Quantum phase transition and long-range order in the ground state of a lattice of pseudospins coupled with optic phonons

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We study a pseudospin-phonon coupling system in its ground state and show that, as a result of the competition between the intrasite pseudospin tunneling and the phonon-induced intersite pseudospin correlation, a quantum phase transition to the pseudospin long-range ordering phase may occur at some critical coupling constant. In the disordered phase the short-range ferroelectric or antiferroelectric correlation gets stronger as the coupling constant increases. The calculated renormalized tunneling matrix element, which is reduced by the pseudospin-phonon interaction, is not as small as the theory of previous authors predicts, that is, the reduction is alleviated by taking into account the retardation effect of the coupling. [S0163-1829(96)02927-X]

Quantum phase transitions (at zero temperature) and long-range ordering in systems with competing interactions have been a subject of great theoretical interest in recent years. The competition may be, e.g., between the electron-electron correlation and the electron-phonon interaction as that in the conventional superconductor-metal phase transition (at zero temperature).[1] In this case the retardation effect of the electron-phonon interaction plays a very important role.[1] In this work we consider a lattice of pseudospins interacting with optic phonons, in which the competing interactions are the intrasite pseudospin tunneling and the phonon-induced intersite pseudospin correlation. The pseudospin-phonon coupling Hamiltonian reads[2–6]

$$H=-\sum_{\mathbf{j}}\Delta_0\sigma_{\mathbf{j}}^x+\sum_{\mathbf{k}}\omega_{\mathbf{k}}\left(b_{\mathbf{k}}^\dagger b_{\mathbf{k}}+\frac{1}{2}\right)+\frac{1}{\sqrt{N}}\sum_{\mathbf{k}}\sum_{\mathbf{j}}g_{\mathbf{k}}\sigma_{\mathbf{j}}^z(b_{-\mathbf{k}}^\dagger+b_{\mathbf{k}})\exp(-i\mathbf{k}\cdot\mathbf{j}),\qquad(1)$$

where N is the number of pseudospins. $b_{\mathbf{k}}^\dagger$ ($b_{\mathbf{k}}$) is the creation (annhilation) operator of phonon mode with frequency $\omega_{\mathbf{k}}$, $\sigma_{\mathbf{j}}^x$ and $\sigma_{\mathbf{j}}^z$ are the Pauli matrices on site $\mathbf{j}$ with bare tunneling matrix element Δ_0. $g_{\mathbf{k}}$ is the pseudospin-phonon coupling constant. This model Hamiltonian has been studied, as a model for the proton-lattice interaction in hydrogen-bonded ferroelectrics, by some authors using various methods, such as the mean-field approximation,[6] the Green's-function technique,[3,4] the variational calculation,[2,7] and the unitary transformation approach.[5] Their interest is concentrated either on the single pseudospin property[2–4,7] or on the long-range ordering induced by phonons.[5,6] Besides, a similar model system, known as a two-level system coupled with a phonon heat bath, has been studied by many authors.[8] The Hamiltonian of it is similar to (1), but without the summation over $\mathbf{j}$, since it is a single impurity problem. In this work, however, we concentrate on the collective properties of pseudospins in both the disordered phase and the long-range ordering one.

When making numerical calculations we assume a two-dimensional square lattice with the following functions of wave vector $\mathbf{k}$:

$$\omega_{\mathbf{k}}(\pm)=\omega_c\sqrt{1-\rho(1\pm\gamma_{\mathbf{k}})/2},\;\; g_{\mathbf{k}}^2=\frac{\alpha}{2}\omega_c^2,\;\; -\pi\leqslant k_x,k_y\leqslant\pi,\qquad(2)$$

where $\gamma_{\mathbf{k}}=(\cos k_x+\cos k_y)/2$. α is a dimensionless constant to measure the coupling strength and $g_{\mathbf{k}}$ is assumed to be $\mathbf{k}$ independent for simplicity. ω_c is the upper limit of phonon frequency and $0<\rho<1$ in (2) measures the size of dispersion. In the following it will be shown that the $\omega_{\mathbf{k}}(+)$ lattice mode may induce a ferroelectric ordering, but $\omega_{\mathbf{k}}(-)$ an antiferroelectric ordering. Throughout this paper we set $\hbar=1$.

Our treatment is based on the unitary transformation approach with

$$H'=\exp(S)H\exp(-S),\qquad(3)$$

$$S=\frac{1}{\sqrt{N}}\sum_{\mathbf{k}}\sum_{\mathbf{j}}\frac{g_{\mathbf{k}}}{\omega_{\mathbf{k}}}\delta_{\mathbf{k}}\sigma_{\mathbf{j}}^z(b_{-\mathbf{k}}^\dagger-b_{\mathbf{k}})\exp(-i\mathbf{k}\cdot\mathbf{j}).\qquad(4)$$

Here we introduce in S a function $\delta_{\mathbf{k}}$ that is $\mathbf{k}$ dependent. The form of it will be determined later. The transformation can be done to the end and the result is

$$H'=H_0+H_I,\qquad(5)$$

$$H_0=\sum_{\mathbf{k}}\omega_{\mathbf{k}}\left(b_{\mathbf{k}}^\dagger b_{\mathbf{k}}+\frac{1}{2}\right)-NV_0-\sum_{\mathbf{j}}\eta\Delta_0\sigma_{\mathbf{j}}^x-\frac{1}{N}\sum_{\mathbf{k}}\sum_{\mathbf{i},\mathbf{j}}\left(\frac{g_{\mathbf{k}}^2}{\omega_{\mathbf{k}}}\delta_{\mathbf{k}}(2-\delta_{\mathbf{k}})-V_0\right)\sigma_{\mathbf{i}}^z\sigma_{\mathbf{j}}^z\exp[i\mathbf{k}\cdot(\mathbf{i}-\mathbf{j})],\qquad(6)$$

$$H_I=-\sum_{\mathbf{j}}\Delta_0\sigma_{\mathbf{j}}^x\left(\coth\left[\frac{2}{\sqrt{N}}\sum_{\mathbf{k}}\frac{g_{\mathbf{k}}}{\omega_{\mathbf{k}}}\delta_{\mathbf{k}}(b_{-\mathbf{k}}^\dagger-b_{\mathbf{k}})\times\exp(-i\mathbf{k}\cdot\mathbf{j})\right]-\eta\right)+\frac{1}{\sqrt{N}}\sum_{\mathbf{k}}\sum_{\mathbf{j}}g_{\mathbf{k}}(1-\delta_{\mathbf{k}})\times\sigma_{\mathbf{j}}^z(b_{-\mathbf{k}}^\dagger+b_{\mathbf{k}})\exp(-i\mathbf{k}\cdot\mathbf{j})-\sum_{\mathbf{j}}\Delta_0 i\sigma_{\mathbf{j}}^y\sinh\times\left[\frac{2}{\sqrt{N}}\sum_{\mathbf{k}}\frac{g_{\mathbf{k}}}{\omega_{\mathbf{k}}}\delta_{\mathbf{k}}(b_{-\mathbf{k}}^\dagger-b_{\mathbf{k}})\exp(-i\mathbf{k}\cdot\mathbf{j})\right]. \quad (7)$$

The last term in H_0 describes a long-range Ising-type interaction between pseudospins, in which

$$V_0=\frac{1}{N}\sum_{\mathbf{k}}g_{\mathbf{k}}^2\delta_{\mathbf{k}}(2-\delta_{\mathbf{k}})/\omega_{\mathbf{k}} \quad (8)$$

is subtracted because it represents a constant self-coupling since $\sigma_{\mathbf{j}}^z\sigma_{\mathbf{j}}^z=1$ and does not contribute to the interaction between pseudospins at different sites.[5] Besides,

$$\eta=\exp\left(-\frac{2}{N}\sum_{\mathbf{k}}g_{\mathbf{k}}^2\delta_{\mathbf{k}}^2/\omega_{\mathbf{k}}^2\right) \quad (9)$$

represents the phonon dressing of the bare tunneling matrix element Δ_0.[2,5]

We treat H_0 as the zeroth order Hamiltonian and H_I the perturbation. So H_I should be as small as possible. One can let $\delta_{\mathbf{k}}\equiv 1$ for all $\mathbf{k}$ and thus the second term in H_I is zero. However, the first order term of $g_{\mathbf{k}}$ still exists in the sinh function. We determine the form of $\delta_{\mathbf{k}}$ as follows. In H_0 the pseudospins and the phonons are decoupled and we have the transverse Ising model for pseudospins.[5,6] The ground state of H_0, denoted as $|g_0'\rangle$, is

$$|g_0'\rangle=|s,0\rangle|p,0\rangle. \quad (10)$$

where $|p,0\rangle$ is the vacuum state of phonons, but an exact solution for $|s,0\rangle$ cannot be obtained. We use the linearized spin-wave approximation[9]

$$\sigma_{\mathbf{j}}^z=B_{\mathbf{j}}^\dagger+B_{\mathbf{j}},\quad i\sigma_{\mathbf{j}}^y=B_{\mathbf{j}}^\dagger-B_{\mathbf{j}},\quad \sigma_{\mathbf{j}}^x=2B_{\mathbf{j}}^\dagger B_{\mathbf{j}}-1, \quad (11)$$

where $B_{\mathbf{j}}$ and $B_{\mathbf{j}}^\dagger$ are boson operators. In this approximation H_0 reads

$$H_0\approx\sum_{\mathbf{k}}\omega_{\mathbf{k}}\left(b_{\mathbf{k}}^\dagger b_{\mathbf{k}}+\frac{1}{2}\right)-NV_0-N\eta\Delta_0+\sum_{\mathbf{k}}2\eta\Delta_0 B_{\mathbf{k}}^\dagger B_{\mathbf{k}}-\frac{1}{2}\sum_{\mathbf{k}}U_{\mathbf{k}}(B_{\mathbf{k}}^\dagger+B_{-\mathbf{k}})(B_{-\mathbf{k}}^\dagger+B_{\mathbf{k}}), \quad (12)$$

where $U_{\mathbf{k}}=2g_{\mathbf{k}}^2\delta_{\mathbf{k}}(2-\delta_{\mathbf{k}})/\omega_{\mathbf{k}}-2V_0$ and $B_{\mathbf{j}}=1/\sqrt{N}\Sigma_{\mathbf{k}}B_{\mathbf{k}}\exp(i\mathbf{k}\cdot\mathbf{j})$. Then H_0 can be diagonalized by the Bogoliubov transformation[9] and the diagonalized H_0 is

$$H_0=\sum_{\mathbf{k}}\omega_{\mathbf{k}}\left(b_{\mathbf{k}}^\dagger b_{\mathbf{k}}+\frac{1}{2}\right)-NV_0-2N\eta\Delta_0+\sum_{\mathbf{k}}E_{\mathbf{k}}\left(B_{\mathbf{k}}^\dagger B_{\mathbf{k}}+\frac{1}{2}\right), \quad (13)$$

where $E_{\mathbf{k}}$ is the energy function of pseudospin excitations

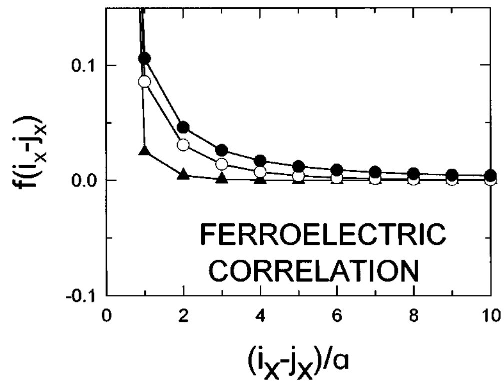

FIG. 1. Ferroelectric correlation $f(i_x-j_x)$ as a function of i_x-j_x with $i_y=j_y$ in the case of $\Delta_0/\omega_c=0.2$, $\rho=0.75$, and $\alpha=0.1$ (solid triangles), 0.2 (empty circles), and 0.2078 (solid circles). a is the lattice constant.

$$E_{\mathbf{k}}=2\sqrt{\eta\Delta_0}[\eta\Delta_0-U_{\mathbf{k}}]^{1/2}. \quad (14)$$

Obviously, we must have

$$\eta\Delta_0-U_{\mathbf{k}}>0 \quad (15)$$

for these excitations to be stable.

The average value of the perturbation H_I [see Eq. (7)] in the ground state $|g_0'\rangle$ is zero. We determine the form of $\delta_{\mathbf{k}}$ by making the matrix element of H_I between $|g_0'\rangle$ and the lowest-lying excited state $B_{\mathbf{k}}^\dagger b_{-\mathbf{k}}^\dagger|g_0'\rangle$ be zero,

$$\langle g_0'|H_I B_{\mathbf{k}}^\dagger b_{-\mathbf{k}}^\dagger|g_0'\rangle=0. \quad (16)$$

The result is

$$\delta_{\mathbf{k}}=\omega_{\mathbf{k}}[\omega_{\mathbf{k}}+2\sqrt{\eta\Delta_0}\sqrt{\eta\Delta_0-U_{\mathbf{k}}}]^{-1}. \quad (17)$$

Thus one can see that $\delta_{\mathbf{k}}<1$ and is $\mathbf{k}$ dependent for H_I to be as small as possible. $\delta_{\mathbf{k}}\equiv 1$ for all $\mathbf{k}$ only when $\Delta_0=0$. Of course, this form of $\delta_{\mathbf{k}}$ is good only when the condition (15) is satisfied. In this case the system is in a disordered phase and the pseudospin correlation function, defined as $f(\mathbf{i}-\mathbf{j})=\langle g|\sigma_{\mathbf{i}}^z\sigma_{\mathbf{j}}^z|g\rangle$, where $|g\rangle$ is the ground state of H, can be derived as follows. In our approximation $|g\rangle\approx\exp(-S)|g_0'\rangle$. By using the linearized spin wave approximation (11) we have

$$f(\mathbf{i}-\mathbf{j})\approx\langle g_0'|\sigma_{\mathbf{i}}^z\sigma_{\mathbf{j}}^z|g_0'\rangle\approx\frac{1}{N}\sum_{\mathbf{k}}\frac{\exp[-i\mathbf{k}\cdot(\mathbf{i}-\mathbf{j})]}{\sqrt{1-U_{\mathbf{k}}/\eta\Delta_0}}. \quad (18)$$

This is a damping or damping-oscillating function of $\mathbf{i}-\mathbf{j}$ because the denominator of the integrand has no real zero-point when the condition (15) is satisfied. Figure 1 shows the ferroelectric correlation $f(i_x-j_x)$. Note that when $\mathbf{i}=\mathbf{j}$ $f(0)=1$. Here the phonon frequency $\omega_{\mathbf{k}}=\omega_{\mathbf{k}}(+)$ is used and thus the maximum of $g_{\mathbf{k}}^2/\omega_{\mathbf{k}}$ is at $\mathbf{k}=\mathbf{Q}=(0,0)$. For $\Delta_0/\omega_c=0.2$ the condition

$$\eta\Delta_0+2V_0=2g_{\mathbf{k}}^2/\omega_{\mathbf{k}}, \quad (19)$$

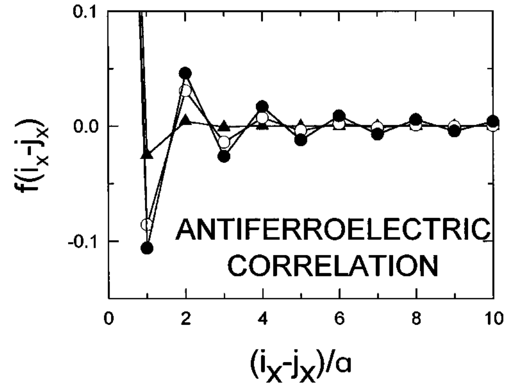

FIG. 2. Antiferroelectric correlation $f(i_x-j_x)$ as a function of i_x-j_x with $i_y=j_y$ in the case of $\Delta_0/\omega_c=0.2$, $\rho=0.75$, and $\alpha=0.1$ (solid triangles), 0.2 (empty circles), and 0.2078 (solid circles).

which determines a phase transition line in the $\Delta_0/\omega_c\sim\alpha$ phase diagram, is satisfied at $\alpha=0.208$. One can see from the figure that the short-range correlation increases with increasing α.

Figure 2 shows the antiferroelectric correlation $f(i_x-j_x)$. Here the phonon frequency $\omega_{\mathbf{k}}=\omega_{\mathbf{k}}(-)$ is used and thus the maximum of $g_{\mathbf{k}}^2/\omega_{\mathbf{k}}$ is at $\mathbf{k}=\mathbf{Q}=(\pi,\pi)$. The phase transition point is still at $\Delta_0/\omega_c=0.2$ and $\alpha=0.208$, since the condition (19) is symmetric under the transformation $k_x\to\pi-k_x$ and $k_y\to\pi-k_y$. However, the correlation function is a damping-oscillating function of i_x-j_x to be different from the ferroelectric one and shows a short-range antiferroelectric correlation.

The $\Delta_0/\omega_c\sim\alpha$ phase diagram, derived from Eq. (19), is shown in Fig. 3. For comparison, the result obtained by assuming $\delta_{\mathbf{k}}\equiv 1$ for all $\mathbf{k}$ is also shown by the dashed line. Our result of $\delta_{\mathbf{k}}<1$ increases the area of the disordered phase compared to that of $\delta_{\mathbf{k}}\equiv 1$, especially when Δ_0/ω_c is larger. We list some numerical results. For $\Delta_0/\omega_c=0.2$ the phase transition point is at $\alpha=0.208$ ($\delta_{\mathbf{k}}<1$) or $\alpha=0.203$ ($\delta_{\mathbf{k}}\equiv 1$); for $\Delta_0/\omega_c=0.35$ it is at $\alpha=0.315$ ($\delta_{\mathbf{k}}<1$) or $\alpha=0.300$ ($\delta_{\mathbf{k}}\equiv 1$).

The ground-state average of the tunneling matrix in the disordered phase $\langle g|\sigma_{\mathbf{j}}^x|g\rangle$ can be calculated as

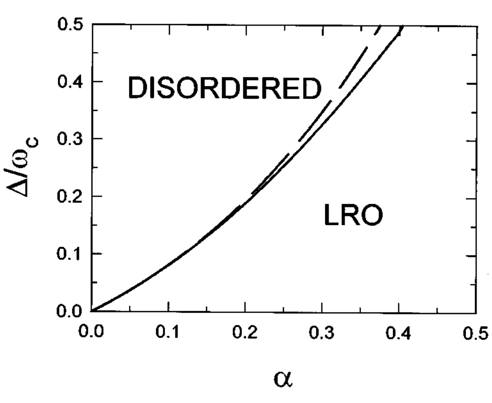

FIG. 3. $\Delta_0/\omega_c\sim\alpha$ phase diagram for the case of $\rho=0.75$. LRO means the long-range ordering phase.

$$\langle g|\sigma_{\mathbf{j}}^x|g\rangle\approx\langle g_0'|\exp(S)\sigma_{\mathbf{j}}^x\exp(-S)|g_0'\rangle \approx 2\eta\left(1-\frac{1}{4N}\sum_{\mathbf{k}}\left[\frac{2\eta\Delta_0}{E_{\mathbf{k}}}+\frac{E_{\mathbf{k}}}{2\eta\Delta_0}\right]\right). \tag{20}$$

The ground-state average of $\sigma_{\mathbf{j}}^z$ in the disordered phase is zero.

When $U_{\mathbf{Q}}$ is greater than $\eta\Delta_0$, we get the ground state of H_0 by a mean field decoupling of the Ising-type interaction[10]

$$H_0=\sum_{\mathbf{k}}\omega_{\mathbf{k}}\left(b_{\mathbf{k}}^\dagger b_{\mathbf{k}}+\frac{1}{2}\right)-NV_0-\sum_{\mathbf{j}}\eta\Delta_0\sigma_{\mathbf{j}}^x -\frac{1}{2N}\sum_{\mathbf{k}}\sum_{\mathbf{i},\mathbf{j}}U_{\mathbf{k}}\exp[i\mathbf{k}\cdot(\mathbf{i}-\mathbf{j})][\sigma_{\mathbf{i}}^z\langle\sigma_{\mathbf{j}}^z\rangle+\sigma_{\mathbf{j}}^z\langle\sigma_{\mathbf{i}}^z\rangle -\langle\sigma_{\mathbf{i}}^z\rangle\langle\sigma_{\mathbf{j}}^z\rangle+(\sigma_{\mathbf{i}}^z-\langle\sigma_{\mathbf{i}}^z\rangle)(\sigma_{\mathbf{j}}^z-\langle\sigma_{\mathbf{j}}^z\rangle)], \tag{21}$$

where $\langle\ \rangle$ means the ground-state averaging $\langle\ \rangle=\langle g_0'|\ |g_0'\rangle$. The mean-field approximation is to neglect the last term in the square brackets. In this approximation,

$$\langle\sigma_{\mathbf{j}}^z\rangle=\sigma_0\exp(i\mathbf{Q}\cdot\mathbf{j}). \tag{22}$$

Here σ_0 is a constant that will be determined self-consistently. For simplicity we assume $\exp(i\mathbf{Q}\cdot\mathbf{j})=\pm 1$, that is, the ferroelectric coupling $[\mathbf{Q}=(0,0)]$ or antiferroelectric coupling $[\mathbf{Q}=(\pi,\pi)]$ between pseudospins. Thus the ground state [Eq. (10)] is a direct product of the on-site ground state $|\mathbf{j},+\rangle$, where

$$|\mathbf{j},\pm\rangle=[(\sqrt{J_{\mathbf{Q}}^2+\eta^2\Delta_0^2}\pm J_{\mathbf{Q}})^2 +\eta^2\Delta_0^2]^{-1/2}\begin{pmatrix}\pm\eta\Delta_0\\ \sqrt{J_{\mathbf{Q}}^2+\eta^2\Delta_0^2}\pm J_{\mathbf{Q}}\end{pmatrix}. \tag{23}$$

Here $J_{\mathbf{Q}}=-U_{\mathbf{Q}}\sigma_0\exp(i\mathbf{Q}\cdot\mathbf{j})$ and $|\mathbf{j},-\rangle$ is the on-site excited state. The self-consistent solution for σ_0 is

$$\sigma_0=[1-\eta^2\Delta_0^2/U_{\mathbf{Q}}^2]^{1/2}. \tag{24}$$

The effect of the perturbation H_I in this long-range ordering phase can be discussed as follows. Its average value in the ground state $|g_0'\rangle$ is zero. $\delta_{\mathbf{Q}}=1$ makes the matrix element $\langle g_0'|H_I b_{-\mathbf{Q}}^\dagger|g_0'\rangle$ be zero. Besides, we determine the form of $\delta_{\mathbf{k}}$ for $\mathbf{k}\neq\mathbf{Q}$ by making the first-order matrix element of H_I between the ground state and the lowest-lying excited state $|\mathbf{j},-\rangle b_{-\mathbf{k}}^\dagger|p,0\rangle$ be zero,

$$\langle p,0|\langle\mathbf{j},+|H_I|\mathbf{j},-\rangle b_{-\mathbf{k}}^\dagger|p,0\rangle=0. \tag{25}$$

The result for $\delta_{\mathbf{k}}$ is

$$\delta_{\mathbf{k}}=\omega_{\mathbf{k}}/(\omega_{\mathbf{k}}+2U_{\mathbf{Q}}). \tag{26}$$

The ground-state average of the tunneling matrix is

$$\langle g|\sigma_{\mathbf{j}}^x|g\rangle\approx\langle g_0'|\exp(S)\sigma_{\mathbf{j}}^x\exp(-S)|g_0'\rangle\approx\eta^2\Delta_0/U_{\mathbf{Q}}. \tag{27}$$

$\langle g|\sigma_{\mathbf{j}}^x|g\rangle$ as a function of α is shown in Figs. 4(a) and 5(a). For comparison, the dashed lines are the results obtained by assuming $\delta_{\mathbf{k}}\equiv 1$ for all $\mathbf{k}$. For convenience, we plot the result for disordered phase and that for long-range ordering phase

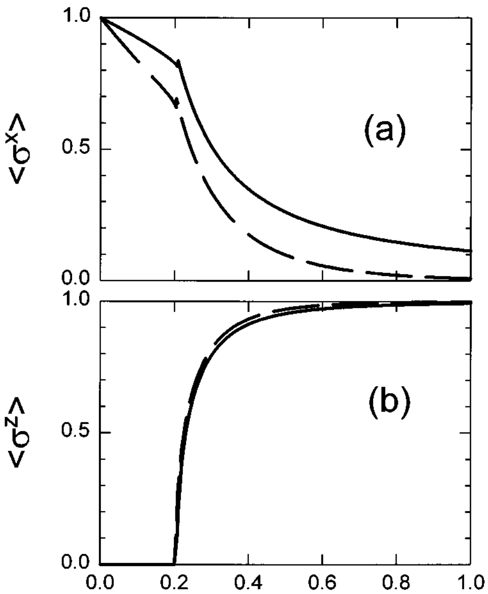

FIG. 4. (a) $\langle g|\sigma^x|g\rangle$ and (b) $\langle g|\sigma^z|g\rangle$ as functions of α for $\Delta_0/\omega_c=0.2$ and $\rho=0.75$.

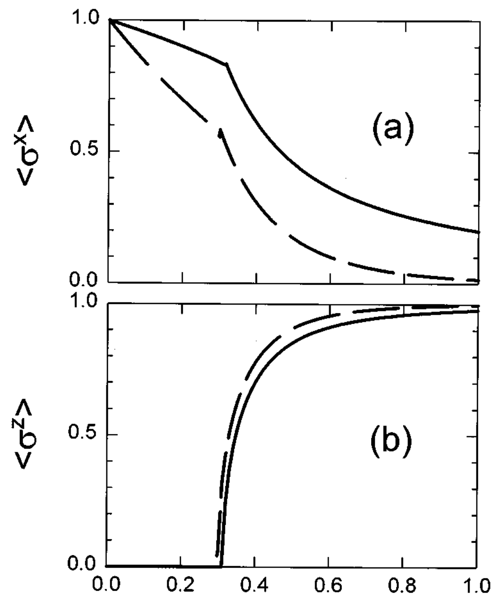

FIG. 5. (a) $\langle g|\sigma^x|g\rangle$ and (b) $\langle g|\sigma^z|g\rangle$ as functions of α for $\Delta_0/\omega_c=0.35$ and $\rho=0.75$.

in the same figure. The difference between the two lines gets larger with increasing α and when α is larger than 0.5 the difference may be more than one order of magnitude. As $\langle g|\sigma_{\mathbf{j}}^x|g\rangle=1$ for $\alpha=0$, we can use $\langle g|\sigma_{\mathbf{j}}^x|g\rangle$ as the renormalization factor for bare tunneling matrix element Δ_0. Our result shows that, although the reduction of the tunneling matrix element by pseudospin-phonon interaction gets larger with the increasing coupling constant α, it is not as small as the theory of $\delta_{\mathbf{k}}\equiv 1$ predicts and the reduction is alleviated by introducing a **k**-dependent $\delta_{\mathbf{k}}$.

We should say something about the physical meaning of $\delta_{\mathbf{k}}\equiv 1$ and our choice of Eqs. (17) and (26). $\delta_{\mathbf{k}}\equiv 1$ means that the phonons can follow completely the intrasite pseudospin tunneling and this should not be the case when the condition $\Delta_0/\omega_{\mathbf{k}}\ll 1$ is not satisfied. $\delta_{\mathbf{k}}<1$ of Eqs. (17) and (26) means that the phonons follow the tunneling motion only partly and there is a retardation effect. According to our result, it is necessary to consider the retardation effect when the ratio $\Delta_0/\omega_{\mathbf{k}}$ is not very small.

$\langle g|\sigma_{\mathbf{j}}^z|g\rangle$ as a function of α is shown in Figs. 4(b) and 5(b). In the disordered phase $\langle g|\sigma_{\mathbf{j}}^z|g\rangle=0$, so one can see where the phase transition point is.

In summary, we have studied a pseudospin-phonon coupling system in its ground state and shown that, as a result of the competition between the intrasite pseudospin tunneling and the phonon-induced intersite pseudospin correlation, a quantum phase transition to the pseudospin long-range ordering phase may occur at some critical coupling constant α. We use a unitary transformation approach in which a **k**-dependent function $\delta_{\mathbf{k}}$ is introduced and the form of it is determined through making the matrix element of H_I between the ground state and the lowest-lying excited state of H_0 be zero. It is shown that in the disordered phase the short-range ferroelectric or antiferroelectric correlation gets stronger as the coupling constant α increases. The calculated renormalized tunneling matrix element, which is reduced by the pseudospin-phonon interaction, is not as small as the theory of $\delta_{\mathbf{k}}\equiv 1$ predicts, that is, the reduction is alleviated by introducing a **k**-dependent $\delta_{\mathbf{k}}$. This fact shows that the retardation effect is important when the ratio $\Delta_0/\omega_{\mathbf{k}}$ is not very small.

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