

In the thermodynamic limit the solutions to equations (2.5) are strings of arbitrary length,[2,26,27]

$$\Lambda_\alpha^{(l),q}=\Lambda_\alpha^{(l)}+i(l+1-2q)/2,\quad q=1,\ldots,l, \tag{2.6}$$

where $\Lambda_\alpha^{(l)}$ is the rapidity for the center of mass of a string involving l spin rapidities and $\alpha=1,...,\zeta_l$ is the running index of the set. The number of strings in each set, ζ_l, is constrained by the total number of spin rapidities $M=\sum_{l=1}^{\infty} l\zeta_l$. These string solutions are inserted into Eq. (2.5), the resulting equations logarithmized and differentiated with respect to $\Lambda^{(l)}$. In the thermodynamic limit we introduce distribution functions for the string rapidities, $\rho_l(\Lambda)$, and similarly the corresponding dressed energy potentials $\varepsilon_l(\Lambda)$, $l=1,2,...$, which satisfy integral equations derived in Ref. 26. According to the Fermi statistics obeyed by the rapidities (required for the wave functions to be linearly independent), states with positive energy potential are empty and those with negative potential are occupied in the ground state. As shown in Refs. 26, 27 all dressed energy potentials ϵ_l are positive for all Λ, except $\varepsilon_1(\Lambda)$ and $\varepsilon_2(\Lambda)$, which can become negative. Hence, the only populated rapidity bands in the ground state are real spin rapidities and two-strings of spin rapidities. The magnetic field acts as the chemical potential for the occupation of these bands. A finite magnetic field raises the energies of the two bands, so that states in the intervals $[-\infty,B_1]$ for ε_1 and $[-\infty,B_2]$ for ε_2 are empty. The two energy potentials satisfy the following coupled Wiener-Hopf integral equations ($l=1,2$):[27]

$$\varepsilon_l(\Lambda)+\sum_{j=1}^{2}\int_{B_j}^{\infty}d\Lambda' K_{lj}(\Lambda-\Lambda')\varepsilon_j(\Lambda')=g_l(\Lambda), \tag{2.7}$$

$$g_l(\Lambda)=lH-\frac{(m_g-m_e)}{L}\,h_l(\Lambda) - \frac{m_e}{\Lambda}\,[h_{l+1}(\Lambda)+\delta_{l\neq 1}h_{l-1}(\Lambda)], \tag{2.8}$$

$$K_{11}(\Lambda)=a_1(\Lambda),\quad K_{22}(\Lambda)=2a_1(\Lambda)+a_2(\Lambda),$$
$$K_{12}(\Lambda)=K_{21}(\Lambda)=a_{1/2}(\Lambda)+a_{3/2}(\Lambda), \tag{2.9}$$

$$a_j(\Lambda)=(j/\pi)/(\Lambda^2+j^2),\quad h_l(\Lambda)=[\pi+2\tan^{-1}(2\Lambda/l)]. \tag{2.10}$$

The integration limits are determined by the zeroes of the energy potentials, $\varepsilon_1(B_1)=0$ and $\varepsilon_2(B_2)=0$. The B_l monotonically increase with the magnetic field with $B_1=B_2=-\infty$ if the field is zero. The relative values of B_1 and B_2 depend on m_e and m_g, i.e., on the crystalline (channel) field splitting.[27]

The ground-state distribution densities for the string rapidities satisfy integral equations with the same integration kernels and limits as Eq. (2.7), but with different driving terms, $f_l(\Lambda)$. The driving terms for the density functions consist of terms for the host and for the impurity. Since the equations are linear, it is convenient to separate ρ_l into host and impurity contributions, $\rho_l(\Lambda)=\rho_l^{\rm host}(\Lambda)+(1/N_e)\rho_l^{\rm imp}(\Lambda)$. The driving terms for $\rho_l^{\rm host}$ and $\rho_l^{\rm imp}$ are given by[27]

$$f_l^{\rm host}(\Lambda)=\frac{m_e}{N_e}\sum_{k=1}^{\min(l,2)} a_{(l+3-2k)/2}(\Lambda)+\frac{m_g-m_e}{N_e}\,a_{l/2}(\Lambda),$$
$$f_l^{\rm imp}(\Lambda)=a_{l/2}(\Lambda+1/J), \tag{2.11}$$

and the populations of the *one* and *two*-strings are $\zeta_l=N_e\int_{B_l}^{\infty}d\Lambda\rho_l(\Lambda)$. The magnetization of the electron gas and the impurity are given by

$$M_{\rm host}=\frac{N_e}{2}-N_e\int_{B_1}^{\infty}d\Lambda\rho_1^{\rm host}(\Lambda)-2N_e\int_{B_2}^{\infty}d\Lambda\rho_2^{\rm host}(\Lambda) =\frac{N_e}{2}\int_{-\infty}^{B_2}d\Lambda\rho_2^{\rm host}(\Lambda),$$
$$M_{\rm imp}=1/2-\int_{B_1}^{\infty}d\Lambda\rho_1^{\rm imp}(\Lambda)-2\int_{B_2}^{\infty}d\Lambda\rho_2^{\rm imp}(\Lambda) =\frac{1}{2}\int_{-\infty}^{B_2}d\Lambda\rho_2^{\rm imp}(\Lambda). \tag{2.12}$$

Note that except for the impurity terms the even- and odd-parity channels contribute with identical terms to the ground-state energy in the thermodynamic limit ($L\to\infty$). The above integral equations are then independent of the parity.

III. FINITE-SIZE CORRECTIONS

In this section we expand the ground-state energy as a power series in L^{-1}

$$E(L)=LE_\infty+E_{\rm imp}+L^{-1}E_{\rm mes}+\cdots, \tag{3.1}$$

where E_∞ is the bulk electron ground-state energy density, i.e., the energy density of the free-electron gas, $E_{\rm imp}$ is the ground-state energy of the impurity, which was studied in Refs. 1–6, 26, 27, and $E_{\rm mes}$ is the mesoscopic (finite-size) correction we are interested in here.

In the Introduction we mentioned that the non-Fermi-liquid behavior is traced back to the singularity in the impurity entropy at $T=H=\Delta=0$. Since finite-size effects of the ground-state energy on a cylinder of perimeter L are related to the properties of the free energy at finite T, we expect the singularity of the entropy of the impurity to reflect in the finite-size expansion to order $E_{\rm mes}$. It is therefore convenient to distinguish three cases, namely (a) Δ is nonzero (two crystalline field split bands giving rise to a Fermi-liquid fixed point), (b) $\Delta=0$ but $H\neq 0$ (one band Fermi-liquid fixed point), and (c) $\Delta=H=0$ when the parafermion sector is relevant (non-Fermi-liquid fixed point).

In all cases the string excitations with string index $l\geqslant 2$ are massive and can be disregarded. The differences among the three cases arise from the behavior of the $l=1$ band corresponding to real spin rapidities and the impurity driving terms.

A. Crystalline field split bands

In this case both bands, i.e., for real spin rapidities and two-strings have a Fermi surface, and the standard methods developed elsewhere[29,30,32,34,38–40] can be followed. There are several contributions to $E_{\rm mes}$, which can be classified as arising from the charge or spin sector of the model. Furthermore both parities (even or odd symmetry) with respect to the impurity site have to be considered. The generalized dressed charge matrix has dimension three, corresponding to the charges (k rapidities), and the $\varepsilon_1(\Lambda)$ and $\varepsilon_2(\Lambda)$ bands for the spin sector. Besides the usual charge-spin separation the Bethe ansatz solution shows that the spin and charge excitations completely decouple, i.e., all entries of the generalized dressed charge matrix referring to the k rapidities, except the diagonal one, are zero. Effectively, we then have to consider only the 2×2 matrix for the spin sector.

For the charge sector we obtain the following finite-size corrections to the ground-state energy of the host

$$E_{\rm mes}^{\rm ch}=(2\pi v_c)\left[\frac{1}{2}\,(\Delta N)^2+2\left(\left\{\frac{\phi}{\phi_0}+\frac{F}{F_0}\right\}+D_c\right)^2 +n_c^+ +n_c^- -\frac{1}{12}\right], \tag{3.2}$$

where v_c is the group velocity of the charges, ΔN represents a change in the total number of electrons, the ϕ and F terms are phase shifts picked up by the wave function due to the presence of magnetic and electric fluxes through the ring, D_c is the backward scattering (change of chirality) quantum number, which can be an integer or a half-integer depending on the initial conditions, and $n_c^{\pm}$ are integer quantum numbers corresponding to particle-hole excitations at the Fermi points for forward and backward moving charges. The symbol $\{x\}$ denotes the fractional part of x to the closest integer. Only the fractional part of the phase shifts are relevant because of the periodicity of the phase discussed in Sec. II.

The finite-size corrections to the ground-state energy of the host arising from the spin sector are[39]

$$E^{S}_{\rm mes}=(2\pi v_{s1})\left[\frac{1}{2}\left(\frac{z_{22}\Delta M_1-z_{12}\Delta M_2}{z_{11}z_{22}-z_{12}z_{21}}\right)^2+n^{+}_{s1}+n^{-}_{s1}-\frac{1}{12}\right]+(4\pi v_{s1})\left[z_{21}\left(\left\{\frac{4F}{F_0}\right\}+D_{s2}\right)+z_{11}\left(\left\{\frac{2F}{F_0}\right\}+D_{s1}\right)\right]^2$$
$$+(2\pi v_{s2})\left[\frac{1}{2}\left(\frac{-z_{21}\Delta M_1+z_{11}\Delta M_2}{z_{11}z_{22}-z_{12}z_{21}}\right)^2+n^{+}_{s2}+n^{-}_{s2}-\frac{1}{12}\right]+(4\pi v_{s2})\left[z_{22}\left(\left\{\frac{4F}{F_0}\right\}+D_{s2}\right)+z_{12}\left(\left\{\frac{2F}{F_0}\right\}+D_{s1}\right)\right]^2, \tag{3.3}$$

where v_{s1} and v_{s2} are the group velocities associated with the two spin-rapidity bands, ΔM_1 and ΔM_2 represent the changes in the total number of rapidities in the respective bands, the terms involving F are the phase shifts due to the radial electric field,[34] D_{s1} and D_{s2} are the backward scattering quantum numbers for spin rapidity bands (they can be integers or half-integers depending on the initial conditions), and $n^{\pm}_{s1}$ and $n^{\pm}_{s2}$ are integer quantum numbers corresponding to the particle-hole excitations within the spin-rapidity bands around the respective Fermi points, z_{ij} is the 2×2 dressed generalized charge matrix relating the Fermi surfaces within the spin sector.

The scattering of the electrons with the impurity also contributes to $E_{\rm mes}$ within the charge and spin sectors[40]

$$E^{\rm imp}_{\rm mes}=(2\pi v_c)[-1/2-M_{\rm imp}]\Delta N_e+(2\pi v_{s1})2M_{\rm imp}\Delta M_{1e}+(2\pi v_{s2})4M_{\rm imp}\Delta M_{2e}. \tag{3.4}$$

The coupling here is only to the electron states with even parity with respect to the impurity site, since the odd states do not couple to the impurity. The impurity is also not affected by the magnetic and electric fluxes enclosed by the ring. The contributions to $E^{\rm imp}_{\rm mes}$ are the dressed scattering phase shifts for electrons off the impurity as given by Friedel's sum rule.

The group velocities for the charges and the spin waves are expected to be equal for a noninteracting gas of electrons. For a linearized dispersion they can be equated to *one*, independent of the bandfilling. Here we limit ourselves to show that the two spin-wave velocities are equal in the absence of a magnetic field. The group velocities should not depend on the magnetic field, since the magnetic field is always much smaller than the rapidity bandwidth and the dispersion is linearized. For $H=0$ the string rapidities fill the entire real axis and the solution of the integral equations satisfied by ε_l and ρ_l can straightforwardly be obtained by Fourier transformation[27]

$$\varepsilon_1(\Lambda)=-2\frac{(m_g-m_e)}{L}\arctan(e^{\pi\Lambda}),$$
$$\varepsilon_2(\Lambda)=-2\frac{m_e}{L}\arctan(e^{\pi\Lambda}), \tag{3.5}$$
$$\rho_1(\Lambda)=\frac{(m_g-m_e)}{N_e}[2\cosh(\pi\Lambda)]^{-1},$$
$$\rho_2(\Lambda)=\frac{m_e}{N_e}[2\cosh(\pi\Lambda)]^{-1}. \tag{3.6}$$

The energy and momentum of a spin-wave excitation, obtained by removing the rapidity $\Lambda^{(l)}_0$ from the rapidity band $l=1,2$, is given by

$$\Delta E^{(l)}_S(\Lambda^{(l)}_0)=|\varepsilon_l(\Lambda^{(l)}_0)|,$$
$$p^{(l)}_S(\Lambda^{(l)}_0)=2\pi\frac{N_e}{L}\int_{-\infty}^{\Lambda^{(l)}_0}d\Lambda\rho_l(\Lambda). \tag{3.7}$$

Low-energy excitations correspond to the limit $\Lambda^{(l)}_0\to-\infty$; expanding Eq. (3.7) for large negative $\Lambda^{(l)}_0$ we obtain that $v_{s1}=v_{s2}=1$, in agreement with the expectations.

The matrix of generalized dressed charges within the spin sector is defined as $z_{i1}=\xi_{i1}(\Lambda=B_1)$ and $z_{i2}=\xi_{i2}(\Lambda=B_2)$ for $i=1,2$. Here $\xi_{ij}(\Lambda)$ satisfies the following integral equations:

$$\xi_{ij}(\Lambda)+\sum_{l=1}^{2}\int_{B_1}^{\infty}d\Lambda' K_{il}(\Lambda-\Lambda')\xi_{jl}(\Lambda')=\delta_{i,j}. \tag{3.8}$$

The matrix of dressed charges represents the interrelation between the Fermi surfaces of the two bands. Note that the dressed charges only depend on (B_2-B_1), but not on B_1 and B_2 independently. Hence, they are only a function of the crystalline field splitting, but are independent of the magnetic field.

B. Zero crystalline field

In the limit $m_e=m_g$ the crystalline field splitting vanishes, $\Delta=0$ and $B_1=\infty$, and the band of real spin rapidities is empty. In this subsection we treat the case of nonzero magnetic field i.e., B_2 is finite and the $\varepsilon_2(\Lambda)$ band has a Fermi surface with particle- and holelike excitations. From the temperature dependence, discussed in Refs. 5, 11, the gap of the $\varepsilon_1(\Lambda)$ band is massive for the finite system, such that the impurity entropy is zero and the ground state is a Fermi liquid.

The mesoscopic finite-size corrections to the ground-state energy again consist of three contributions. $E_{\rm mes}=E^{\rm ch}_{\rm mes}+E^{S}_{\rm mes}+E^{\rm imp}_{\rm mes}$ with $E^{\rm ch}_{\rm mes}$ still given by Eq. (3.2). The contribution of the spin sector now only refers to the ε_2 band

$$E^{S}_{\rm mes}=(2\pi v_{s2})\left[\frac{1}{2}\left(\frac{\Delta M_2}{z}\right)^2+2z^2\left(\left\{\frac{4F}{F_0}\right\}+D_{s2}\right)^2+n^{+}_{s2}+n^{-}_{s2}-\frac{1}{12}\right], \tag{3.9}$$

and $E^{\rm imp}_{\rm mes}$ is given by

$$E^{\rm imp}_{\rm mes}=(2\pi v_c)[-1/2-M_{\rm imp}]\Delta N_c+(2\pi v_{s2})4M_{\rm imp}\Delta M_{2c}. \tag{3.10}$$

The generalized dressed charge is a scalar defined as $z=\xi(B_2)$ with $\xi(\Lambda)$ satisfying the integral equation

$$\xi(\Lambda)+\int_{B_2}^{\infty}d\Lambda' K_{22}(\Lambda-\Lambda')\xi(\Lambda')=1. \tag{3.11}$$

The solution of this Wiener-Hopf integral equation is straightforward and yields $z=1/2$ independent of the magnetic field. Note that the group velocities are still equal to 1.

C. Parafermion sector

As discussed in the Introduction for $\Delta=H=0$ the impurity entropy is singular at $T=0$. This singular behavior reflects in the finite-size excitation spectrum from the ground state. In particular, in this limit the energy band ε_1 vanishes identically in the thermodynamic limit, but the impurity driving term couples to this band to order $1/L$. Furthermore the string hypothesis, Eqs. (2.4) and (2.6), is only valid in the strict sense in the thermodynamic limit. These two limits interfere with the usual procedure to evaluate the finite-size corrections to the ground-state energy and a straight calculation starting from the Bethe ansatz equations (2.5) was therefore so far not possible. From the field theory point of view the problem consists of extracting the correct spectrum for the $Z(2)$ parafermions.

An alternative technique was proposed by Fujimoto and Kawakami,[41] who noticed that the overscreened impurity case can be described in terms of the restricted solid-on-solid model (RSOS) coupled to the impurity. The problem is reduced to the effective transfer matrix of multikinks with a level $p=1$ impurity scattering matrix on site $(N+1)$

$$T^q(u)=W^{1,q}(u|\{\sigma_{j+1}\})\cdots W^{1,q}(u|\{\sigma_N\})\times W^{p,q}_{\rm imp}(u|\{\sigma_{N+1}\})W^{1,q}(u|\{\sigma_1\})\cdots W^{1,q}(u|\{\sigma_{j-1}\}), \tag{3.12}$$

where $W^{1,q}$ is the fused face weight of the host system

$$W^{1,q}(u|\{\sigma\})=\prod_{k=0}^{q-2}s_k^{-1}(u)\sum_{\sigma'}\prod_{k=1}^{q}W[u+(k-1)\lambda|\{\sigma'\}] \tag{3.13}$$

in terms of the face weights of the RSOS model with a spectral parameter u. Here $\lambda=\pi/4$, $\{\sigma\}$ is the set of spins around the face, and $s_k(u)=\sin[u+(k-j)\lambda]/\sin(\lambda)$ for the host. The fused face weight of level $p=1$ for the impurity is defined in a similar way, but with $s_k^{\rm imp}(u)=\sin(u+k\lambda)/\sin(\lambda)$.

Using the method to calculate the finite-size corrections of the RSOS model developed in Ref. 42, Fujimoto and Kawakami[41] obtain for the $Z(2)$ parafermion sector with level $p=1$ impurity

$$E^{\rm par}_{\rm mes}=2\pi v_{s2}\left(\frac{j(j+1)}{4}-\frac{(m+1)^2}{8}+{\rm const}\right), \tag{3.14}$$

where $m=2j$ (mod 2) and $j=0$, 1/2, 1.[41,42] These fusion rules are identical to those postulated in Ref. 6. $E^{\rm par}_{\rm mes}$ is to be added to the normal finite-size corrections arising from the charge and spin sectors. The impurity contributes with $-\pi v_c\Delta N_e$ to the former (the impurity absorbs one charge, so that the scattering phase shift is $\pi/2$), and the contribution of the spin sector cancels the second term in Eq. (3.14). Hence, if the two orbital channels are equally populated, $m_e=m_g$, non-Fermi-liquid properties arise and the finite-size spectrum has a parafermion sector with central charge $c=c_{\rm WZW}-1=1/2$, where $c_{\rm WZW}$ is the central charge of the two-level SU(2) Wess-Zumino-Witten model.[6,41]

IV. RESULTS

In this section we analyze the physical implications of the finite-size corrections to the ground-state energy. First we present our results for the Aharonov-Casher interference pattern in the persistent spin current and then we discuss correlation functions, in particular the resistivity due to the scattering of the electrons off the impurity.

A. Persistent currents

Persistent currents arise from the phase shifts ϑ picked up by the wave function as a consequence of the gauge invariance of the electromagnetic fields. To study the persistent currents it is then sufficient to keep in Eqs. (3.2) and (3.3) only the terms involving the fluxes ϕ and F, and setting all quantum numbers equal to zero.[34] The flux-dependent terms are (with $v_c=v_{s1}=v_{s2}=v$)

$$\Delta E(\phi,F)=\frac{4\pi v}{L}\left(\left\{\frac{\phi}{\phi_0}+\frac{F}{F_0}\right\}\right)^2+\frac{4\pi v}{L}\left[\left(z_{21}\left\{\frac{4F}{F_0}\right\}+z_{11}\left\{\frac{2F}{F_0}\right\}\right)^2+\left(z_{22}\left\{\frac{4F}{F_0}\right\}+z_{12}\left\{\frac{2F}{F_0}\right\}\right)^2\right]. \tag{4.1}$$

We first discuss the persistent charge current (Aharonov-Bohm effect) as a function of the magnetic flux ϕ for $F=0$. The dependence of the energy on ϕ is parabolic, so that the persistent current has a sawtooth shape with periodicity ϕ_0. The parity of the sawtooth depends on the charge backward scattering quantum number D_c, which can be zero or one-half. The parity determines whether the contribution to the magnetic susceptibility is paramagnetic or diamagnetic.

The persistent spin current (Aharonov-Casher effect) shows more interesting features. First, for $\phi=0$ and as a function of F, Eq. (4.1) shows three different periodicities, namely F_0, $F_0/2$, and $F_0/4$. The term with periodicity F_0 arises from the charge sector and is decoupled from the other terms. The persistent current is then the superposition of sawtoothlike oscillations of periodicity F_0 and the oscillations arising from the spin sector. Second, the spin sector has two Fermi surfaces, corresponding to simple spin rapidities and two-strings, which are nontrivially coupled by the matrix of dressed charges because of the crystalline field splitting. The overall periodicity of the contribution from the spin sector has periodicity $F_0/2$. The dependence of $\Delta E(\phi=0,F)$ on F is piecewise parabolic. The parabolas can be concave or convex depending on the initial conditions,[43] i.e., the backward scattering quantum numbers D_{s1} and D_{s2}. The relative

amplitude of these oscillations strongly depends on the crystalline field. $\Delta E(\phi=0,F)$ displays jumps as a function of F. These discontinuities of the energy are, however, microscopic, of the order of the uncertainty of the energy according to Heisenberg's principle. The discontinuities are the consequence of two interacting (i.e., in general z_{12} and z_{21} are nonzero) Fermi seas, both responding to the same ''chemical potential,'' namely the magnetic field. We found similar jumps in special spin chain systems.[43]

The persistent spin current, which is the derivative of $\Delta E(\phi=0,F)$ with respect to F, consists of segments of constant slope separated by δ-function-like singularities where $\Delta E(\phi=0,F)$ has jumps. These singularities could be interpreted as ''supercurrents,'' necessary to generate the discontinuities of the energy. The discontinuities disappear at any finite temperature, since the temperature suppresses the higher harmonic content. In a similar fashion a relaxation time (e.g., due to impurities) would spoil the coherence of the wave function and smear the jumps (as the Dingle temperature in the de Haas–van Alphen effect). On the other hand, we could define the spin-current operator as a derivative of the Hamiltonian (rather than the spin current via the energy) and consider its ground-state expectation value. In this case the same oscillations of the sawtooth form are obtained except for the δ-function-like singularities.

If $\Delta=0$ the bands are not split and the flux dependence of the ground-state energy is given by Eqs. (3.2) and (3.9). The Aharonov-Bohm interference pattern is unchanged, but the Aharonov-Casher oscillations are the superposition of two sawtooth of periodicities F_0 and $F_0/4$, respectively. The energy has no discontinuities in this case, since there is only one spin rapidity band.

It is worthwhile to place our results for the persistent currents into the general context of the n-channel Kondo problem of an arbitrary impurity of spin S. Depending on the relative values of n and S three situations may arise (see Introduction): (i) if $S>2n$ the impurity is said to be undercompensated and an effective spin $S-2n$ remains in the ground state, (ii) if $S=2n$ the impurity is compensated into a singlet, and (iii) if $S<2n$ the impurity is overcompensated giving rise to non-Fermi-liquid behavior. A crystalline field splitting can modify this classification.[26] Since the magnetic and electric fluxes do not couple to the impurity to order L^{-1} in the energy, the results discussed above remain valid for all situations. In particular, if the crystalline field splits the n-fold degeneracy of the orbits into two submultiplets of degeneracy n^* and $(n-n^*)$, the spin current oscillations arising from the spin sector will have periods $F_0/(2n^*)$ and $F_0/(2n)$, respectively. Note that the parafermion sector is decoupled from the field fluxes and does not at all affect the persistent currents.

Consider a mesoscopic metallic ring coupled to a quantum dot. As a function of an external voltage V between the quantum dot and the ring, electrons may be localized or delocalized in the dot, an effect analogous to the Coulomb blockade. As a consequence the backward scattering quantum numbers change and may give rise to a change in parity in the Aharonov-Casher response. Similar effects are expected if two metallic rings are coupled to the a quantum dot. Now, in addition to a change in the number of localized electrons in the dot, electrons can be transferred from one ring to the other, opening additional possibilities for changes in parity of the Aharonov-Bohm-Casher oscillations.[44,45]

B. Zero-temperature resistivity

While the external field fluxes couple to the states of odd parity with respect to the impurity site, the impurity behavior is determined by states with even parity. Only electrons in states with even parity are scattered by the impurity. The incoming and outgoing conduction states are related by the S matrix. Due to the contact potential (δ-function) interaction the S matrix is momentum independent and only a function of energy. The S matrix can be characterized by the scattering phase shift and the resistivity expressed in terms of the phase shifts for electrons at the Fermi level via Friedel's sum rule.

The phase shifts δ are obtained from the impurity terms of the finite-size corrections to the ground-state energy, Eq. (3.4). Andrei[46] has shown this for the traditional spin-1/2 Kondo impurity. Similar procedures have been used to obtain the zero-temperature magnetoresistivity for the Anderson impurity and the Coqblin-Schrieffer model.[47] The T matrix is defined by the one-electron Green's function

$$G_{kk'n\sigma}(\omega)=G^0_{kn\sigma}(\omega)+G^0_{kn\sigma}(\omega)T_{n\sigma}(\omega)G^0_{k'n\sigma}(\omega), \tag{4.2}$$

which depends only on the energy because of the contact potential. The S matrix is related to the T matrix by

$$S=1-iT=e^{2i\delta}. \tag{4.3}$$

The propagator Eq. (4.2) for $\omega=0$ yields then the phase shift at the Fermi level. The propagator annihilates (creates) one electron at $t=0$ and creates (annihilates) it at time t. The time evolution is then given by the additional hole (electron) state at the Fermi level. When it propagates through the ring it causes a change in phase of $L[E(N_e)-E(N_e-1)]$, where $E(N_e)$ is the energy of the N_e-particle state. The phase shift (2δ) is given by the above quantity modulo 2π.

The scattering phase shift depends in principle on the spin and the orbital band. Consider first an up-spin electron in an even-parity state removed from the system. Hence, $\Delta N_e=-1$, while $\Delta M_{1e}=\Delta M_{2e}=0$, since no spin is reversed. The scattering phase shift for an up-spin electron is then

$$\delta_\uparrow=\pi[1/2+M_{\rm imp}], \tag{4.4}$$

where $M_{\rm imp}$ is the magnetization of the impurity. Two cases have to be distinguished in the case we remove a down-spin electron. Then $\Delta N_e=-1$ and the number of reversed spins has to be reduced by one. If the electron is taken from an orbitally unpaired state, $\Delta M_{1e}=-1$ and $\Delta M_{2e}=0$. The scattering phase shift is then [with the same sign definition as in Eq. (4.4)]

$$\delta^{\rm unp}_\downarrow=\pi[1/2-M_{\rm imp}]. \tag{4.5a}$$

On the other hand, if the electron is removed from the orbitally paired state, then $\Delta M_{2e}=-1$ (which flips two spins) and $\Delta M_{1e}=1$ (which restores one spin flip). The scattering phase shift for this case is again

$$\delta_{\downarrow}^{p}=\pi[1/2-M_{\mathrm{imp}}]. \tag{4.5b}$$

Hence, the scattering phase depends only on the magnetization. In the absence of a magnetic field the magnetization of the impurity is zero and a phase shift of $\delta=\pi/2$ is obtained as for the standard Kondo impurity. This corresponds to the resonance scattering expected for a singlet ground state.

Assuming that the ring has a width much larger than the atomic scale but much smaller than the length of the ring L, we may assume the electrons move as plane waves along the ring. Only the s-wave component of the plane wave is scattered by the impurity (when expanded about the impurity site). The magnetoresistivity is then given by[46,47]

$$\rho(H,\Delta)=2\rho_0/[\sin^{-2}(\delta_{\uparrow})+\sin^{-2}(\delta_{\downarrow})], \tag{4.6}$$

since the resistivities of up-spin and down-spin electrons add in parallel. The resistivity is maximum in zero field and monotonically decreases with increasing magnetization. In particular, as M_{imp} tends to its saturation value 1/2, the magnetoresistivity tends to zero, since the phase shifts become π and 0, respectively.

The magnetoresistance is also a function of the crystalline field splitting Δ via the magnetization. The dependence of M_{imp} on the crystalline field splitting (and on the magnetic field) has been discussed in Refs. 26, 27. Note that the above discussion of the resistivity holds only for a singlet ground state, i.e., when either $\Delta\neq 0$ or $H\neq 0$. The scattering phase shift is still $\pi/2$ if $\Delta=H=0$, so that the resistivity is given by the unitarity bound. Hence, the $T=0$ resistivity is continuous as a function of Δ and H. The parafermion sector of the excitation spectrum, however, affects the temperature dependence at low T. According to Ludwig and Affleck[6] if $\Delta=H=0$ the resistivity decreases with T as

$$\rho(T)=\rho_0[1-(T/T_K)^{1/2}], \tag{4.7a}$$

while in the Fermi-liquid regime (Δ and/or H are nonzero) where the parafermion sector is massive the resistivity depends on T as

$$\rho(T)=\rho(H,\Delta)[1-\alpha(T/T_K)^2], \tag{4.7b}$$

with α being a function of H and Δ, which is expected to change sign for large magnetic fields. Hence, the temperature dependence of the resistivity is singular at the point $\Delta=H=0$. This corresponds to the singular behavior in the entropy discussed in the Introduction.

V. CONCLUDING REMARKS

We considered a spin-1/2 impurity embedded into an electron gas of *two* orbital channels interacting via spin exchange. The exchange is isotropic and the same for both channels. The symmetry between the two orbitals is broken by a crystalline field that changes the population of the bands. The crystalline field is a relevant parameter that quenches the overcompensated fixed point into one with Fermi-liquid properties, suppressing the logarithmic singularities in the susceptibility and the specific heat if the splitting Δ is nonzero.[26,27] Instead the susceptibility is finite and the low-T specific heat is proportional to T. The value of χ and γ strongly depend on Δ.

Within the framework of Bethe's ansatz the ground state for $\Delta=0$ is determined by the band of two-strings.[2–5,10–12] The crystalline field populates in addition, the band of simple spin rapidities, thus dramatically changing the physical properties of the impurity.[27]

We derived the finite-size corrections to the ground-state energy from the Bethe ansatz equations for this system. There are several contributions to the tower structure of excitations,[39] which can be classified as arising from the charges, the simple spin rapidities and the two-strings. They are characterized by several quantum numbers for each class, i.e., the change in the number of ''particles,'' the backward scattering (across the Fermi surface) quantum numbers, and the particle-hole excitations about the Fermi level. The scattering phase shifts (Friedel's sum rule) due to the scattering of the electrons off the impurity also contribute to order L^{-1}.[40] We distinguished three different situations: (a) Two spin rapidity bands contributing to the finite-size effects ($\Delta\neq 0$), (b) only two-strings contributing to the finite-size corrections ($\Delta=0$ but $H\neq 0$), and (c) the parafermion spectrum which is nonmassive only if $\Delta=H=0$.

The external magnetic-field flux enclosed by the ring couples to the charge of the electrons and gives rise to a phase shift proportional to the flux. Similarly, the magnetic moment (spin) of the electron couples to a radially directed electric field, generated by a charged string passing through the center of the ring, giving rise to an additional phase shift. These phases, modulo 2π, also contribute to the finite-size corrections of the ground-state energy, being additive to the backward scattering quantum numbers.[29,34] The periodicity in the phase causes oscillations in the energy as a function of the fluxes, i.e., an Aharonov-Bohm-Casher interference pattern.

The persistent field current due to the magnetic-field flux has a simple sawtooth shape. The persistent spin current (Aharonov-Casher effect), on the other hand, has in general a more complicated pattern and displays three periodicities, namely F_0, $F_0/2$, and $F_0/4$. In particular, the matrix of generalized dressed charges couples the periods $F_0/2$ and $F_0/4$, so that the interference pattern consists of a periodic arrangement of two straight segments of constant (but in general different) slope,[43] superimposed to a sawtooth with periodicity F_0. The parity of the sawtooth and the sign of the slopes of the segments depends on the initial values of the backward scattering quantum numbers. If $\Delta=0$ real spin rapidities have no Fermi surface and do not contribute, so that only oscillations of periods F_0 and $F_0/4$ are present.

Our Fermi-liquid analysis of the $T=0$ resistivity through phase shifts is only valid if either $\Delta\neq 0$ and/or $H\neq 0$. In this case the ground state is a singlet and in zero magnetic field the scattering phase shift is $\pi/2$. The magnetic-field dependence of the resistivity is completely determined by the impurity magnetization. The magnetoresistivity decreases monotonically with the field. The dependence on the crystalline field splitting is only implicit through the dependence of M_{imp} on Δ. For $\Delta=H=0$ the resistivity is still given by the unitarity bound, so that at $T=0$ the magnetoresistance is a continuous function of Δ and H. The low-temperature dependence, on the other hand, is singular at $\Delta=H=0$, consequence of the parafermion sector,[6] which is nonmassive if $\Delta=H=0$ but massive otherwise. Such singular behavior is not unexpected in view of the singularity of the impurity

entropy, discussed in the Introduction.

The most relevant applications of the two-channel Kondo model are the quadrupolar Kondo effect[16,17] (Δ is the splitting of the Γ_8 quartet) and electron-assisted tunneling of an atom in a potential well[18–20] (Δ corresponds to an external magnetic field). An experimental realization is the differential resistance of metal point contacts containing structural disorder.[21] Another possible application of the model could be a quantum dot coupled to two equal metallic rings of the same length L (here Δ represents the electrostatic potential difference between the two rings).

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