Magnetotransport coefficients in a two-dimensional SiGe hole gas

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Magnetotransport coefficients have been measured, at temperatures down to 30 mK, in a single-side p-doped $Si_{0.88}Ge_{0.12}$ strained quantum well. Odd filling factors dominate the transport, indicating a large g factor, and the coefficients are found to be extremely insensitive to the in-plane component of magnetic field. An effective mass of $(0.30_5 \pm 0.01)m_0$ was deduced from the temperature dependence. The coefficients are discussed in terms of semiclassical transport theory. At low magnetic fields both longitudinal and Hall coefficients could be well explained in this way, in contrast to the situation in GaAs-based heterostructures. Small deviations from theory at higher fields are attributed to localization and Landau-level mixing effects.
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INTRODUCTION

The transport properties of two-dimensional (2D) hole gases in SiGe differ from the much more widely known example of the two-dimensional electron gas (2DEG) in GaAs in several important respects. One is a g factor sufficiently large that the spin splitting is comparable to the Landau-level spacing.[1] Another is the domination of the transport by large-angle scattering processes. This means, for example, that the low-field Hall resistivity ρ_{xy} in these hole gases differs qualitatively from that of electrons in GaAs 2DEG's. Also an insulating phase is sometimes observed at filling factors $\nu \leqslant 2$.[2,3] Many features of this phase are not yet clearly understood; for example, in some cases it is clearly controlled by density (appearing for densities less than about 4×10^{15} m^{-2}) and in another case it is enhanced by tilting the magnetic field.[4] It is not clear what controls the existence of this phase and whether it is the same factors that lead to an insulating phase in Si metal-oxide-semiconductor field-effect transistors (MOSFET's) both at $B=0$ and at filling factor $\nu=1.5$.[5]

More generally, for a phenomenon that has been studied for over 20 years,[6,7] the behavior of the longitudinal and Hall conductivities in 2D systems away from quantum Hall plateaus is not as well understood as might be expected. In the strongly localized regime scaling theories work well,[8] but when localization does not dominate the theoretical picture is less complete. For example, there seems to be no theory in the literature that explains quantitatively the low-field Hall resistivity in high-mobility GaAs-based 2DEG samples. By contrast, the results presented here for SiGe 2D hole gases can be explained quite accurately at low fields and with a relatively small error at higher magnetic fields.

EXPERIMENT

The sample, grown in an UHV chemical vapor deposition system, involved an n^- Si substrate, a buffer layer of nominally intrinsic Si, a 40-nm well of strained $Si_{0.88}Ge_{0.12}$, a 6-nm spacer layer of i-Si, and a 30-nm layer of Si doped at 2×10^{24} m^{-3} with boron. Hall bar samples, 100 μm wide, were prepared by wet etching and Ohmic contacts formed by the annealing (at 400 °C) of evaporated aluminum. The sample density was 5.9×10^{15} m^{-2} with a low-temperature mobility (μ_{tr}) of 0.68 m^2/V s. Compared to GaAs-based 2DEG's the mobility is low, but, because of the large-angle nature of the scattering and the larger effective mass of the holes, the Landau-level broadening is roughly comparable to that of a GaAs-based 2DEG with a mobility of 30 m^2/V s. The experimental results are extremely reproducible both for different sets of contacts on the same Hall bar and also for different samples from the same wafer.

Figure 1 shows the Hall and longitudinal resistivities, measured at a temperature of approximately 30 mK using a measuring current of 1 nA (which, it was established, produced no detectable heating). At low fields the longitudinal magnetoresistance contains both positive and negative contributions of a few percent. These are qualitatively like the results reported in Ref. 9 and following the approach taken there these are attributed to a combination of weak localization and other effects.

An obvious feature of the data in Fig. 1 is the dominance in ρ_{xx} of minima associated with odd filling factors. This is a well-established property of the p-type SiGe systems,[1–4] where the g factor is sufficiently large that the spin splitting is comparable to the Landau-level spacing so the $N_L\uparrow$ state is

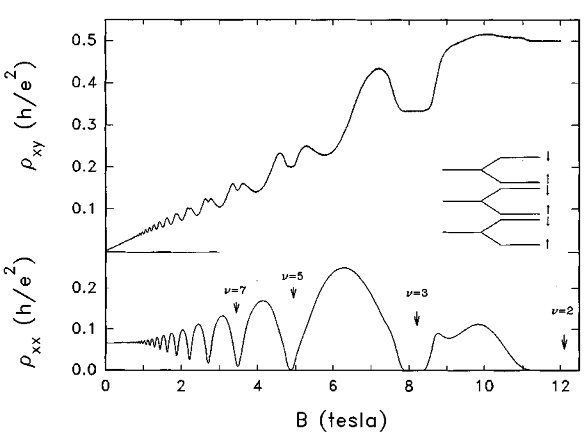

FIG. 1. Resistivity coefficients, in units of h/e^2, measured at a temperature of approximately 30 mK. The inset shows, schematically, the large spin splitting that leads to zeros in ρ_{xx} at odd filling factors ν.

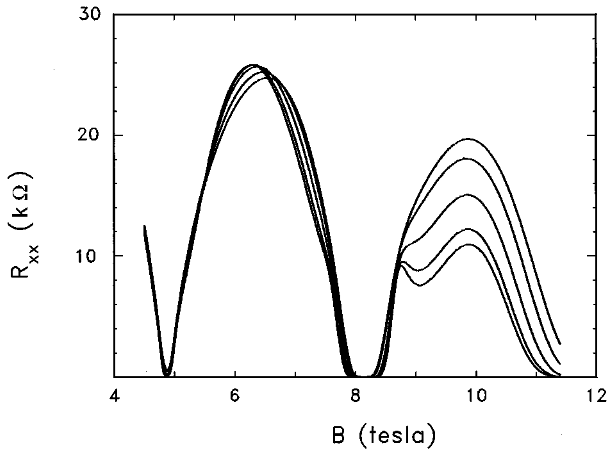

FIG. 2. Temperature dependence of the longitudinal resistance (R_{xx}) between approximately 4 and 12 T. The temperatures are 100, 200, 325, 560, and 750 mK. At 10 T the resistivity is a maximum for $T=750$ mK; at 6.5 T it is a maximum for $T=100$ mK.

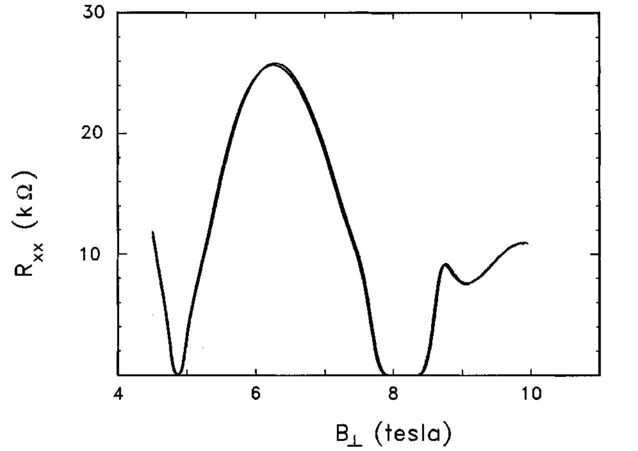

FIG. 3. Longitudinal resistance, measured at 100 mK, as a function of the perpendicular component of magnetic field $B_\perp$ for tilt angles of 0° and 34°.

very close in energy to the $(N_L+1)\downarrow$ state (here $\uparrow$ and $\downarrow$ denote, respectively, the upper and lower spin states and N_L is the Landau level index). This situation is illustrated schematically in the inset to Fig. 1. The splitting between the two spin states is only fully resolved for fields above 8 T, with the $\nu=2$ quantum Hall state clearly defined for both resistivity components, but there are indications of incipient removal of the spin degeneracy between $\nu=3$ and 5 in the form of shoulders on the sides of the ρ_{xx} peaks.

The Hall data ρ_{xy} are a little more complicated than is normally seen in high-mobility 2DEG samples. At low temperatures well-defined plateaus are seen at $\nu=2$ and 3, but at lower fields the oscillations are essentially in antiphase to the oscillations in ρ_{xx}. Generally this sample behaves very similarly to that discussed by Dunford *et al.*[2] with the exception, presumably because of the higher density, that there is no indication of an insulating phase.

Effective-mass values were extracted from the temperature dependence of the oscillations between about 1 and 2 K. Measurements made for fields between 1.1 and 2.4 T gave $m^*/m_0=0.30_5\pm0.01$. At higher magnetic fields, there was little temperature dependence except for the peak between $\nu=2$ and 3 (see Fig. 2). There a strong temperature dependence (and also current dependence) was observed. This is in the opposite sense to that which would be expected for an insulating phase.

The magnetoresistance was found to be extremely *insensitive* to tilting the magnetic field (except for the obvious $B\cos\theta$ dependence). In Fig. 3, which shows two curves for tilt angles of 0° and 34°, the effect of tilt is essentially undetectable. For tilt angles up to 47° the largest change observed was a reduction in height of the $\nu=4$ peak by about 4%. This behavior is well explained in terms of strain and confinement, which both split the heavy hole–light hole degeneracy so the ‘‘heavy hole,’’ with $|M_J|=\frac{3}{2}$, lies well above the ‘‘light hole’’ and is strongly decoupled.[10] In these circumstances, consideration of the appropriately generalized Luttinger-Kohn Hamiltonian shows that the ‘‘spin’’ splitting between the $M_J=\pm\frac{3}{2}$ states depends only on the perpendicular component of magnetic field.

DISCUSSION

A clearer picture of the relatively complicated ρ_{xy} behavior is obtained if the resistivies are inverted to obtain conductivities, shown in Fig. 4. The dashed lines are the standard classical expressions for σ_{xx} and σ_{xy} using the known density and a transport mobility $\mu_{\rm tr}=0.69$ m^2/V s. The structure in both σ_{xx} and σ_{xy} can clearly be attributed to deviations from these expressions, which are in the opposite sense for the two components. These deviations are associated with Landau-level structure in the density of states. This type of behavior is predicted by semiclassical transport theory,[11,12] according to which

$$\sigma_{xx}=\frac{n_{\rm eff}e^2\tau_{\rm tr}}{m^*(1+\omega_c^2\tau_{\rm tr}^2)}, \tag{1}$$

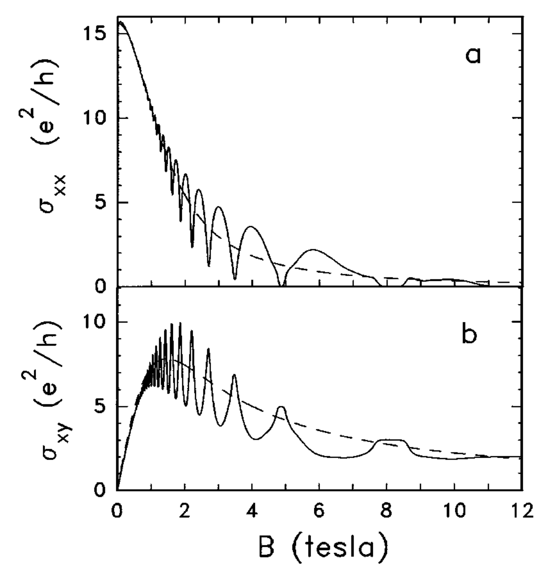

FIG. 4. Conductivity coefficients, in units of e^2/h, obtained by inverting the data in Fig. 1: (a) σ_{xx} and (b) σ_{xy}. The dashed lines are the classical curves $\sigma_{xx}=n_0e\mu_{\rm tr}/(1+\mu_{\rm tr}^2B^2)$ and $\sigma_{xy}=\mu_{\rm tr}B\sigma_{xx}$, calculated with $n_0=5.9\times10^{15}$ m^{-2} and $\mu_{\rm tr}=0.69$ m^2/V s.

where ω_c is the cyclotron frequency and τ_{tr} is the transport lifetime. This is the standard classical expression for σ_{xx}, but with the density of carriers n_0 replaced by an effective density n_{eff} and the zero-field lifetime τ_0 $(=m^*\mu_{tr}/e)$ replaced by τ_{tr}, where

$$n_{eff}=(g/g_0)n_0, \quad \tau_{tr}=(g_0/g)\tau_0. \tag{2}$$

Here g is the density of states at the Fermi energy and g_0 the corresponding zero-field value. The zero-field conductivity $\sigma_0=n_0e^2\tau_0/m^*$. The Hall conductivity is given by two terms

$$\sigma_{xy}=\sigma_{xy}^{(1)}+\sigma_{xy}^{(2)} \tag{3a}$$

$$=e[\partial n/\partial B]_{E_F}+\omega_c\tau_{tr}\sigma_{xx}, \tag{3b}$$

where the first term was introduced by Streda[13] and the second is the normal diffusive term. For small deviations Δg of the density of states from the zero-field value these equations give

$$\sigma_{xx}=\frac{\sigma_0}{1+\omega_c^2\tau_0^2}\left[1+\frac{2\omega_c^2\tau_0^2}{1+\omega_c^2\tau_0^2}\,\frac{\Delta g(E_F)}{g_0}\right] \tag{4}$$

and

$$\sigma_{xy}=\frac{\sigma_0\omega_c\tau_0}{1+\omega_c^2\tau_0^2}\left[1-\frac{3\omega_c^2\tau_0^2+1}{\omega_c^2\tau_0^2(1+\omega_c^2\tau_0^2)}\,\frac{\Delta g(E_F)}{g_0}\right]. \tag{5}$$

The deviations associated with Δg are in the opposite sense for the two components of σ, as seen experimentally in Fig. 4.

A quantitative test of this behavior can be made by examining Dingle plots, shown in Fig. 5. The amplitude of the resistivity oscillations ($\Delta\rho_{xx}$ and $\Delta\rho_{xy}$), obtained by inverting Eq. (4) and (5), should then be given by[11,12]

$$\Delta\rho_{xx}=2\mu_{tr}B\Delta\rho_{xy}=4\rho_0\ \exp(-\pi/\mu_qB), \tag{6}$$

where ρ_0 $(=1/\sigma_0)$ is the zero-field resistivity and μ_q the quantum mobility, is defined (by analogy with the transport mobility μ_{tr}) as $e\tau_q/m^*$, where $\tau_q=\hbar/2\Gamma$ and Γ is the Landau-level broadening. When plotted in the appropriate fashion both components should lie on the same straight line as is indeed the case. A linear fit to the data, constrained to go through the point 4 at $1/B=0$, confirms that the prefactor in Eq. (6) is given correctly and has a slope corresponding to $\mu_q=0.72$ m^2/V s. There are many examples in the literature of good Dingle plots for the longitudinal resistivity,[14] but this appears to be the first time such a clear agreement between theory and experiment has been reported for the oscillations in the Hall resistivity. This can be traced to the fact that most results are obtained in $GaAs/Ga_{1-x}Al_xAs$ heterojunctions where the ratio μ_{tr}/μ_q is typically 10 or more. By contrast, here this ratio is approximately one. This means that for a given deviation Δg, i.e., for a given value of $\omega_c\tau_q$, the value of $\omega_c\tau_0$ $(=\mu_{tr}B)$ is much smaller so the oscillations in σ_{xy}, which, for large $\omega_c\tau_0$ vary as $(\omega_c\tau_0)^{-3}$, are much larger and more clearly observable. Qualitatively the experimental behavior observed here agrees well with earlier work reported for electrons in Si MOSFET's, where the scattering is also large angle.[6,7]

At higher magnetic fields, when the minima in ρ_{xx} approach zero, Δg is no longer small and Eqs. (4) and (5) are

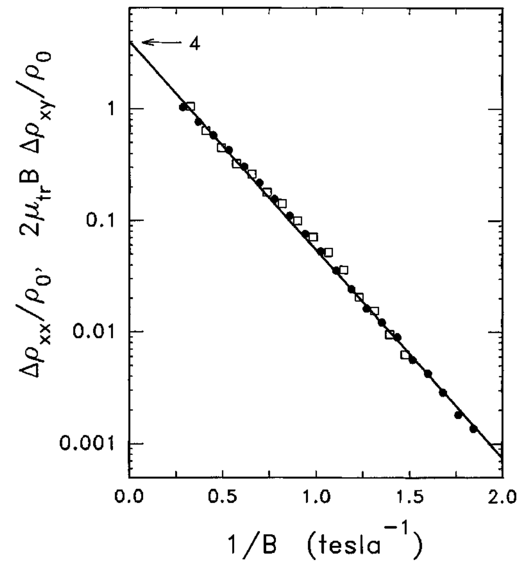

FIG. 5. Dingle plots for the amplitude of the oscillations in the normalized longitudinal resistivity ($\Delta\rho_{xx}/\rho_0$, solid points) and Hall resistivity ($2\mu_{tr}B\Delta\rho_{xy}/\rho_0$, open squares). The straight line is the fit to Eq. (6) with one adjustable parameter $\mu_q=0.72$ m^2/V s.

not expected to be valid. In high-mobility GaAs-based samples there is evidence[15] that with separated Landau levels (and for $\omega_c\tau_0>1$) the conductivity is correctly given by a ''quantum diffusion'' equation,[16] i.e.,

$$\sigma_{xx}=e^2R_{cycl}^2g(E_F)/2\tau_{tr}, \tag{7}$$

where R_{cycl} is the cyclotron radius and τ_{tr} is defined in Eq. (2). This describes a diffusion process for scattering between cyclotron orbits with a mean free path (R_{cycl}) corresponding to the average spacing between orbit centers. For a semielliptical density of states (with a width proportional to $B^{1/2}$) this model predicts peak values of σ_{xx} given by[7]

$$\sigma^{peak}=(2/\pi)g_s(\nu/2)(\tau_q/\tau_0)(e^2/h), \tag{8}$$

with $g_s=1$ or 2 according to whether or not the spin splitting is resolved. For short-range scatterers (i.e., $\tau_q=\tau_0$) this reduces exactly to the expression originally derived by Ando and Uemera.[16] For smooth, random potentials, the density of states is Gaussian,[17] with a width also proportional to $B^{1/2}$. In this case the peak value of σ_{xx} is given by a similar expression but without the prefactor $(2/\pi)$. Figure 6 shows experimental values of σ^{peak}, for ν between 4 and 10, compared with Eq. (8) using $g_s=2$ and with the same expression without the factor $2/\pi$. The peaks all lie within about 10% of the values predicted by Ando and Uemera. The small deviations seen are to be expected at large ν because the Landau levels start to overlap, and at small ν because spin splitting starts to appear. Thus the experimental values of σ_{xx} can be explained quantitatively both at low fields, when several Landau levels overlap, and at high fields, when the Landau levels are well separated.

For the Hall term testing the validity of Eqs. (1)–(3) when $\Delta g/g_0$ is not small is a little more complicated. However, values of g/g_0 can be extracted from σ_{xx}, using Eqs. (1) and (2), and these can then used to obtain σ_{xy} from Eq. (3) provided that a suitable expression for $[\partial n/\partial B]_{E_F}$ can be ob-

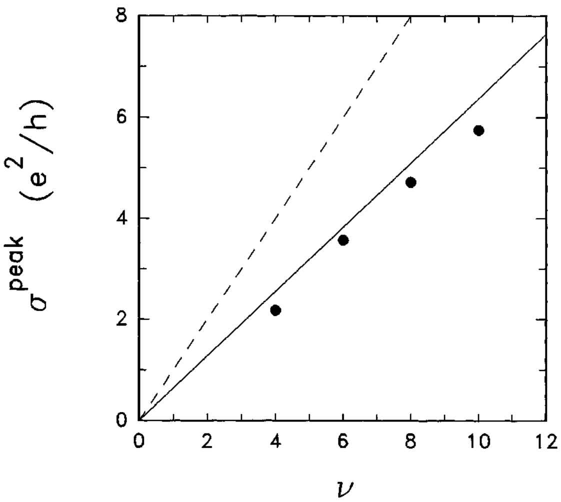

FIG. 6. Peak values of σ_{xx} compared with the predictions of Eq. (8) (solid line) when the spin degeneracy $g_s=2$ and $\tau_0=\tau_q$. This theoretical result, for a semielliptical density of states, was derived in Ref. 16 for short-range δ-potential scatterers. The dotted line represents the corresponding expression for a Gaussian density of states, which is appropriate for smooth, random potentials (i.e., for $\tau_0 \gg \tau_q$).

tained. To a good first approximation this is just $-(n_0/B)(\Delta g/g_0)$ [cf. Eq. (9) below]. The result of this procedure is shown in Fig. 7, plotted against $1/B$ for clarity. At low and medium fields the calculated values of σ_{xy} provide a satisfactory description of the amplitude of the oscillations, although the background term is slightly incorrect. This occurs mainly because the small extra magnetoresistance at low fields means that a single value for $\mu_{\rm tr}$ cannot simultaneously describe the background for both the Hall and longitudinal terms. At higher fields (above about 3 T) significant deviations appear. In part this is because localization effects have not been considered: on quantum Hall plateaus, when the Fermi level is pinned in the gap between the density of states peaks, $g(E_F)=0$ and $\sigma_{xy}^{(2)}$ vanishes. The Hall conductivity is then $\sigma_{xy}^{(1)}$, which should be equal to the constant, plateau value. However, when localization is neglected in Eqs. (1)–(3) $\sigma_{xy}^{(1)}$ is given on the plateaus, incorrectly, by en_0/B and

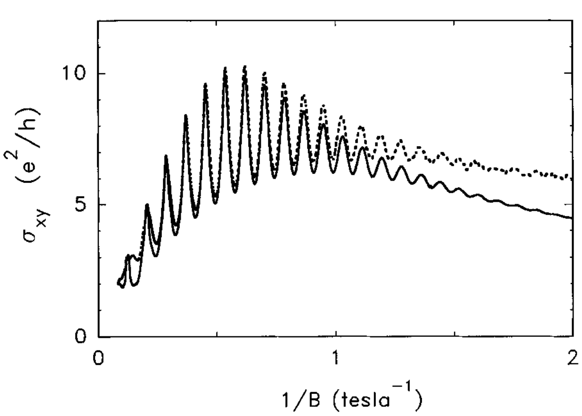

FIG. 7. Calculated values of σ_{xy} derived from σ_{xx} (dashed line) compared with experimental values (solid line), plotted as a function of $1/B$.

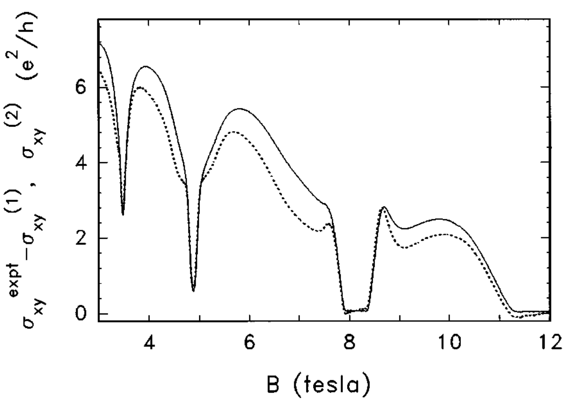

FIG. 8. Comparison between values of $\sigma_{xy}^{\rm expt}-\sigma_{xy}^{(1)}$ (dotted curve) and $\sigma_{xy}^{(2)}$ (solid curve) with $\sigma_{xy}^{(1)}$ and $\sigma_{xy}^{(2)}$ derived from experimental values of σ_{xx}. The deviations around the Hall plateaus are attributed to localization and those around the maxima of the density-of-states peaks to Landau-level mixing.

varies with magnetic field. These deviations are relatively small. Much larger errors are seen *between* the plateaus where localization is unimportant. In this case some other phenomenon must be invoked to explain the behavior.

The discrepancies are actually somewhat smaller than they appear. This is because the two terms $\sigma_{xy}^{(1)}$ and $\sigma_{xy}^{(2)}$ in Eq. (3) are of the opposite sign but roughly the same magnitude. This can be seen in Fig. 8, where $\sigma_{xy}^{\rm expt}-\sigma_{xy}^{(1)}$ is compared with $\sigma_{xy}^{(2)}$. The differences are relatively small, typically less than 20% of each term. A possible source of these differences is ΔE_F, the deviation from the zero-field value that occurs as the Fermi energy moves through the Landau levels. This adds correction terms to the simple expression for $[\partial n/\partial B]_{E_F}$, which depend, in detail, on the exact shape of the Landau levels. Using a specific model of Gaussian broadened Landau levels, with a width proportional to $B^{1/2}$,[17] gives

$$\left[\frac{\partial n}{\partial B}\right]_{E_F} = -\frac{n_0 \Delta g}{B g_0} - \frac{g \Delta E_F}{B} + \sum_N \frac{g_N}{2B}(E_F - E_N), \qquad (9)$$

where g_N denotes the density of states at the Fermi energy for each spin-resolved Landau level N and E_N is the energy of the center of the Landau level. For Landau levels with a different form of broadening similar terms appear, but with different relative strengths. For a single symmetric spin-resolved Landau level (or for two spin-unresolved levels), ΔE_F vanishes at half filling, i.e., at maxima of the density of states and at the same time $E_F=E_N$. The second and third terms in Eq. (9) therefore both vanish at the center of the Landau levels. Similarly, for overlapping levels, when spin is partially resolved, ΔE_F is again zero at half filling. Also, because the density of states in each of the levels is the same (at least for symmetrical Landau levels) and the centers of the levels are symmetrically placed about E_F, the third term again vanishes. Oscillations in the Fermi level can therefore explain *asymmetry* around the center of a Landau level for the deviations of σ_{xy} (e.g., as seen around 4 and 6 T in Fig. 8), but they cannot explain the discrepancies in magnitude.

The deviations seen in Fig. 8 occur either on Hall plateaus, where, as explained above, they are readily understandable in terms of localization, or well away from the quantum Hall plateaus, when localization effects are much less important. They are largest in the middle of the Landau level and small, or even nonexistent, near the edges. They seem to be closely associated with the shoulders on the side of the peaks that appear when the Fermi energy moves out of a single, spin-resolved, Landau level and into two overlapping levels. This suggest they might be caused by Landau-level mixing, which could be particularly important here because, with the large g factor, the mixing occurs between levels with different values of N_L and different cyclotron radii. For example, in the $\nu=4$ peak where the $N_L=1$ and 2 Landau levels are involved, the value of R_{cycl}^2 [which appears in Eq. (7)] differs by a factor of almost 2 for the two levels.

An interesting feature of the results is how well the standard semiclassical description of the transport coefficient works. Even at the lowest temperatures localization is important only over a very limited field range and there is little or no sign of scaling behavior. This is a little unexpected because what are probably the best examples of scaling measurements have been obtained in a $\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As}$-InP heterostructure with a relatively low mobility (16 000 cm^2/V s) in which short-range scattering dominated.[8] In $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ samples the absence of scaling behavior, except at the lowest temperatures, has been attributed to the long-range potential fluctuations (small-angle scattering) in this system,[18] so, using this argument, the SiGe hole system might be expected to show scaling at relatively high temperatures, but this is not the case.

Because of the onset of spin resolution the peak between $\nu=2$ and 3 behaves rather differently from those at lower fields. The dip around 9 T might be associated with a fractional quantum Hall effect feature at $\nu=8/3$, but this is unlikely, not only because of the low mobility of this sample but also because the dip moves in position as the temperature is raised. It seems more reasonable to identify the structure with a shoulder, similar to but more developed than those seen at lower fields, and Landau-level mixing effects. On the high field side there is no shoulder because, in contrast to the $\nu=3$ zero, the spin must be resolved at $\nu=2$. Exchange effects may well play a strong role in this process and model calculations suggest that if this is the case a boot-strapping effect could drive the transition. This would explain the strong temperature dependence of the peak shown in Fig. 2.

One interesting question is whether the (exchange-enhanced) spin splitting is sufficiently large that the spin-split levels cross so the system is spin polarized for $\nu\leqslant 2$. It is tempting to speculate that such a situation, and an associated instability, might play some role in the dependence on density of the presence or absence of an insulating phase for $\nu<2$. However, without a more detailed theoretical understanding of the situation this question cannot be answered from the results presented here.

CONCLUSIONS

Magnetotransport measurements made in a strained SiGe two-dimensional hole gas are found to be in good general agreement with earlier measurements. In particular the g factor is found to be comparable to the Landau-level spacing and large-angle scattering processes dominate the transport properties. Also, in tilted magnetic fields the magnetoresistance was found to be extremely insensitive to the in-plane component of the field, which confirms that the carriers are in the $|M_J|=\frac{3}{2}$ state, well decoupled from other levels. At low magnetic fields the transport coefficients are explained quantitatively by semiclassical transport theory. While good agreement has been observed previously for the longitudinal resistivity, this appears to be the first time that such a clear confirmation has been achieved for the Hall resistivity. This occurs because large-angle scattering dominates in this system. At high magnetic fields the peak values of σ_{xx} are in good quantitative agreement with the same theoretical model, but deviations are observed for σ_{xy}. These are relatively small and are attributed to (a) the onset of localization effects and (b) Landau-level mixing effects. Localization appears to be relatively unimportant in this sample, for reasons that are not clear. Landau-level mixing is likely to be important because the large spin splitting mixes states with different Landau-level indices. It is possible that exchange effects play an important role in forming the spin-resolved $\nu=2$ state.

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changes sign when the charge on the carriers changes sign and when the magnetic-field direction is reversed.
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