Resonant reflection, cooling, and quasitrapping of ballistic electrons by dynamic potential barriers

O. A. Tkachenko and D. G. Baksheyev
Novosibirsk State University, Novosibirsk 630090, Russia

V. A. Tkachenko
Institute of Semiconductor Physics, Novosibirsk 630090, Russia

(Received 9 July 1996)

Resonances of full elastic and inelastic reflection of particles from one or several high-frequency barriers $V(x)\cos\omega t$ have been found. For the resonances to be observed the x component of the energy of incident particles $\hbar^2k_x^2/2m^*$ must be close to $\hbar\omega$ and the smoothing of the barriers' edges should not exceed $\sqrt{\hbar/(m^*\omega)}$. These requirements can be realized by an inclined impingement of ballistic electrons on dynamic barriers in a two-dimensional electron gas of a $GaAs/Al_{0.3}Ga_{0.7}As$ heterostructure. [S0163-1829(96)05743-8]

Recent experimental studies of electron transmission through ring[1] and double-slit[2] lateral semiconductor nanostructures have confirmed that the electron phase in spatially separated channels can efficiently be controlled by stationary magnetic and electric fields. In this work we predict that the interaction of a free ballistic electron with a submicrometer region of a sharply nonuniform high-frequency (hf) field can also lead to the full modulation of the transmission coefficient as a result of resonant interference between different channels of the quasienergetical wave function. It will be shown analytically that in case there are no stationary obstacles a one-dimensional rectangular dynamic barrier $V\Theta(x)\Theta(d-x)\cos\omega t$ with the amplitude $V\ll\hbar\omega$ can either transmit or elastically fully reflect the particles of the energy $E=\hbar^2k_x^2/2m^*=\hbar\omega$, depending on the value k_xd. The effects appear to be more various for several equally spaced dynamic barriers. By means of numerical simulation we show that under certain conditions the window of almost full inelastic electron reflection with photon emission at $(E-\hbar\omega)/E\ll 1$ occurs and quasitrapping of electrons to the levels with small negative energies E_i induced by the hf field at the frequencies $\hbar\omega_i>E$ is possible when the inelastic channel with energy $E-\hbar\omega_i=E_i$ is closed. Resonances of the latter type are characterized by strong localization of the particles in the hf field region, and resemble the capture-escape resonances in the photon-assisted electron scattering on a heterostructure quantum well[3] or a nonuniform constriction.[4]

To observe these effects, the smoothing of the edges of the hf barriers should not exceed a quarter the de Broglie wavelength λ of a ballistic electron, that is, the barriers must be sufficiently sharp. Such barriers can be created by a few narrow strip gates placed extremely closely to each other above a uniform region of two-dimensional electron gas (2DEG) between emitter and collector quantum point contacts in a $GaAs/Al_{0.3}Ga_{0.7}As$ heterostructure. A similar arrangement of split and strip gates has already been used in magnetotransport measurements[2,5] or discussed in connection with development of electrostatic barriers with abrupt edges.[6] The high-frequency voltage can be applied to the gates by means of a broadband antenna exposed to microwave irradiation, which is analogous to experiments on photon-assisted electron transmission through a quantum point contact,[7] quantum wire blocked with a barrier,[8] or quantum dot.[9] However, unlike Refs. 2,5,7–9, in our case the constant voltage at the strip gates must make stationary potential uniform. Then the fluctuation potential of random distribution of charged impurities will be entirely screened by the electrons of the 2DEG and the electron mean-free path will exceed 10 μm.

Generally, transmission of particles through a hf region is multichanneled, that is, real or virtual emission and absorption of n quanta of the hf field is possible, $n=0, \pm1, \pm2, \ldots$. When the energy of incident particles is close to zero or $\hbar\omega$, the central role is played by the channel near the bottom of the continuum. Getting to this channel, an electron becomes ''ultracold'' and interacts with the places of rapid change of the hf potential as with stationary δ barriers. At low hf field amplitudes only three coupled channels are important, with energies $\hbar\omega$, 0, $-\hbar\omega$. Then the problem of transmission of the particles through the simplest hf field obstacles like a rectangular step[10] or rectangular barrier of the height $V/\hbar\omega\ll 1$ can be solved analytically.

For a particle impinging on the hf barrier $U(x,t)=V\Theta(x)\Theta(d-x)\cos\omega t$ from the left, the wave function obeying the time-dependent Schrödinger equation is given by

$$\Psi=\begin{cases} e^{ik_0x-i(E/\hbar)t}+\sum r_ne^{-ik_nx-i(E_n/\hbar)t}, & x<0\\ \sum (a_ne^{ik_nx}+b_ne^{-ik_nx})Se^{-i(E_n/\hbar)t}, & 0\leq x\leq d\\ \sum t_ne^{ik_n(x-d)-i(E_n/\hbar)t}, & x>d \end{cases}$$

where $E_n=E+n\hbar\omega$, $\hbar k_n=\sqrt{2m^*E_n}$, and $S=\exp[-i(V/\hbar\omega)\sin\omega t]=\Sigma J_m(V/\hbar\omega)\exp(-im\omega t)$, where J_m is a Bessel function.

By matching the wave function at the points $x=0$ and $x=d$, we obtain the relation on the amplitudes of reflection r_0, r_n, and transmission t_0, t_n

$$J_m[k_m\pm k_0]=-\sum_n J_{m-n}[r_n(k_m\mp k_n)-t_n(k_m\pm k_n)e^{\mp ik_md}]. \tag{1}$$

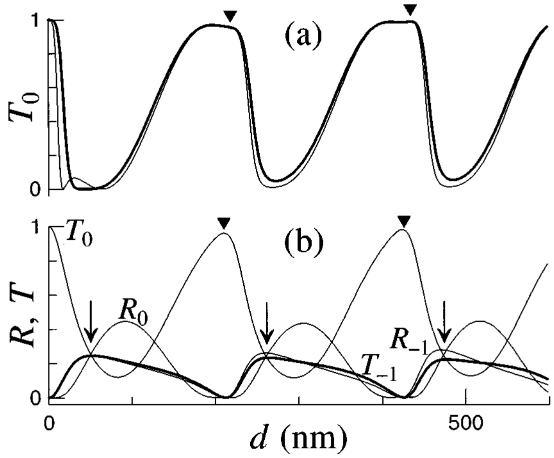

FIG. 1. The coefficients of elastic ($n=0$) and inelastic ($n=-1$) reflection (R_0, R_{-1}) and transmission (T_0, T_{-1}) versus the thickness of rectangular hf barrier d near the opening threshold of the channel $n=-1$ ($\hbar\omega=E$) at $E=0.5$ meV ($m^*=0.067m_e$, $\lambda\approx 213$ nm) and $V/\hbar\omega\approx 0.2$. The triangles and arrows denote the values $d=m\lambda$ and $d=(m+1/4)\lambda$, respectively. (a) The elastic transmission coefficient for two nearby frequencies $\hbar\omega>E$ (bold curve for $\kappa=\kappa_{max}$ and thin one for $\kappa=0.4\kappa_{max}$). (b) The transmission and reflection coefficients for a frequency $\hbar\omega<E$.

In the three-channel approximation for $E\approx\hbar\omega$ the indices n and m take the values $0, -1, -2$. Thus (1) is the system of six linear equations.

Consider the solutions of this system in the limit of small $\delta\equiv J_1^2/J_0^2$. When $E<\hbar\omega$ the channel $n=-1$ is closed, transmission is elastic, and the coefficient T_0 is given by

$$T_0=\frac{(\gamma k_0\delta-2\kappa)^2}{(\gamma k_0\delta-2\kappa)^2+\delta^2k_0^2(1-\cos k_0d)^2}, \tag{2}$$

where $\gamma=(\sin k_0d+e^{-k_0d}-1)$ and $\hbar\kappa=\sqrt{2m^*|E-\hbar\omega|}$. It is seen that $T_0=1$ when $k_0d=2\pi m$ [Fig. 1(a), marked with triangles]. Besides, $T_0=0$ when $\kappa=\gamma=0$. For large widths of the barrier $\exp(-k_0d)\to 0$, and T_0 becomes zero at $k_0d=\pi/2+2m\pi$. The resonances of full reflection correspond to virtual transitions to the bottom of the continuum and back to the elastic channel.

T_0 can also turn to zero at $\gamma>0$ and $\kappa=\gamma k_0\delta/2$. With E fixed, κ takes extrema when $\cos k_0d=\exp(-k_0d)$. Since $\kappa\geqslant 0$, we look for solutions of the equation only on the intervals $[2\pi m, 2\pi m+\pi/2]$. The largest maximum is $\kappa_{max}\approx 0.12k_0\delta$ for $k_0d\approx 1.3$, so that virtual transitions of the particles occur to the channel with the small negative energy $E_{-1}=E-\hbar\omega\approx -0.014E\delta^2$. For $V/\hbar\omega\approx 0.2$ and $E=0.5$ meV this case is shown in Fig. 1(a) with a bold curve (effective mass $m^*=0.067m_e$). When $\kappa<\kappa_{max}$, there are two close values of d for which T_0 turns to zero. For comparison, the thin curve in the Fig. 1(a) represents $T_0(d)$ for $\kappa=0.4\kappa_{max}$.

When $E\geqslant\hbar\omega$ the channel $n=-1$ is open and transmission coefficients T_0 and T_{-1} are given by

$$T_0=(4k_{-1}^2+\gamma^2\delta^2k_0^2)P^{-1}, \tag{3}$$

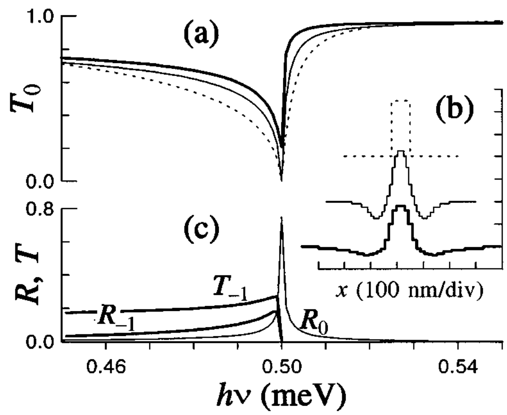

FIG. 2. (a) Frequency dependence of the elastic transmission coefficient T_0 at $E=0.5$ meV for rectangular and smoothed hf barriers represented in (b) by lines of the same kind. (c) The coefficients of reflection R_0, R_{-1} and inelastic transmission T_{-1} versus frequency for the barrier shown at the bottom of (b).

$$T_{-1}=[2k_0\delta k_{-1}(1-\cos k_0d)]P^{-1}, \tag{4}$$

$$P=[2k_{-1}+k_0\delta(1-\cos k_0d)]^2+\gamma^2\delta^2k_0^2.$$

From (3) and (4) it follows that the hf barrier becomes fully transparent, $T_0=1$ and $T_{-1}=0$, when $k_0d=2m\pi$ [Fig. 1(b), marked with triangles]. In contrast, if the width of the barrier is an integer and a quarter wavelength $k_0d=2m\pi+\pi/2$ and $k_{-1}=(k_0\delta/2)[1+\exp(-2k_0d)]^{-1/2}$, the coefficient T_{-1} reaches its maximum value $T_{-1}=[2+2\sqrt{1+\exp(-2k_0d)}]^{-1}\approx 1/4$. At these parameters the reflection into inelastic channel $n=-1$, as well as transmission and reflection in elastic channels, is also about 25% each [Fig. 1(b), marked with arrows]. Thus, 50% of particles undergo transition to the channel $n=-1$ and decrease of energy to almost zero. When the energy of the incident particles is close to zero, the transmission coefficient for the particles with absorption of a quantum is found from Eq. (4) by using the principle of detailed equilibrium $T_{+1}(E)=T_{-1}(E+\hbar\omega)$.

The calculations with the help of MULTI-$\hbar\omega$ (Ref. 11) show that analogous results are obtained when the edges of the barrier are smoothed over the length $\sim\lambda/4$ that is 53 nm for $E=0.5$ meV and $m^*=0.067m_e$ for electrons in GaAs (Fig. 2). Notice that taking into account high-frequency antiphase oscillations of the 2DEG charge near the barrier [Fig. 2(b)] does not negate the main theoretical results, namely, the possibility of full elastic reflection of ballistic electrons [Fig. 2(a)] and the sharp cooling that accompanies the opening of the inelastic channel [Fig. 2(c)].

It is natural to expect that the influence of the hf region on a moving particle is enhanced by replacing one hf barrier with a few barriers, as a result of multiple electron reflections from their boundaries. This hypothesis was checked by multichannel numerical calculations. Figure 3 represents the result of modeling of transmission of ballistic electron through six hf barriers of the width $\approx\lambda/4$ spaced from each other by $\approx\lambda/2$ at $\hbar\omega=0.5$ meV. Barriers were assumed to oscillate in phase (standing wave). It's worth noting that both types of barriers, rectangular or smoothed with account of real charge oscillations in the 2DEG, produce similar results. The graph

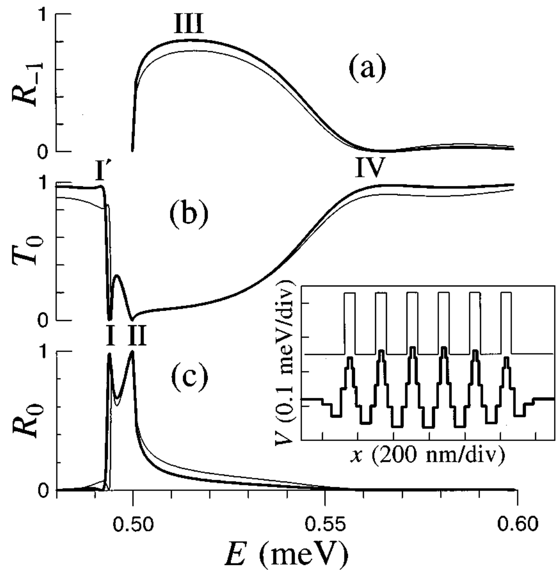

FIG. 3. Results of calculation for transmission of ballistic electrons through a series of hf barriers. Dependences of elastic transmission T_0, elastic and inelastic reflection R_0, R_{-1} coefficients versus the energy of incident electrons E at $\hbar\omega = 0.5$ meV for $V(x)$ shown in the inset by lines of the same kind. For the smoothed barriers $T_0 + R_0 + R_{-1} \approx 1$. The peaks marked with roman numerals are brightened with coordinate distributions shown in Fig. 4.

$T_0(E)$ shows two dips, I and II, down to almost zero at the energies below 0.5 meV [Fig. 3(b)] and respective sharp peaks of elastic reflection up to 100% [Fig. 3(c)]. The first dip in transmission is preceded by the peak I′ like Fano resonances. Time averaged probability densities for energies I and II that correspond to the minima of T_0 (maxima of R_0) become much greater than in the incident wave and resemble corresponding distributions for the first and second bound states in a well [Fig. 4(a)]. However, particles are not really trapped; it is rather quasitrapping, or a wholly coherent process of virtual transition of the particle from the state of

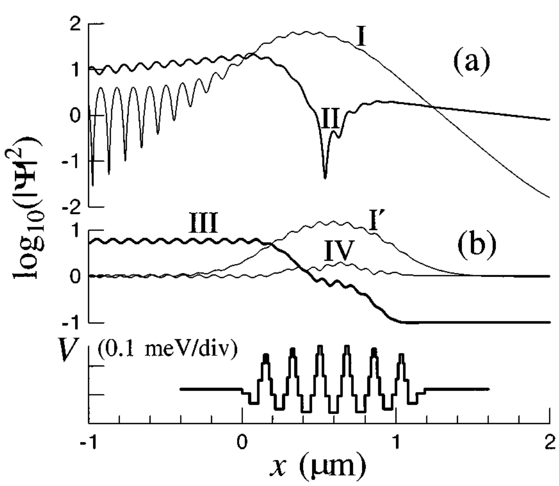

FIG. 4. The time-averaged probability density against the plot of the amplitude of the hf potential (at the bottom) for the peaks of T_0, R_0, and R_{-1} marked with respective roman numerals in Fig. 3. Exponential decrease of curve III with x inside the hf region is analogous to evanescent waves in a crystal at Bragg reflection.

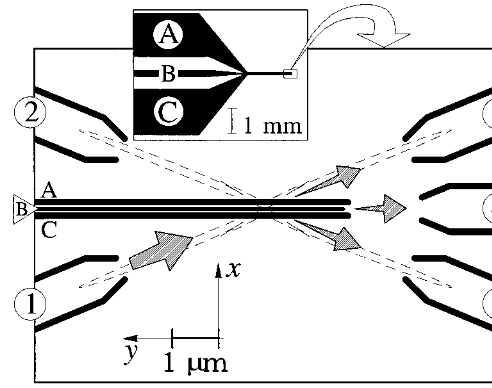

FIG. 5. Suggested scheme of experiment on the base of an $Al_{0.3}Ga_{0.7}As/GaAs$ heterostructure. Solid lines denote the surface gates that make static and dynamic modulation of potential. The hf barrier across the base is created by means of the strip gates A, B, and C. Gate B is driven with the hf voltage from the antenna (shown on the top) exposed to 120 GHz microwave irradiation. (Without irradiation these gates make no obstacle for the electrons.) Electron flows from injector 1 to collector 3–5 contacts are shown by arrows. The dotted line represents the calculated directivity diagram for the flows of ballistic electrons of energy $E_F + \hbar\omega$ generated by the hf barrier of the middle profile from Fig. 2(b).

continuous spectrum to the quasibound state of negative energy induced by an alternating field and back to the continuum. The state of full transmission I′ exhibits, like I, localization of particles inside the region of the standing wave of potential [Fig. 4(b)]. Thus, several hf barriers can act as one shallow stationary potential well with uniform high-frequency field.

With increasing energy, when the channel of inelastic transmission with emission of a quantum $\hbar\omega$ is opened, the reflection coefficient $R_{-1}(E)$ exhibits a wide 80% maximum which is marked III [Fig. 3(a)]. It should be noted that the dependences $R_{-1}(E+\hbar\omega)$ and $R_1(E)$ are identical according to the principle of detailed equilibrium; thus the reflection of particles of a small energy with absorption of a quantum $\hbar\omega$ also reaches 80% at the energy value that is $\hbar\omega$ below point III.

The effects shown in Figs. 3 and 4 have the interference nature and, therefore, are sensitive to the changes of the width and spacing of the barriers. For instance, when the two parameters are $\lambda/2$, electron motion through the series of the hf barriers becomes almost free.

To test experimentally the effects predicted here, including the cooling of ''fast'' electrons and heating of the ''slow'' ones, a possible scheme is suggested, as shown in Fig. 5. As the typical energy of ballistic electrons of a 2DEG is $E \sim 3$–4 meV, an inclined impingement of the particles on the barriers is necessary to increase the admissible smoothing of their edges, which always emerges in experimental realization. If the impingement angle is about 70° to the normal of the slit then the transverse energy component decreases to 0.5 meV and respective wavelength λ becomes over 200 nm, so that the smoothing $\sim\lambda/4$ of the edges of the hf barriers should make the effects observable at the frequency of a tunable microwave source $\nu \approx 120$ GHz.

Shown in Fig. 5, the arrangement of quantum point contacts allows one to detect directly both reflected and transmitted particles, as well as the particles that lost their normal momentum component after injection into the base and went on moving along the gates. A scheme of this process is shown by the arrows. Moreover, this arrangement also allows one to detect the particles that belonged initially to the 2DEG, moved with Fermi velocity almost along a tangent to the barriers and, after absorption of a photon, had received the normal momentum component. The angular distribution of the ballistic particles appearing in this process is represented by four directed flows and all the point contacts shown in Fig. 5, except the fourth one, could serve as a collector to register such particles.

By analogy with Ref. 5, to inject ballistic electrons with kinetic energy ≈ 4 meV into the base, it is biased by the voltage a bit above 1 mV with respect to the emitter. Passing through the multimode emitter quantum point contact of the resistance $R \approx 1$ kΩ the current $I \approx 0.1$ μA determines the energy spread for ballistic electrons $\Delta E = eIR \approx 0.1$ meV. Thus the minimal height of the barrier of the emitter quantum point contact is close to 4 meV. The barriers of almost the same height are set at the entrance to collector contacts 3 and 5 to provide for registering the elastically transmitted and reflected ballistic electrons. For contact 4 the corresponding barrier must be 0.5 meV lower. With this setup the particles that have emitted a quantum $h\nu$ and are moving along the hf barriers still have an energy 0.5 meV above the Fermi level in the base ($E_F \approx 3$ meV). The potential barrier of the collector quantum point contact 2 must be about 3.3 meV high to pass the ‘‘heated’’ particles that initially belonged to the 2DEG.

In each case, the hits of ballistic electrons past the collectors are registered by the voltage drops between the base and the regions 2, 3, 4, and 5, that is by means of the respective photovoltages.

In summary, we have solved analytically and numerically the time-dependent Schrödinger equation for dynamic potential barriers when stationary potential is uniform. We have found strong interference oscillations of their transparency which are caused by stimulated electron transitions to the bottom of the continuum. With changing the width of the single hf barrier the period of these oscillations is the *wavelength* of the incident electrons, not *half the wavelength* as in Fabry-Pérot type interferometers. Modulation of transparency in our case is *full* like that in double-slit and ring interferometers, in contrast to *partial* modulation at a stationary barrier and/or well. The finite ‘‘crystal’’ of hf barriers can produce windows of almost full inelastic and elastic reflection corresponding to forbidden and allowed quasienergetical bands. In the 2DEG of GaAs/$Al_{0.3}Ga_{0.7}As$ structures the dynamic barriers could serve as a beam splitter, trap, or energy converter for ballistic electrons. They can also be a source of sharply directed beams of ballistic electrons.

The authors wish to thank V. K. Ignatovich, M. Budantsev, S. Studenikin, and I. Panaev for valuable discussions. This work is supported by the program ‘‘Universities of Russia’’ and Russian Foundation for Basic Research (Grant No. 95-02-04583a).

[1] A. A. Bykov *et al.*, JETP Lett. **60**, 809 (1994).
[2] A. Yacoby *et al.*, Phys. Rev. Lett. **73**, 3149 (1994).
[3] R. A. Sacks and A. Szoke, Phys. Rev. A **40**, 5614 (1989).
[4] L. Y. Gorelik *et al.*, Phys. Rev. Lett. **73**, 2260 (1994).
[5] X. Ying *et al.*, Appl. Phys. Lett. **65**, 1154 (1994).
[6] J. P. Lu, X. Ying, and M. Shayegan, Appl. Phys. Lett. **65**, 2320 (1994).
[7] R. A. Wyss *et al.*, Appl. Phys. Lett. **63**, 1522 (1993).
[8] C. Karadi *et al.*, J. Opt. Soc. Am. B **11**, 2566 (1994).
[9] R. H. Blick *et al.*, Appl. Phys. Lett. **67**, 3924 (1995).
[10] O. A. Tkachenko, V. A. Tkachenko, and D. G. Baksheyev, Phys. Rev. B **53**, 4672 (1996).
[11] D. G. Baksheyev, V. A. Tkachenko, and O. A. Tkachenko, in *Physics, Chemistry and Application of Nanostructures*, edited by V. E. Borisenko *et al.* (Belarusian State University of Informatics and Radioelectronics, Minsk, Belarus, 1995), pp. 268–270.