Cyclotron resonance and spin states in $GaAs/Ga_{1-x}Al_xAs$ heterojunctions: Experiment and theory

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The cyclotron resonance of high-mobility $GaAs/Al_xGa_{1-x}As$ heterojunctions displays both a temperature and Landau-level occupancy ν dependence. For occupancies below $\sim 1/10$, the behavior is identical to that found in high-purity bulk GaAs with spin splitting of the resonance. The position of the two peaks changes only slightly as the temperature is raised from 0.1 to 2 K. For $\frac{1}{10} \leqslant \nu \leqslant \frac{1}{6}$, the peak position and relative peak intensity of the two peaks shifts radically as the temperature is raised or the density varied. At integral and greater occupancies only a single cyclotron resonance peak is observed whose position changes very little with temperature. Finally, the fractional regime with occupancies between $\frac{1}{6}$ and 1 shows only a single cyclotron resonance with a slight temperature dependence and provides no evidence that the many-body interactions responsible for the fractional quantum Hall effect influence the cyclotron resonance. The experimental cyclotron resonance behavior can be explained by a recently published theory by Cooper and Chalker which models the system in terms of the Coulomb interaction and the thermal population of both spin states. A rigorous comparison between data taken over a wide range of densities and temperatures and the theoretical model establishes the validity of this explanation. [S0163-1829(96)03843-X]

I. INTRODUCTION

Cyclotron resonance (CR) has proved to be a very useful experimental technique in the study of the two-dimensional electron system (2DES) in high-mobility $GaAs/Ga_{1-x}Al_xAs$ heterojunctions, providing valuable information on, e.g., carrier scattering mechanisms, electron-optic phonon coupling effects, and the shape and width of the confining potential. Recently, however, CR has also been used as an indirect probe of electron-electron interactions in the 2DES at very low Landau-level filling factor $\nu(=N_s h/eB)$, i.e., conditions of low carrier density N_s and/or high magnetic field B under which electrical transport measurements become very difficult or even impossible (see, e.g., Refs. 1–3). Most of the latter studies focus on manifestations of effects due to the Zeeman splitting of individual Landau levels. CR is a spin-conserving effect and because the g factor in GaAs is energy dependent, the observation of two distinct transitions might be expected when both spin-split halves of the lower Landau level involved are populated.[2] Initial experiments[4] looked for the expected splitting of CR in the region $1<\nu<2$; however, no splitting was observed. This negative observation was interpreted as the result of electron-electron interactions, and later in this paper we shall demonstrate that this surmise is correct. However, similar experiments in the extreme quantum limit revealed a splitting that was occupancy and temperature dependent,[5,6] only becoming resolved for $\nu<1/6$. This second peak was initially cited as evidence for the magnetically induced Wigner solid.[5,7] Further work suggested that the origin of the two peaks was due to the spin splitting of the Landau levels rather than formation of the Wigner crystal.[1–3] The unusual temperature and density dependence, however, could not be explained by a single-particle picture. One possibility suggested to explain this behavior[3] was electron-electron interactions[8] or mode coupling.[9] In a detailed explanation, however, Cooper and Chalker (CC) (Ref. 10) considered the extreme quantum limit and modeled the 2DES as two compressible states composed of electrons in the different spin orientations. The cyclotron motions of the spin states couple through the Coulomb interaction which is a function of the total electron density. The temperature of the system sets the population of the two spin states through the Boltzmann distribution function. With the system's temperature and electron density as the only entered variables, the CC calculation gave computationally generated spectra which reproduced the features of the experimental data for similar occupancies and temperatures.

In this paper, we report a systematic study of the CR of the 2DES over a filling factor range that extends from $\nu=1/17$ up to occupancies greater than 2. Changes in the occupancy are made by using a series of high-mobility heterojunctions and with continuous illumination from a visible laser, which results in a dynamic depopulation of the 2DES. The temperature of the system is varied from 0.05 to 14 K through the use of several experimental configurations including a specially modified Helium dilution refrigerator, ca-

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pable of holding temperatures below that at which the electrons are predicted to condense into an ordered solid or the Wigner crystal.[11] These experimental results demonstrate that the CC model not only accurately predicts the observed behavior of the CR in the extreme quantum limit but also allows the mean position of the CR to be used as a direct measure of the spin polarization, even above integral occupancies.

The paper is organized as follows. Section II provides a brief review of cyclotron resonance in the 2DES and introduces the elements of the CC model which are necessary to interpret the experimental data presented. Section III describes the experimental apparatus, and Sec. IV provides a systematic comparison between the data obtained and the predictions of the CC model. A summary is given in Sec. V.

II. CYCLOTRON RESONANCE OF 2D SYSTEMS AND THE COOPER-CHALKER MODEL

Cyclotron resonance is detected through the resonant absorption of far-infrared radiation at the cyclotron frequency given by $\omega_c = eB/m^*$, where m^* is the single carrier effective mass. Illumination with the far-infrared constitutes a long wavelength (small k) perturbation of the electron gas or plasma. The collective response of an electron plasma to such a disturbance occurs at the plasma frequency, ω_p. For bulk semiconductors, the plasma frequency approaches a finite value as k goes to zero. In two dimensions, however, $\omega_p \to 0$ when $k \to 0$. When a 2D electron system is placed in a magnetic field, Landau quantization of the energy levels occurs and the plasma excitation modes occur close to integral multiples of ω_c. At long wavelengths, the lowest of these modes is plasmonlike and is referred to as the magnetoplasmon. The magnetoplasmon mode is equivalent to the formation of a *magnetic exciton,* which would correspond to the energy required to promote an electron from an occupied Landau level to an unoccupied Landau level leaving a hole behind in the lower level. As a result, the dispersion relation of a 2DES in a perpendicular magnetic field can be expressed as

$$E(k) = \hbar\omega_c + \delta E(k), \tag{1}$$

with $\hbar\omega_c$ the cyclotron energy and $\delta E(k)$ the energy of the magnetoexciton which is due to the correlation between electrons. Pure cyclotron resonance corresponds to the zero k-vector limit of this dispersion.

For a system that is translationally invariant, cyclotron resonance is independent of any electron-electron interactions: the system resonates at a single frequency $\omega_c = eB/m^*$. This result is known as Kohn's theorem[12] and follows from the fact that the long wavelength electric field couples only to the center of mass motion. Kohn's theorem leads to the conclusion that $\delta E(k) \to 0$ as $k \to 0$. This has been confirmed by several calculations of the magnetoplasmon mode made by various authors.[13,14] However, translational invariance can be broken by dopant impurities,[14,15] lattice defects or nonparabolicity of the conduction band,[16] which could cause the $k \neq 0$ modes to mix with the cyclotron mode at $k=0$. Mode mixing can lead to changes in the effective mass and resonance linewidth. An early example of this behavior occurred in silicon inversion layers, where a large increase of the CR effective mass was observed where a population of higher subbands with a different effective mass occurred due to higher temperatures[17] or with the application of uniaxial strain which lowers the higher effective mass subbands.[18] These data were explained using either mode coupling theory[9] in which the different masses were coupled through mutual Coulomb interactions, or by electron-electron scattering[8] in which the CR modes merge due to motional narrowing in the limit of strong scattering.

Predictions of possible Wigner crystallization at low filling factors have led to very recent studies of the magnetoplasmon where the magnetic length is kept small relative to the interparticle spacing. Iordanskii and Muzykantskii[19] have predicted additional structure in the absorption spectrum due to the pinning of the periodic crystal potential by potential fluctuations. Côté and MacDonald find that for $\nu \leqslant \frac{1}{3}$ exchange effects become small and the strength of the collective excitation decreases as the occupancy is reduced[20].

The most recent theoretical approach to explain the CR spectra of a two 2DES has been made by Cooper and Chalker.[10] Their formalism is a marked departure from previous studies in that they considered a *compressible* regime where Coulomb interaction between excitations of the two different spin states is taken into account. MacDonald and Kallin only considered incompressible systems where a Landau level (or spin state) was completely filled. The calculation of CC has since been extended to include broadening effects on the CR from radiation damping and thermal motion of the electrons.[21] Similar theory has also been used to explain behavior of the CR where electron-electron interactions between the ground and higher thermally populated Landau levels causes shifts in the resonance position.[22]

The CC model places electrons on the sites of a triangular lattice and assigns a spin to each with the spin population set by the physical temperature of the system through the Boltzmann distribution function. The negative g factor of GaAs results in the spin ↑ electrons lying lowest in energy, making this the majority spin state. At a fixed magnetic field, minority spins undergo CR at a lower frequency, which may be thought of as arising from a slightly larger cyclotron mass. The two spin states couple with each other through the Coulomb force; the strength of the interaction, I, is proportional to the carrier density ($I \propto N^{3/2}$) and is given by

$$I = \frac{e^2 l^2}{8\pi\epsilon\epsilon_0 a^3}\,\frac{1}{\hbar\,\delta\omega_c} = \frac{e}{\pi\epsilon\epsilon_0\kappa}\left(\frac{\sqrt{3}\nu e}{8hB}\right)^{3/2}, \tag{2}$$

where $a (=[(\sqrt{3}/2)N_s]^{-1/2})$ is the interparticle spacing, l is the Larmor radius, and $\delta\omega_c (=\kappa B^2)$ is the difference in energy of the two transitions. Beginning with these assumptions, CC calculate the normal modes of an interacting two-component plasma with frequencies near the cyclotron frequency. It is important to stress that the CC model is an absolute calculation of the energy levels of an interacting plasma that is dependent on only two experimental parameters: the spin population determined from the system temperature and the interaction strength determined from the density.

It was also shown that the detailed numerical results could be simulated by a simplified model in which each electron was coupled equally to all others, with a coupling constant inversely proportional to the total number of particles. Analytic expressions for the two supported frequency modes from this model enable a quantitative evaluation of the experimental data within the CC framework. The simplified model supports two optically active modes which represent the in-phase and out-of-phase oscillations of the two spin populations, with frequencies

$$\omega_{\rm in}=[\alpha-1-\sqrt{(1-\alpha)^2+4\alpha p}]/2, \tag{3}$$

$$\omega_{\rm out}=[\alpha-1+\sqrt{(1-\alpha)^2+4\alpha p}]/2, \tag{4}$$

which are measured relative to the majority spin frequency in units of the splitting $\delta\omega_c$, and normalized so that $\delta\omega_c=0{,}1$ for the majority and minority spins, respectively. The oscillator strengths are given by

$$S_{\rm in}=1-S_{\rm out}=\frac{\omega_{\rm out}+p}{\omega_{\rm out}-\omega_{\rm in}}, \tag{5}$$

where p is the fractional proportion of minority spins. A comparison of the simple model to the full numerical calculation requires that the coupling constant $\alpha=11.034I$ in order to match the results for the two calculation procedures.

CC conclude their theory by postulating that the weighted mean cyclotron frequency is a direct measure of the relative population of the two different spin states. Although presented in Ref. 10 as applicable for occupancies within the extreme quantum limit, this result has been extended and shown to be valid for *any* filling fraction within the assumptions (1) that there is no disorder within the system and (2) no coupling of the Landau levels.[23]

A move from the CC terminology to the experimental parameters is straightforward. Experimental results are presented in terms of temperature, carrier density, and the splitting between the two resonance positions. The temperature in Kelvin determines the proportion of electrons with minority spin using the Fermi-Dirac distribution function,

$$p=\frac{e^{-E_z/k_BT}}{1+e^{-E_z/k_BT}}, \tag{6}$$

where the Zeeman energy $E_z=\mu_B(g^*+g'B)B$. B is measured in tesla and $g'=0.004T^{-1}$ represents the slight field dependence of the g factor. The total carrier density N_s enters the interaction strength I with $I\propto N_s^{3/2}$. The splitting of the CR corresponds to

$$\omega_{\rm in}-\omega_{\rm out}=\sqrt{(1-\alpha)^2+4\alpha p}. \tag{7}$$

There is a critical value of $\alpha=1$, corresponding to the density at which the splitting will go to zero at low temperatures, and which will give a single strong collective resonance at higher temperatures. This will be used at a later stage to define a characteristic occupancy ν_c which separates a radical difference in the system's response to changes in temperature.

III. EXPERIMENTAL DETAILS

The experiments described in this paper consist of monitoring the transmission of radiation through GaAs/Ga$_{1-x}$Al$_x$As heterojunctions as a function of either frequency or magnetic field, using either a far-infrared laser or Fourier transform spectrometer. A helium dilution refrigerator, ^{3}He system, and standard 4 K cryostat were alternately used in order to take readings over a temperature range from 0.05 to 14 K. The samples consist of a series of high-mobility GaAs/Ga$_{1-x}$Al$_x$As heterojunctions grown by molecular beam epitaxy at Philips Research Laboratory, Redhill.[24] The samples had spacer layer thicknesses of 4800, 2400, 1600, and 1200 Å with carrier densities after illumination from 3 to 12×10^{10} cm^{-2}. Reduction of the density by up to 70% was also achieved *in situ* by using continuous illumination from a red HeNe laser carried to the sample by a 1 mm silica optical fiber. With energy greater than the (Ga,Al)As band gap, the light depopulates the 2DES;[25] a power density of 10^{-3} W/cm^2 reduced the lower bound on the carrier concentration to 1×10^{10} cm^{-2}. The power density was altered through the use of externally placed neutral density filters. Even the dilution refrigerator was reasonably tolerant of this additional source of heat, holding the base temperature for all but the largest power densities, with a full HeNe power of 0.5 mW.

Electrical contacts allowed simultaneous transport measurements to be made on the high density sample, enabling a precise comparison of optical and transport measurements of the carrier density. Conversion of absorption strength to carrier concentration was made using the classical Drude absorption formula.

IV. EXPERIMENTAL RESULTS

The majority of the experimental data to be presented correspond to Landau-level occupancies ν less than 1/6, where changes in the carrier density and temperature of the 2DES have been shown to dramatically affect the cyclotron resonance spectra.[1–3] This is also the occupancy region which is directly addressed by the CC model.[10]

As has been mentioned in Sec. I, the spin splitting only manifests itself in CR measurements because of the energy dependence of the g factor, which changes the magnitude of the splitting for successive Landau levels. This leads to two possible transitions for an electron going from the $N=0$ to $N=1$ Landau level. The predicted field location of the spin↑ and spin↓ transitions is carrier concentration dependent due to band nonparabolicity and can be approximated by the formula,

$$m^*=m_0\left[1-\frac{2K_2}{E_g}(E_{\rm CR}+\langle T_z\rangle)\right], \tag{8}$$

where K_2 is a parameter describing the slope of the effective mass when plotted as a function of energy, m_0 is the band edge effective mass, $\langle T_z\rangle$ is the kinetic energy due to the shape of the 2DES confinement potential, E_g the GaAs band gap, and $E_{\rm CR}=\hbar\omega_c\pm g^*\mu_BB$ the cyclotron energy.[26–29] The splitting has a characteristic B^2 dependence because the magnetic field enters both the cyclotron separation between the Landau levels and the g factor difference between the ground and first Landau level.[26,27] If the electrons do not interact and behave as a classical gas of single particles, the CR spectrum of a 2DES should be identical to that found in high-purity, low carrier concentration bulk GaAs. Bulk GaAs

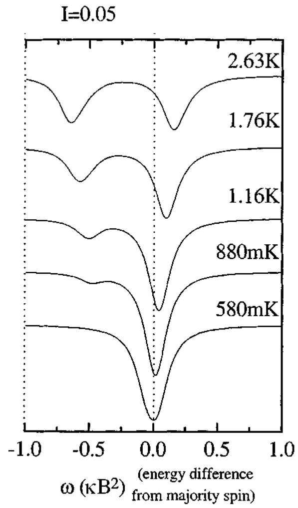

FIG. 1. Cooper and Chalker generated spectra for an interaction strength $I=0.05$ match experimental CR traces for an occupancy $\nu=\frac{1}{12}$.

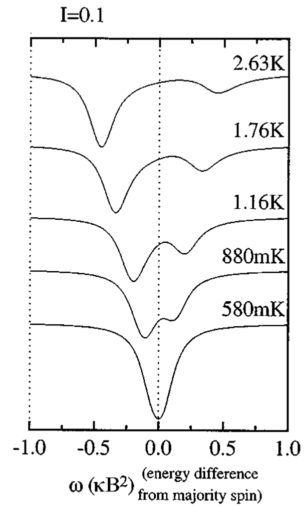

FIG. 2. Cooper and Chalker generated spectra for an interaction strength $I=0.1$ match experimental CR traces for an occupancy $\nu=\frac{1}{9}$.

has a $N=0$ g factor of ~ -0.4 resulting in two spin-conserving transitions split by 60 mT (3 K) at 13 T with the spin $\uparrow$ state lowest in energy.

A. The extreme quantum limit, $\nu\leqslant\frac{1}{6}$

We now discuss separately the temperature and density dependence and fit the data to the predictions of the Cooper and Chalker model.

1. Temperature dependence

The temperature dependence was established by using the ^{3}He system in conjunction with either the Fourier transform spectrometer or far-infrared laser. Experiments with the spectrometer were performed at constant magnetic field while the frequency was swept; the reverse situation applies for measurements with the laser. The magnetic field axis on the graphs for the laser data are reversed in order to preserve the peak ordering between the different experimental configurations. Three sets of data are presented which illustrate the temperature dependence for different occupancies below $\nu=\frac{1}{6}$. Matched with each series of experimental spectra are theoretical CR spectra generated by the CC model for similar temperatures and densities.

Figure 1(a) from Ref. 10 shows CR traces as a function of temperature for a very low occupancy of $\frac{1}{12}$. Most of the carriers are seen to reside in the lower field peak at base temperature. As the sample temperature is raised, the intensity of the higher field peak is increased until the strength of both peaks is roughly equal by 1.7 K. Drude model analysis of the absorption spectra confirms that the total carrier density remains constant while the intensity transfer takes place. The resonance peak positions vary little with increasing temperature. The absolute positions of both resonances are close to the field positions of the spin-split transitions seen in bulk GaAs, with the splitting reduced by $\sim$20% from the bulk value. Previous work has demonstrated that the correspondence is exact ($\pm$5 mT) in the low density limit where there would be no correction for band nonparabolicity arising from confinement of the electrons at the interface.[1] Similar behavior at very low occupancies has been observed by other researchers over a much larger temperature range.[3] Figure 1 is the CR spectra generated by the CC model for an interaction strength of 0.05.

The temperature dependence of the CR at the lowest occupancy of 1/12 can be adequately described within a single-particle picture: the peak positions are close to the spin transitions seen in high-purity GaAs with only weak shifts in peak positions with temperature and the relative populations of the spin states follow simple Boltzmann statistics such that the system is spin polarized at low temperatures.[1]

If the 2DES carrier density is raised slightly to 3.4×10^{10} cm^{-2} to give an occupancy of $\frac{1}{9}$, a very different response to temperature is observed [Fig. 1(b) from Ref. 10]. The base temperature of 0.37 K produces only a single peak with a position that corresponds to the lower energy spin$\uparrow$ transition in bulk GaAs corrected for band nonparabolicity. As the temperature is raised, the single peak splits into two resonances which continue to move apart at higher temperatures. Gradually, most of the intensity is shifted into the higher field peak which moves upward in field by a distance which is close to half the single-particle splitting. The magnitude of the splitting between the peaks at the intermediate temperature of 1.7 K is significantly less than the expected splitting given by the single-particle g factor for these fields. Another study at similar occupancies[3] has shown that further increases in temperature result in virtually all the oscillator strength shifting over to the high field peak by 13 K. Figure 2 is the CR spectra produced by the CC model for an interaction strength of 0.1.

An increase in the occupancy to $\frac{1}{6}$, where only a single peak is seen in the CR spectra, also changes the temperature dependence. Figure 3(a) shows a series of CR spectra as a function of temperature at 14 T with an occupancy of $\sim\frac{1}{6}$ ($N_s=5\times10^{10}$ cm^{-2}). The Fourier transform spectrometer was used for this series of data. The single CR peak shifts to lower energy as the temperature is raised from 0.36 to 4.0 K. The peak position at base temperature corresponds to the

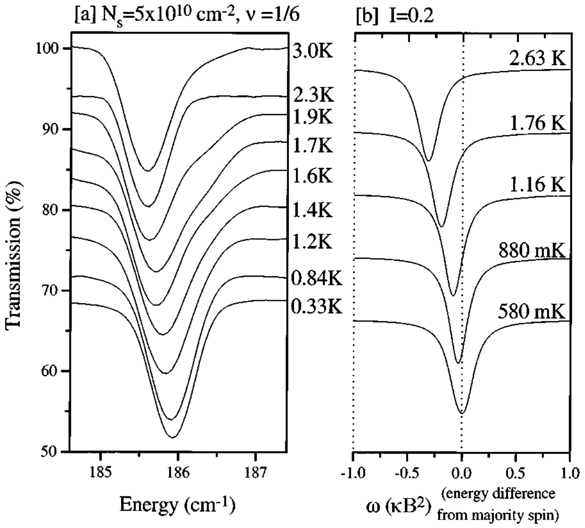

FIG. 3. (a) Experimental CR traces for an occupancy $\nu = \frac{1}{6}$ are matched with (b) Cooper and Chalker generated spectra for an interaction strength $I = 0.2$.

spin ↑ transition if an allowance is made for the carrier concentration dependent nonparabolicity concentration. The highest temperature sees the single peak shifted lower in energy by approximately half of the predicted single-particle splitting. Experiments in the dilution refrigerator where the base temperature was extended to below 0.15 K show a similar temperature dependence for these low occupancies. Figure 3(b) is the CR spectra produced by the CC model for an interaction strength of 0.2.

The best fits between the experimental data and the theory were obtained for values of the interaction strength of $I = 0.05$, 0.1, and 0.2 which are slightly lower than the values deduced from the experimental densities which give $I = 0.1$, 0.15, and 0.26 from Eq. (2). This discrepancy is attributed to the finite thickness of the 2DES which is thought to reduce the strength of the electron-electron interaction. Later in the paper when the simplified analytical model is used for a detailed comparison of theory and experiment, it is shown that a reduction in the interaction strength by a factor of 0.65 provides the best fit. This is consistent with earlier efforts to model the fractional quantum Hall effect,[30] which supported a reduction in the strength of the electron-electron interaction in a 2DES by factors of order $0.5 - 0.7$. No adjustment was made in the conversion from experimentally recorded temperatures to p, the population of the higher energy minority spin state. In the low temperature limit, the CC theory faithfully reproduces the experimental result of complete spin polarization with all the electrons in the lowest energy spin ↑ state located at $\omega = 0$.

The theoretical CR spectra produced by the CC calculation are seen to match the experimental results for each of the three representative occupancies in the quantum limit. We believe that this is good evidence for the validity of the model. At low temperatures, the peak positions of the 2DES CR correspond to the bulk GaAs spin transitions if the band nonparabolicity correction of Eq. (8) is taken into account. An additional indicator that spin effects are responsible for the split resonance is that the magnitude of the splitting at low densities is identical to that of the bulk at similar magnetic fields. Pulsed magnetic field measurements of heterojunctions with the same origin as those used in this study have shown that the splitting continues to follow accurately the characteristic B^2 dependence of bulk GaAs for fields up to 90 T.[6,31] The possibility of impurity shifted CR, which was thought to have influenced earlier measurements on less pure samples,[32,33] can be discounted due to the highly systematic dependence of resonance positions and intensities on carrier density. This correspondence is continuous from one sample to the next, and independent of other factors such as spacer layer thickness, illumination states and cool-down history.

Consideration of these facts justifies the acceptance of the idea that the two CR peaks in the low temperature and occupancy limit result from the spin ↑ and spin ↓ transitions. Away from low temperature and occupancies (i.e., $\nu = 1/9 - 1/6$, $T > 700$ mK), the CR spectra is due to the *interaction* between the spin states; the observed peaks are neither spin ↑ or spin ↓, but a hybridization of the two, with intensities and positions accurately predicted by the CC model.

2. Density dependence

As mentioned earlier, the CC model predicts that the weighted mean position of the cyclotron resonance provides a measure of the spin polarization of the system. To test this hypothesis rigorously, we plotted the weighted mean position of the cyclotron resonance from $0.36 - 8$ K for four different densities: 1.6, 2.1, 2.6, and 3.5×10^{10} cm^{-2}. The weighted mean field position B_{mean} is given by

$$B_{\text{mean}} = \frac{I_+ B_+ + I_- B_-}{I_+ + I_-}, \tag{9}$$

where $I_+, (B_+), I_-, (B_-)$ are the normalized intensities (field positions) of the spin ↑ and spin ↓ resonances. If the spin population is governed by single-particle Boltzmann statistics, then $I_- = I_+ + e^{-E_z/k_B T}$ where E_z is the Zeeman energy of the $N = 0$ Landau level. Substituting $\Delta = B_- - B_+$, the weighted mean then becomes

$$B_{\text{mean}} = B_+ + \frac{\Delta e^{-E_z/k_B T}}{1 + e^{-E_z/k_B T}}. \tag{10}$$

For each density in Fig. 4 the field positions of the high field (circles) and low field (squares) spin transitions as well as the weighted mean field position (triangles) determined from Eq. (9) is plotted. The dotted line is drawn from Eq. (10) and is seen to follow the experimentally determined weighted mean. This is confirmation that the spin polarization is given solely by the single-particle Boltzmann factor for these occupancies. Raising the temperature increases the population of the upper spin state which causes the single peak, equivalent to the weighted mean, to shift up to half of the predicted single-particle spin splitting. The predilection of the weighted mean to follow a Boltzmann dependence continues for densities up to 10×10^{10} cm^{-2}, where only a single peak is ever seen in the CR spectrum (Fig. 5). It is now clear that the position of the single peak represents a weighted average of the temperature driven population of the two spin states.

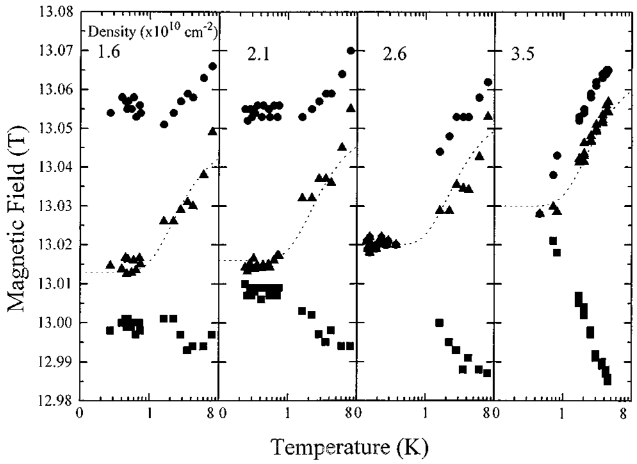

FIG. 4. Temperature dependence of the CR for the four densities. The field positions of the high field (circles) and low field (squares) spin transitions as well as the weighted mean field position (triangles) determined from Eq. (9) are plotted. The dotted line is drawn from Eq. (10), which shows the expected behavior of the weighted mean if the spin population is given by the Boltzmann distribution function.

B. Analysis using the simplified model

Visual comparison of the experimental traces and the predictions of the CC model shows the excellent qualitative agreement between the two spectra. It would be preferable, however, if there was a way to combine data taken at different densities and temperatures and present it using the CC formalism. The analytic expressions derived from the simplified model allow for such a comparison. The process involves several steps. Initially seven different density values were selected spanning the range from $(3.5-0.5)\times 10^{10}$ cm^{-2}. For each density, the magnitude of the spin splitting for a given temperature was recorded. These points were then fitted to Eq. (7) to derive a value of α which is the interaction strength of the simplified model. Experimental and theoretically determined interaction strengths are shown to match over a large density range.

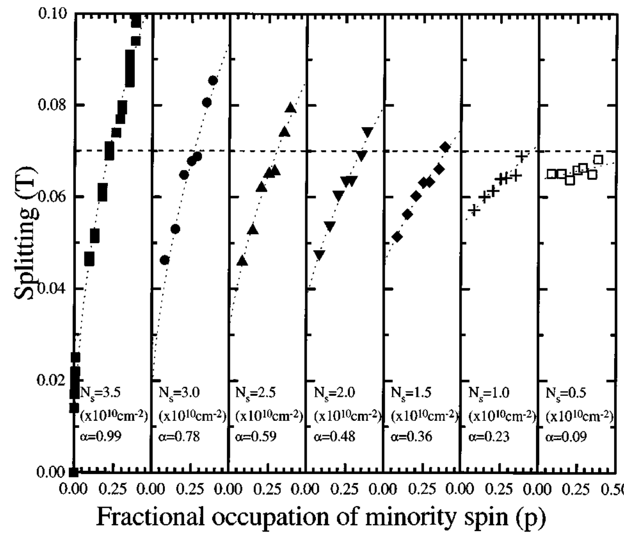

FIG. 6. Temperature dependence of the two cyclotron resonance spin-split transitions measured at 13 T, for a series of densities. The splitting is fitted using Eq. (7) with the spin population calculated from the Boltzmann factor. The horizontal dashed line shows the single-particle spin splitting which is 70 mT.

Figure 6 plots the magnitude of the splitting for data from 0.3 to 8 K as a function of fractional occupation of minority spin, assuming that the value of p is given only by the Boltzmann factor calculated using Eq. (6). The splitting is fitted to Eq. (7) with α as the fitting parameter. The interaction strength $I=(\alpha/11.034)$ is then plotted against the experimental density in Fig. 7. The dashed line shows the values for α calculated from Eq. (2) assuming a reduction in the electron-electron interaction strength by a factor $F=0.65$. This reduction was used as a fitting parameter and agrees with the estimated effect of the finite thickness of the 2DES in the experiment. This confirms the prediction of CC that the interaction strength follows a $N_s^{3/2}$ over a range of densities from $(0.5-10)\times 10^{10}$ cm^{-2}.

The simplified model also leads to the concept of a characteristic occupancy $\nu_c(T)$, which can be used to explain the

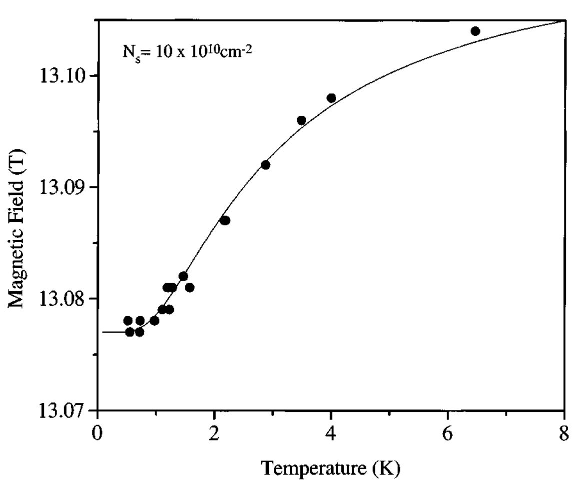

FIG. 5. Temperature dependence of the CR for a density of 10×10^{10} cm^{-2}. The single peak continues to follow a Boltzmann dependence.

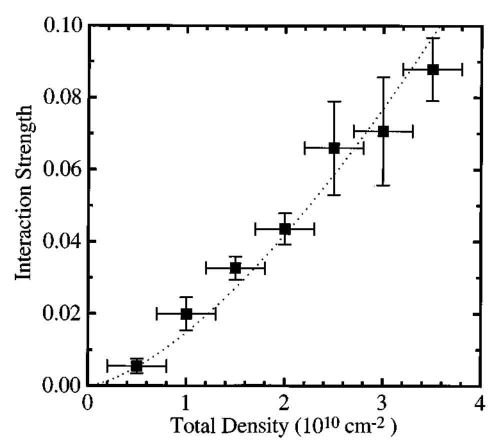

FIG. 7. The fitted values for the interaction strength given by $I=\alpha/11.034$ are plotted as a function of density deduced from the splittings shown in Fig. (6). The dashed line shows the predictions of Eq. (2) if the interaction strength is scaled down by a factor of 0.65.

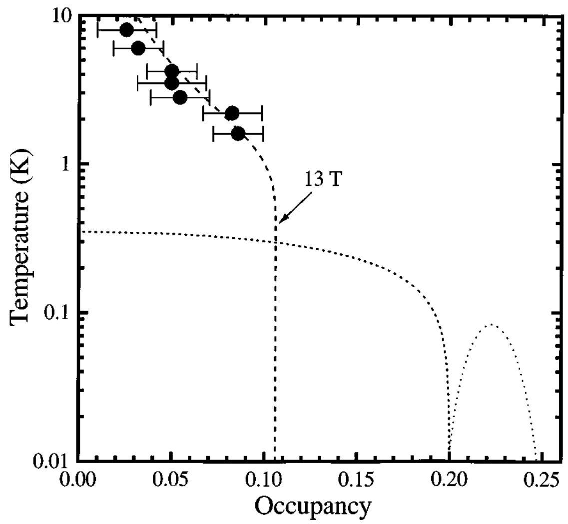

FIG. 8. A diagram of the 2DES in the low temperature and density limit with the dashed line showing the theoretically predicted characteristic occupancy for GaAs at 13 T. The points are taken from an experimental determination of the minimum splitting for a given temperature. The dotted line is a schematic picture of the Wigner solid phase as proposed in Refs. 34 and 35.

temperature dependence for differing occupancies. $\nu_c(T)$ is defined as the occupancy at which the observed splitting is a minimum for a given temperature [$\alpha=1-2p$ from Eq. (7)], which also corresponds to the point of equal resonance intensity ($S_{\text{in}}=S_{\text{out}}$).

$$\nu_c=\left(\frac{\pi\epsilon\epsilon_0\kappa(1-2p)}{11.034e\hbar F}\right)^{2/3}\frac{8hB}{\sqrt{3}e}=8.14\times10^{-3}(1-2p)^{2/3}B. \tag{11}$$

$F=0.65$ is included to reflect the reduction in interaction strength due to the softening of the Coulomb potential by the wave function extent in the z direction. For GaAs at 13 T, Eq. (11) gives $\nu_c(T)=0.106(1-2p)^{2/3}$.

A diagram of the 2DES in the low temperature and density limit is shown in Fig. 8, which includes experimentally determined values for $\nu_c(T)$. There is excellent agreement between the theoretical line and experimental points. The Wigner solid boundary deduced from earlier measurements is also shown.[34,35] Although $\nu_c(T)$ does not represent a qualitative change in the system or its excitations, it does indicate a special point in the gradual crossover of the magnetoplasmon modes from a weakly interacting electron gas at very low occupancies to a strongly correlated electron liquid at larger occupancies. For filling factors less than the characteristic occupancy, two CR peaks are seen with a splitting close to that seen in bulk GaAs. The intensity of the carriers in each peak as a function of temperature is satisfactorily explained by single-particle statistics indicating a gaslike excitation. The low temperature and low occupancy region is characterized by almost complete spin polarization with the majority of the carriers in the lower energy spin$\uparrow$ state. Occupancies greater than ν_c correspond to a strongly interacting system, characterized by the interplay between the Coulomb force and Zeeman energy which results in the striking temperature dependence.

C. The fractional regime $\frac{1}{6}\leqslant\nu\leqslant1$

The fractional regime is considered separately because for this range of filling factors it is known that the strong electron-electron interactions result in the formation of a liquid state responsible for the fractional quantum Hall effect (FQHE). Transport data showing several well-defined FQHE states in this region and predictions for a narrowing of the linewidth at fractional occupancies[36] led researchers to try and correlate shifts in cyclotron resonance positions or changes in absorption linewidths to filling factors associated with the FQHE. In reports on high mobility GaAs heterojunctions, Seidenbusch *et al.*[37] and Rikken *et al.*[38] have suggested that there was a small oscillation in the CR linewidth at 4.2 K, a temperature which is rather high for observation of the FQHE. These studies have been countered by other workers[32,39,40] who could find no significant effects. Taken as a whole, experimental results of this controversial topic provide no *conclusive* evidence for the observation of the FQHE by CR, although the majority of experiments have been performed between 1 and 2 K, where the FQHE features are not particularly strong.

The CR for fractional occupancies does continue to produce only a single sharp resonance which is in contrast to the expected bulk GaAs-like behavior. At low temperatures (i.e., 330 mK) the system would be expected to be almost completely spin polarized with a spin-flip energy in the field range 5–13 T of 1.5–3.5 K. However, the collective electron ground state of the system may in fact be mixed spin in nature. This is specifically true for the FQHE state at $\nu=\frac{2}{3}$, where it was both calculated[41] and shown experimentally[42] that the ground state of the system was spin *unpolarized* for fields less than 2 T. One of the general predictions of the CC model is that the weighted mean of the resonance position is insensitive to the electron-electron interactions and should reflect the mean spin composition of the underlying ground state. Thus a change from a polarized single-particle ground state, away from the FQHE state, to the unpolarized state could be accompanied by a shift to lower energy of the CR peak by up to half of the single-particle spin splitting. This would be within the range $0.1-0.6\ \text{cm}^{-1}$ or a corresponding effective mass enhancement of 0.1–0.3 %. Given the extremely narrow CR linewidths of the Philips samples and experimental resolution of the Fourier transform spectrometer such shifts should be detectable.

Figure 9 shows the results of a cyclotron resonance experiment performed on a sample with a carrier density $8.2\times10^{10}\ \text{cm}^{-2}$ up to a field of 12.5 T and a temperature 330 mK. These data were taken with the Fourier transform spectrometer for fixed values of field. The field range includes the fractions $\nu=\frac{2}{3}$ to $\nu=\frac{1}{3}$, where strong features in transport have been observed in these samples. The spectra are regularly spaced with little variance in their absorption strengths or linewidths indicating the absence of any strong influence on the CR as the fractions are traversed. The CR effective mass and linewidth which are extremely sensitive measures of any departure from ideal cyclotron behavior, are plotted out in Fig. 10. The small oscillations present in the linewidth are considerably smaller than experimental errors which arise primarily due to the spectrometer resolution of $0.1\ \text{cm}^{-1}$. Figure 10(b) shows the cyclotron effective mass

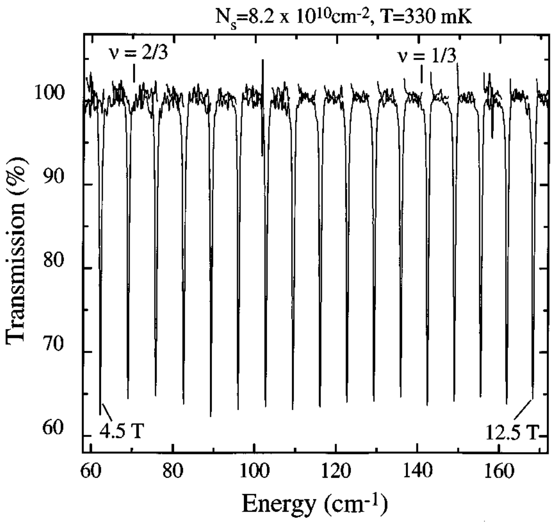

FIG. 9. CR in the fractional quantum Hall regime as recorded using the Fourier transform spectrometer. The sample was at 330 mK. The fractions 2/3 and 1/3 are marked where strong features are seen in transport measurements.

and this again shows no discernible features. However the influence of the conduction band nonparabolicity and the polaron interaction, which manifest themselves in an increasing effective mass with energy (i.e., magnetic field), act to mask any small oscillations. In Fig. 10(c), the effects of nonparabolicity have been normalized out using Eq. (8). Deviations from the norm are seen to be less than 0.05% which corresponds to less than 1/10 of the CR linewidth at $\nu=\frac{1}{3}$.

The dilution refrigerator used in conjunction with the far-infrared laser provided the capability to extend this work down to 100 mK. No anomalies in the effective mass are seen at fields corresponding to fractional filling factors.[2]

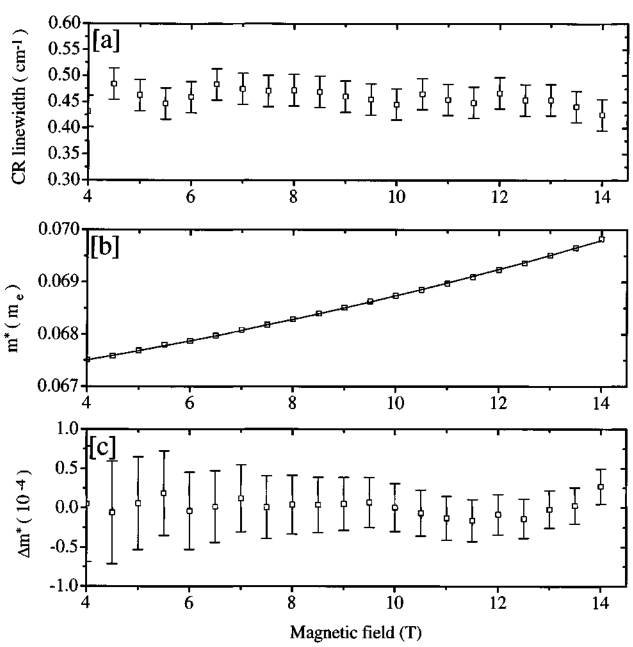

FIG. 10. CR linewidth (a), effective mass (b), and effective mass corrected for nonparabolicity (c) from the spectra shown in Fig. 9.

Taken together, the ^{3}He and dilution refrigerator results provide strong evidence that the ground state of the 2DES contained within these samples is spin polarized throughout the FQHE region at fields above 4 T.

While not finding any strong influence of the FQHE states, CR measurements in this regime[1,6] show a systematic shift in resonance position as the temperature is varied. This temperature dependence appears to have an onset at $\nu\sim\frac{1}{3}$ and becomes more pronounced towards smaller filling factors. Changing the temperature from 2 K to 300 mK for $\nu=\frac{1}{3}-\frac{1}{6}$ resulted in a continuous shift of the resonance position downwards in field. The total shift in position was up to half of the single-particle spin splitting. This behavior suggests that at occupancies below $\frac{1}{3}$ the composition of the ground state only becomes spin polarized at temperatures corresponding to a spin polarization of the underlying single-particle states.

D. Integral filling regime $\nu>1$

Experiments on very high-mobility GaAs/Al_xGa_{1-x}As heterojunctions for $1<\nu<2$ failed to observe a splitting of the CR despite experimental linewidths that were considerably smaller than the expected splitting.[4] At the time, observation of a single sharp resonance was attributed to plasma mode mixing or hybridization of the two spin transitions, resulting in a single coupled resonance although as mentioned earlier, mixing of the spin states was predicted to be weak and only activated by disorder.[16]

Occupancies greater than one indicate that the lowest spin state is filled. Designation of the system as compressible or incompressible is not straightforward, however, as an occupancy of $\frac{3}{2}$ could be formed by combining a completely filled (incompressible) lower energy state plus half-filled upper spin or through two partially filled (compressible) states (e.g., $1+0.5$ or $0.9+0.6$). The calculations of MacDonald and Kallin[16] show that mode coupling between the magnetoplasmon and CR is negligible for completely filled Landau levels. This would suggest that the spin states are not fully occupied even in the integral filling regime and are instead composed of a superposition of compressible states.

If this is true, then the CC model can continue to provide an insight into the CR in this higher density regime. The single peak is a reflection of the strength of the interaction between the two different spin transitions given by Eq. (2) and the weighted mean position provides a measure of the relative spin populations.

V. CONCLUSIONS

The technique of cyclotron resonance has shown to be uniquely suited to probing the spin composition of the 2DES in large perpendicular magnetic fields. Millikelvin experiments have confirmed that even at the lowest achieved sample temperatures there is a significant population of both spin states of the lowest Landau level. A theoretical calculation of CR spectra by CC convincingly reproduces experimental results, using a model that considers the interaction between interacting, compressible spin states. The temperature dependence is accounted for in terms of the thermal population of the two spin states, without invoking exchange interaction. Changes in the spectra with occupancy are seen to be the result of changing from single-particle behavior at low densities to a single mode dominated by Coulomb interactions at higher densities.

Comparison between experiment and the CC model clearly shows that there is now a good quantitative understanding of the cyclotron excitations of the interacting two-dimensional system. These excitations can be divided by a characteristic occupancy which separates single-particle behavior at low temperatures from a system with coupled excitations at higher densities. The characteristic occupancy is a function of the spin population and the carrier density. Previous suggestions that anomolies in the CR might be due to formation of the Wigner crystal, a spin-pairing mechanism, or a combination of a solid and liquid phase are shown to be inadequate in light of the Cooper and Chalker model which successfully models the system behavior.

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