Vortices at planar defects in layered superconductors

A. Gurevich
Applied Superconductivity Center, University of Wisconsin, Madison, Wisconsin 53706

M. Benkraouda and John R. Clem
Ames Laboratory and Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011

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We propose a self-consistent nonlocal approach for the description of vortices in layered supeconductors that contain planar defects parallel to the layers. The model takes account of interlayer Josephson coupling and of a reduced maximum Josephson current density j_0' across the defect as compared to j_0 for other interlayer junctions. Analytical formulas that describe the structure of both static and moving vortices, including the nonlinear Josephson core region, are obtained. Within the framework of the model, we have calculated the lower critical field H_{c1}, vortex mass M, viscous drag coefficient μ, and the nonlinear current-voltage characteristic $V(j)$ for a vortex moving along planar defects. It is shown that for identical junctions ($j_0'=j_0$) our approach reproduces results of Clem, Coffey, and Hao [Phys. Rev. B **42**, 6209 (1990); **44**, 2732 (1991); **44**, 6903 (1991)] for μ, M, and H_{c1}. In the opposite limit $j_0'\ll j_0$, our model gives an Abrikosov vortex with anisotropic Josephson core described by a nonlocal Josephson electrodynamics. A sign change in the curvature of $V(j)$ is shown to occur due to a crossover between underdamped ($T\ll T_c$) and overdamped $T\simeq T_c$ dynamics of interlayer junctions as the temperature T is increased. Implications of the results on the c-axis current transport in high-T_c superconductors are discussed. [S0163-1829(96)02042-5]

I. INTRODUCTION

The discovery of high-temperature superconducting (HTS) oxides has renewed considerable interest in the properties of vortex structures in highly anisotropic superconductors.[1] Characteristic features of these materials include the noncollinearity of flux lines and applied magnetic field **H**,[2–4] fragmentation of line vortices into quasi-two dimensional pancake vortices,[5] and formation of kinks on the flux lines if **H** is not parallel to the symmetry axes.[6] The structure of a single fluxon essentially changes for **H** parallel to the superconducting planes as well, due to weak interplane coupling, which results in a strong deformation of the vortex core[7–12] and additional intrinsic pinning.[13]

Depending on the degree of anisotropy, different theoretical models are used for the description of vortex structure. At moderate anisotropy, the linear London theory is used, which ignores the layered structure of a superconductor but takes into account its anisotropy via an effective mass tensor.[2–4] This approach, valid as long as the coherence length ξ remains much larger than the atomic length scales, enables one to describe the distribution of screening currents everywhere in the vortex except in the vicinity of the normal core, where one has to invoke nonlinear equations for the superconducting order parameter.

In highly anisotropic layered superconductors or artificial superlattices the interlayer superconducting coupling can be so weak that the coherence length in the direction perpendicular to the layers ξ_c becomes smaller than the interlayer spacing s. In this case, the discreteness of the superconductor at the atomic level dramatically affects the structure of the vortex core.[7–12] There are several models that take account of the layered structure of a superconductor, for example, the Lawrence-Doniach (LD) model, which describes a stack of thin superconducting layers coupled by the Josephson interaction.[14] Another model is the S-N-S superlattice consisting of alternating superconducting (S) and normal (N) layers.[8–12,15] An important feature of such models is that, due to weak Josephson interlayer coupling, the maximum supercurrent density j_0 between the layers is much smaller than the intralayer depairing current density j_d. As a result, the maximum current density that can be locally generated by a vortex parallel to the layers is limited by j_0, and therefore the magnitude of the order parameter remains the same in all layers, including the layers closest to the vortex axis. This enables one to use linear two-dimensional (2D) London electrodynamics within the layers and retain only the most essential Josephson nonlinearity of the interplane current density $j_0\sin\varphi_n$. The resulting equations that describe the distributions of $\mathbf{j}(\mathbf{r})$ and $\mathbf{b}(\mathbf{r})$ in the vortex can be expressed in terms of an infinite set of coupled sine-Gordon equations for the gauge-invariant phase differences $\varphi_n(\mathbf{r})$ ($n=0,\pm1,\pm2,\ldots$) across the junctions between superconducting layers.[16,17] These nonlinear difference equations, based only on the Josephson relations and Maxwell equations, are insensitive to the microscopic mechanism of superconductivity.

Although obtaining single-vortex solutions of the equations for $\varphi_n(\mathbf{r})$ is a fairly complicated mathematical problem, a qualitative description of a vortex parallel to the layers proves to be similar to that of an Abrikosov (A) vortex.[18] Namely, there is an outer region of circulating screening currents which flow around the vortex axis and decay exponentially in the directions parallel (y axis) and perpendicular (z axis) to the layers. Here the phases $\varphi_n(\mathbf{r})$ change smoothly over the interlayer spacing s, and the LD model reduces to the London equation with some effective penetration depths λ_y and λ_z. Furthermore, there is a central nonlinear core region where $\varphi_n(\mathbf{r})$ varies significantly over the length $\sim s$, with the phases $\varphi_n(\mathbf{r})$ on the different layers being coupled via the long-range (over scales $\lambda \gg s$) magnetic field $\mathbf{b}(\mathbf{r})$. Here the discreteness of the superconductor becomes essential, and the long-range interaction of $\varphi_n(\mathbf{r})$ across the different layers ultimately results in a nonlocal relation between the interlayer current $\mathbf{j}(\mathbf{r})$ and the phase gradient $\nabla \varphi_n(\mathbf{r})$.

Such a nonlocality caused by the long-range magnetic coupling occurs if the phase $\varphi(\mathbf{r})$ changes on a scale shorter than that of $\mathbf{b}(\mathbf{r})$. This, for example, can occur in a single high-j_c Josephson contact in the strong-coupling limit for which the Josephson penetration depth λ_J becomes smaller than λ and the local Josephson electrodynamics based on the sine-Gordon equation becomes invalid. In this case the contact is described by integral equations of nonlocal Josephson electrodynamics[19–23] which can also describe the interaction of A vortices with planar crystalline defects.[24] Another example is weak links in thin films of thickness $d \ll \lambda$, where the Josephson nonlocality is strongly enhanced by the large penetration depth $\lambda_{\mathrm{eff}} = \lambda^2/d$ due to long-range magnetic fields outside the film.[25–29] Therefore, the Josephson nonlocality results from a long-range magnetic field $\mathbf{b}$, which can be due to both superconducting properties of the contact and the sample geometry. A similar situation occurs for vortices in layered superconductors, since the Josephson core size can be comparable with the interlayer spacing s and nonlocality occurs if $s \ll \lambda$,[16,30] which holds in HTS oxides, if one treats s as a distance between the CuO_2 planes ($s \sim 10$ Å), or typical planar defects, say, twins in the ab planes or stacking faults perpendicular to the c axis ($s \sim 100$–1000 Å).[31]

Due to the short coherence length $\xi_c < s$ in layered HTS oxides, planar defects parallel to the ab planes can strongly reduce interlayer coupling, thus limiting the current flow along the c axis and enhancing the magnetic field penetration along the defects. This may be essential for the interpretation of current transport along the c axis in HTS oxides, which often contain numerous planar defects parallel to the ab planes (stacking faults, intergrowths of low-T_c phases, etc.)[31] At small ξ_c, these defects can strongly reduce the local maximum supercurrent j_0' as compared to j_0, thus becoming a significant limiting factor for the c-axis current transport, which may be determined by vortex dynamics and pinning along defects rather than by the inherent interlayer Josephson coupling. Recently, this problem has attracted much attention due to the observation of an intrinsic Josephson effect which may clarify the nature of the interlayer coupling in HTS oxides.[32] The c-axis current transport can also play an important role in determining the current-carrying capability of Bi-based tapes.[33,34]

In this paper we apply the nonlocal Josephson electrodynamics to describe both a static and a moving vortex at planar defect in layered superconductors, assuming that the vortex and the defects are parallel to the planes, and that the critical current density across the defect j_0' differs from the interlayer critical current density j_0. This case, for example, corresponds to a vortex parallel to the ab plane in HTS oxides in the presence of a stacking fault or a vortex in a superconducting superlattice in a longitudinal magnetic field. The paper is organized as follows.

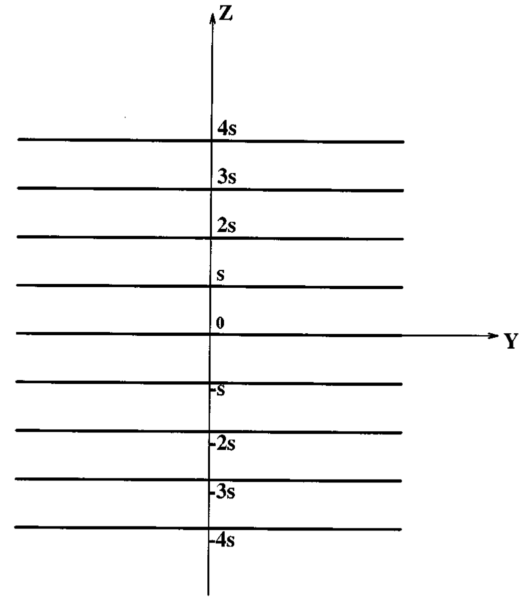

FIG. 1. Sketch of the geometry of the layered superconductors.

First we obtain a set of coupled nonlinear integral equations which describe the distribution of the phase differences $\varphi_n(\mathbf{r})$ across the layers. Then we propose a self-consistent mean-field approach, which enables one to obtain static and moving single-vortex solutions that describe both the circulating currents and the vortex core. For a stack of identical layers ($j_0' = j_0$) this approach in the static case reproduces the results of Clem, Coffey, and Hao,[8,9] and decribes a crossover between an Abrikosov vortex with a highly anisotropic Josephson (J) core and a pure J vortex if j_0' is decreased from $j_0' \sim j_0$ to $j_0' \ll j_0$. Making use of these solutions, we calculate the lower critical field H_{c1}, vortex mass, mobility, and the nonlinear voltage-current characteristics for a vortex moving along a planar defect. Implications for the c-axis transport in layered HTS oxides are discussed.

II. MODEL AND MAIN EQUATIONS

We model a layered superconductor as a periodic infinite stack of thin nonsuperconducting (N) layers of thickness $d \ll s$ embedded in an anisotropic superconducting (S) matrix characterized by the uniaxial tensor of magnetic penetration depths $\lambda_{\alpha\beta}$ (Fig. 1). One of the orthogonal principal axes z of $\lambda_{\alpha\beta}$ is perpendicular to the layers, which are situated at $z_n = ns$, $n = 0, \pm 1, \pm 2, \ldots$, and the principal values of $\lambda_{\alpha\beta}$ are $\lambda_{zz} = \lambda_c$ and $\lambda_{xx} = \lambda_{yy} = \lambda_b$, respectively. The subscripts on these principal values of λ correspond to the supercurrent direction. For example, induced screening current flowing in the superconducting matrix only in the z direction decays exponentially over the length λ_c. It is assumed that the N layers can be considered as planar Josephson contacts, all layers but the central one ($n = 0$) having the same critical current density j_0. The layer with $n = 0$ represents a planar defect which has a critical current density j_0' smaller than j_0. We are interested in the dependence of the properties of a vortex localized at the defect upon j_0' and the parameters of the superconductor. Before deriving the integral equations for the phase distribution $\varphi_n(\mathbf{r})$, we consider the linear screening in the layered superconductor shown in Fig. 1 and obtain formulas for effective magnetic penetration depths λ_y and λ_z, needed later on.

A. Linear screening

In this section we calculate the macroscopic magnetic penetration depths λ_y and λ_z in the case of the magnetic field $\mathbf{b}$ directed along the x axis (parallel to the layers in Fig. 1). For instance, if $b(y,z)$ changes only along the z axis, such that the current density flows only parallel to the y axis, the penetration depth λ_y just coincides with λ_b, since the thickness of the N layers is much less than that of the S layers.[1] We next consider the case for which $b(y,z)$ is periodic along the z direction $[b(y,z)=b(y,z+s)]$, but decays exponentially along the y axis over the penetration depth λ_z associated with currents perpendicular to the N layers which depend on j_0. To calculate λ_z, we consider variations of $b(y,z)$ described by the London equation

$$\lambda_c^2\frac{\partial^2 b}{\partial y^2}+\lambda_b^2\frac{\partial^2 b}{\partial z^2}-b=-\frac{\phi_0}{2\pi}\sum_n \frac{\partial\varphi_n}{\partial y}\,\delta(z-ns), \qquad (1)$$

where $\varphi_n(y)$ is the gauge-invariant phase difference of the superconducting order parameter across the nth Josephson contact. Here the right-hand side of Eq. (1) ensures the well-known boundary condition on the N layers,[35]

$$\frac{\partial\varphi_n}{\partial y}=\frac{8\pi^2\lambda_b^2}{c\phi_0}[j_y(y,ns+0)-j_y(y,ns-0)]. \qquad (2)$$

Indeed, if we integrate Eq. (1) over z from $z=ns-0$ to $z=ns+0$ and use the continuity of $\mathbf{b}$ at $z=ns$, we obtain

$$\frac{\partial b(y,ns+0)}{\partial z}-\frac{\partial b(y,ns-0)}{\partial z}=-\frac{\phi_0}{2\pi\lambda_b^2}\frac{\partial\varphi_n}{\partial y}. \qquad (3)$$

By expressing $\partial b/\partial z$ in Eq. (3) in terms of j_y by the Maxwell equation $\partial b/\partial z=4\pi j_y/c$, we recover Eq. (2). Here the thickness of the N layers is assumed to be negligible compared with s and λ_c. We also Fourier transform Eq. (1) in y, taking into account that for $b(y,z+s)=b(y,z)$ the gauge-invariant phase differences $\varphi_n(y)$ are the same for all n. In terms of the Fourier components

$$b_k(z)=\int_{-\infty}^{\infty}b(y,z)e^{-iky}\,dy, \quad \varphi_k=\int_{-\infty}^{\infty}\varphi(y)e^{-iky}dy, \qquad (4)$$

Eq. (1) and the boundary condition (3) take the form

$$\lambda_b^2 b_k''-(1+\lambda_c^2k^2)b_k=-\frac{ik\phi_0}{2\pi}\varphi_k\sum_n \delta(z-ns), \qquad (5)$$

$$b_k'(ns+0)-b_k'(ns-0)=-\frac{ik\phi_0}{2\pi\lambda_b^2}\varphi_k, \qquad (6)$$

where the prime denotes differentiation with respect to z. Since the solutions of Eqs. (5) and (6) are periodic in z, we consider only the domain $0<z<s$, for which

$$b_k(z)=-\frac{ik\phi_0\varphi_k\cosh q(z-s/2)}{4\pi\lambda_b^2 q\sinh(qs/2)}, \qquad (7)$$

with $q=(1+k^2\lambda_c^2)^{1/2}/\lambda_b$. From Eq. (7), we can calculate the Fourier transform of the z component of the current density $\mathbf{j}$, $j_{zk}(z)=-ikcb_k(z)/4\pi$, and evaluate it at $z=0$ or $z=s$, where $j_z(0)$ should be equal to the Josephson current density $j_J=j_0\sin\varphi$ through the contact. In the regime of linear screening ($|j_J|\ll j_0$), we have $j_J=j_0\varphi_k$; by equating $j_{zk}(0)$ to $j_J(k)$, we obtain the self-consistency condition

$$-\frac{k^2\phi_0 c}{16\pi^2\lambda_b^2 q\tanh(qs/2)}=j_0. \qquad (8)$$

Although in Eq. (4) we have allowed for arbitrary values of k, Eq. (8) has in fact only two roots $k=\pm i/\lambda_z(s)$, which correspond to the solutions which vary exponentially along the y direction over the penetration depth $\lambda_z(s)$. While $\lambda_z(s)$ can be determined numerically from Eq. (8) for arbitrary s, it is more convenient to express this relationship in terms of the inverse function $s(\lambda_z(s))$, given by

$$s(\lambda_z)=\frac{\lambda_z\lambda_b}{\sqrt{\lambda_z^2-\lambda_c^2}}\ln\left[\frac{\lambda_z\sqrt{\lambda_z^2-\lambda_c^2}+\lambda_J^2}{\lambda_z\sqrt{\lambda_z^2-\lambda_c^2}-\lambda_J^2}\right], \qquad (9)$$

where

$$\lambda_J=\left(\frac{c\phi_0}{16\pi^2\lambda_b j_0}\right)^{1/2}. \qquad (10)$$

Now we consider some limiting cases. In the weak-coupling limit $\lambda_z\gg\lambda_c$, Eq. (9) yields

$$\lambda_z=\lambda_J\coth^{1/2}\left(\frac{s}{2\lambda_b}\right). \qquad (11)$$

Hence it follows that the length $\lambda_z(s)$ decreases as s increases, approaching λ_J at $s\gg\lambda_a$. At small $s\ll\lambda_b$, Eq. (11) gives[8,36]

$$\lambda_z=\left(\frac{c\phi_0}{8\pi^2 s j_0}\right)^{1/2}. \qquad (12)$$

Equation (11) is valid if $\lambda_J\gg\lambda_c$.[1] Using Eq. (10), we find that the inequality $\lambda_J>\lambda_c$ can be written in the form of $j_0<j_l$, where

$$j_l=\frac{c\phi_0}{16\pi^2\lambda_c^2\lambda_b}. \qquad (13)$$

For $j_0>j_l$ the Josephson penetration depth (λ_J) becomes smaller than λ_c, which corresponds to a nonlocal Josephson electrodynamics for a single contact.[19]

B. Nonlinear equations for φ_n

In this section we derive integral equations for $\varphi_n(y)$ that take into account the long-range magnetic coupling of the layers and the nonlinearity of the Josephson interlayer currents. This can be done by means of the Green function $G(\mathbf{r},\mathbf{r}')$ of Eq. (1),

$$G(\mathbf{r},\mathbf{r}')=\frac{1}{2\pi\lambda_b\lambda_c}K_0\left(\left[\frac{(y-y')^2}{\lambda_c^2}+\frac{(z-z')^2}{\lambda_b^2}\right]^{1/2}\right), \tag{14}$$

where K_0 is a modified Bessel function. From Eqs. (1) and (14), we obtain the field distribution $b(y,z)$ in the form

$$b(y,z)=\frac{\phi_0}{4\pi^2\lambda_b\lambda_c}\times\sum_m\int_{-\infty}^{\infty}K_0\left(\left[\frac{(y-u)^2}{\lambda_c^2}+\frac{(z-ms)^2}{\lambda_b^2}\right]^{1/2}\right)\frac{\partial\varphi_m}{\partial u}du. \tag{15}$$

In order to get the equations for $\varphi_n(y)$, we use the Maxwell equation

$$\frac{c}{4\pi}\frac{\partial b(y,ns)}{\partial y}=j_{0n}\sin\varphi_n+\frac{\hbar}{2eR}\frac{\partial\varphi_n}{\partial t}+\frac{C\hbar^2}{4e^2}\frac{\partial^2\varphi_n}{\partial t^2}, \tag{16}$$

where the right-hand side represents the sum of the Josephson, resistive, and displacement current densities through the nth N layer, R and C are the specific interlayer resistance and capacitance, respectively, and $-e$ is the electron charge. Substituting Eq. (15) into Eq. (16) and integrating by parts, we arrive at the following equations for $\varphi_n(y)$:

$$\frac{\partial^2\varphi_n}{\partial\tau^2}+\eta\frac{\partial\varphi_n}{\partial\tau}=\frac{l_0}{\pi}\sum_m\int_{-\infty}^{\infty}K_0\left(\left[\frac{(y-u)^2}{\lambda_c^2}+\frac{(n-m)^2s^2}{\lambda_b^2}\right]^{1/2}\right)\times\frac{\partial^2\varphi_m}{\partial u^2}du-\sin\varphi_n. \tag{17}$$

Here $\tau=t\omega_J$ is the dimensionless time, $\omega_J=(2ej_0/\hbar C)^{1/2}$ is the Josephson plasma frequency, $\eta=1/RC\omega_J$ is the dimensionless damping constant due to the resistive currents, and l_0 is the characteristic nonlinear screening length,

$$l_0=\frac{c\phi_0}{16\pi^2j_0\lambda_b\lambda_c}. \tag{18}$$

When deriving Eq. (17), we assumed that all contacts are identical. In the case of the N layers with different j_{0n}, the stationary equations for $\varphi_n(y)$ become

$$\frac{l_0}{\pi}\sum_m\int_{-\infty}^{\infty}K_0\left(\left[\frac{(y-u)^2}{\lambda_c^2}+\frac{(n-m)^2s^2}{\lambda_b^2}\right]^{1/2}\right)\frac{\partial^2\varphi_m}{\partial u^2}du=\beta_n\sin\varphi_n, \tag{19}$$

where $\beta_n=j_{0n}/j_0$, and j_0 is the mean value of j_{0n}. We shall use Eq. (19) for the description of a vortex on a single-defect N layer $(n=0)$, which has a reduced critical current density $j_0'<j_0$. Notice that Eqs. (17)–(19) can be obtained from the variational principle

$$\eta\frac{\partial\varphi_n}{\partial\tau}=-\frac{\delta S}{\delta\varphi_n}, \tag{20}$$

where $S=\int \pounds\, d\tau$ is the dimensionless action, and the Lagrangian $\pounds$ is given by

$$\pounds=-\frac{l_0}{2\pi}\sum_{n,m}\int_{-\infty}^{\infty}dy_1\int_{-\infty}^{\infty}dy_2\times K_0\left(\left[\frac{(y_1-y_2)^2}{\lambda_c^2}+\frac{(n-m)^2s^2}{\lambda_b^2}\right]^{1/2}\right)\frac{\partial\varphi_m(y_1)}{\partial y_1}\frac{\partial\varphi_n(y_2)}{\partial y_2}-\sum_n\int_{-\infty}^{\infty}dy\left[\beta_n(1-\cos\varphi_n(y))-\frac{1}{2}\left(\frac{\partial\varphi_n(y,\tau)}{\partial\tau}\right)^2\right]. \tag{21}$$

Here the first term in the right-hand side of Eq. (21) describes the magnetic energy and the kinetic energy of superconducting currents in S layers. The diagonal terms $(m=n)$ correspond to self-energy, while the off-diagonal terms describe the magnetic interactions between different layers. The second term describes the Josephson energy, and the third one corresponds to the energy of the electric field stored in the Josephson contacts. The Lagrange form can be useful when describing dynamics of Josephson vortices at low temperatures, for example, for calculation of quantum flux creep in layered superconductors.[37]

C. Mean-field approach

In the general case, the nonlinear integral equation (19) is very complicated, and so we consider here a mean-field approach which enables one to obtain analytical single-vortex solutions for $\varphi_n(y)$ and $b(y,z)$. We consider here the most interesting case $s\ll\lambda_{b,c}$, for which the magnetic field $b(y,z)$ varies smoothly over the interlayer spacing, and the vortex has two characteristic regions similar to those of an A vortex. First there is a core region much smaller than $\lambda_{b,c}$, where the phase $\varphi_n(\mathbf{r})$ changes over the length $\sim s$, and the current density is of the order of j_0. Here both the discreteness of the superconductor and the nonlinearity of the interlayer Josephson currents become very important. Furthermore, there is a region of circulating screening currents which decay exponentially over the lengths λ_y and λ_z along the z and y axes, respectively. In this region the phase differences $\varphi_n(\mathbf{r})$ are small and change slowly over the interlayer spacing, and $\mathbf{j}(\mathbf{r})$ is much smaller than j_0. This region can be described by the linear London equation with the above effective penetration depths λ_y and λ_z,

$$\lambda_z^2\frac{\partial^2b}{\partial y^2}+\lambda_y^2\frac{\partial^2b}{\partial z^2}-b=-\frac{\phi_0}{2\pi}\frac{\partial\varphi}{\partial y}\delta(z), \tag{22}$$

where the right-hand side of Eq. (22) results from the vortex core located at the central defect layer $(n=0)$ and $\varphi(y)=\varphi_0(y)$. Since the phase $\varphi(y)$ in the vortex changes from 0 to 2π as y runs from $-\infty$ to ∞,[8] the derivative $\partial\varphi/\partial y$ in Eq. (22) can be replaced by $2\pi\delta(y)$ at distances from the core much larger than its size. Then Eq. (22) reduces to the well-known London equation for an A vortex in an anisotropic uniform superconductor. However, there is a qualitative difference between the normal core of an A vortex and the Josephson core of the vortex in the layered superconductors. For an A vortex, the core appears because of the suppression of the superconducting order parameter, since the screening current density at its center attains j_d.[18] By contrast, $\mathbf{j}(\mathbf{r})$ in the vortex in layered superconductors is limited by the Josephson interlayer j_0, which is always much smaller than j_d. As a result, the normal core is absent and the order parameter $\Delta(\mathbf{r})$ remains constant everywhere in the vortex including the core region, which is now defined as a domain of significant variation of the phases $\varphi_n(\mathbf{r})$, where $j(\mathbf{r})\sim j_0$.

These qualitative features of layered superconductors enable one to propose the following self-consistent mean-field description of the vortex based on the exact Eq. (19). Let us consider the central defect layer $(n=0)$, where $j(\mathbf{r})$ is maximum at a planar weak link embedded in a continuous anisotropic superconductor with the effective penetration depths λ_y and λ_z. In other words, we take into account the most essential nonlinearity of the Josephson current across the central layer and replace the rest of the sample by an effective medium that provides a linear magnetic screening of circulating currents outside the core, where the phases $\varphi_n(\mathbf{r})$ are small and vary smoothly over scales of order s. Therefore, the equation for $\varphi(y)$ can simply be obtained from Eq. (19) by retaining only the term with $n=0$ and replacing λ_b and λ_c by λ_y and λ_z, respectively. When describing the structure of the core, one can further simplify Eq. (19) by taking into account that the phase $\varphi(y)$ sharply decays over the core size l, which is much smaller than λ_c. In this case the Bessel function $K_0(|y-u|/\lambda_c)$ in Eq. (19) changes much more slowly than the derivative $\partial^2\varphi(u)/\partial u^2$ and thereby can be replaced by its expansion at small argument, $K_0(r)=\ln(2/r)-\mathbf{C}$, where $\mathbf{C}=0.577$ is the Euler constant.[38] Then the equation for $\varphi(y)$ takes the form

$$\frac{l}{\pi}\int_{-\infty}^{\infty}\ln\left(\frac{\lambda_z}{|y-u|}\right)\frac{\partial^2\varphi}{\partial u^2}\,du=\sin\varphi. \tag{23}$$

Here λ_z under the logarithm does not affect the solutions of Eq. (23), which satisfy the boundary conditions $\partial\varphi/\partial y=0$ at $y=\pm\infty$. The length l in Eq. (23) is

$$l=\frac{c\phi_0}{16\pi^2\lambda_y\lambda_z j_0'}\approx\frac{s\lambda_z j_0}{2\lambda_y j_0'}, \tag{24}$$

where j_0' is the critical current density across the defect layer. Likewise, one can obtain the field distribution $b(y,z)$ at the distance $(r\ll\lambda_{y,z})$ from the core in the form

$$b(y,z)=-\frac{\phi_0}{8\pi^2\lambda_y\lambda_z}\int_{-\infty}^{\infty}\left[\ln\left(\frac{z^2}{4\lambda_y^2}+\frac{(y-u)^2}{4\lambda_z^2}\right)+2\mathbf{C}\right]\frac{\partial\varphi}{\partial u}\,du. \tag{25}$$

The above expansion of the kernel $K_0(y-u)$ implies that at $l\ll\lambda_z$ the London screening of the circulating currents does not affect the structure of the vortex core described by the solution of Eq. (23). In this case the relation between the magnetic field $b(y,z)$ and the phase gradient $\partial\varphi/\partial y$ becomes nonlocal; that is, the field $b(y)$ at the point y is determined not only by the value of $\partial\varphi/\partial y$ at the same point but by the values $\partial\varphi/\partial u$ within the domain $|y-u|<l$ as well. Such a nonlocality results from the coupling of different N layers by the long-range magnetic field $b(y,z)$. In the opposite limit $l\gg\lambda_z$ the phase $\varphi(u)$ changes slowly as compared to the kernel $K_0(y-u)$, which thus can be replaced by $\pi\delta(y-u)$. In this case Eqs. (19) turn into decoupled sine-Gordon equations which describe noninteracting N layers in anisotropic superconductor, and Eq. (15) gives the known formula $b(y)=(\phi_0\partial\varphi/\partial y)/4\pi\lambda_c$ of local Josephson electrodynamics.[35] Notice also that Eq. (23) turns out to be similar to the equation which describes dislocations in crystalline potential. This analogy can be quite useful, enabling the use of extensive results of dislocation theory[39] to obtain solutions which describe vortex structures.[22,23]

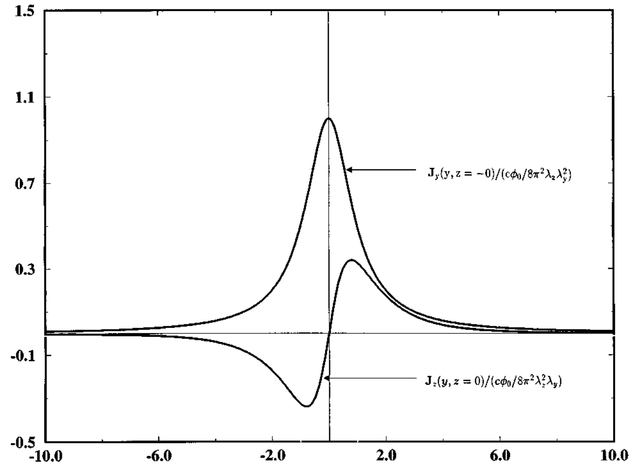

FIG. 2. The current distributions $j_y(y,z=0)$ and $j_z(y,z=+0)$ at the central layer for $l_\perp=\lambda_y$.

The nonlinear integral equation (23) has the following single-vortex solution[19]

$$\varphi(y)=\pi+2\tan^{-1}\left(\frac{y}{l}\right), \tag{26}$$

which describes the structure of the Josephson core. The nonlocal screening length l defines the characteristic core size along the layers. For identical layers, Eq. (26) was also obtained in Refs. 7 and 8 by another method. Substituting Eq. (26) into Eq. (25) and performig the integration, we find $b(y,z)$ in the region $y^2/\lambda_z^2+z^2/\lambda_y^2\ll 1$ in the form

$$b(y,z)=-\frac{\phi_0}{4\pi\lambda_y\lambda_z}\left\{\ln\left[\frac{y^2}{4\lambda_z^2}+\left(\frac{l}{2\lambda_z}+\frac{|z|}{2\lambda_y}\right)^2\right]+2\mathbf{C}\right\}. \tag{27}$$

The components of the current density $\mathbf{j(r)}$ are given by

$$j_y=\frac{c}{4\pi}\frac{\partial b}{\partial z}=-\frac{c\phi_0\lambda_z}{8\pi^2\lambda_y}\frac{(l_\perp+|z|)\operatorname{sgn}(z)}{[y^2\lambda_y^2+(l_\perp+|z|)^2\lambda_z^2]}, \tag{28}$$

$$j_z=-\frac{c}{4\pi}\frac{\partial b}{\partial y}=\frac{c\phi_0\lambda_y}{8\pi^2\lambda_z}\frac{y}{[y^2\lambda_y^2+(l_\perp+|z|)^2\lambda_z^2]}, \tag{29}$$

where $l_\perp=l\lambda_y/\lambda_z$. The distributions of $j_y(y,0)$ and $j_z(y,0)$ on the central layer are shown in Fig. 2. The field distribution $b(y,z)$ and the current lines corresponding to contours of constant $b(x,y)$ are shown in Figs. 3(a) and 3(b).

Using Eqs. (12) and (24), one can write

$$l_\perp=\frac{c\phi_0}{16\pi^2\lambda_z^2 j_0'}\approx s\frac{j_0}{2j_0'}. \tag{30}$$

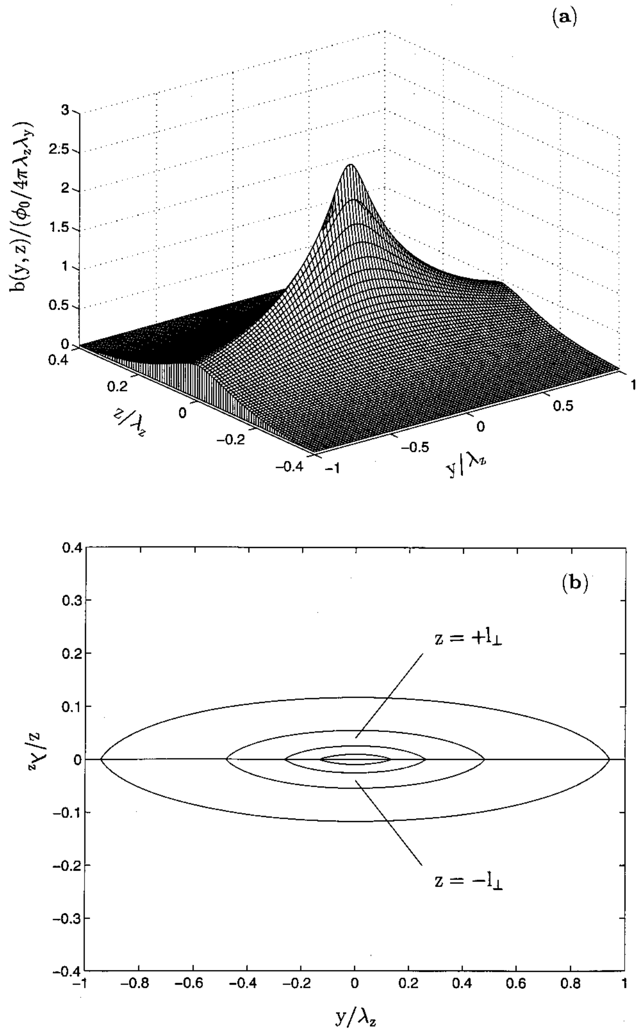

FIG. 3. (a) The field distribution $b(y,z)$ in the yz plane. (b) The current lines in the vortex-region in the case of $j_0' \ll j_0$, $l=0.1\lambda_z$, and $\lambda_z/\lambda_y=7$.

The phase φ_n with $n\neq 0$ can be calculated from the condition $j_z(ns)=j_0\varphi_n$, where $j_z(ns)$ is given by Eq. (29). From Eqs. (12) and (29), we obtain

$$\varphi_n(y)=\frac{s\lambda_z\lambda_y y}{y^2\lambda_y^2+(l_\perp+s|n|)^2\lambda_z^2}. \tag{31}$$

For $l^2/\lambda_z^2 \ll y^2/\lambda_z^2+z^2/\lambda_y^2 \ll 1$, Eqs. (27)–(29) reduce to the well-known formulas for the field distribution in an A vortex in an anisotropic superconductor with the penetration depths λ_y and λ_z. However, in contrast to an A vortex, the magnetic field $b(y,z)$ and the current density $j(y,z)$ in the center of the vortex in layered superconductors remain finite, and Eqs. (27)–(31) describe not only the circulating screening currents in the vortex, but also the core region. As it follows from Eqs. (26), (28), and (29), the phase gradient $\partial\varphi/\partial y$ and the components $j_y(y,0)$ and $j_z(y,0)$ decay over the length l which determines the core size along the layers. In layered HTS oxides the core length l given by Eq. (24) can be much larger than s, since $\lambda_z \gg \lambda_y$, and $j_0' < j_0$.

Let us now consider the change of the current-density components $j_y(y,z)$ and $j_z(y,z)$ in the direction perpendicular to the layers. As seen from Eqs. (28) and (29) and Fig. 2, the values $j_y(y,ns)$ and $j_z(y,ns)$ are maximum at $y=0$ and $y=y_m=\pm(l_\perp+s|n|)\lambda_z/\lambda_y$, respectively. Using Eq. (30), we find that the maximum values of j_y and j_z on the layers at $y>0$ are given by

$$j_y^{\max}(n)=-\frac{2\lambda_z}{\lambda_y}\frac{j_0'\mathrm{sgn}(n)}{1+2|n|j_0'/j_0}, \tag{32}$$

$$j_z^{\max}(n)=\frac{j_0'}{1+2|n|j_0'/j_0}. \tag{33}$$

Here both $j_y^{\max}(n)$ and $j_z^{\max}(n)$ decrease over the length $l_\perp = sj_0/2j_0'$. For identical layers ($j_0'=j_0$), we get $l_\perp=s/2$.[8,9] However, in the presence of the planar defect the length $l_\perp$ increases as j_0' decreases, becoming much larger than s for $j_0' \ll j_0$.

Since both l and $l_\perp$ are much smaller than λ_y and λ_z, one can generalize Eq. (27) by taking into account the whole region of the screening currents in the vortex. An interpolation formula which gives correct asymptotics of $b(y,z)$ both at small and large distances from the core can be written as

$$b(x,y)=\frac{\phi_0}{2\pi\lambda_z\lambda_y}K_0\left[\left(\frac{y^2}{\lambda_z^2}+\frac{(l_\perp+|z|)^2}{\lambda_y^2}\right)^{1/2}\right]. \tag{34}$$

At $y^2/\lambda_z^2+z^2/\lambda_y^2 \ll 1$, Eq. (34) reduces to Eq. (27). In the opposite case of large distances, the core structure becomes unimportant. Then one can put $l_\perp=0$ in Eq. (34), which gives $b(y,z)$ of an anistropic A vortex. Notice that in the vicinity of the vortex core, the self-consistent equation (34) somewhat differs from

$$b(y,z)=\frac{\phi_0}{2\pi\lambda_z\lambda_y}K_0\left[\left(\frac{y^2}{\lambda_z^2}+\frac{z^2}{\lambda_y^2}+\frac{l_\perp^2}{\lambda_y^2}\right)^{1/2}\right], \tag{35}$$

which has been obtained in Refs. 1, 8, and 9 within the framework of a variational approach. This results in a difference in φ_n with $n\neq 0$ given by Eq. (31) as compared to the formulas of Refs. 8 and 9, although $\varphi(y)$ on the central layer described by Eq. (26) coincides with that of Refs. 8 and 9.

Formulas (27)–(29) allow a clear geometrical interpretation,[19] if one takes account of the fact that for $l_\perp=0$ they describe an A vortex. Hence, it follows that at $l_\perp\neq 0$, the current lines in the half-plane $z>0$ coincide with that of a fictitious A vortex placed at the point $z=-l_\perp$. Likewise, the field $b(x,y)$ for $z<0$ is given by a fictitious vortex at $z=l_\perp$ [see Fig. 3(b)]. For $z \gg l_\perp$, such a field configuration reduces to that of an A vortex, and the discreteness of the superconductor manifests itself only in the core region $|z|<l_\perp$ and $|y|<l$. Notice that for the planar weak link ($j_0' \ll j_0$) the distance $2l_\perp$ between the fictitious A vortices is much larger than s; thus, the effective-medium approach holds everywhere, including the vortex core. However, for identical layers ($j_0'=j_0$), the distance $2l_\perp$ becomes of the order s. Here, the discreteness of the superconductor becomes essential and the structure of current lines in the core region changes as shown in Ref. 9. In this case, which has been considered in detail in Refs. 8 and 9, the above geometrical interpretation of $b(y,z)$ is valid only qualitatively.

To conclude this section, we discuss the validity of the mean-field approach in which the central layer is described exactly and the rest of the sample is replaced by a continuous effective medium which provides a linear magnetic screening. As seen from Eq. (33), the maximum current density across the layers is limited by the value j_0'. Thus at $j_0' \ll j_0$ one can always linearize the interlayer Josephson current $j_0 \sin\varphi_n$, obtaining the above formulas for the linear London screening with effective penetration depths λ_y and λ_z. As for identical layers, we have $j_z^{\max}(n) = j_c/(1+2|n|)$, and so the linear approximation, strictly speaking, becomes invalid since j_z across the layers with $n \neq 0$ remains of order j_0. However, even in this limiting case the maximum values of j_z across the neighboring layers are numerically small ($j_z^{\max} = j_0/3, j_0/5, \ldots$ at $n=1,2,\ldots$, respectively). The latter enables one to use the effective-medium approach near the vortex core even at $j_0' = j_0$.

III. LOWER CRITICAL FIELD H_{c1}

In order to calculate the lower critical field $H_{c1} = 4\pi F/\phi_0$, we consider the free energy of a single vortex localized on the planar defect, $F = F_m + F_J$, where F_m is the free energy associated with the magnetic field energy and the kinetic energy of the screening superconducting currents,

$$F_m = \int\int \left[b^2 + \lambda_z^2 \left(\frac{\partial b}{\partial y}\right)^2 + \lambda_y^2 \left(\frac{\partial b}{\partial z}\right)^2 \right] \frac{dy\,dz}{8\pi}, \tag{36}$$

and F_J is the Josephson-coupling free energy,

$$F_J = \frac{\hbar j_0'}{2e} \int (1 - \cos\varphi)\, dy, \tag{37}$$

where φ is determined by Eq. (26). Here F_J, which can be regarded as the vortex-core energy, does not contain contributions from layers with $n \neq 0$. These terms are automatically taken into account in the electromagnetic part of F_m by replacing λ_b by λ_y and λ_c by λ_z. We calculate F_m by expressing $b(y,z)$ in Eq. (36) via $\varphi(y)$ by using Eq. (25). In the mean-field approximation ($m=0, \lambda_b = \lambda_y, \lambda_c = \lambda_z$) this yields

$$F_m = \frac{\phi_0^2}{64\pi^4 \lambda_y \lambda_z} \int_{-\infty}^{\infty} K_0 \left[\frac{|y_1 - y_2|}{\lambda_z} \right] \frac{\partial\varphi}{\partial y_1} \frac{\partial\varphi}{\partial y_2}\, dy_1\, dy_2 . \tag{38}$$

It is convenient to write Eq. (38) in terms of the Fourier components $\varphi'(k)$ of the phase gradient $\partial\varphi/\partial y$,

$$F_m = \frac{\phi_0^2}{128\pi^4 \lambda_y} \int_{-\infty}^{\infty} \frac{|\varphi'(k)|^2}{\sqrt{1 + k^2\lambda_z^2}}\, dk. \tag{39}$$

For a single vortex the Fourier transform of the derivative of Eq. (26) gives

$$\varphi'(k) = 2\pi \exp(-l|k|). \tag{40}$$

Substituting Eq. (40) into Eq. (39) and performing the integration,[38] one finds

$$F_m = \frac{\phi_0^2}{32\pi \lambda_y \lambda_z} \left[\mathbf{E}_0\left(\frac{2l}{\lambda_z}\right) - N_0\left(\frac{2l}{\lambda_z}\right) \right], \tag{41}$$

where $\mathbf{E}_0(u)$ and $N_0(u)$ are the Weber and Neumann functions, respectively. Equation (41) can be simplified if the core size l is much smaller than λ_z. Then $u = 2l/\lambda_z \ll 1$ and one can use the following asymptotic expansions $\mathbf{E}_0 \sim u$ and $N_0(u) \approx 2[\ln(u/2) + \mathbf{C}]/\pi$, where $\mathbf{C} = 0.577$ is the Euler constant. Hence

$$F_m = \frac{\phi_0^2}{16\pi^2 \lambda_y \lambda_z} \left[\ln\left(\frac{\lambda_z}{l}\right) - \mathbf{C} \right]. \tag{42}$$

In order to calculate the Josephson energy F_J, we substitute Eq. (26) into Eq. (37) and obtain

$$F_J = \frac{\hbar j_0'}{e} \int_{\infty}^{\infty} \sin^2 \frac{\varphi}{2}\, dy = \frac{\pi\hbar j_0' l}{e}. \tag{43}$$

By adding F_m and F_J, we obtain

$$H_{c1} = \frac{\phi_0}{4\pi\lambda_y\lambda_z} \left[\ln\left(\frac{\lambda_z}{l}\right) + \gamma \right], \tag{44}$$

where $\gamma = 1 - \mathbf{C} = 0.423$. At $s \ll \lambda_y$, one can use Eqs. (12) and (24) to express H_{c1} in the form

$$H_{c1} = \frac{\phi_0}{4\pi\lambda_y\lambda_z} \left[\ln\left(\frac{\lambda_y j_0'}{s j_0}\right) + \gamma_1 \right], \tag{45}$$

with $\gamma_1 = \ln 2 + 1 - \mathbf{C} = 1.116$. Notice that H_{c1} depends logarithmically on j_0', the maximum supercurrent across the defect layer.

For identical layers ($j_0' = j_0$), Eq. (45) reduces to that obtained by Clem *et al.*[9] by another method (numerical calculations of H_{c1} have been carried out in Ref. 40). If $j_0' \ll j_0$, there are several different regimes. For $j_0' < j_l \sim j_0 s/\lambda_y$ the core length l along the layers becomes of the order of λ_z and the vortex considered above turns into a Josephson vortex in an anisotropic superconductor for which $H_{c1} \sim \phi_0/\lambda_J\lambda_y$.[35] The increase of j_0' above j_l results in the transformation of the Josephson vortex into an Abrikosov vortex but with a Josephson core, which can be considered as an intermediate stage between the pure Josephson vortex and the Abrikosov vortex with a normal core, which appears at $j_0' \sim j_0 \sim j_d$.[19]

IV. VORTEX MASS AND MOBILITY

The above results allow us to calculate the inertial vortex mass M and the viscous drag coefficient μ. Let the vortex move along the y axis with a constant velocity v much smaller than the Swihart velocity $c_s = \lambda_J \omega_J$. This causes the inductive voltages V_n across the N layers,

$$V_n = \frac{\hbar}{2e} \frac{\partial\varphi_n}{\partial t} = -\frac{\hbar v}{2e} \frac{\partial\varphi_n}{\partial y}, \tag{46}$$

where $\varphi_n(y,t) = \varphi_n(y - vt)$. As a result, the kinetic energy of the moving vortex, $Mv^2/2$, is determined by the total energy of the electric field stored in the N-layers:[41,42]

$$M=\frac{1}{v^2}\sum_n C_n\int_{-\infty}^{\infty}V_n^2dy=\left(\frac{\hbar}{2e}\right)^2\sum_n C_n\int_{-\infty}^{\infty}\left(\frac{\partial\varphi_n}{\partial y}\right)^2dy, \quad (47)$$

where C_n is the specific capacitance of the nth contact and $\varphi_n(y)$ is the static distribution of the phase differences. Substituting Eqs. (26) and (31) into Eq. (47), we obtain

$$M=\frac{\pi\hbar^2}{2e^2}\left[\frac{C_0}{l}+\frac{2C\lambda_y}{s\lambda_z}f\left(\frac{j_0}{2j_0'}\right)\right], \quad (48)$$

where

$$f(\alpha)=\frac{1}{8}\sum_{n=1}^{\infty}\frac{1}{(n+\alpha)^3}, \quad (49)$$

and C_0 and C are the specific capacitances of the central and the remaining identical layers, respectively. The function $f(\alpha)$ can be expressed in terms of the Riemann zeta function $\zeta(3,\alpha)$.[38] The asymptotics of $f(\alpha)$ are

$$f\left(\frac{1}{2}\right)=\frac{7}{8}\zeta(3)-1=0.052 \quad (j_0'=j_0), \quad (50)$$

$$f(\alpha)=\frac{1}{16\alpha^2}, \quad \alpha\gg 1. \quad (51)$$

Substituting s given by Eq. (9) into Eq. (48), we can express M in terms of measured macroscopic parameters. For instance, in the case $s\ll\lambda_y$, for which $\lambda_z(s)$ is given by Eq. (12), Eq. (48) becomes

$$M=\frac{8\pi j_0'\phi_0\lambda_y\lambda_z}{c^3}[C_0+2\alpha C f(\alpha)], \quad \alpha=\frac{j_0}{2j_0'}. \quad (52)$$

As seen from Eq. (52), the main contribution to M comes from the central layer.[8,9] The contribution from the layers with $n\neq 0$ is maximum for identical layers ($j_0'=j_0,\beta=1/2$) and rapidly decreases as j_0' decreases. For the case $j_0'=j_0$ and $C_0=C$, Eq. (52) reduces to that obtained by Coffey and Clem,[41] except for a difference in the values of f [$f(1/2)=0.113$ in Ref. 41] due to the above-mentioned difference in the vortex core structure given by Eq. (34) as compared to Eq. (35) used in Ref. 41. As a result, the value of M given by Eq. (52) is smaller than that of Ref. 41 by $\approx 5\%$. At $j_0'\ll j_0$, the vortex mass is entirely determined by the central layer and linearly increases with j_0', similar to that of a single high-j_c Josephson contact in a continuous superconductor.[19] For an Abrikosov vortex which corresponds to strong interlayer coupling, $l_\perp\sim\xi$, Eq. (52) describes the electromagnetic contribution to M, though in this case there is a much larger contribution to M due to electrons localized in the normal core.[43]

Likewise, one can calculate the viscous drag coefficient μ. Let the vortex move at a constant velocity $v\ll c_s$. Then one can obtain μ by equating the total power disspated in the vortex to μv^2, which yields[8,44,45]

$$\mu=\left(\frac{\hbar}{2e}\right)^2\sum_n\frac{1}{R_n}\int_{-\infty}^{\infty}\left(\frac{\partial\varphi_n}{\partial y}\right)^2dy, \quad (53)$$

where R_n are specific resistances of the N layers. By comparing Eqs. (47) and (53), we see that μ can be obtained from the above formulas for M by replacing C_n by $1/R_n$. Hence

$$\mu=\frac{\pi\hbar^2}{2e^2}\left[\frac{1}{lR_0}+\frac{2\lambda_y}{s\lambda_z R}f\left(\frac{j_0}{2j_0'}\right)\right]. \quad (54)$$

For $s\ll\lambda_y$, this takes the form

$$\mu=\frac{8\pi j_0'\phi_0\lambda_y\lambda_z}{c^3}\left[\frac{1}{R_0}+2\alpha\frac{f(\alpha)}{R}\right], \quad (55)$$

where R_0 and R are the linear resistivities of the central and the remaining identical layers. As for M, the main contribution to μ comes from the central layer, with the other layers giving a much smaller contribution, which becomes negligible for $j_0'\ll j_0$. By comparing Eqs. (52) and (55), we see that the dependence of μ upon j_0' and s coincides with that of M. For identical layers Eq. (55) reduces to the formula obtained by Clem and Coffey,[8] except for the difference in f. At $j_0'\ll j_0$, Eq. (54) also reproduces the formula for μ which has been obtained within the framework of nonlocal Josephson electrodynamics for a vortex localized at a high-j_c planar defect in a continuous superconductor.[19]

V. NONLINEAR RESISTIVITY

The linear vortex mobility μ calculated in the previous section corresponds to small velocities $v\ll c_s$ for which the difference in structure of the moving and static vortex does not affect μ. In this case the vortex velocity is proportional to the driving Lorentz force, which implies an Ohmic voltage-current (V-I) characteristic. For large v, the structure of the Josephson core can strongly change with v, which in turn gives rise to a nonlinear V-I characteristic due to the dependence of μ on v. In this section we consider this effect for a vortex moving along a planar defect with $j_0'\ll j_0$ and with a uniform transport current $j(t)$ flowing perpendicular to the defect plane. The phase distribution $\varphi_n(y,t)$ for the moving vortex is therefore described by the set of dynamic integral equations represented by Eq. (17). In the mean-field approach they reduce to a single integral equation which describes $\varphi(y,t)$ at the central layer:

$$\eta\frac{\partial\varphi}{\partial\tau}=\frac{l}{\pi}\int_{-\infty}^{\infty}\ln\left(\frac{\lambda_z}{|y-u|}\right)\frac{\partial^2\varphi}{\partial u^2}du-\sin\varphi+\beta, \quad (56)$$

where $\beta(t)=j(t)/j_0'$ is the dimensionless transport current density, which generally depends on time. Here we restrict ourselves to the overdamped case $\eta\gg 1$ in the absence of pinning, neglecting for simplicity the displacement and resistive currents through all layers but the defect one. The latter allows one to use the static effective λ_y and λ_z in Eq. (56). As was shown in Refs. 21 and 46, the solution of Eq. (56) has the form

$$\varphi(y,t)=\theta(t)+\pi+2\tan^{-1}\left[\frac{y-u(t)}{L(t)}\right], \quad (57)$$

where the functions $\theta(t)$, $u(t)$, and $L(t)$ obey the following set of ordinary differential equations:

$$\dot{\theta}+\sin\theta=\beta(t), \tag{58}$$

$$\dot{L}+L\cos\theta=l, \tag{59}$$

$$\dot{u}=-L\sin\theta. \tag{60}$$

Here the overdot implies differentiation with respect to the dimensionless time t/τ_0, where $\tau_0=\phi_0/2\pi cR_0j_0'$ is the Ohmic time constant. Equation (58) describes the relaxation dynamics of $\theta(t)$ in an overdamped Josephson junction.[35] Equations (59) and (60) describe the dynamics of the Josephson-core width $L(t)$ and velocity $v(t)=-du/dt$, respectively. As seen from Eq. (57), the shape of the phase core, $\varphi(y,t)$ in the overdamped case, remains self-similar for any ac transport current $j(t)$, since the time dependence manifests itself only in the scaling functions $L(t)$ and $u(t)$. In addition, Eq. (58) turns out to be decoupled from Eqs. (59) and (60), which makes the nonlinear dynamics of the vortex fully integrable for any given $\theta(t)$.[46] The distributions of $b(y,z,t)$, $j_y(y,z,t)$, and $j_z(y,z,t)$ for the moving vortex can be obtained from the static Eqs. (28) and (29) by replacing l by $L(t)$ and y by $y-u(t)$.

Now let us consider a particular case of a dc current ($\dot{\theta}=0$, $\theta=\sin^{-1}\beta$) for which Eqs. (58)–(60) become algebraic. Then the core width $L(\beta)$ and velocity $v(\beta)$ are given by

$$L=\frac{l}{\sqrt{1-\beta^2}}, \tag{61}$$

$$v=\frac{v_0\beta}{\sqrt{1-\beta^2}}, \tag{62}$$

where

$$v_0=\frac{c^2R_0}{8\pi\lambda_y\lambda_z}. \tag{63}$$

These formulas generalize those obtained in Refs. 21 and 46 for high-j_c Josephson contacts in isotropic superconductors to the case of layered materials. As seen from Eq. (61), the core width L increases with j, diverging at $j=j_0'$ [this divergence is eliminated by a nonzero interlayer capacitances C_n (Ref. 21)]. Unlike the Lorentz contraction of the Josephson vortex for low dissipation (see, e.g., Ref. 35), the increase of $L(v)$ is a specific feature of the overdamped case.[21] In turn, the dependence of the core width L on v gives rise to a nonlinear *V-I* characteristic of a layered superconductor if **H** is parallel and **j** is perpendicular to the layers.

At low magnetic fields, for which the Josephson cores of neighboring vortices do not overlap, the voltage V at the defect layer can be obtained from Faraday's law, namely, $V=\phi_0 v/ca(H)$. Here $a(H)\gg l$ is the spacing between vortices and $v(j)$ is given by Eq. (62). Hence the *V-I* characteristic can be written in the form

$$V=\frac{Rj}{\sqrt{1-(j/j_0')^2}}, \tag{64}$$

where the linear resistivity R is given by

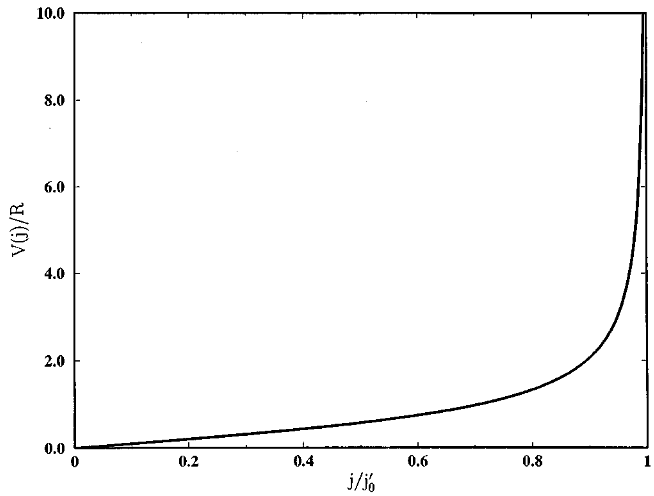

FIG. 4. The nonlinear *V-I* characteristic described by Eq. (64) for the overdamped case.

$$R=\frac{c\phi_0R_0}{8\pi\lambda_y\lambda_z a(H)j_0'}=R_0\frac{j_0}{j_0'}\frac{s}{a}\frac{\lambda_z}{\lambda_y}. \tag{65}$$

As seen from Eq. (64), the nonlinear $V(j)$ has an upward curvature (Fig. 4). The field dependence of the resistivity $R(H)$ is mostly determined by $a(H)$. If H_{c1} for the defect layer is much smaller than the bulk H_{c1}^b, the lattice spacing $a\sim\lambda_y\ln[H_{c1}/(H-H_{c1})]$ for $H\approx H_{c1}$ and $a=\phi_0/2\lambda_yH$ for $H_{c1}\ll H<H_{c1}^b$. In the latter case, we obtain the linear field dependence $R\propto H$, similar to that of the Bardeen-Stephen model.[45]

Now we consider the periodic ac signal $j(t)=j_a\cos\omega t$ for the quasistatic regime $\omega\tau\ll 1$, for which one can substitute $j(t)$ in Eq. (62). Then a simple integration of Eq. (62) yields the following time dependence of the vortex displacement $u(t)$:

$$u(t)=\frac{v_0}{\omega}\ln\frac{j_a\sin\omega t+\sqrt{j_0'^2-j_a^2\cos^2\omega t}}{\sqrt{j_0'^2-j_a^2}}. \tag{66}$$

This formula describes periodic oscillations of the vortex position $u(t)$ from $-u_m$ to u_m. The amplitude u_m is given by

$$u_m=\frac{v_0}{2\omega}\ln\frac{j_0'+j_a}{j_0'-j_a}. \tag{67}$$

For small ac signals $j_a\ll j_0'$, the value $u_m=j_av_0/\omega j_0'$ linearly increases with j_a and is inversely proportional to the j_0' of the defect layer. Notice that since $v_0\propto j_0^{1/2}$, the amplitude u_m decreases as the anisotropy factor λ_z/λ_y is increased.

Using Eq. (62), we can now calculate the mean power dissipated by the oscillating vortex, $Q=\phi_0\langle v(t)j(t)\rangle/c$, where $\langle\ \rangle$ stands for the time averaging over the period $2\pi/\omega$. We obtain

$$Q=Q_0[K(\beta_0)-E(\beta_0)]. \tag{68}$$

Here $Q_0 = 2\phi_0 v_0 j_0'/\pi$, and $K(\beta_0)$ and $E(\beta_0)$ are complete elliptic integrals of the first and second kinds, respectively, where $\beta_0 = j_a/j_0'$ is the modulus of K and E.[38] For $j_a \ll j_0'$, the dissipated power $Q(j_a) = \phi_0 v_0 j_a^2/2cj_0'$ increases quadratically with j_a, and diverges logarithmically as j_a approaches j_0'.[46]

VI. CONCLUSION

We have considered the Abrikosov vortex with anisotropic Josephson core localized at a planar defect in layered superconductors. We propose a self-consistent mean-field approach, which enables one to calculate the vortex structure analytically, including the core region. We have obtained formulas for the lower critical field H_{c1}, the vortex mass M, and the nonlinear mobility $\mu(v)$, which take into account both the discreteness of the superconducting matrix and the parameters of the defect layer. The absence of the normal core considerably reduces H_{c1}, M, and μ compared with an Abrikosov vortex.

The reduced H_{c1} leads to preferred flux penetration along planar defects, which, for example, can manifest itself as vortex chains localized at the defects in decoration[47,48] and magneto-optical[49–51] experiments. This can be due to the strong deformation of the vortex core caused by weak interlayer coupling and the small j_0' across the defect. As a result, the core becomes highly anisotropic, its size l along the layers essentially exceeding both the transverse core length $l_\perp$ and the interlayer spacing. This can give rise to a highly anisotropic vortex pinning force **f** with respect to the current-flow direction. The component of the pinning force parallel to the defect plane is much smaller than the perpendicular one.[24] As a result, the defect becomes a channel for easier flux motion.[19] In addition, the reduction of the vortex mass and the viscous drag coefficient due to the absence of the normal core strongly enhances the probability of quantum vortex tunneling in a pinning potential as compared with Abrikosov vortices. This can considerably facilitate temperature independent quantum flux creep in HTS oxides.

The above features may be essential for the interpretation of current transport along the c axis in HTS oxides, which often contain numerous planar defects parallel to the ab planes (stacking faults, intergrowths of low-T_c phases, etc.)[31] Because of the short coherence length along the c axis, these defects can strongly reduce the local j_0' as compared to j_0 and thus become a limiting factor for the c-axis current transport, now determined by vortex dynamics and pinning along the defects. In this case the measurement of $V(j)$ along the c axis enables one to extract $R_0(T)$ and $j_0'(T)$, whose temperature dependence can clarify the character of the superconducting coupling across the defects. For instance, at $j \ll j_0'$, the V-I curve is linear, which allows one to obtain the $R_0(T)$ dependence from Eq. (65). At the same time, by measuring $V(j)$ at $j \simeq j_0'$, one can extract j_0' with the help of Eq. (64). Notice here that the character of the $V(j)$ at $j \simeq j_0'$ can change significantly with T because of strong temperature dependence of the McCumber parameter $1/\eta(T) = C_0 R_0 \omega_J$. As a result, a transition occurs from the overdamped regime $\eta \gg 1$ at $T \simeq T_c$ to the underdamped regime $\eta \ll 1$ at $T \ll T_c$ due to a strong decrease of the density of normal quasiparticles which provide the ohmic dissipation in Josephson contacts. In turn, the change in $\eta(T)$ strongly affects the form of $V(j)$, which has an upward curvature at $\eta \gg 1$ [see Eq. (64)] and a downward curvature at $\eta \ll 1$ typical for underdamped Josephson contacts.[25]

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