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Magnetic vortex mass in two-dimensional easy-plane magnets

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The dynamical mass of a vortex in a two-dimensional easy-plane magnetic model is calculated from the translational mode spectrum and an effective force constant for the vortex in a finite lattice system. A significant feature of this method is that the mass can be calculated for both in-plane and out-of-plane vortices. [S0163-1829(96)01845-0]

I. INTRODUCTION

Nonlinear vortex excitations in models for layered or two-dimensional (2D) magnets[1] have attracted much study, not only for their role in the thermodynamics of the Berezinskii-Kosterlitz-Thouless vortex-unbinding transition,[2,3] but more recently because their microscopic dynamic behavior is not fully understood. The dynamics of individual vortices continues under study for ferromagnets (FM's)[4,5] and antiferromagnets (AFM's),[6,7] with attention to external and gyrotopic force terms, dissipation terms, and mass terms in appropriate equations of motion. Specifically, a question of great importance that we consider here is whether vortices have dynamics that requires an effective mass. The value of the vortex effective mass, if present, will relate to vortex motion in thermal equilibrium and vortex rms velocity. The results may also be relevant for vortex fluctuation experiments in related systems such as high-temperature superconductors.[8]

We consider 2D magnetic models with easy-plane anisotropy, where the spins have an energetic preference to lie in the xy plane, with smaller z components, depending on the strength of the anisotropy. A vortex position is defined by the location of a local singularity in the in-plane (xy) components of the spin field. If the anisotropy is weak, there is an associated nonzero out-of-plane (z) spin structure that peaks at the vortex center and falls off away from the vortex center.[9,10] If one supposes that this ''out-of-plane'' vortex spin structure (in continuum theory) is fixed as the vortex moves with constant velocity $\vec{V}=\dot{\vec{X}}$, then the response of the vortex position $\vec{X}$ to an external force $\vec{F}$ (due to applied field or other vortices) is associated with a topological charge of the vortex, or gyrovector, $\vec{G}=G\hat{e}_z$, according to an equation of motion derived by Thiele[11] for domain walls and applied by Huber[12] to dynamics of vortices:

$$\vec{F}+\vec{G}\times\vec{V}=0. \tag{1.1}$$

In closely related work, dynamic properties of vortices in continuum easy-plane magnets were analyzed by Nikiforov and Sonin[13] by considering a solvability condition for the inhomogeneous linear equation for the corrections to the vortex structure that are due to uniform vortex motion. They also evaluated the momentum balance in the absence of external forces. From both calculations they concluded, consistently with Eq. (1.1), that a vortex in an easy-plane ferromagnet does not move $(\vec{V}=0)$ in the absence of an external force. It is ''frozen'' in the medium and effectively appears as if it had infinite mass. This conclusion applies to vortices with nonzero gyrovector or, as stated in Ref. 13, vortices in which the magnetization at the vortex core is nonzero even in the absence of magnetization at infinity.[14] On the other hand, the gyrovector is zero for vortices in an antiferromagnet[15,16] (unless there is an applied field,[6,7] which is not considered here), and free AFM vortices can move with arbitrary velocity in the absence of a force.[17]

In the presence of strong enough easy-plane anisotropy, however, the FM vortex spin structure is planar,[18,19] spins have only small deviations out of the easy plane caused by the motion,[20] and the gyrovector vanishes. Then Eq. (1.1) makes no statement about $\vec{V}$ in the absence of a force, and is inapplicable when there is a force. A modification of its derivation[21,22] to allow for a time-dependent velocity, including the possibility that the vortex spin structure changes as the vortex velocity changes, leads to an additional mass times acceleration term on the right-hand side:

$$\vec{F}+\vec{G}\times\vec{V}=M\dot{\vec{V}}. \tag{1.2}$$

This equation was derived in a continuum limit. In the absence of $\vec{G}$ it still has a dynamics. A similar equation has been applied by Ivanov and Stephanovich[23] to calculate the effective mass for localized magnetic vortices in a 2D *easy-axis* FM. Here we consider whether the nonlocalized vortices in an *easy-plane* model move with such an effective mass. We consider a lattice system of finite extent and study the results as a function of increasing system size.

In a simulation for an individual vortex near the center of a small circular *lattice* system, we show that Eq. (1.2) gives a good description of the vortex core motion and can be used to evaluate the mass. The force is a linear central force provided by the combination of a boundary condition and the effect of the discrete lattice itself, with the lattice force being dominant. Through an energy minimization procedure we evaluate an effective force constant K that describes the central force. The gyrovector is evaluated from a lowest-order finite-difference approximation to its continuum definition. Most importantly, the mass is evaluated in a direct way by using the eigenfrequencies of the translational modes of the vortex. Numerical diagonalization[24] is used to determine the complete spectrum of small-amplitude oscillations of the spin degrees of freedom, i.e., the spin waves in the presence of the vortex. The translational modes are identified, and produce either linear or orbital motions of the vortex center, depending on the absence or presence of the gyrovector, respectively. As these motions are the *same* as those that Eq. (1.2) predicts under a linear central force, it is possible to use the translation mode frequencies together with the effective force constant K to deduce the mass. The mass can be found in this way for $\vec{G}=0$ as well as for $\vec{G}\neq 0$. The calculation is applied to FM and AFM models, although we concentrate on the FM system because of the more interesting dynamics due to the gyrovector in the absence of a field. Comparing the results with a prediction of continuum theory for moving vortices,[21,22] the mass found here is rather large and does not generally increase as $\ln R$, where R is the system radius.

II. MODEL SYSTEM

Consider the 2D classical easy-plane FM spin model with anisotropy parameter $\lambda<1$ and lattice Hamiltonian

$$H=-J\sum_{\mathbf{n},\mathbf{a}}\left(S^x_{\mathbf{n}}S^x_{\mathbf{n+a}}+S^y_{\mathbf{n}}S^y_{\mathbf{n+a}}+\lambda S^z_{\mathbf{n}}S^z_{\mathbf{n+a}}\right), \tag{2.1}$$

where the subscript $\mathbf{n}$ labels the lattice site at position $\mathbf{r_n}=(x_{\mathbf{n}},y_{\mathbf{n}})$, and $\mathbf{a}$ labels the set of displacements to the nearest neighbors. The equations of motion and static FM vortex solutions as a function of λ for $0\leqslant\lambda<1$ are well known.[9,10,20] Further general properties of vortex excitations are reviewed in Ref. 4. The spin variables are of fixed length, and can be described by in-plane and out-of-plane angles $\phi_{\mathbf{n}}$ and $\theta_{\mathbf{n}}$ via

$$\vec{S}_{\mathbf{n}}=S(\cos\theta_{\mathbf{n}}\cos\phi_{\mathbf{n}},\cos\theta_{\mathbf{n}}\sin\phi_{\mathbf{n}},\sin\theta_{\mathbf{n}}). \tag{2.2}$$

The vortex solution with charge $q=\pm 1$ centered at position (x_v,y_v) is described by

$$\phi_{\mathbf{n}}=q\tan^{-1}\left(\frac{y_{\mathbf{n}}-y_v}{x_{\mathbf{n}}-x_v}\right). \tag{2.3}$$

The center position of an individual vortex is a well-defined quantity, determined by the singularity in $\vec{\nabla}\times\vec{\nabla}\phi$, and may fall anywhere between lattice sites. There is no general closed form solution for the out-of-plane angle θ, except that for λ less than a critical value $\lambda_c\approx 0.704$ for the square lattice[19] the stable *in-plane* vortex has $\theta=0$. Above this critical value, θ becomes a nonzero function localized on the vortex center with a length scale of $r_v=\frac{1}{2}\sqrt{\lambda/(1-\lambda)}$. In continuum theory, at the core of the out-of-plane vortex the spin is along either the positive or negative z axis:

$$\theta(x_v,y_v)=p\frac{\pi}{2}, \tag{2.4}$$

with polarization $p=\pm 1$. The product of the vortex charge and its polarization in the form

$$\vec{G}=2\pi pq\hat{e}_z \tag{2.5}$$

defines the *gyrovector* $\vec{G}$ (in continuum limit), which is the net topological charge (or cover of the spins on the unit sphere). $\vec{G}$ plays an interesting role in the dynamics when the vortex is considered to move as a particle. The in-plane vortices ($\lambda<\lambda_c$) have zero gyrovector. For a static vortex in the AFM system on a square or hexagonal lattice, the spins on an individual sublattice take the same form as this structure for the FM vortex, with opposite phases between the sublattices. Thus there is no particular distinction in the static structures, critical λ, and energies for FM and AFM vortices.

For numerical calculations, we consider a finite system, taken to be a circle of radius R cut out of a square lattice, with the center of the circle at the center of a unit cell. A vortex-Dirichlet boundary condition is applied, by setting spins on the square lattice just outside the circle to lie in the xy plane, with directions as given by the static in-plane vortex, Eq. (2.3). The resulting effect of this boundary condition is that it forces the lowest-energy position of the vortex to be at the center of the system. Small displacements $\vec{X}$ away from the center involve an energy increase. In fact, for displacements much less than the lattice spacing, the potential is close to harmonic, and can be described by an effective linear force acting on the vortex,

$$\vec{F}=-K\vec{X}, \tag{2.6}$$

where K is the effective force constant. For larger displacements, the potential involves a periodic contribution due to the lattice and a background contribution due to the boundary condition. The constant K can be considered to include contributions from both effects measured at the center of the system; however, the lattice contribution is dominant here.

If the vortex were to behave as an ordinary particle with mass M moving under the influence of this force, the resulting motion would be the same as that of a two-dimensional harmonic oscillator. This would involve two translational modes of oscillation corresponding to x- and y-direction motions, with equal frequencies given by $\omega=\sqrt{K/M}$. We can see if this is the case for vortices by determining the translational spin-wave modes of oscillation of the vortex.

III. TRANSLATION MODE SPECTRUM

The microscopic calculation of the small-amplitude spin motions has already been carried out in Refs. 24 and 25, using the system and boundary conditions as specified here. It is important to note that it is a *microscopic* calculation, as opposed to a collective coordinate calculation, and considers the full dynamical motions of the spins themselves. There is no set of reduced degrees of freedom or reduced coordinates used. A vortex with $q=p=+1$ is initially placed at the center of the system, and then the small-amplitude spin motions are determined by a numerical diagonalization of the *spin* equations of motion linearized about the vortex. A typical spectrum of a few of the lowest eigenmode frequencies of a FM vortex for $R=19$ is shown in Fig. 1, as a function of the anisotropy constant λ. (This system has 1124 sites.) The solid circles are used to indicate where the translation mode is doubly degenerate, for $\lambda<\lambda_c$. The splitting of this and other mode degeneracies has been found to be associated with the crossover from in-plane to out-of-plane vortices at $\lambda=\lambda_c$.[24]

The eigenfunctions for the translation and other modes were discussed in Refs. 24–26. We identified the transla-

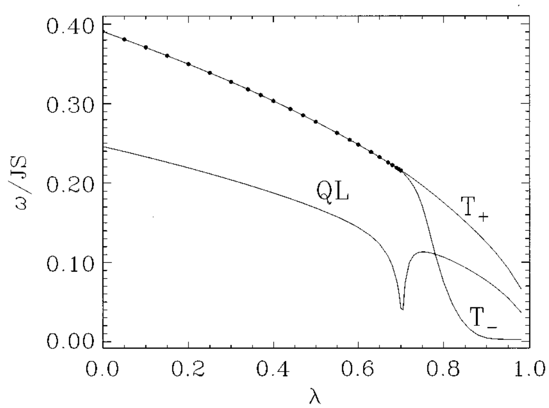

FIG. 1. The frequencies of the three lowest modes for a vortex with gyrovector $\vec{G}=+2\pi\hat{e}_z$ in a system with radius $R=19$, as a function of anisotropy parameter λ. Solid circles indicate where the translation modes are degenerate. QL indicates the quasilocal mode associated with the in-plane vortex instability. T_+ indicates the translational mode that produces counterclockwise orbital motion; T_- produces clockwise orbital motion. The translational modes' degeneracy splits at $\lambda_c \approx 0.704$, where the in-plane vortex crosses over into an out-of-plane vortex.

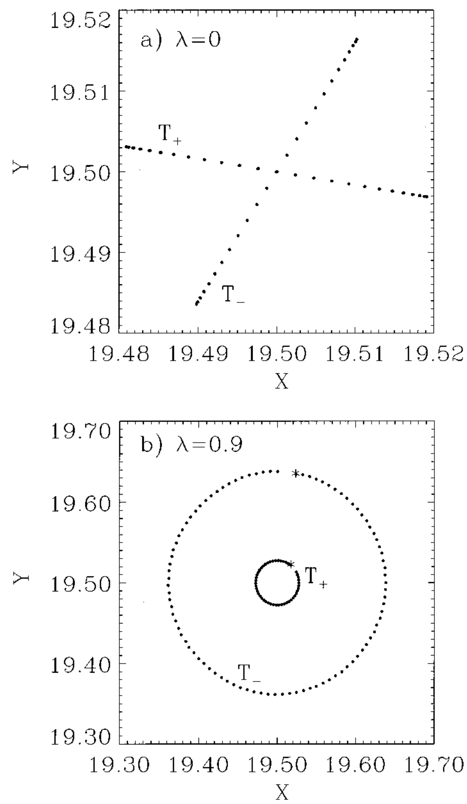

FIG. 2. Typical orbital motions of the FM vortex center associated with the translational modes T_+ and T_-, with amplitude [Eq. (3.1)] $A=0.5$. (a) For an in-plane vortex, $\lambda=0.0$. Points shown represent one full period ($T\approx 16/JS$) for these modes, with time increment $\Delta t=0.3216/JS$. (b) For an out-of-plane vortex, $\lambda=0.9$. For T_+, the time increment is $\Delta t=1.0/JS$, period $T\approx 50/JS$; for T_-, the time increment is $\Delta t=150/JS$, period $T\approx 1410/JS$. The asterisks show the starting points.

tional modes (T_+ and T_- in the figures) by viewing the motions of the spins that result when a given spin-wave eigenfunction is added to the original vortex structure. The time-dependent spins are taken to be

$$\vec{S}_{\mathbf{n}}(t)=\vec{S}_{\mathbf{n}}^{\,0}+A\vec{\sigma}_{k,\mathbf{n}}e^{-i\omega_k t}, \tag{3.1}$$

where the superscript 0 refers to the static vortex, $\vec{\sigma}_{k,\mathbf{n}}$ is the normalized eigenfunction for mode k with frequency ω_k, and $A \ll 1$ is a small amplitude. In particular, one can observe the spin motions in the central core region of the vortex and use their instantaneous directions to estimate the position of the vortex center. A least-squares fit of the four core spins to the form in Eq. (2.3) (scheme due to Schnitzer[27]) can be used to evaluate (x_v, y_v) to a precision of less than 1% of the lattice constant. By this definition, most of the modes do not result in any motion of the vortex core. The translational modes are identified to be the two lowest modes that do produce motion of the vortex center, and additionally, the structures of their eigenfunctions vary as $e^{\pm im\chi}$, with azimuthal quantum number $m=1$, where χ is the azimuthal polar coordinate.

As a numerical check, we also used the expression in Eq. (3.1) at $t=0$ as the initial condition for fourth-order Runge Kutta numerical integration of the spin dynamics equations of motion. For small amplitudes $A \ll 1$, the resulting orbits and periods are consistent with the time evolution expressed in Eq. (3.1). Using a larger amplitude $A=0.5$, typical motions that result from the two translation modes for the FM $R=19$ system are shown in Fig. 2. For $\lambda < \lambda_c$ [in-plane vortices, Fig. 2(a)], the degenerate modes result in linear motion of the vortex center, along two different directions, and the dynamics is just like that for an ordinary massive particle. Interestingly, the motion's amplitude is quite small even for this fairly large mode amplitude. Note that appropriate linear combinations of these two degenerate modes could be used to construct new modes that produce motions in exactly perpendicular directions, although this is not necessary here. For $\lambda > \lambda_c$ [Fig. 2(b)], the two nondegenerate translation modes produce circular vortex motion in opposite senses with different frequencies. The motion's amplitude is larger than that seen in Fig. 2(a) for an in-plane vortex. For a vortex with an "up" or positive gyrovector ($\vec{G}=+2\pi\hat{e}_z$), the mode T_+ that produces counterclockwise orbital motion has azimuthal quantum number $m=+1$, and the larger frequency, ω_+. The mode T_- that produces clockwise orbital motion has $m=-1$, and the smaller frequency, ω_-. If the sign of $\vec{G}$ is reversed, then the circulations and azimuthal quantum numbers are reversed. In contrast to in-plane vortices, the presence of nonzero $\vec{G}$ for out-of-plane vortices leads to interesting nonclassical particle behavior.

IV. COLLECTIVE COORDINATE MASS

Vortices have been considered to have a dynamical equation of motion that describes the time-dependent center position $\vec{X}(t)$.[5] Analysis shows that the vortex spin structure is velocity dependent[20] and asymmetric about a line through the vortex center parallel to the velocity. This implies that there must also be a mass times acceleration term in the equation of motion[21] for the center, as in Eq. (1.2). Generally, the effective mass M is a tensor,[28,5] but a scalar M is sufficient in the leading approximation.

For a vortex near the center of the circular system already described, with a linear restoring force given in Eq. (2.6), a particular solution to Eq. (1.2) is circular motion:

$$\vec{X}(t)=X(\cos\omega t,\sin\omega t)=X\hat{e}_r. \tag{4.1}$$

The velocity and acceleration can be written in terms of the angular frequency $\vec{\omega}=\omega\hat{e}_z$ as

$$\vec{V}=\vec{\omega}\times\vec{X}=\omega\hat{e}_\chi, \tag{4.2a}$$

$$\dot{\vec{V}}=\vec{\omega}\times\vec{V}=-\omega^2\vec{X}=-\omega^2X\hat{e}_r. \tag{4.2b}$$

With the gyrovector as $\vec{G}=G\hat{e}_z$, all terms in Eq. (1.2) have only radial components, leading to

$$\omega^2-\frac{G}{M}\omega-\frac{K}{M}=0. \tag{4.3}$$

Provided $M\neq 0$ and $G\neq 0$ (out-of-plane vortex), there are two distinct solutions

$$\omega_\pm=\frac{1}{2M}(G\pm\sqrt{G^2+4KM}). \tag{4.4}$$

Assuming $M>0$ and $K>0$, we have $\omega_+>0$ and $\omega_-<0$, regardless of the sign of G. In this notation, ω_+ is a counterclockwise or positive sense of rotation solution, and ω_- is a clockwise or negative sense of rotation solution. On the other hand, if $M=0$, then there is only one orbital solution $\omega=-K/G$, provided $G\neq 0$. Finally, if $G=0$, as in the in-plane vortex, then

$$\omega_+=-\omega_-=\sqrt{K/M}, \tag{4.5}$$

corresponding to two orbital solutions at equal rates but in opposite senses.

Under the assumption of this collective coordinate representation of the vortex position, the two solutions ω_+ and ω_- must be identified as the fundamental translational modes found in Sec. III by numerical diagonalization. It is especially clear because the modes T_+ and T_- in Figs. 1 and 2 have counterclockwise and clockwise orbital motions, respectively, just as the ω_+ and ω_- solutions here. When $G=0$, for $\lambda<\lambda_c$, T_+ and T_- are degenerate, and their opposite frequencies should be identified with those in Eq. (4.5). When $G\neq 0$, the frequencies of T_+ and T_- should be identified with ω_+ and ω_-, respectively, in Eq. (4.4). The main difference here compared to Sec. III is that it is important to include the sense of rotation caused by the T_+ and T_- modes in the signs of their frequencies, with counterclockwise being positive and clockwise being negative.

Therefore, we can use the frequencies from the numerical diagonalization to estimate the mass, provided the force constant K is known, which is determined below. The coefficients in Eq. (4.3) relate to the frequencies according to

$$\omega_+\omega_-=-\frac{K}{M}\rightarrow M=\frac{-K}{\omega_+\omega_-}, \tag{4.6a}$$

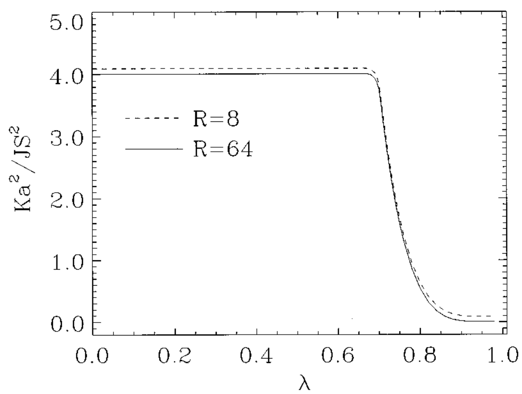

FIG. 3. The effective force constant K [Eq. (2.6), FM or AFM] vs anisotropy parameter λ for system radii indicated.

$$\omega_++\omega_-=\frac{G}{M}\rightarrow M^*=\frac{G}{\omega_++\omega_-}. \tag{4.6b}$$

This presents two possibly conflicting expressions for the vortex mass. Equation (4.6b), however, cannot be applied for the in-plane vortex because both the numerator and denominator are zero. Expression (4.6a) will apply to any vortex.

V. FORCE CONSTANT AND GYROVECTOR

In order to apply the above expressions for the mass, the gyrovector and force constant must be evaluated for vortices in the finite circular square lattice system. The force constant K was determined in a quasistatic approximation, by enforcing a desired displacement X of the vortex from the system center, and then relaxing the spins to their minimum-energy configuration (similar to scheme in Ref. 24). The vortex position was constrained by fixing the in-plane angles $\phi_{\mathbf{n}}$ for the four core sites of the vortex according to Eq. (2.3). The other sites in the system had no constraints; the vortex-Dirichlet boundary condition [also Eq. (2.3)] was applied to extra sites outside the system as described in Sec. II. The minimum-energy configuration was found by iterating the process of setting each spin to lie in the direction of the effective field due to its neighbors. Then K was estimated from the second derivative of the energy with respect to the vortex displacement, for a set of small displacements out to 0.2 lattice constants. Because the static AFM vortex structure on one sublattice is the same as that for the FM vortex, both have the same energy and effective force constant.

Representative results for K are shown in Fig. 3 for two system sizes. For the in-plane vortices, the force constant is approximately independent of the anisotropy, with a value $K\approx 4JS^2/a^2$, where a is the lattice constant. The development of out-of-plane spin components for $\lambda>\lambda_c$ apparently is associated with a much smaller force constant for out-of-plane vortices. Because K has only a very weak dependence on the system size, the dominant force on the vortex must come from effects due to the discrete lattice.

For the discrete lattice system, the definition of the gyrovector $\vec{G}$ is not completely clear. In the continuum definition, the S^z spin component at the vortex core is either zero or $\pm S$ [Eq. (2.4)], leading to $G=0$ or $G=\pm 2\pi$ for in-plane and out-of-plane vortices, respectively. For the discrete lat-

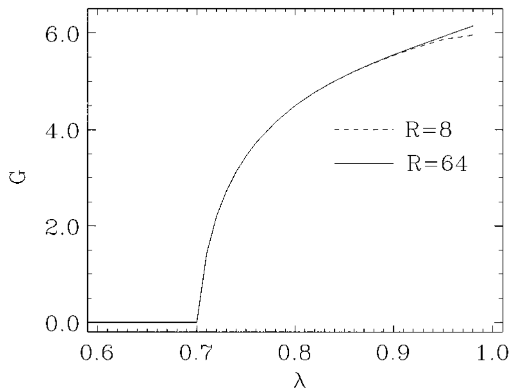

FIG. 4. Discrete FM vortex gyrovector G vs anisotropy parameter λ for system radii indicated.

tice, the vortex core does not fall on a lattice site, and so in a real sense, especially for λ just above λ_c, the spins of the out-of-plane vortex do not cover 2π steradians. Rather, the four sites surrounding the vortex center can have quite small components $S^z \approx pS$ with $|p|<1$, and G is effectively reduced to a low value $G \approx 2\pi p < 2\pi$. This behavior is approximately represented by using the lowest-order symmetrical finite difference approximation for G, on a square lattice,[22]

$$\vec{G}=(2S)^{-2}\sum_{\mathbf{n}} \vec{S}_{\mathbf{n}}\cdot(\vec{S}_{\mathbf{n}+\mathbf{a}}-\vec{S}_{\mathbf{n}-\mathbf{a}})\times(\vec{S}_{\mathbf{n}+\mathbf{b}}-\vec{S}_{\mathbf{n}-\mathbf{b}}), \tag{5.1}$$

where $\mathbf{a}=a\hat{e}_x$ and $\mathbf{b}=a\hat{e}_y$ are the lattice basis vectors. The sum is effectively over triple products of spins in all possible triangular plaquettes of the lattice. This definition for $\vec{G}$ approaches zero smoothly as $\lambda \to \lambda_c$ from above, a behavior that makes it reasonable to be used in Eq. (4.6b) for the vortex mass estimate, because the denominator $(\omega_+ + \omega_-)$ there also goes to zero in this limit. The behavior of G versus λ for the FM vortex is shown in Fig. 4.

VI. FM VORTEX MASS

Results for the FM mass as defined by Eq. (4.6a) are shown for a range of system sizes with $R=5$ up to $R=19$ in Fig. 5(a). The corresponding results from the alternative expression Eq. (4.6b), valid only for the out-of-plane vortex, are shown in Fig. 5(b). For λ not too close to λ_c, and not too close to 1, the two expressions have a reasonable agreement, and the collective coordinate description for the vortex translational motion in terms of a mass and force constant is self-consistent. Unfortunately, there is no such check of self-consistency for the in-plane vortices. However, the application of Eq. (4.6a) for the in-plane vortices is straightforward and does not require an appropriate definition of G for the discrete system, and its interpretation as a 2D harmonic oscillator is simple. Equation (4.6a) certainly is the prefered method to calculate the mass for both types of vortices.

A simple continuum description of the FM vortex[20] suggests that the spin structure perturbations due to nonzero velocity are proportional to $\vec{V}$ and decay away from the vor-

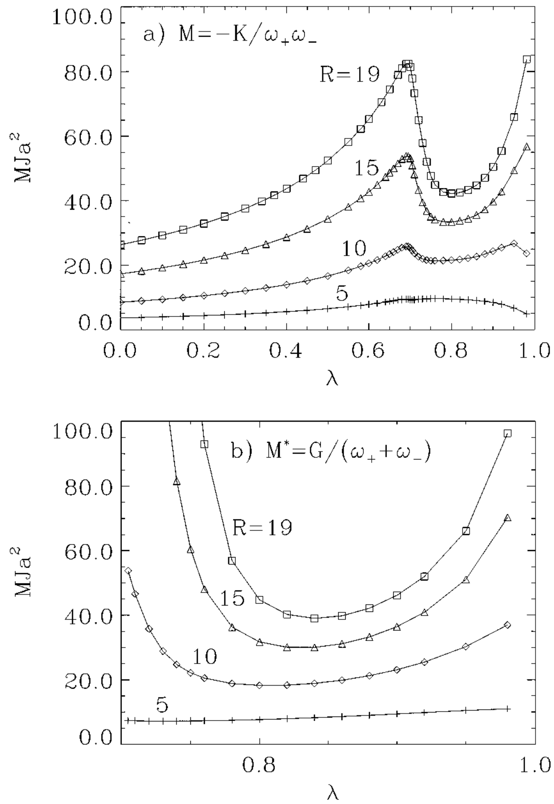

FIG. 5. FM vortex mass M vs anisotropy parameter λ for system radii indicated, obtained (a) from $M=-K/(\omega_+\omega_-)$ and (b) from $M^*=G/(\omega_+ + \omega_-)$, for $\lambda > \lambda_c$ only.

tex center as $1/r$. This implies that the mass should increase with the logarithm of the system size[21,22] in the same manner as the energy of a static vortex. The prediction is

$$M \approx \frac{\pi q^2}{4Ja^2(1-\lambda)} \ln(R/a_0). \tag{6.1}$$

Continuum theory, however, requires a short-distance core cutoff for integrals at $r=a_0$. The cutoff appropriate for the square lattice system can be determined to be $a_0 \approx 0.24a$ by analyzing the static vortex energy and fitting it to the standard continuum expression $E_0=\pi JS^2\ln(R/a_0)$ for a range of system radii R (Fig. 6). Then some typical masses from Eq. (6.1) for system radii from $R=5$ to $R=19$ are $2.3<MJa^2<3.5$ for $\lambda=0$, and $12<MJa^2<17$ for $\lambda=0.8$, values considerably smaller than those of Fig. 5. Although we do not have data to large enough radius to decide the asymptotic trend of the mass with R, we can make the following statements. For the in-plane FM vortex, especially near $\lambda=0$, the mass grows much faster than $\ln R$, possibly with a power law $M \propto R^{\alpha}$ with α slightly less than 2. For the out-of-plane vortex, for example, $\lambda \approx 0.8$, the mass also grows faster than $\ln R$, but with a power closer to $\alpha=1$.

VII. AFM VORTEX MASS

For AFM vortices the gyrovector is zero and the translation modes are degenerate for all values of λ.[25] For these reasons, the AFM vortex mass was evaluated only by Eq. (4.6a). The force constant as described above (e.g., Fig. 3)

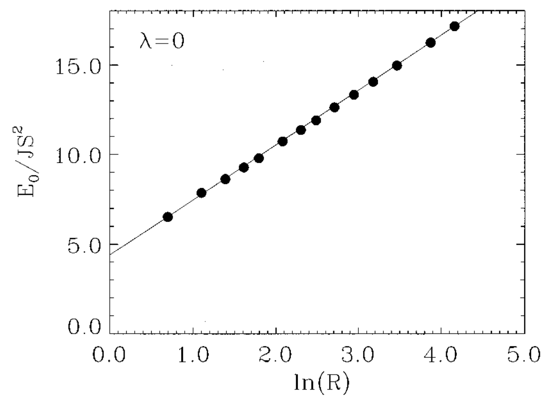

FIG. 6. Static vortex energy (FM or AFM) for $\lambda=0$, vs system radius R. The solid line is a fit to the expression $E_0/JS^2=\tilde{\pi}\ln(R/a_0)$, with slope $\tilde{\pi}=3.06$ and cutoff $a_0=0.24a$.

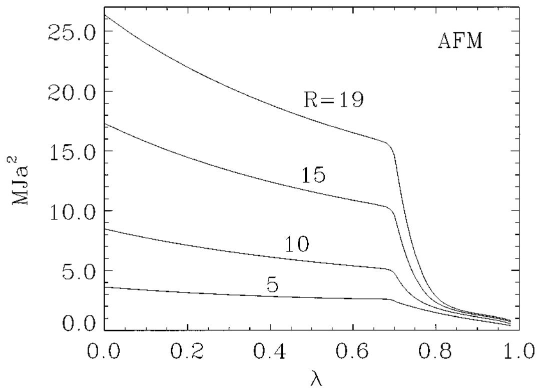

FIG. 8. AFM vortex mass M versus anisotropy parameter λ for system radii indicated, obtained from $M=-K/(\omega_+\omega_-)$.

was applied. The necessary translation mode frequencies are shown in Fig. 7. AFM in-plane vortices have translation mode frequencies similar to those for FM in-plane vortices (especially at $\lambda=0$); however, the translation mode frequencies for AFM out-of-plane vortices are considerably larger than ω_- for FM out-of-plane vortices. As a result, the AFM vortex mass, shown in Fig. 8, is similar to that for in-plane FM vortices near $\lambda=0$, but quite a bit smaller than that for out-of-plane FM vortices. The most striking feature of Fig. 8 is that there is only a very weak dependence of the mass on the system size for the out-of-plane AFM vortices; it is possible that the mass converges to a finite limit as $R\rightarrow\infty$. Furthermore, the out-of-plane vortex mass appears to be approaching zero for λ approaching 1, the isotropic limit. We should note that even at the data points $\lambda=0.98$ on these curves, the vortex core structure decays away on the length scale $r_v=3.5a$, which means that finite-size effects are strong only in the $R=5$ system.

VIII. CONCLUSIONS

We used the translation mode spectrum of a single vortex at the center of a finite lattice system to show that magnetic vortices have dynamics that requires a mass term, as in Eq. (1.2), and we evaluated the mass as a function of anisotropy

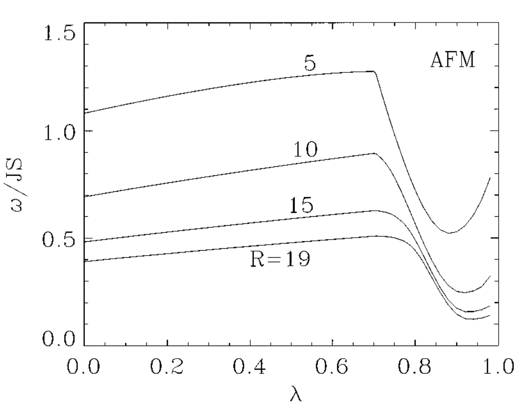

FIG. 7. Translation mode frequencies vs anisotropy parameter λ for AFM vortices in system radii indicated.

for FM and AFM vortices. The force acting on the vortex in this calculation is primarily due to the pinning of the lattice, with a weaker effect due to the boundary condition. The fundamental small-amplitude periodic motions of the vortex core implied by the translation modes in response to this force are clockwise and counterclockwise orbital motions around the system center. The frequencies of these motions are degenerate when the vortex gyrovector is zero, i.e., for in-plane FM vortices and for AFM vortices. The combination of nonzero gyrovector and mass for out-of-plane FM vortices leads to nondegenerate translation modes: different orbital rates for clockwise and counterclockwise motions.

One interesting aspect of the translation mode spectrum is present independently of the calculation of the vortex mass. When a normalized translational mode is excited on a (FM or AFM) vortex with a certain amplitude, the resulting amplitude of the translational motion it produces is considerably larger for out-of-plane vortices than for in-plane vortices (Fig. 2). There is not much difference in the motion amplitudes for FM and AFM *in-plane* vortices; however, *out-of-plane* AFM vortices obtain greater motion amplitudes than their FM counterparts. The conclusion is that, for example, in thermal equilibrium, it is easier for out-of-plane vortices to move over distances approaching or even greater than one lattice constant than it is for in-plane vortices. This is partly because the lattice pinning potential is weaker at larger λ, where the vortex spin structure has smaller spatial gradients. Equivalently, we can interpret the easier motion of out-of-plane vortices as due to the associated weaker force constant K. This easier motion for out-of-plane vortices in the FM system could be partly responsible for the shorter vortex lifetime found for out-of-plane vortices compared to in-plane vortices in a recent simulation by Dimitrov and Wysin.[29]

Although we can only study a limited-size system, due to numerical diagonalization memory limitations, some trends of the mass dependence on increasing system radius are apparent, and the differences between FM and AFM vortices are clear. For a particular anisotropy λ, the AFM vortex mass is smaller than the FM vortex mass. This is similar to the case of 1D solitons.[30] The FM vortex mass increases with system radius faster than ln$R$, with the strongest size dependence associated with in-plane vortices. In contrast, the in-plane AFM vortex mass also increases faster than ln$R$, while the out-of-plane AFM vortex mass may reach a finite limit with increasing system radius. The much smaller mass for AFM vortices, especially for $\lambda > \lambda_c$, is partially consistent with the conclusions drawn by Nikiforov and Sonin[13] concerning ease of vortex motion for AFM vortices but not for FM vortices. However, because an AFM vortex in a lattice system will always experience some pinning force, even a freely translating AFM vortex is not possible except perhaps at $\lambda \to 1$.

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