Drag effect for a bilayer charged-Bose-gas system

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The Coulomb-drag effect, which has previously been considered theoretically and experimentally for a system of two electron gas layers, is studied for a system of two charged-Bose-gas (CBG) layers within an analytically solvable model. We consider a bilayer CBG in the presence of counterflow which, in superfluid mixtures and in spatially separated superconductors, leads to a drag effect. We investigate the effects of counterflow on the collective excitations, interaction energy, screened interactions, induced charge densities, and plasmon density of states in the bilayer CBG. These quantities, many of which have not been considered in connection with Coulomb drag in a bilayer electron gas, show how the many-body properties are affected, and thus provide additional insights into the drag effect. [S0163-1829(96)03244-4]

I. INTRODUCTION

The discovery of high-T_c superconductivity in a class of materials containing CuO_2 planes has led to a great interest in the many-body properties of layered electronic systems. At the same time, advances in modern semiconductor technology have made possible the fabrication of double-layer and multilayer structures. These systems exhibit interesting physical effects, and provide a testing ground for our theoretical understanding of the electronic correlations in low dimensions.[1] Superconductivity in double-layer cuprate materials has also been reported.[2] The charge carriers obeying Bose statistics, namely, a charged Bose gas (CBG), is a useful model in the study of many-body effects,[3] and has recently been linked to a mechanism of superconductivity in some high-T_c superconductors. The latest collection of experimental results indicates that not all high-Tc superconductors display d-wave symmetry in their order parameter. Some show an s-wave symmetry, which makes it conceivable that a phonon-based mechanism such as the formation of bipolarons may be operative. Microscopic models of superconductivity with charged bosons formed by strong electron-phonon and electron-electron exchange interactions have been proposed.[4,5] CBG systems studied under various theoretical techniques[6,7] have recently gained renewed attention.

An interesting Coulomb coupling mechanism between two spatially separated superconducting layers or one-dimensional wires was studied by Duan and Yip[8] and Duan.[9] Their prediction of a strong supercurrent drag resulting from the Coulomb interaction has been experimentally explored,[10] and recently observed in normal metal-superconducting films.[11] The superconducting nature of one or both of the layers makes this effect somewhat different from the dissipative Coulomb drag mechanism, as observed in semiconducting quantum wells.[12] Motivated by these considerations, in this paper we investigate, in a bilayer CBG system, the analog of the drag effect as discussed by Duan.[9] We present some results for and insights into the Coulomb drag effect between two layers containing CBG or electron gas. We have recently been studying various aspects of double-layer CBG systems, in particular their collective excitations.[13] Furthermore, our model system offers some analytical facility at a computational level, since in obtaining the collective modes of a bilayer system we do not resort to low-frequency, small-wave-vector approximations. It is expected that the drag effect for CBG systems may have relevance to actual high-T_c materials where a Bose liquid picture is applicable. This also opens the possibility of an experimental observation of a CBG drag effect in a bilayer of high-T_c superconductors.

The rest of this paper is organized as follows. After some brief preliminaries on a single-layer CBG in Sec. II, we discuss the behavior of collective excitations of a double-layer CBG in the presence of counterflow in Sec. III. Using exact plasmon-dispersion relations, we calculate the zero-point energy difference which leads to the drag effect. In Secs. IV, V, and VI, we study the screened interactions, induced charge densities, and plasmon density of states, respectively, in double-layer CBG, with an emphasis on the counterflow effects. We discuss our results in Sec. VII, and conclude with a summary.

II. PRELIMINARIES

For our model we first need to consider some relevant physical quantities of a single-layer CBG. The excitation spectrum of a two-dimensional Bose gas within the Bogoliubov approximation is given by $\omega(q)/E_s=(x+x^4)^{1/2}$, where $E_s=q_s^2/2m$ and $x=q/q_s$. Here we use the definition[5] for the screening wave vector $q_s^3=32\pi e^2 n_0(T)m/\epsilon_0$, in which $2e$ and m are the boson charge and mass, respectively, and ϵ_0 is the dielectric constant of the background. $n_0(T)$ is the temperature-dependent condensate density. The Bogoliubov approximation is valid for $r_s=(16m^2e^4/\pi n\epsilon_0)^{1/2}<1$, and for temperatures around $T\simeq 0$. Nevertheless, in an approximate analysis one may extend the mean-field expression up to $T\simeq T_c$ by taking into account the depletion of the condensate at non-zero temperatures $T \leqslant T_c$. Fisher and Hohenberg[14] showed through a renormalization-group analysis that in the dilute limit, $n_0(T)=n(1-T/T_c)$. In the following discussion we assume that a bilayer system consists of two such condensates interacting via long-range Coulomb forces.

III. DOUBLE-LAYER CHARGED-BOSE-GAS WITH COUNTERFLOW

It will be of interest to consider a model for the drag effect, which is amenable to exact analytic manipulations. This is a two-layer CBG system (in the absence of any disorder) with current flows in each layer, characterized by their velocities V_1 and V_2. Decomposing this current flow into two parts:— (i) ''center-of-mass'' flow $V=(V_1+V_2)/2$, and (ii) ''counterflow'' $2v=V_1-V_2$— we may write[9]

$$\omega-qV_1=(\omega-qV)-qv,$$
$$\omega-qV_2=(\omega-qV)+qv.$$

Since the role of the center-of-mass flow is like a Galilean transformation, we concentrate on the counterflow part.[15] The first physical quantity we calculate is the collective modes of the system in the presence of counterflow. The dispersion relations of these will later be used also to study other quantities. For simplicity we consider only the random-phase approximation (RPA). The equation that gives the collective modes in the presence of counterflow is

$$[(y+x\tilde{v})^2-(x+x^4)][(y-x\tilde{v})_+^2-(x+x^4)]-x^2e^{-2x\tilde{d}}=0\,, \qquad (1)$$

in which $y=\omega/E_s$ and $x=q/q_s$. We have also defined $\tilde{v}=2mv/q_s$ and $\tilde{d}=dq_s$. The collective modes of the system can be calculated analytically,

$$\omega_\pm(q;v)=\{x^2\tilde{v}^2+(x+x^4)\pm[4\tilde{v}^2x^2(x+x^4)+x^2e^{-2x\tilde{d}}]^{1/2}\}^{1/2}. \qquad (2)$$

Note that for $\tilde{v}=0$ (no counterflow), the collective modes reduce to the familiar result[7,13] $\omega_\pm(q)=[x+x^4\pm xe^{-x\tilde{d}}]^{1/2}$. The long-wavelength limit (also $\tilde{d}\to 0$) of the dispersion relations with counterflow are

$$\omega_+\simeq\sqrt{2}x^{1/2}+\sqrt{2}(3\tilde{v}^2-\tilde{d})x^{3/2}/4+\mathcal{O}(x^{5/2}),$$
$$\omega_-\simeq(\tilde{d}-\tilde{v}^2)^{1/2}x+\frac{(\tilde{d}/2-\tilde{v}^2)^2-d^2/2}{(\tilde{d}-\tilde{v}^2)^{1/2}}x^2+\mathcal{O}(x^3), \qquad (3)$$

showing the optical (ω_+) and acoustic (ω_-) nature of the collective excitations. These are to be compared with our earlier results in the absence of counterflow.[13] In general, $\omega_+(q;v)$ and $\omega_-(q;v)$ will be hardened and softened, respectively, compared to the zero-current modes. We note that for large enough counterflow velocity v, the out-of-phase mode (ω_-) acquires damping, and can only propagate above a critical wave vector q_c. This critical wave vector, which depends on v and interlayer separation d, is calculated from

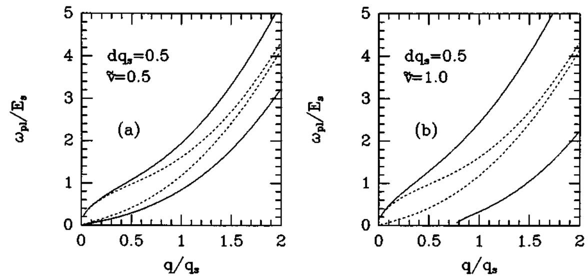

FIG. 1. Plasmon dispersions of a double-layer CBG in the presence of counterflow (solid lines) for $\tilde{d}=0.5$ and (a) $\tilde{v}=0.5$ and (b) $\tilde{v}=1$, compared with $\tilde{v}=0$ case (dotted lines).

$$v^2x_c^2+x_c+x_c^4-[4v^2x_c^2(x_x+x_c^4)+x_c^2e^{-2x_cd}]^{1/2}=0\,. \qquad (4)$$

In Fig. 1(a) we show the collective modes $\omega_\pm(q)$ in a double-layer CBG with (solid lines) and without (dotted lines) counterflow for $\tilde{d}=dq_s=0.5$ and $\tilde{v}=2mv/q_s=0.5$. For a large enough counterflow the acoustic plasmon mode (ω_-) starts off at a critical wave vector q_c, as depicted in Fig. 1(b) for $\tilde{d}=0.5$ and $\tilde{v}=1$. The above results are similar to the behavior of collective modes of a double-layer system in the presence of collisional broadening.[13] The main difference here is that the cutoff wave vector appears only beyond a critical counterflow velocity, whereas in the case of a disordered system damping will always be present for any finite amount of disorder. In the case with counterflow we have the possibility of going from a nondissipative drag effect to a dissipative one. It should be remarked that this damping arises due to a counterflow or drag, but not due to disorder. Such a damping has not previously been noted in the literature, to our knowledge. A similar behavior is also expected in systems obeying Fermi statistics with the counterflow (see the discussion in Sec. VII). We show the dependence of q_c on $\tilde{v}$ in Fig. 2.

Using the results for the dispersion relations, we now calculate the zero-point-energy (per unit area) change due to counterflow. This change is given by[9]

$$\Delta E^0=\tfrac{1}{2}\sum_q\,[\Delta\omega_++\Delta\omega_-], \qquad (5)$$

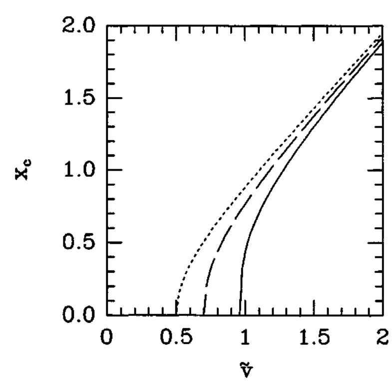

FIG. 2. The critical wave vector q_c below which out-of-phase plasmons (ω_-) do not propagate as a function of counterflow velocity. Solid, dashed, and dotted lines indicate interlayer spacings $\tilde{d}=1$, 0.5, and 0.25, respectively.

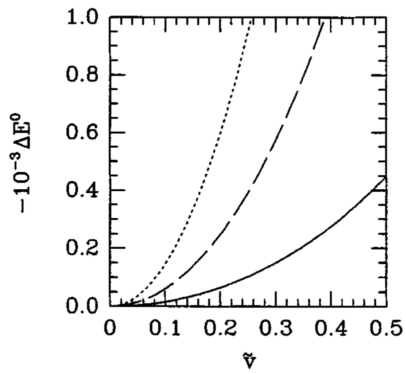

FIG. 3. The zero-point energy of a double-layer CBG as a function of the counterflow velocity $\tilde{v}$ for interlayer spacings $\tilde{d}=2$ (solid), $\tilde{d}=1$ (dashed), and $\tilde{d}=0.5$ (dotted).

where $\Delta\omega_{\pm}=\omega_{\pm}(q;v)-\omega_{\pm}(q;0)$. Considering only the acoustic mode ω_- and integrating up to $q\sim 1/d$, we first obtain a crude estimate, $\Delta E^0\approx-(q_s^2/24\pi)\tilde{v}^2/\tilde{d}^{7/2}$ for the zero-point energy which is valid to leading order in $\tilde{v}$. As may be seen in Fig. 1, $\Delta\omega_+$ and $\Delta\omega_-$ individually diverge when integrated over all wave vectors, but the sum in Eq. (5) yields a convergent answer when the full dispersion relations [Eq. (2)] are used. Our numerical results ΔE^0 as a function of counterflow for various layer separations are shown in Fig. 3. The main conclusions of this section are as follows. (i) ΔE^0 is negative, implying that the total energy of the bilayer CBG system is lowered by the counterflow. (ii) ΔE^0 is in leading order proportional to $\tilde{v}^2$. (iii) The dependence of ΔE^0 on the layer separation is to leading order $\sim\tilde{d}^{-7/2}$. The former two results are also shared with a bilayer (or double-wire) electron-gas system.[8,9,16] The last result, on the other hand, depends on dimensionality as well as on statistics. For two parallel wires Rojo and Mahan[17] find $\Delta E^0\sim d^{-2}$, whereas Duan[9] reports $\Delta E^0\sim d^{-3}$ for a double-layer system.

The physical significance of the dependence of the zero-point energy on counterflow velocities may be understood as follows. We can combine ΔE^0 with the kinetic energy of the charged bosons ($\frac{1}{2}n_0mv^2$ for each layer) to obtain an effective free energy, F. The partial derivative of F with respect to the velocity in a given layer yields the current density j in the same layer. For instance, the current density in the second layer becomes

$$j_2=\left(\tfrac{1}{2}n_0-\frac{q_s^2}{24\pi\tilde{d}^{7/2}}\right)\tilde{v}_2+\frac{q_s^2}{24\pi\tilde{d}^{7/2}}\tilde{v}_1, \qquad (6)$$

where we have denoted the drift velocities v_1 and v_2 in layers 1 and 2, respectively. The above argument shows that the current in the second layer will depend on the velocities in both layers. This is called the drag effect, as is well known in two-component superfluid systems,[18] and as has recently been discussed by Duan[9] for spatially separated electronic superconductors.

IV. SCREENED INTERACTIONS

A test charge in an interacting charged Bose system is screened at large distances and the screened potential

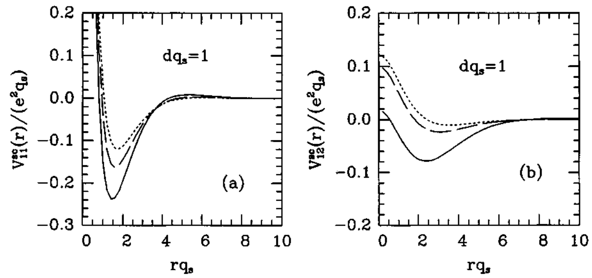

FIG. 4. (a) The intralayer and (b) interlayer screened interactions in the presence of counterflow for $\tilde{v}=0.1$ (dotted), 0.5 (dashed), and 0.75 (solid).

$V^{sc}(r)$ exhibits a different behavior than the bare Coulomb potential. The screening properties of single and double-layer CBG were studied by Gold[19] and by Tanatar and Das[13], respectively, in the absence of counterflow. For a double-layer system the screened interactions are written in matrix notation $v_{ij}^{sc}(q)=\Sigma_k v_{ik}(q)[\varepsilon^{-1}(q)]_{kj}$, where the elements of the static dielectric function are defined as $\varepsilon_{ij}(q)=\delta_{ij}-v_{ij}(q)\chi_{ii}^0(q)$, in the RPA. The real-space expressions for the screened interactions in the presence of counterflow are obtained by Fourier transformation,

$$V_{11}^{sc}(r)=(e^2q_s)\left\{\frac{1}{\tilde{r}}-\int_0^\infty dx\,\frac{(1+e^{-x\tilde{d}})J_0(x\tilde{r})}{1+x^3-x\tilde{v}^2+e^{-x\tilde{d}}}\right\}, \qquad (7)$$

$$V_{12}^{sc}(r)=(e^2q_s)\int_0^\infty dx\,\frac{x^2(x^2-\tilde{v}^2)^2e^{-x\tilde{d}}J_0(x\tilde{r})}{(1+x^3-x\tilde{v}^2)^2-e^{-2x\tilde{d}}}, \qquad (8)$$

where $J_0(x)$ is the zeroth-order Bessel function of the first kind. The above expressions reduce to the previously reported[13] results as $\tilde{v}\to 0$. In Fig. 4 we show the intralayer and interlayer screened interactions for various counterflow velocities. Dotted, dashed, and solid lines indicate $\tilde{v}=0.1$, 0.5, and 0.75, respectively, at $\tilde{d}=1$. The short-range attractive part in V_{11}^{sc} and V_{12}^{sc} is enhanced by the presence of counterflow. This exemplifies a physical manifestation of the drag effect. Such an attractive interaction may have some interesting consequences for superconductivity in these systems.

V. INDUCED CHARGE DENSITIES

Further insights into the drag effect in a bilayer CBG system can be obtained by considering the ratio of the induced charge densities in individual layers when the collective modes are excited. Using the linear-response theory we relate the induced charge densities $\sigma_{1,2}(q,\omega)$ in layers 1 and 2, respectively, to $V_{\rm total}^{1,2}(q,\omega)$, which includes the external and effective potentials from the interactions, as[20]

$$\sigma_1(q,\omega)=\chi_{11}^0V_{\rm total}^1(q,\omega),\ \text{ and }\ \sigma_2(q,\omega)=\chi_{22}^0V_{\rm total}^2(q,\omega),$$

with

$$\sigma_1(q,\omega)=\chi_{11}^0(q,\omega)[v_{11}(q)\sigma_1(q,\omega)+v_{12}(q)\sigma_2(q,\omega)],$$
$$\sigma_2(q,\omega)=\chi_{22}^0(q,\omega)[v_{12}(q)\sigma_1(q,\omega)+v_{22}(q)\sigma_2(q,\omega)], \qquad (9)$$

where the intralayer and interlayer bare Coulomb interactions are given by $v_{11}(q)=v_{22}(q)=2\pi e^2/q$, and $v_{12}(q)=v_{21}(q)=v_{11}(q)e^{-qd}$, respectively. At resonance, namely, for a nonzero response for vanishing external potential, $\omega=\omega_{\pm}(q)$, and the ratio of induced charge densities is calculated from the above relations. In the absence of counterflow ($\tilde{v}=0$) and assuming identical layers, we simply have $(\sigma_1/\sigma_2)_{\pm}=\pm 1$, reflecting the in-phase and out-of-phase oscillations of the charges. When there is counterflow we explicitly obtain

$$\left(\frac{\sigma_1}{\sigma_2}\right)_{\pm}=\frac{xe^{-x\tilde{d}}}{(y\pm x\tilde{v})^2-(x+x^4)}\Bigg|_{y=\omega_{\pm}}. \tag{10}$$

The presence of counterflow renders the ratio of induced charge densities $\tilde{v}$ dependent. In Fig. 5, we show σ_1/σ_2 as a function of q for a bilayer CBG with $\tilde{d}=0.5$ and $\tilde{v}=1$.

VI. PLASMON DENSITY OF STATES

The plasmon density of states has been found useful in interpreting the photoelectron spectra in layered materials, particularly high-T_c superconductors.[21] We calculate the plasmon density of states by using the relation $\rho_{\rm pl}(\omega)=\Sigma_q\delta(\omega-\omega_{\pm}(q;v))$, in the presence of counterflow. We find

$$\rho_{\rm pl}(\omega)=\frac{m}{\pi}\frac{2yx_0}{2x_0\tilde{v}^2+1+4x_0^3\pm\dfrac{6\tilde{v}^2x_0^2(1+2x_0^2)+x_0(1-x_0d)e^{-2x_0\tilde{d}}}{[4\tilde{v}^2x_0^2(x_0+x_0^4)+x_0^2e^{-2x_0\tilde{d}}]^{1/2}}}, \tag{11}$$

where x_0 is the solution of $y=\omega_{\pm}(x_0,v)$ in which we have used the scaled quantities $y=\omega/E_s$ and $x_0=q_0/q_s$. In Fig. 6 we display the density of states for plasmon branches ω_- (upper curves) and ω_+ (lower curves) in a double-layer CBG with $\tilde{d}=0.5$. The dotted lines refer to the case without counterflow (i.e., $\tilde{v}=0$), and the solid lines correspond to a counterflow velocity $\tilde{v}=0.2$. Even in the absence of counterflow, we observe that the acoustic mode exhibits a peak around $\omega/E_s\sim 1$. When the counterflow is present, we note that the charges oscillating out of phase (ω_- mode, upper curves) are influenced much more than their in-phase counterparts.

VII. DISCUSSION

In the preceding sections we studied effects of a counterflow or drag on various properties of a double-layer CBG system. As stated earlier, to our knowledge many of these properties have not previously been studied in the context of a drag effect. However, we would like to make some comparison of our findings on the collective excitations with those of Duan and Yip[8] and Duan,[9] who discussed the phenomenon of supercurrent drag (due to Coulomb interaction) in double-wire and double-layer superconducting systems, respectively. For this purpose we may consider a bilayer electron gas and use an approximate response function for a single layer (in the presence of counterflow), namely,

$$\chi^0_{ii}(q,\omega)=\frac{m}{\pi}\frac{v_{Fi}^2q^2/2}{\omega^2-v_{Fi}^2q^2/2}, \tag{12}$$

for $q\ll k_{Fi}$, $v_{Fi}^2q^2\ll\omega\ll E_{Fi}$ and $i=1$ and 2. v_{Fi}^2 in the above equation is the Fermi velocity in ith layer. Within the RPA the collective modes of a double-layer system are then obtained by solving $\mathrm{Det}[\chi^{-1}]=0$, where

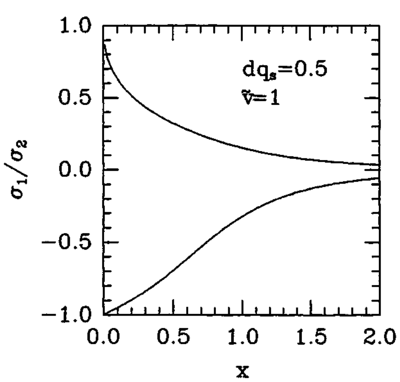

FIG. 5. The ratio of the induced charge densities at resonance frequencies $\omega_{\pm}(q,v)$ for $\tilde{d}=0.5$ and $\tilde{v}=1$. The upper and lower curves indicate $=+$ and $-$, respectively.

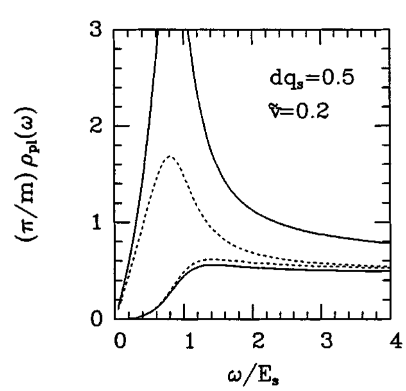

FIG. 6. The plasmon density of states in a double-layer CBG with (solid lines) and without (dotted lines) counterflow. The upper and lower curves represent the out-of-phase (ω_-) and in-phase (ω_+) plasma oscillations, respectively. We take $\tilde{d}=0.5$ and $\tilde{v}=0.2$.

$$[\chi(q,\omega)]^{-1}=\begin{pmatrix}[\chi_{11}^{0}(q,\omega)]^{-1}-v_{11}(q) & -v_{12}(q)\\ -v_{21}(q) & [\chi_{22}^{0}(q,\omega)]^{-1}-v_{22}(q)\end{pmatrix}. \quad (13)$$

For the Coulomb interaction we have $v_{11}(q)=2\pi e^2/q$ and $v_{12}(q)=v_{11}(q)e^{-qd}$, and assuming identical layers with drift velocities V_1 and V_2 in layers 1 and 2, respectively, one obtains the determinant equation given by Duan [Eq. (3.15) in Ref. 9]. The original derivation of Duan[9] was based on the hydrodynamic considerations (continuity equation) and Maxwell's equations. Presumably due to the approximations involved in the density-response function, the drag effect thus obtained appears to be dissipationless. On the other hand we find that, in our case, if the counterflow velocity $\tilde{v}$ exceeds a critical value the acoustic plasmon enters the particle-hole continuum at a very small wave vector q_c, and undergoes Landau damping. Such a damping mechanism may render the drag effect discussed by Duan[9] dissipative. It may be possible to observe this dissipation for large enough counterflow velocities in an experiment similar to that performed by Huang, Bazán, and Bernstein.[10] We illustrate the qualitative changes on the plasmon dispersions in an electronic system brought about by increasing $\tilde{v}$ in Fig. 7. We note that for small velocities, the plasmon dispersions are similar to those without the counterflow [see Fig. 7(a)]. At large $\tilde{v}$, the acoustic plasmon mode is affected much more than the optical mode, and exhibits the drastic behavior shown in Fig. 7(b).

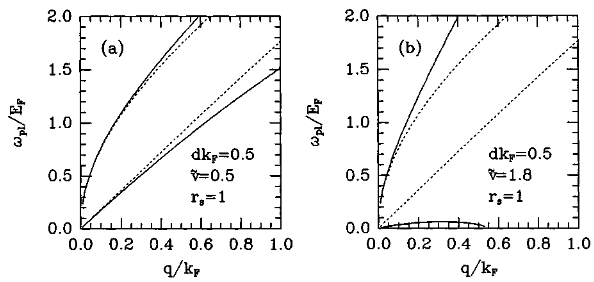

FIG. 7. The plasmon dispersions in a double-layer electron gas in the presence of counterflow (solid lines) for $\tilde{d}=0.5$ and (a) $\tilde{v}=0.5$ and (b) $\tilde{v}=1.8$, compared with $\tilde{v}=0$ case (dotted lines).

VIII. CONCLUDING REMARKS

We have presented our study of the drag effect in a bilayer CBG system. Instead of considering the transresistance which has been investigated extensively for bilayer electron-gas systems, we have studied a number of other quantities such as collective excitations, screened interactions, induced charge densities, and plasmon densities of states for our system in the presence of a counterflow or drag. These many-body quantities are affected by the drag to varying degrees. The choice of a bilayer CBG system offers analytical and computational advantages, and furnishes us with a simple model to study various interesting aspects of the Coulomb drag effect. Some experimental implications of our findings have also been pointed out.

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