Electron interference due to localization paths in an Aharonov-Bohm ring

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The Aharonov-Bohm effect has been investigated in mesoscopic rings fabricated from GaAs/Al$_x$Ga$_{1-x}$As heterostructures constructed with a corrugated metal gate on the rings. In a certain range of the electrostatic gate voltages, two different h/e magnetoresistance oscillations have been observed near zero magnetic field, $|B|<B_c$. These behaviors are consistent with combining results of the magnetostatic and electrostatic Aharonov-Bohm effects of the electron wave function coherently circling around the ring. *Random* phases caused by the different transverse modes and their multiple localization paths reduce the interference intensity resulted from the multiple localization paths as $B>B_c$. [S0163-1829(96)05524-5]

The phase of electrons in an electrical device can be controlled by the electrostatic potential and/or the magnetic vector potential that the traveling electrons experience in the conducting path. Changes in the magnetic flux in a ring structure lead to the well-known magnetostatic Aharonov-Bohm (AB) oscillations,[1] whereas changes in the gate potential lead to the electrostatic quantum interference effect.[2–6]

In recent years, there have been studies on the electrostatic quantum interference effect in the electron-wave interferometers of the AB geometry.[3–6] Since the electrostatic potential disturbs the carrier density,[3–5] carrier trajectories,[5,6] and the number of conduction modes[4,5] as well as the electrostatic AB phase, the results from the experimental studies are not clear enough to extract meaningful information on the electron's behavior in such devices. Theoretically, the existence of two different interference conditions due to the electrostatic AB effect in a one-dimensional ring has been predicted in the ballistic transport regime;[7] one is caused by the conventional interference of transmitted electrons, and the other arises due to the interference of an electron traveling completely around the ring and interfering with itself at the point of entry.

The earlier experiments in disordered normal-metal rings displayed only *one* set of h/e magnetoresistance oscillations[1] (and their phase shift in electrostatic AB effect experiment[6]), whose behaviors agree well with the theoretical considerations based on the multichannel scattering properties[8] or ensemble averaging in the conducting paths.[9] The number of channels (or transverse modes) at the Fermi energy is large in the metal ring, $N_T>1000$, so that the multichannel effect and the interchannel scatterings (or mode mixing effect) occuring everywhere around the ring particularly randomize the phase of the electron in the electron's multiple circulations around the ring, that is, ''localization type interference.''[7] The experimental results clearly demonstrated that the h/e magnetoresistance oscillations originated purely from the phase difference between two arms of the ring. On the other hand, in a typical quasi-one-dimensional wire such as a GaAs/Al$_x$Ga$_{1-x}$As-based quantum wire, the current is carried by only a few ($N_T<10$–20) channels at the Fermi energy (at 17 mK) with minimal scattering. The mode mixing effect mainly occurs at the irregular part of the wire width, e.g., at the junction parts for the current splitting or recombining in a ring structure, and the effect is supposed to be small. When the magnetic field penetrates the annulus of the ring, two multiple localization paths typically enclose different amounts of flux, then no longer oscillate with the same phase in the sum over paths. So we expect that the localization-type interference fringes are possibly exhibited in the transport measurement of the ballistic quasi-one-dimensional ring under low magnetic fields.

Using a laterally corrugated gate structure[2] we have been able to obtain experimental results about the formation of two different h/e oscillations within a small magnetic field range with the variation of the gate potential, and the electrostatic interference fringes induced by the gate potential change in a corrugated gate GaAs/Al$_x$Ga$_{1-x}$As-based mesoscopic ring structure.

The conducting path of a two-dimensional electron gas (2DEG) has been fabricated using electron beam lithography and subsequent chemical etching on a modulation-doped GaAs/Al$_x$Ga$_{1-x}$As ($x=0.3$) heterostructure grown by molecular-beam epitaxy. The average ring diameter is 1.9 μm, the width of the conducting path is 0.3 μm in the lithographic length, and the distance between the measurement probes across the ring is 3.8 μm. The heterostructures were

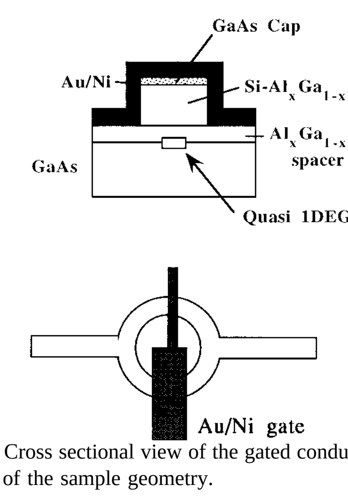

FIG. 1. (a) Cross sectional view of the gated conducting path. (b) Schematic diagram of the sample geometry.

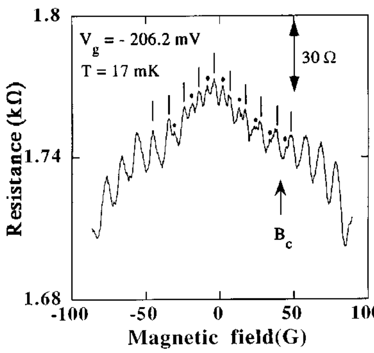

FIG. 2. Magnetoresistance spectrum at $V_g = -206.2$ mV. Dots represent the newly formed h/e oscillations, while bars represent the background h/e oscillations.

wet etched down to a 2DEG layer to form a well-shaped pathway. Lateral depletion further reduces the conducting width. To form the gate, electron-beam lithography and lift-off techniques were employed. The second step defined the corrugated Au/Ni gate; the metal gate length is 0.2 μm on one conducting path of the ring and 1 μm on the other. Figure 1 shows the schematic diagram of the fabricated sample.

The carrier concentration and mobility in this substrate at 1.5 K, as deduced from the measurement of Shubnikov–de Haas oscillation in a two-dimensional bar (50×150 μm), were found to be $n = 3.2 \times 10^{11}$ cm^{-2} and $\mu = 5.5 \times 10^5$ cm^2 V^{-1} s^{-1}, respectively. We measured longitudinal resistances in the applied magnetic fields and gate voltages by using the lock-in technique and 10-nA driving current at 17 mK. We found periodic magnetoresistance oscillations that are in direct correspondence with the penetration of the flux $\phi_0 = h/e$ through the average area of the annulus. This result demonstrates that the mesoscopic ring was well fabricated.

Figure 2 shows the magnetoresistance spectrum when the gate voltage (V_g) is -206.2 mV. The h/e magnetostatic AB oscillations are exhibited in the range of $|B| > \sim 40$ G. However, two sets of oscillations with the h/e period are clearly seen near zero magnetic field; for approximately -40 G $< B < +40$ G, they are shifted by π with respect to each other. The two sets of oscillation peaks are almost equal in magnitude, so as to make the period of the resistance oscillations *appear* to be $h/2e$ as a whole. However, we rule out that these oscillations are due to the interference effect caused by the time-reversal paths around the ring, because the appearance of the $h/2e$ oscillations is dependent on the gate voltage [see Fig. 3(a)].[10] Since the pair of electron waves in the time-reversal paths acquires the same amount of the phase shift by the gate potential in the sample structure, the $h/2e$ oscillations between the time-reversal paths should be independent of the gate voltages.

In careful magnetoresistance measurements at different gate voltages from -195.3 to -238.2 mV, we have observed changes of the two sets of oscillations, which are shown in Fig. 3(a). In the magnetic field range of -40 G $< B < +40$ G, the peak positions of both h/e oscillations do not change as the gate voltage varies, but only their amplitudes alternate. Newly formed oscillations begin to appear at the troughs of the background oscillations at $V_g = -195.3$ mV. As we increase the gate voltage negatively, the new

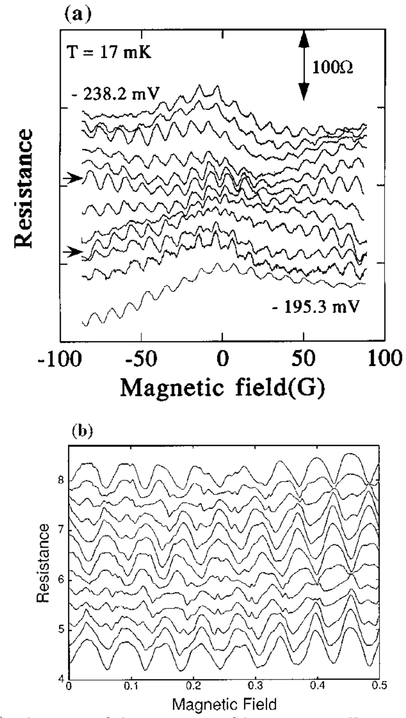

FIG. 3. (a) Changes of the new set of h/e AB oscillations. Difference in the applied gate voltage between the successive curves is ~ 4 mV. Magnetoresistance curves indicated by the upper and lower arrows are at $V_g = -218.3$ and -206.2 mV, respectively. (b) A theoretical calculation of the magnetoresistances with different electrostatic AB phase ϕ. The magnetic field is in the unit of $\hbar c/eW^2$, and ϕ changes by $\pi/12$ with $\phi = 0$ for the bottom curve. In the calculation, the number of the conducting channels is 10 and the temperature averaging at $k_B T/(\hbar^2/2m^* W^2) = 0.5$ has been done. The unit of the resistance is h/e^2.

peaks gradually grow while the background peaks shrink as much. At -206.2 mV, the two sets of peaks are almost equal in magnitude. If the gate voltage increases further, the new oscillations continue to grow and they finally take over the background oscillations at -218.3 mV. Therefore, while the gate voltage changes by 23 mV, we have a half cycle with respect to the alternation between the new and the background oscillations. The initial background oscillations reappear at -238.2 mV, so that one period of the background oscillations is ~ 43 mV. The formation of two sets of h/e oscillations was also observed near zero magnetic field ($B < B_c \sim 40$ G; B_c is the crossover magnetic field) with respect to the gate voltages in a different range.

The experimental behaviors of two h/e oscillations below B_c, as shown in Fig. 3(a), are in good qualitative agreement with the previous theoretical study, where the influence of the electrostatic potential on the magnetic AB effect has been investigated in the one-dimensional structure.[11] However, the crossover behavior, that the two sets of h/e oscillations alternating with the gate potential cease to exist beyond B_c, is not seen in the one-dimensional study. We have therefore considered the effect of many channels in the sample structure. The Hamiltonian for an electron moving ballistically in the structure is considered in the vector potential of the perpendicular magnetic field $\mathbf{B} = (0,0,B)$ and the electrostatic potential $V(x,y)$, given by infinity at the wire walls, constant value at the gate, and zero otherwise. The gate is modeled as a simple potential rise of constant height V_g/E_0, where $E_0 = \hbar^2/2m^* W^2$, where W is the width of the wires. We

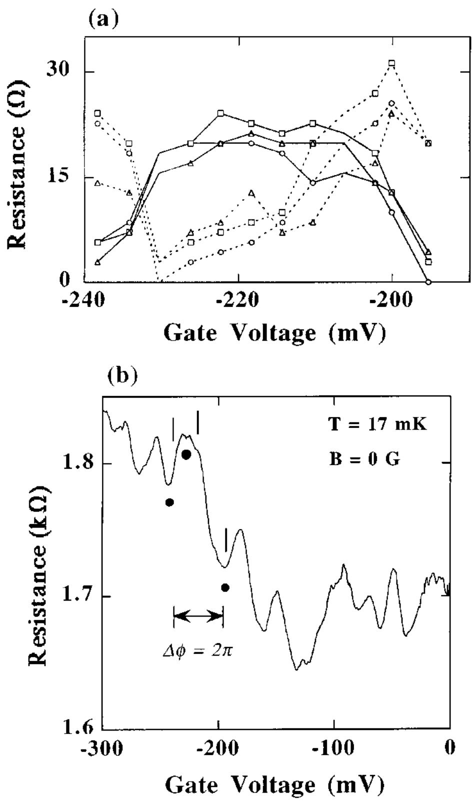

FIG. 4. (a) The resistance changes of the amplitude of the new and background h/e peaks vs the gate voltages; The solid lines are the changes when $B=-19$ G (○), -8 G (□), and 3 G (△) for the new h/e peaks and the dotted lines are the changes when $B=-13$ G (○), -3 G (□), and 7 G (△) for the background h/e peaks. The oscillatory component of the resistance is the height from the overall baseline in each magnetoresistance spectrum. (b) The resistances as a function of gate voltage at $B=0$. Dots represent the constructive destructive interferences caused by the electrostatic AB phase shift; $V_g=-244$, -226, and -197 mV. Bars represent the gate voltages where the constructive/destructive interference effects occur in Fig. 3.

have numerically solved the resultant time-independent Schrödinger equation by the standard mode-matching technique.[12]

The result in Fig. 3(b) of our realistic calculation for ten conducting channels in the system together with the finite temperature averaging demonstates that, for a certain magnetic field range near zero, new h/e oscillations are formed and compete with background h/e oscillations with respect to the varying electrostatic AB phase, but that beyond the magnetic field range, the alternating feature disappears. That is, our calculation shows the same crossover behavior with respect to the magnetic field as our experimental result, which is an importantly different feature from the one-dimensional study.[7,11,13]

Although our numerical calculation exhibits the behavior of our experiment results well, we still need to devise a simple model by which the important feature of the behavior can be explained easily. In our ring structure, the ballistic nature of the transport leads us to suppose that the electrons in the ring annulus can circle, clockwise or counterclockwise, the ring more than once. That is, we should consider the interference effect by the localization paths as well as the Mach-Zehnder–type paths (which are composed of only transmitted-electron paths in two arms). Then, we can assume that the backscattering effect and the mode-mixing effect are small in the source and drain junction sides, and that the total interference intensity can be represented by the addition of the Fabry-Pérot–type resonances (multiple reflection effects with the clockwise circulating wave and with the counterclockwise circulating wave) and their interferences; the first-order term in these interference effects between two Fabry-Pérot resonances is the Mach-Zehnder–type interference.

The two Fabry-Pérot resonances contain the phase terms induced by the magnetic flux and the electrostatic gate potential, $\Delta\phi=(e/\hbar)\int \pm B ds + V_{\mathrm{eff}} dt$, where the integration is done over the area enclosed by one turn of the circulation and the duration time in the gate potential. The positive and negative signs in the magnetic flux term represent the clockwise and counterclockwise circulations, respectively. V_{eff} is the effective gate potential that the electrons experience under the gate. It is obvious that whenever the electrostatic phase shift term equals $2\pi n$ (n is an integer), the resonance conditions in the two Fabry-Pérot paths give rise to one type of the h/e oscillations in the magnetoresistance measurements. Otherwise, the two sets of h/e oscillations are seen when varying the gate voltages, and the peak positions of the oscillations are not changed. We attribute the behaviors of the new and background h/e oscillations shown in Fig. 3(a) to the combined effects of two Fabry-Pérot resonances shifted in the opposite direction by the effective gate potential in the magnetoresistance measurements. In this analysis we treat the inverse of the total transmittance of two Fabry-Pérot resonances as the resistance of the sample. These analytic considerations also imply that the interference intensities between two Fabry-Pérot resonances are small in $B < B_c$.

Since it is only by having a fixed relative phase that the multiple localization paths give rise to the resonance, the resonance effect is vanished by the field in the annulus; the wave functions of the electrons are extended in the finite wire width, the relative phases between successive multiple localization paths are not fixed. On the other hand, the Mach-Zehnder–type h/e paths never have a fixed relative phase, even at zero field, and so their contribution is unaffected. Moreover, there are several transverse modes that enclose different amounts of the flux; the resonance effects contributed by different transverse modes decrease as the field increases. We estimate the value of B_c of the sample by a simple argument. B_c is the crossover magnetic field where the particular behaviors of two Fabry-Pérot resonances are averaged out to yield the simple Mach-Zehnder–type oscillations. Hence, at B_c, the phase difference between the successive localization paths of each mode or the localization paths of different modes becomes at least 2π, i.e., $2\pi RWB_c \sim \phi_0$,[14] where R is the average radius of the ring, $1\,\mu$m, and W is the width of the conducting wire, 80 nm; we calculated the width from the full width at half maximum of the Fourier spectrum of the h/e magnetoresistance.[15] Then, $B_c \sim 80$ G, which is in good agreement with the experimental value. Above $B > B_c$, we did not observe two different sets of h/e oscillations. It should be noted that the shift of the conventional h/e oscillations, $B > B_c$, is not unidirectional but rather oscillating by the applied gate voltages.[4,6,16] The question of such phenomena requires further study; we suspect that the higher-order terms in the interference between two Fabry-Pérot resonances plays an important role.

Next we show the measurements for observation of electrostatic AB oscillations. Figure 4(a) shows the consonant changes of the amplitude of the new and background h/e peaks versus the gate voltages, which are deduced from the magnetoresistance measurements in Fig. 3(a). The competition and the oscillatory change of the amplitudes support the simple argument of two Fabry-Pérot resonances. The measured resistances as a function of the gate voltage at zero magnetic field are depicted in Fig. 4(b). The measurements have been done in the range of low gate voltages without any mode depopulations in between; total resistance ($\sim$1.7 K Ω) and the resistance change (less than 0.2 KΩ) indicate that the number of transverse modes is not changed in this gate voltage range. The overall resistance increases slowly as the gate voltage increases negatively. The resistance curve exhibits oscillatory behaviors, whose amplitudes are $\sim$30 Ω [the amplitude of the h/e magnetoresistance oscillations of Fig. 3(a) is $\sim$30 Ω]. As a source of the oscillations, we first rule out the multiple reflection effect by the gate potential steps since its intensities are negligibly small in the measurement range.[2]

The resistance curve with the electrostatic potential sweep alone cannot unambiguously identify the source of the oscillations, which leads us to turn to the magnetoresistance spectra at constant gate voltages in Fig. 3(a). We observed that the phase differences are π between two successive magnetic h/e oscillations at the selected gate voltages, $V_g = -195.3$ mV, -218.3 mV, and -238.2 mV, which are also represented by bars in the case of $B=0$ in Fig. 4. [The magnetoresistance spectra displayed in Fig. 3(a) are for $V_g = -195.3$ mV ~ -238.2 mV in Fig. 4.] The results of the constructive interference condition ($V_g = -195.3$, -238.2 mV) and the destructive interference condition ($V_g = -218.3$ mV) from the magnetoresistance spectra are coincident with those from the electrostatic spectrum, that is, $V_g = -197$, -244, and -226 mV, respectively. The period of $\sim$47 mV in the electrostatic resistance oscillations is in good agreement with the period of $\sim$43 mV estimated in Fig. 3(a). These results imply that the electrostatic phase shift between the successive resistance minima in the electrostatic measurement is 2π, at least in the range of the gate voltages discussed above.

The electrostatic AB phase shift is induced by the time-dependent scalar potential, while no force is exerted on the electron wave function. When we apply the gate voltage, the electrons experience classical electrostatic forces whose direction is perpendicular to the 2DEG layer. Thus the small gate voltages do not affect the electron trajectory as much. The Fermi wave vector, however, changes a little, which results in a change of the phase for the quantum state of the electron. (This type-2 electrostatic AB effect is described in Ref. 17.) The electrostatic AB phase acquired in the gate potential is given by $\Delta\phi = e/\hbar V_{\mathrm{eff}} \langle \tau_t \rangle = \Delta(kL)$, where $\langle \tau_t \rangle$ is the harmonic mean of the traveling time through the gate potential, k is the wave vector under the gate, and L is the gate length.[18] This electrostatic AB phase has been automatically taken into account in our calculation of the resistance spectra in Fig. 3(b).

In summary, we have investigated the electrical transport properties of a GaAs/Al_xGa_{1-x}As-based mesoscopic ring structure in the presence of magnetic flux and electrostatic potential. *New* h/e AB oscillations near zero magnetic field have been observed, $B < B_c$. We find that these phenomena are well explained by the combined behavior of the magnetic and electrostatic Aharonov-Bohm effects occurring simultaneously in the multiple localization paths of electrons.

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