# **Dynamics of a single hole in the two-dimensional** *t***-***J* **model in the presence of a magnetic field and the composite nature of quasiparticles**

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The density of state for the single hole in the two-dimensional *t*-*J* model is studied in the presence of a constant and homogeneous magnetic field by means of exact calculations for small systems and spin-wave perturbative calculations for larger systems. As the field is turned on, the energy of the ground state is *reduced* by an amount proportional to parameter *t* and depending weakly on *J*. This result, together with the behavior of the Drude weight of the doped *t*-*J* model, yields strong support for the idea that the dynamics of charge carriers is governed by parameter *t*. This idea, confronted to the existence in the spectral density of the hole of a quasiparticle peak dispersing with *J*, constitutes evidence for the composite nature of quasiparticles. [S0163-1829(96)05925-5]

# **I. INTRODUCTION**

The discovery of high- $T_c$  superconductivity has renewed interest in the electronic and magnetic properties of doped Mott insulators. A central question in the theory of these systems concerns the nature of low-energy excitations. In particular, it is important to know whether the spin and charge of a hole doped into a quantum antiferromagnet are deconfined as in the one-dimensional case, $\frac{1}{1}$  or whether the spin and the charge remain bound together, leading to an electronlike quasiparticle. $^{2}$ 

In Ref. 3, the case was made that existing numerical works<sup>4,5</sup> for the two-dimensional  $(2D)$  *t*-*J* model, which provides a simplified description of copper oxide planes, constitute evidence of the decay of the hole into more elementary excitations: On one hand, the analysis of the spectral density of the hole points to the existence of a quasiparticle with a dispersion proportional to parameter *J*. On the other hand, coupling the charge of the hole to an external electromagnetic field and examining the response of the system to this external field points to a dynamics governed by parameter *t*. In particular, exact calculations for small systems lead to a Drude weight proportional to the density of holes and to parameter *t*. These numerical results are inconsistent with the picture of the hole as a well-definite quasiparticle excitation with spin 1/2 and charge-*e*, but quite consistent with the idea that the hole breaks up into two constituents, one carrying the charge and dispersing with *t* and another one dispersing with *J*. The properties of the constituents of the hole evidenced by the numerical analysis present striking similarities with those of neutral spin- $1/2$  (spinon) and spinless charge- $e$  (holon) excitations of the spin liquid state<sup>6</sup> of the 2D*t*-*J* model. The picture of the quasiparticle which emerges from this analysis is that of a spinon-holon bound state.

In this note, we study the issue of spin-charge separation by examining the dynamics of a single hole in the presence of a constant and homogeneous magnetic field applied perpendicular to the 2D system. We consider only the diamagnetic coupling of the external field with the charged particles. Using exact calculations for small systems, we find that the

change in ground-state energy due to the application of the magnetic field is proportional to *t* and depends weakly on parameter *J*, giving further evidence that the excitation carrying the charge of the hole and coupling to the external field disperses with *t*. This, together with the presence of a lowenergy peak dispersing with  $J$  in the spectral density,<sup>3</sup> gives support to the idea of the decay of the hole into more elementary constituents.

Surprisingly, the ground-state energy of the system with a single hole is *reduced* by the introduction of magnetic field. Again, this is inconsistent with the quasiparticle picture of the hole, in which the ground-state energy is expected to increase in the presence of a magnetic field due to the quasiparticle cyclotron energy. A possible explanation for this energy reduction can be formulated using the gauge theory of the  $t$ -*J* model,<sup>6</sup> which allow us formally to define the decay products of the hole as spinons and holons.

The numerical evidence for the decay of the hole described in Ref. 3 rests on the assumption that the independence in parameter  $J$  of quantities such as  $(i)$  the Drude weight or (ii) the ground-state energy difference in the presence and absence of an external magnetic field observed in exact calculations for small systems is a generic effect which survives in the bulk limit. In order to see if the weak dependence in parameter  $J$  of the second quantity  $(ii)$  observed in our exact calculations is a finite-size effect, we evaluate this quantity for larger systems by means of perturbative spinwave calculations.<sup>7</sup> The perturbative results show a reasonable agreement with exact results for small systems, and confirm the weak dependence in parameter *J* of the ground-state energy difference in the presence and absence of an external magnetic field.

The study of strongly correlated fermions systems in the presence of an external magnetic field has been initiated prior to this work. The Hall coefficient of the repulsive 2D Hubbard model was evaluated using the quantum Monte Carlo method in Ref. 8, and using exact calculations in Ref. 9. In the latter, the sign of the Hall coefficient was used as a criterion for the deconfinement of spin and charge excitations.

# **II. DENSITY OF STATE AND GROUND-STATE ENERGY**

We consider here the properties of a single hole in the *t*-*J* model defined by the Hamiltonian

$$
\mathcal{H} = P_G \bigg[ \sum_{\langle jj \rangle \sigma} t_{jl} c_{j\sigma}^{\dagger} c_{l\sigma} \bigg] P_G + \frac{J}{2} \sum_{\langle jj \rangle} \mathbf{S}_j \cdot \mathbf{S}_l, \tag{1}
$$

where  $P_G$  is the Gutzwiller projector which filters out states containing doubly occupied sites, where the sum  $\langle jl \rangle$  is performed over near-neighbor pairs with each pair counted twice to maintain hermiticity and where the hopping parameter  $t_{il}$  contains a phase factor describing the coupling of the electron's charge to an external homogeneous magnetic field. Taking the lattice bound length to be unity, this hopping parameter is expressed in the Landau gauge as

$$
t_{jl} = te^{2\pi i n_\phi j_y (j_x - l_x)},\tag{2}
$$

where  $n_{\phi}$  is the number of flux quanta per plaquette, and where  $j_x$  and  $j_y$  are the integer coordinates of site  $j$ .

Here we are interested in the density of state for the single hole, which is defined as

$$
D_{\sigma}(j,\omega) = -\frac{1}{\pi} \Im \mathcal{G}_{\sigma}(j,j,\omega), \tag{3}
$$

where the hole propagator  $\mathcal{G}_{\sigma}(j, j, \omega)$  is given by

$$
\mathcal{G}_{\sigma}(j,l,\omega) = \left\langle \Psi_0 \middle| c_{j\sigma}^{\dagger} \frac{1}{\omega + E_0/\hbar - \mathcal{H} + i\eta} c_{l\sigma} \middle| \Psi_0 \right\rangle, \quad (4)
$$

where  $|\Psi_0\rangle$  denotes the undoped ground state of the system with energy  $E_0$ . Here and in the following, we set  $\hbar = 1$ . The density of state defined in Eq.  $(3)$  is in fact independent of site position *j* and spin  $\sigma$ , and will hereafter be denoted by  $D(\omega)$ .

The density of state is evaluated using exact calculations for small systems subject to periodic boundary conditions in both x and y directions. The  $t$ -J Hamiltonian of Eq. (1) is diagonalized in the subspace generated by Slater determinants made up of single-electron wave functions  $\psi(j,\sigma)$  defined in the whole 2D lattice and satisfying periodicity conditions  $T_{(m,n)}(\psi) = \psi$  and  $T_{(n,m)}(\psi) = \psi$ , where *m* and *n* are positive integers both even or both odd, $^{10}$  and where

$$
\mathcal{T}_{(l_x,l_y)} = e^{\pi i n_\phi l_x l_y} \sum_{j\sigma} e^{2\pi i n_\phi j_x l_y} c_{j+l\sigma}^\dagger c_{j\sigma} \tag{5}
$$

is a magnetic translation operator $11$  which commutes with the Hamiltonian of Eq.  $(1)$ . In order to find wave functions satisfying such periodicity conditions simultaneously, the operators  $T_{(m,n)}$  and  $T_{(n,m)}$  must commute, which in turn implies that  $N \times n_{\phi}$  must be an integer, where  $N = m^2 + n^2$  is the number of sites in the cluster. This last condition expresses the quantization of magnetic flux passing through the cluster.

Figure 1 displays the density of state  $D(\omega)$  obtained by exact diagonalization for a small system for  $n_b = 0$ , 1/16, and 1/4. The spectrum consists in a strong low-energy peak, called the quasiparticle peak,<sup>4,5</sup> plus a  $7t$ -wide continuum at higher energy. The hole visible at zero field in the middle of the continuum progressively disappears as the magnetic field



FIG. 1. Exact results for a square 16-site cluster  $(m=4, n=0)$  for the density of states of a single hole, as defined by Eq.  $(3)$ , and for  $J=0.2$ . The three panels correspond to different magnetic-field strength. Both frequency  $\omega$  and parameter *J* are in units of  $t$ . The broadening parameter  $\eta$  appearing in Eq. (4) is taken to be equal to 0.005.

is turned on, while the width of the continuum remains approximately constant. Also, the quasiparticle peak is shifted to lower energies as the magnetic field is increased.

Figures 2 and 3 display the difference in ground-state energy in the presence and absence of a magnetic field for systems with a single hole as a function of field strength and for different values of parameter *J*. The ground-state energy is reduced by about 0.3*t* as  $n_{\phi}$  goes from 0 to 1/2. The behavior of the ground-state energy for other values of  $n_{\phi}$ can be inferred from the symmetries  $n_{\phi} \rightarrow n_{\phi} + 1$  and  $n_{\phi} \rightarrow n_{\phi}$ .

Note the small dependence of this energy difference in *J*. This is inconsistent with the quasiparticle picture of the hole. Indeed, the analysis of the low-energy behavior of the single-hole spectral density<sup>4,5,3</sup> in terms of a quasiparticle leads to a quasiparticle mass proportional to the inverse of parameter *J*. The difference in ground-state energy in the presence and absence of field, given by the cyclotron energy of the quasiparticle, should therefore be positive and proportional to *J*.

On the other hand, the low sensitivity of this energy difference in parameter *J* is consistent with the idea that the hole breaks up into two constituents, one dispersing with *J* and another one dispersing with *t* and carrying the charge of the hole. Indeed, in the latter case, the magnetic field should only couple to the second constituent, leading to an energy change proportional to *t*.



FIG. 2. Exact results for a square 16-site cluster for the groundstate energy of a single hole as a function of the number  $n<sub>\phi</sub>$  of flux quanta per plaquette, and for various values of parameter *J*. For each curve, the energy reference is taken to be the ground-state energy of the system in the absence of magnetic field, and for the corresponding value of *J*. The energy is given in units of *t*.



FIG. 3. Exact results for a tilted square 18-site cluster  $(m=n=3)$  for the ground-state energy of a single hole. Same caption as in Fig. 2.

A possible explanation for the decrease in ground-state energy occurring as the magnetic field is switched on can be formulated using the gauge theory of the  $t$ -*J* model.<sup>6</sup> This theory allows us formally to define two constituents for the hole in the 2D *t*-*J* model: A spinless excitation carrying the charge of the hole and dispersing with  $t$  (called the holon) and a neutral spin- $1/2$  excitation dispersing with  $J$  (called the spinon). The fields of matter associated with these excitations are coupled to a gauge field which, in the saddle-point approximation corresponding to the flux phase, leads to half a flux quantum per plaquette at low doping. In this picture, the moving holon experiences a net magnetic field which is given by the sum of the external field and of the mean gauge field. As the external field is varied from zero to half a flux quantum per plaquette, the net field experienced by the holon varies from half a flux quantum to zero flux quantum per plaquette, leading to a reduction in the holon kinetic energy.

However, this simple picture is too crude to be quantitatively accurate. Indeed, in the aformentioned saddle-point approximation, the dynamics of the single holon is described by the mean-field Hamiltonian<sup>6</sup>

$$
\overline{\mathcal{H}}_b = -\sum_{\langle j,k\rangle} t_{jk} \overline{\chi}_{jk} b_j^{\dagger} b_k, \qquad (6)
$$

where  $b_j^{\dagger}$  is the creation operator for holon excitations, and where  $\frac{\partial}{\partial x}$  is the creation operator for holon excitations, and where  $\frac{\partial}{\partial x}$  is the saddle-point solution for the gauge field, whose modulus approximately equals 0.95 and whose phase describes a fictitious magnetic field with half a flux quantum per plaquette. Using Eq.  $(6)$ , the reduction in the singleholon ground-state energy occurring as the external field is varied from zero to half a flux quantum per plaquette is given by  $2(2-\sqrt{2})t$ , which is about four times larger than our estimated value. However, this discrepancy might be attributed to the fluctuations of the gauge field, which are neglected here.

This explanation for the energy decrease occurring as the magnetic field is switched on is supported by the following argument: The physical meaning of this gauge field can be established by considering a single hole in a system consisting of a single plaquette with a site at each corner. In this system, the total spin can only take the values  $S=1/2$  and  $S=3/2$ . The case  $S=3/2$  corresponds to the ferromagnetic instability occurring for a small value of *J*/*t*. Let us take  $J=0.3t$ , for which the ground state has total spin  $S=1/2$ . Following Ref. 12, we now consider  $S=1/2$  wave functions in which the hole is fixed at some site, and which can be represented as

$$
3^{-1/2}\left[\begin{pmatrix} 0 & \uparrow \\ \uparrow & \downarrow \end{pmatrix} + \gamma \begin{pmatrix} 0 & \downarrow \\ \uparrow & \uparrow \end{pmatrix} + \gamma^2 \begin{pmatrix} 0 & \uparrow \\ \downarrow & \uparrow \end{pmatrix}\right], (7)
$$

where  $\gamma = e^{-(2/3)\pi i}$  and where 0,  $\uparrow$ , and  $\downarrow$ , respectively, denote empty sites, sites with spin-up electrons, and sites with spin-down electrons. Using the restricted basis given by state (7) and other states obtained from  $\pi/2$  rotations about the center of the plaquette, the Hamiltonian can be expressed as

$$
\mathcal{H} = \begin{pmatrix}\nJ & -\beta t & 0 & -\beta t \\
-\beta t & -\frac{J}{2} & -\beta t & 0 \\
0 & -\beta t & -\frac{J}{2} & -\beta t \\
-\beta t & 0 & -\beta t & -\frac{J}{2}\n\end{pmatrix},
$$
\n(8)

where  $\beta = \gamma e^{i(\pi/2)n} \phi$ , and where  $n_{\phi}$  is the number of the flux quanta of external magnetic field threading the plaquette. It can easily be verified that the Hamiltonian of Eq.  $(8)$  corresponds to a single particle moving around a plaquette threaded by  $n_{\phi} - \frac{1}{3}$  flux quanta. Here, the  $\frac{1}{3}$  fraction of flux quantum corresponds to the contribution of the gauge field. The Hamiltonian of Eq. (8) is invariant under  $\pi/2$  rotations about the center of the plaquette, and its states can be classified using *s*, *p*, and *d* symmetries. The ground state has a symmetry of  $p$  type for all values of  $n_{\phi}$ , and its energy is given by

$$
E_{\rm gs} = -2t \cos\left[\pi\left(\frac{n_\phi}{2} - \frac{1}{6}\right)\right] - \frac{J}{2}.\tag{9}
$$

Using  $J=0.3t$ , this leads to  $E_{gs}=-1.882t$  for  $n<sub>\phi</sub>=0$ , and to  $E_{gs}$ = -2.082*t* for  $n_{\phi}$ = 1/2. The energy decrease occuring as the external field is varied from  $n_{\phi}=0$  to  $n_{\phi}=1/2$  is due to the fact that the *net* field (external field plus gauge field) to which the particle is coupled is smaller in the case  $n_{\phi} = 1/2$ . The validity of this picture for the system with four sites is confirmed by the good agreement of the above energy estimates, obtained using a restricted basis, with the true ground-state energies given by  $E_{gs} = -1.888t$  for  $n<sub>\phi</sub> = 0$ , and by  $E_{gs} = -2.091t$  for  $n_{\phi} = 1/2$ . It is interesting to observe that the ground state for larger systems with a single hole shows correlations similar to those of the four-site state described by Eq.  $(7)$ , in the sense that the ground-state expectation value of the projector associated with states in which the hole is located at the corner of a given plaquette and in which the spins located at the three other corners are in a  $S=1/2$  state is always close to 1.

#### **III. SELF-CONSISTENT BORN APPROXIMATION**

In order to check if the properties of the hole observed in the presence of a magnetic field using exact calculations for small systems survive in the limit of large systems, we now evaluate the one-hole density of state and ground-state energy by means of perturbation theory based on the linear spinwave approximation.<sup>7</sup> Following Ref. 13 we rewrite the  $t$ -*J* Hamiltonian of Eq.  $(1)$  using the "slave fermion" representation  $c_{j\sigma}^{\dagger} = f_j b_{j\sigma}^{\dagger}$ , where  $f_j$  is a slave fermion and  $b_{j\sigma}^{\dagger}$ is a Schwinger boson. Working to lowest order in 1/*S* about the Neel state consisting of spin-down electrons in sublattice 1 and spin-up electrons in sublattice 2, one obtains<sup>13</sup>

$$
\mathcal{H} = \mathcal{H}_J + \mathcal{H}_t, \qquad (10)
$$

where the Heisenberg and hopping parts, respectively, are given by

$$
\mathcal{H}_J = \frac{SJ}{2} \sum_{\langle jl \rangle} \left[ b_j^\dagger b_j + b_l^\dagger b_l + b_j^\dagger b_l^\dagger + b_j b_l \right] \tag{11}
$$

and

where

$$
\mathcal{H}_t = \sqrt{2S} \sum_{\langle jl \rangle} t_{jl} f_j \dot{f}_l^{\dagger} [b_j^{\dagger} + b_l], \qquad (12)
$$

 $b_l \equiv \begin{cases} b_{l\uparrow} & \text{if } l \in 1 \\ b_{l\downarrow} & \text{if } l \in 2. \end{cases}$ 

The Heisenberg part of the Hamiltonian can be easily diagonalized by Fourier and Bogoliubov transformation, leading to

$$
\mathcal{H}_J = \sum_{\mathbf{k}} \ \Omega_k \beta_k^{\dagger} \beta_k \tag{13}
$$

and

$$
\mathcal{H}_{t} = \left(\frac{2S}{N}\right)^{1/2} \sum_{\langle j l \rangle k} t_{j l} f_{j l}^{\dagger} \left[ (e^{i\mathbf{k} \cdot \mathbf{j}} u_{k} + e^{i\mathbf{k} \cdot \mathbf{l}} v_{k}) \beta_{k}^{\dagger} + (e^{-i\mathbf{k} \cdot \mathbf{l}} u_{k} + e^{-i\mathbf{k} \cdot \mathbf{j}} v_{k}) \beta_{k} \right],
$$
\n(14)

where *k* denotes wave vectors in the Brillouin zone of the lattice,  $\mathbf{j}=(j_x,j_y)$ ,

$$
\Omega_k = z S J \sqrt{1 - \gamma_k^2}.
$$
\n(15)

 $z = 4$  is the coordination number, and  $\gamma_k = z^{-1} \sum_{\delta} e^{i\mathbf{k} \cdot \delta}$ , with  $\delta$  taking values  $(1,0), (-1,0), (0,1)$  and  $(0,-1),$ 

$$
\begin{bmatrix} \beta_k \\ \beta_{-k}^+ \end{bmatrix} = \begin{bmatrix} u_k & -v_k \\ -v_k & u_k \end{bmatrix} \begin{bmatrix} b_k \\ b_{-k}^+ \end{bmatrix},
$$

$$
u_k = \left[ \frac{1 + (1 - \gamma_k^2)^{1/2}}{2(1 - \gamma_k^2)^{1/2}} \right]^{1/2},
$$

$$
v_k = -sgn(\gamma_k) \left[ \frac{1 - (1 - \gamma_k^2)^{1/2}}{2(1 - \gamma_k^2)^{1/2}} \right]^{1/2},
$$

and where

$$
b_k = \frac{1}{N^{1/2}} \sum_j e^{i\mathbf{k} \cdot \mathbf{j}} b_j.
$$

We now focus on the fermion propagator

$$
G(j,l,\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \langle T[f_j(t)f_l^{\dagger}(0)] \rangle, \qquad (16)
$$

which provides an approximate description of the true hole propagator, $^{13}$  and write the Dyson equation

$$
G = [\omega + i\,\eta - \Sigma\,]^{-1},\tag{17}
$$

where *G* and  $\Sigma$ , respectively, stand for  $G(j, l, \omega)$  and  $\Sigma(j,l,\omega)$  taken at fixed frequency  $\omega$  and considered as  $N \times N$  matrices, and where the self-energy  $\Sigma(i, l, \omega)$  is given in the self-consistent Born approximation by



FIG. 4. Pertubative results for a square 16-site cluster for the density of states of a single hole. Same caption as in Fig. 1.

$$
\Sigma(j,l,\omega) = \frac{1}{N} \sum_{\delta_1 \delta_2 k} t_{ll+\delta_1} t_{j-\delta_2 j} e^{i\mathbf{k} \cdot \mathbf{l} - i\mathbf{k} \cdot \mathbf{j}} (u_k + e^{i\mathbf{k} \cdot \delta_1} v_k)
$$

$$
\times (u_k + e^{i\mathbf{k} \cdot \delta_2} v_k) G[j - \delta_2, l + \delta_1, \omega - \Omega(k)],
$$
(18)

where  $\delta_1$  and  $\delta_2$  are vectors connecting nearest neighbors.

Figures 4 and 5 display the density of state for the single hole evaluated by iterating Eqs.  $(17)$  and  $(18)$ . Again, the total width of the spectrum remains approximately unchanged as the magnetic field is turned on. The low-energy quasiparticle peak and the higher-energy string resonances<sup>1</sup> visible in Fig. 5 can be identified, respectively, as the ground state and excited states of the pair formed by the spinon and the holon, which are bound together by a *string* potential.<sup>6,3</sup> This binding of the spinon-holon pair can be described within the gauge theory of the *t*-*J* model as resulting from the *confinement*<sup>6,14</sup> of the gauge field to which spinon and holon fields are coupled.

Figures 6 and 7 display the perturbative results for the difference in ground-state energy in the presence and absence of a magnetic field as a function of field strength. Note the reduction in ground-state energy occurring as  $n_{\phi}$  is varied from 0 to 1/2. In contrast to exact results, perturbative results for this energy difference show a (weak) dependence in parameter *J*.

Figure 8 displays the ground-state energy of systems with a single hole as a function of parameter *J* for  $n<sub>φ</sub>=0$  and  $n_b=1/2$ . Let us consider the perturbative results for  $N=256$ , for which finite-size effects are negligible. These results are well fitted by



FIG. 5. Pertubative results for a square 36-site cluster  $(m=6, n=0)$  for the density of states of a single hole. Same caption as in Fig. 1.



FIG. 6. Perturbative results for a square 16-site cluster for the ground-state energy of a single hole. Same caption as in Fig. 2.



FIG. 7. Perturbative results for a square 36-site cluster for the ground-state energy of a single hole. Same caption as in Fig. 2.

$$
E_0 = \beta t + \gamma (J/t)^{2/3} t, \tag{19}
$$

with  $\beta = -3.102$  and  $\gamma = 1.727$  for  $n_b = 0$  (Ref. 15) and with  $\beta = -3.102$  and  $\gamma = 1.727$  for  $n_{\phi} = 0$  (Ker. 15) and parameter *J* visible in Figs. 6 and 7 results from the depen-<br> $\beta = -3.400$  and  $\gamma = 2.033$  for  $n_{\phi} = 1/2$ . The dependence in dence of the forefactor  $\gamma$  of



FIG. 8. Perturbative results for the ground-state energy of a single hole as a function of parameter *J* for  $n_{\phi} = 0$  and  $n_{\phi} = 1/2$  and for various system sizes. The energy is given in units of *t*. The dashed line corresponds to Eq.  $(19)$ .



FIG. 9. Perturbative results for a square 64-site cluster  $(m=8, n=0)$  for the ground-state energy of a single hole. Same caption as in Fig. 2. The error bars result from the finite broadening parameter  $\eta$ =0.0025 used in Eq. (17).

dence of the forefactor  $\gamma$  of the second term in the right-hand side of Eq. (19) in magnetic-field strength. The latter term can be regarded as the binding energy<sup>3</sup> of the spinon-holon pair bound by a string potential  $V(r) = \alpha J |\mathbf{r}|$ , described by the Hamiltonian

$$
H = \frac{1}{2m} \Delta + \alpha J |\mathbf{r}| + \beta t,\tag{20}
$$

where  $m$  is the mass of the  $\ell$ light) holon in orbit around the (heavy) spinon. The forefactor  $\gamma$  appearing in Eq. (19) is proportional to  $m^{1/3}(\alpha J)^{2/3}$ . Assuming that the string tension  $\alpha$  is not affected by the presence of the magnetic field, the dependence of the spinon-holon binding energy in magneticfield strength must be attributed to the dependence of the holon mass *m* in field strength, in the manner

$$
\frac{m(n_{\phi} = 0)}{m(n_{\phi} = 1/2)} \approx 1.63,
$$
\n(21)

where  $m(n_{\phi}=1/2)$  and  $m(n_{\phi}=0)$ , respectively, denote the holon mass in presence of half a flux quantum per plaquette, and in the absence of external field.

Equation  $(21)$  is in reasonable agreement with the prescription of the gauge theory of the  $t - J$  model<sup>6</sup> for the mass of the holon. In the cases  $n_{\phi}=0$  and  $n_{\phi}=1/2$  the Hamiltonian of Eq.  $(6)$  leads to band bottom effective masses whose ratio  $m(0)/m(1/2) = \sqrt{2}$ , which is in reasonable agreement with Eq.  $(21)$ . Note that the reduction in holon mass occurring as the external field is varied from zero to half a flux quantum per plaquette is qualitatively consistent with the explanation for the reduction of the single-hole ground-state energy in terms of the reduction of the kinetic energy of the holon. In the bulk limit and in the limit  $J\rightarrow 0$ the difference in ground-state energy between the cases  $n_{\phi}=0$  and  $n_{\phi}=1/2$  is equal to 0.3*t*, which is consistent with the exact results.

The limit of low magnetic field is of particular interest. If the size of the spinon-holon bound pair is much smaller than the magnetic length, the spinon-holon pair may be regarded as a quasiparticle with a definite cyclotron frequency. This may explain the increase in ground-state energy observed in Fig. 9 as the external field is varied from  $n_b=0$  to  $n_{\phi}$ =0.08.

In conclusion, we have examined the effect of coupling a single hole to a constant and homogeneous magnetic field by means of exact calculations for small systems and perturbative spinwave calculations for larger systems. These calculations are in reasonable agreement with each other, and indicate that the ground-state energy of the hole is decreased by about 0.3*t* as the field is varied from zero to half a flux quantum per plaquette. The fact that this energy decrease is proportional to *t* and depends weakly on parameter *J* indicates that the excitation carrying the charge of the hole and coupling to the external field disperses with *t*. This result, together with the presence<sup>3</sup> of a low-energy peak dispersing with *J* in the spectral density of the hole, constitutes evidence that the hole decays into two constituents, one carrying the charge of the hole and dispersing with *t* and the other one dispersing with *J*. The energy decrease observed as the field is varied from zero to half a flux quantum per plaquette can be explained qualitatively using the gauge theory of the *t*-*J* model, which provides a formal description of the decay products of the hole.

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