Dynamical susceptibility of a thermally excited neutral Fermi-liquid film

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We perform a detailed study of the dynamical susceptibility of two-dimensional Fermi gases and liquids, in terms of the momentum and energy transferred to the system by an external probe, and both at zero and at finite temperatures. The response of a noninteracting system is computed and analyzed together with that of a liquid subjected to a monopole interaction, paying special attention to the Landau limit. The disappearance of Pauli blocking associated with thermal effects is examined in the frame of the collisional Landau regime. All results are compared to the equivalent ones obtained for a three-dimensional Fermi liquid. [S0163-1829(96)09333-2]

I. INTRODUCTION

In the past decades there have been several experiments with ^{3}He adsorbed on graphite and on bulk ^{4}He.[1,2] With such substrates the two-dimensional (2D) signatures of these thin films have been clearly established.

Heat-capacity results presented for ^{3}He adsorbed on graphite allow their identification as a fluid: Above a few kelvin degrees the heat capacity approaches k_B per atom, resembling a 2D gas behavior,[3] while between roughly 3 and 50 mK the Fermi system is degenerate and the measured heat capacity is almost proportional to the temperature.[4] Greywall and Busch have reported a sharp break in the trend of the data at 3.2 mK, below which the decrease of the heat capacity with temperature is faster.[5] Measurements of magnetic susceptibility and NMR relaxation times at temperatures between 0.4 and 4 K suggest a weakly interacting 2D gas behavior.[6] Within a hard-core model, these experimental results are very well fitted by the 2D Landau Fermi-liquid approach for finite temperatures developed by Havens-Sacco and Widom.[7]

Determinations of the surface tension[8,9] and the velocity of surface sound propagating in ^{3}He-^{4}He mixtures[9] show that the two-dimensional ^{3}He layer formed at the bulk ^{4}He-vacuum interface behaves as an almost ideal 2D gas of Fermi quasiparticles. The binding energy at the surface and the effective mass of this 2D gas are 2.2 K and 1.45 ^{3}He atomic masses, respectively, and the effective interaction between the quasiparticles is found to be very weak and predominantly repulsive at large distances.[9] In Ref. 10, a calculation of transport coefficients in a two-dimensional Fermi-liquid, using the Landau Fermi-liquid theory, predicts the thermal conductivity, spin diffusion, and transverse viscosity for these liquid ^{3}He submonolayers.

Films of ^{3}He are also present in ^{3}He-^{4}He mixtures films when the atoms of ^{3}He are adsorbed on thin films of ^{4}He, which are themselves either partially or fully adsorbed on a solid substrate. These ^{3}He coverages constitute objects of research by themselves, since their properties are substantially different from those of layers on the surface of bulk ^{4}He. Although there is some evidence of the two-dimensional behavior of these layers,[11–13] their description becomes more complicated as long as the ^{4}He film thickness is increased,[14–17] since the ^{3}He inside the film may behave either as a 2D or 3D system.

The existence of systems closely behaving as a two-dimensional Fermi liquid has forced the extension of the available theories of Fermi liquids to the lower dimensionality involved. In addition to the works already mentioned, this goal has been pursued by several authors giving rise to calculations of parameters of a Fermi gas with hard-core interactions,[18] thermodynamic and frequency-dependent magnitudes of a charged 2D Fermi gas,[19] and quasiparticle energies and effective interaction in the frame of Landau theory.[20,21] More recently, the possibility of mapping 2D quantum fluids into classical ones that interact according to potentials in phase space, rather than in configuration space, has been discussed.[22]

In spite of the existence of several studies concerning the dynamical response of 2D electronic systems,[23,24] the spectrum of collective excitations in 2D neutral Fermi liquids has not been investigated to a deep extent, neither from the experimental nor from the theoretical viewpoint. Then, the purpose of this work is to perform a detailed study of the dynamical susceptibility of a two-dimensional Fermi system. Such an investigation includes the analysis of the free gas and of the liquid at arbitrary nonvanishing temperatures. As pointed out in various contexts concerning Fermi liquids (see, for example, Ref. 25), whenever such a system is thermally excited, Pauli blocking effects are smoothed away due to the presence of finite collision rates. It is then important to manage as well the response to external fields in the collisional regime. To accomplish these purposes, the present paper is structured as follows. In Sec. II we present a detailed calculation of the exact dynamical susceptibility of a 2D Fermi gas as a function of the momentum and energy transferred by an external probe, with the temperature as a parameter. In Sec. III we investigate the low-momentum–low-energy case, in other words, the so-called Landau limit.[26,27] The response of a two-dimensional liquid characterized by a monopole interaction is analyzed in Sec. IV both for arbitrary momentum and energy and for the Landau limit. Section V contains an abridged reference to a recently developed formalism for the response of a 3D liquid in the collisional Landau regime;[28] this formulation is here adapted to the lower dimensionality and its consequences are examined at

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zero and at finite temperatures. The conclusions are summarized in Sec. VI.

II. RESPONSE FUNCTION OF A TWO-DIMENSIONAL FERMI GAS

The response function per unit area A of a two-dimensional Fermi liquid at arbitrary temperature T is

$$\chi(\mathbf{q},\omega)=-\frac{1}{A}\sum_{\mathbf{p}}\langle\mathbf{p}|\hat{O}|\mathbf{p}+\mathbf{q}\rangle|^2\frac{n_{\mathbf{p}}-n_{\mathbf{p}+\mathbf{q}}}{\varepsilon_{\mathbf{p}+\mathbf{q}}-\varepsilon_{\mathbf{p}}-\hbar\omega-i0^+}, \tag{2.1}$$

where $\hat{O}$ is the transition operator, $\mathbf{q}$ and ω are the momentum and energy transferred by the probe, respectively, $\varepsilon_{\mathbf{p}}$, is the energy of the particles having momentum, $\mathbf{p}$ and $n_{\mathbf{p}}$ is the Fermi distribution. If the system is a free Fermi gas, the transition matrix elements are unity, the single-particle (SP) spectrum is just the kinetic one, and the dynamical response is the free 2D susceptibility χ_0. As in the 3D case,[29] the latter can be exactly computed at zero temperature. For this sake, we introduce the dimensionless variables $x=q/p_F$, $\eta=p/p_F$, $y=\hbar\omega/\varepsilon_F$, and $\tau=k_BT/\varepsilon_F$, where p_F and ε_F, respectively, are the momentum and energy at the Fermi surface of a gas of fermions at $T=0$. Furthermore, let α_1 and α_2 be

$$\alpha_1=\frac{1}{2}\left(\frac{y}{x}-x\right) \tag{2.2}$$

and

$$\alpha_2=\frac{1}{2}\left(\frac{y}{x}+x\right). \tag{2.3}$$

In terms of these variables, after carrying the angular integration, the dynamical susceptibility of the particle-hole (p-h) continuum reads, at any temperature,

$$\begin{aligned}\chi_0(x,y,\tau)=\frac{2N(0)}{x}\Bigg[&\operatorname{sgn}(\alpha_2)\int_0^{|\alpha_2|}d\eta\frac{\eta n(\eta)}{\sqrt{\alpha_2^2-\eta^2}}\\&-\operatorname{sgn}(\alpha_1)\int_0^{|\alpha_1|}d\eta\frac{\eta n(\eta)}{\sqrt{\alpha_1^2-\eta^2}}\Bigg]\\&-i\frac{2N(0)}{x}\Bigg[\int_{|\alpha_2|}^{\infty}d\eta\frac{\eta n(\eta)}{\sqrt{\eta^2-\alpha_2^2}}\\&-\int_{|\alpha_1|}^{\infty}d\eta\frac{\eta n(\eta)}{\sqrt{\eta^2-\alpha_1^2}}\Bigg],\end{aligned} \tag{2.4}$$

where $N(0)=gm/2\pi\hbar^2$, with g the spin degeneracy, is the density of states per unit area on the Fermi surface, and $\operatorname{sgn}(x)=|x|/x$.

At zero temperature, the occupation numbers $n(\eta)$ in Eq. (2.4) are step functions and the integrals must be performed observing the three different possibilities that may relate x to y, namely, (i) $|\alpha_1|<1$, $|\alpha_2|<1$; (ii) $|\alpha_1|<1$, $|\alpha_2|>1$; and (iii) $|\alpha_1|>1$, $|\alpha_2|>1$. According to these relationships, we obtain

$$\frac{\chi_0}{N(0)}=\begin{cases}1-\frac{1}{x}\left(\sqrt{\alpha_1^2-1}-\sqrt{\alpha_2^2-1}\right) & \text{for } y\leqslant-2x-x^2,\\ 1-\frac{1}{x}\left(\sqrt{\alpha_1^2-1}+i\sqrt{1-\alpha_2^2}\right) & \text{for } -2x-x^2\leqslant y\leqslant\operatorname{sgn}(2-x)(-2x+x^2),\\ 1-\frac{i}{x}\left(\operatorname{sgn}(2-x)\sqrt{1-\alpha_2^2}-\sqrt{1-\alpha_1^2}\right) & \text{for } |y|\leqslant\operatorname{sgn}(2-x)(2x-x^2),\\ 1-\frac{1}{x}\left(\sqrt{\alpha_2^2-1}-i\sqrt{1-\alpha_1^2}\right) & \text{for } \operatorname{sgn}(2-x)(2x-x^2)\leqslant y\leqslant 2x+x^2,\\ 1-\frac{1}{x}\left(\sqrt{\alpha_2^2-1}-\sqrt{\alpha_1^2-1}\right) & \text{for } 2x+x^2\leqslant y.\end{cases} \tag{2.5}$$

This result formally coincides with that obtained by Stern[23] for the polarizability of a two-dimensional electron gas.

Straightforward computation of the dynamical structure factor $S(\mathbf{q},\omega,T)$,

$$S(\mathbf{q},\omega,T)=\frac{1}{A}\sum_{\mathbf{p}}\langle\mathbf{p}|\hat{O}|\mathbf{p}+\mathbf{q}\rangle|^2 n_{\mathbf{p}}(1-n_{\mathbf{p}+\mathbf{q}})\times\delta(\varepsilon_{\mathbf{p}+\mathbf{q}}-\varepsilon_{\mathbf{p}}-\hbar\omega), \tag{2.6}$$

for the free Fermi gas permits us to verify that

$$\operatorname{Im}\chi_0(\mathbf{q},\omega,T)=-\pi[S(\mathbf{q},\omega,T)-S(\mathbf{q},-\omega,T)], \tag{2.7}$$

as expected.

If the liquid is thermally excited, the integral in Eq. (2.4) cannot be done analytically, opposite to the case of the 3D system.[29–31] A numerical integration leads to the results displayed in Fig. 1, where the imaginary and real parts of $\chi_0(x,y,\tau)$ are plotted in the left and right columns, respectively, as functions of positive y for different values of τ and for $x=0.5$, 1, and 2.3. As we can see in this figure, the response function extends over a larger range of frequencies y as x increases at fixed τ or as τ increases at fixed x. Likewise, the amplitude of both $\operatorname{Re}\chi_0$ and $\operatorname{Im}\chi_0$ diminish. Furthermore, all cusps that correspond to matching different

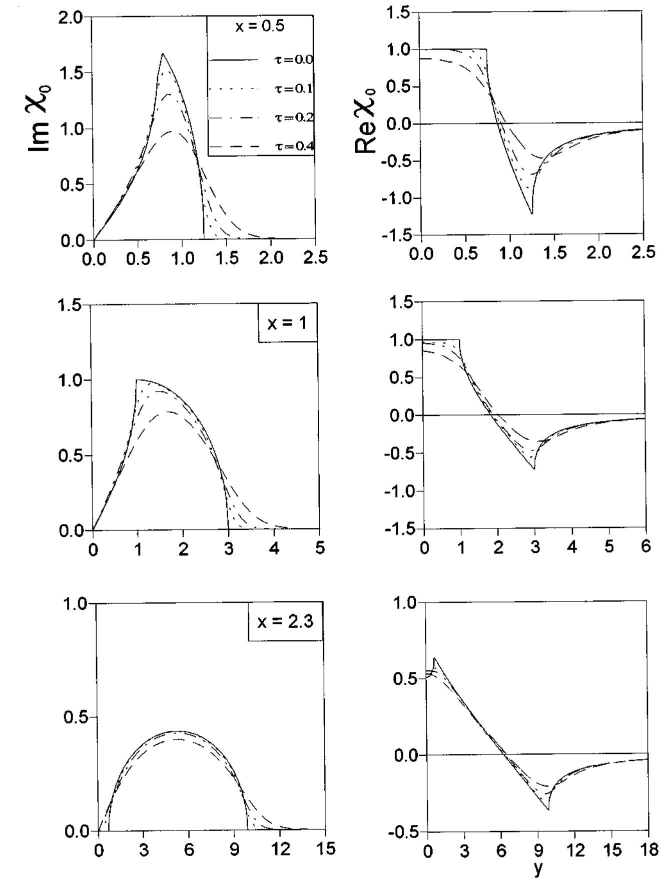

FIG. 1. The susceptibility $\chi_0/N(0)$ of a free 2D Fermi gas as a function of energy for different values of the transferred momentum and temperature. The left and right columns, respectively, show the imaginary and real parts of the response of the p-h continuum. All variables and parameters are dimensionless as defined in text.

y domains are smoothed away for nonvanishing values of τ. It is worth mentioning that the same behavior is observed in the 3D case;[29–31] in particular, it is interesting to keep in mind that for the temperatures and transferred momenta under consideration, the latter are much more efficient than thermal smoothing to reduce the p-h strength in the gas.

III. LANDAU LIMIT

The Landau limit corresponds to exposing the liquid to an almost homogeneous ($q \ll p_F$) and static ($\hbar\omega \ll \varepsilon_F$) perturbing field, however with finite dimensionless phase velocity $s = m^*\omega/qp_F$, where m^* is the effective mass at the Fermi level. Consequently, at $T=0$ the dynamical susceptibility in this limit can be obtained from the general one in Eq. (2.5). On the other hand, this dynamical susceptibility can be straighforwardly calculated starting from Landau's kinetic equation (LKE), just like in the 3D case.[26,27] In two dimensions, the LKE remains formally unchanged; however, the multipole expansions of the effective interaction and population fluctuations must be taken in terms of the orthogonal angular functions in two dimensions, $e^{i\alpha}$. These multipole expansions now read

$$\delta n_{\mathbf{p}} = \sum_{\mathbf{p}} \delta n_l(p) e^{il\alpha} \tag{3.1}$$

for the population fluctuations and

$$f_{\mathbf{pp}'} = \sum_l f_l(p,p') e^{il\alpha_{pp'}} \tag{3.2}$$

for the effective interaction. Here α denotes the angle between $\mathbf{p}$ and $\mathbf{q}$, while $\alpha_{pp'}$ corresponds to that between $\mathbf{p}$ and $\mathbf{p}'$.

One then needs to redefine the response matrix[27,21]

$$\Omega_{ll'}(s) = \frac{1}{2\pi}\int_0^{2\pi} d\alpha \frac{\cos\alpha}{\cos\alpha - s - i0^+} e^{i(l'-l)\alpha}. \tag{3.3}$$

In particular, the response function for the 2D Fermi gas in the Landau limit can be expressed in terms of Ω_{00} as

$$\chi_0^L(s,0) = N(0)\Omega_{00}(s) \tag{3.4}$$

for $T=0$ and

$$\chi_0^L(s,\tau) = \frac{1}{A}\sum_{\mathbf{p}} \left(-\frac{\partial n_{\mathbf{p}}}{\partial \varepsilon_{\mathbf{p}}}\right)\Omega_{00}[s(p)] \tag{3.5}$$

for finite T, where $s(p) = sp_F/p$. Notice, anyway, that for any l we have $\Omega_{ll} = \Omega_{00}$.

In the zero-temperature case, explicit calculation of the integral in Eq. (3.3) gives

$$\chi_{ll'}^L(s) = N(0)\left\{\delta_{ll'} - i\frac{s}{2\sqrt{1-s^2}}[(s+i\sqrt{1-s^2})^{l'-l} - (s-i\sqrt{1-s^2})^{l'-l}]\right\} + N(0)i\frac{s}{\sqrt{1-s^2}} \times \cos[(l'-l)\arccos s] \tag{3.6}$$

for $s<1$ and

$$\chi_{ll'}^L(s) = N(0)\left[\delta_{ll'} - \frac{s}{\sqrt{s^2-1}}(s-\sqrt{s^2-1})^{l'-l}\right] \tag{3.7}$$

for $s>1$. We then obtain the Landau limit of the dynamical susceptibility,

$$\chi_0^L(s,0) = N(0)\left[1 - \frac{s}{\sqrt{s^2-1}}\Theta(s-1) + i\frac{s}{\sqrt{1-s^2}}\Theta(1-s)\right], \tag{3.8}$$

with $\Theta(x)$ the usual step function.

For finite temperatures, no closed analytical expression can be written for the integral over $\mathbf{p}$ in Eq. (3.5), which is thus numerically solved. This has to be contrasted with the 3D case,[28] where an exact expression exists for the imaginary part of the susceptibility, and the real part can be cast as an infinite summation over residues of the integrand at the thermal poles of the Fermi occupation numbers $n_{\mathbf{p}}$.[28,29]

In Fig. 2 we show the free response function in the Landau limit, given by Eqs. (3.4) and (3.5), for different temperature values going from 0 to 0.2 in units of the Fermi energy. The thermal effects are the same as in the general case (Fig. 1), with a noticeable loss of strength for temperatures up to $0.2\varepsilon_F$. For $T=0$, a divergence occurs in the imaginary part as $s=1$ as we can visualize in Eq. (3.8). The

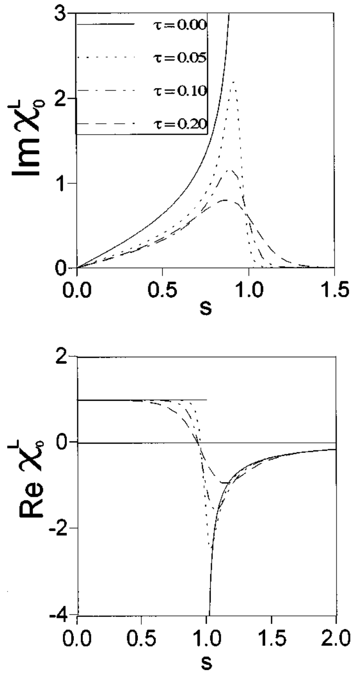

FIG. 2. The imaginary and real parts (above and below, respectively) of the p-h response in the Landau limit [in units of $N(0)$] for different temperatures, as a function of the dimensionless phase velocity.

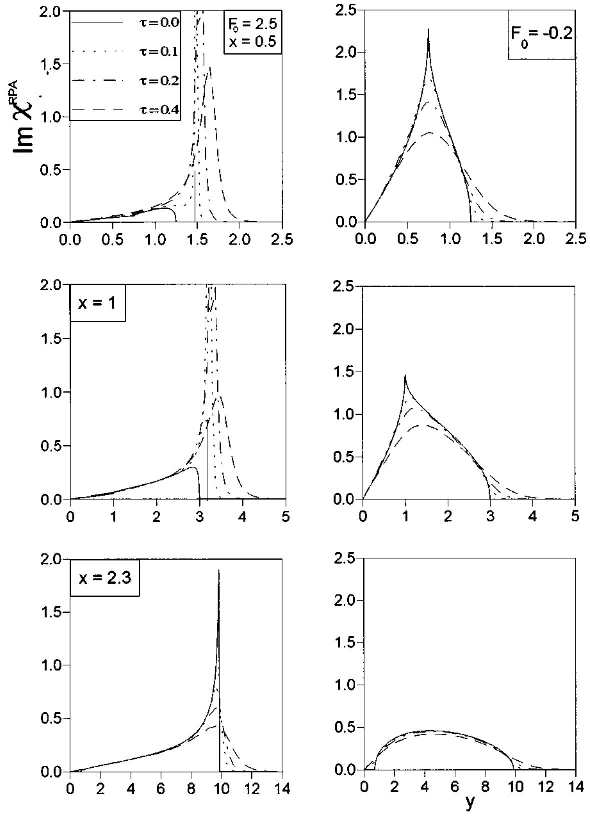

FIG. 3. The imaginary part of the dynamical susceptibility in the liquid for interaction strengths $F_0=2.5$ (left) and -0.2 (right). Transferred momentum and temperature are the same as in Fig. 1. The ordinate of each collective state corresponds to the residue of $\mathrm{Re}\chi$ at the pole.

latter is indeed a peculiarity of the 2D Fermi gas, since it does not occur in the 3D system. The appearance of this singularity is related to the angular integration that gives rise to the response matrix (3.3); since the integration variable is the angle itself, rather than $\cos\alpha$ as in the 3D gas, the branching point at $s=1$ is a power, rather than a logarithmic, one.

IV. MONOPOLE MODEL

If the liquid is in a collisionless regime and subjected to a quasiparticle interaction with constant momentum-momentum matrix elements, i.e., in the monopole interaction model, the response function is obtained in terms of the free one as

$$\chi(x,y,\tau)=\frac{\chi_0(x,y,\tau)}{1+f_0\chi_0(x,y,\tau)}, \tag{4.1}$$

with f_0 the monopolar Landau parameter in the spin channel under consideration. We must keep in mind that according to the surface sound data,[9] at low densities the value of this parameter is close to zero but possesses a large uncertainty, as high as 200%. As in the 3D case, this expression can be easily derived in the Landau limit;[26,27] for finite momentum and energy transfer, the same holds within the random phase approximation (RPA) frame for thermally excited Fermi liquids.[32]

We then realize that a collective mode appears, for given x, at an energy y that solves the equation

$$\chi_0(x,y,\tau)=-\frac{1}{f_0}. \tag{4.2}$$

Let us first analyze the properties of the collective poles for finite transferred momentum. Figure 3 displays the imaginary part of the response for the same momenta shown in Fig. 1 for the free gas. The left and right columns, respectively, show results obtained with $F_0=2.5$ and -0.2, $F_0=N(0)f_0$ being the dimensionless Landau parameter. For repulsive interactions, the collective peak moves towards slightly higher energies, while the width increases to a significant amount; however, for a transferred momentum as high as $x=2.3$, no collective state is present at $T=0$ and we only observe the smearing of the p-h continuum and broadening of the resonance in the vecinity of the p-h cutoff. This is due to the fact that for this value of x, the minimum of $\mathrm{Re}\chi_0$ is higher than $-1/f_0$. If we consider an attractive interaction, no collective state may appear and we verify the disappearance of the cusp in the p-h strength as the temperature increases, similarly to the case of the free gas. Moreover, the maximum moves towards larger energies with increasing x. It should be remarked that the effect of the interaction is to concentrate and redistribute the strength in the continuum, giving rise to a sharper cusp and to a curvature change at its right-hand side.

In the Landau limit, taking into account Eq. (3.8), we easily find the phase velocities that solve Eq. (4.2) at zero temperature, namely,

$$s_0(F_0)=\frac{1+1/F_0}{\sqrt{(1+1/F_0)^2-1}}. \tag{4.3}$$

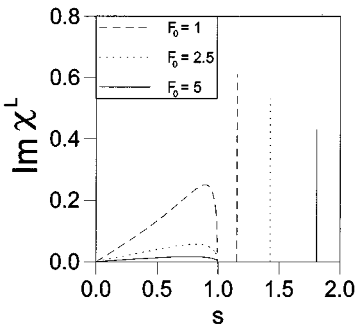

FIG. 4. Zero-temperature strength in the Landau limit for several positive values of the interaction strength.

Equation (4.3) tells us that in the limit $F_0 \gg 1$, $s_0(F_0) \approx \sqrt{F_0/2}$. Notice that F_0 is reduced by a factor 1/3 in the 3D case.[26]

The imaginary part of the response function at $T=0$ is plotted in Fig. 4 for positive values of F_0. We observe that a collective mode appears at a phase velocity $s_0>1$, but in contrast to the 3D case,[28] there is no damped resonance with $s_0<1$. We can also verify that as indicated by Eq. (4.3), $s_0(F_0)$ is an increasing function of F_0. As in the general case, no collective mode exists for negative values of F_0; in Fig. 5 the distorsion of the p-h continuum at $s<1$ is shown for F_0 between -1 and 0. For lower values of F_0 the system becomes unstable. We realize that as F_0 approaches zero from negative values, the location of the maximum is shifted towards $s=1$, while the intensity increases until the divergence of the free response appears. If the interaction strength decreases further into negative values, the peak drifts approaching $s=0$ as F_0 approaches -1.

On the other hand, Reχ at $s=0$ also diverges at $F_0=-1$; this can be easily understood realizing that from Eqs. (3.8) and (4.1), we have, at $T=0$, the static compressibility

$$\chi = \frac{N(0)}{1+F_0}, \tag{4.4}$$

which exhibits, at $F_0=-1$, the singularity that indicates the onset of instabilities against density fluctuations.

In Figs. 6 and 7 the imaginary part of χ in the Landau limit is shown for several values of reduced temperature τ and for $F_0=2.5$ and -0.2, respectively. As the temperature increases for a fixed F_0, the p-h spectrum spreads beyond

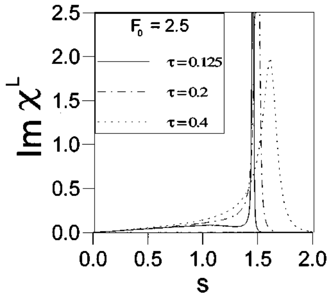

FIG. 6. Landau limit of the dynamical susceptibility (imaginary part) for a given repulsive interaction and several temperatures.

$s=1$, causing the collective mode to become broader and to merge in the thermally extended p-h continuum. Again, the effect of increasing temperature is essentially the same as for the 3D liquid.

V. COLLISIONAL REGIME IN THE LANDAU LIMIT

In Ref. 28 we present the full formalism leading to the construction of the density-density and temperature-density response of a 3D Fermi liquid in the collisional Landau regime. This scheme makes room for an arbitrary momentum dependence of the effective interaction between quasiparticles; the main results of the procedure are summarized in the Appendix for a simplified situation, namely, the case in which the expansion amplitudes $f_l(p,p')$ of the effective interaction are independent—or weakly dependent—upon the indicated momenta.

Although the system of equations (A1)–(A3) is cumbersome to solve in a general case, it can be considerably simplified in the monopole model, since in that situation one can extract a closed expression for the density fluctuation eliminating δT from the two conservation laws (A2) and (A3). The density-density response is then

$$\chi = \frac{\delta\rho}{\delta U} = \frac{\chi^{sc}}{1+f_0\chi^{sc}}, \tag{5.1}$$

with the screened resposne

$$\chi^{sc} = \frac{A}{i\alpha A/\tau_0 + \omega B/\omega' N_{00}}, \tag{5.2}$$

being here

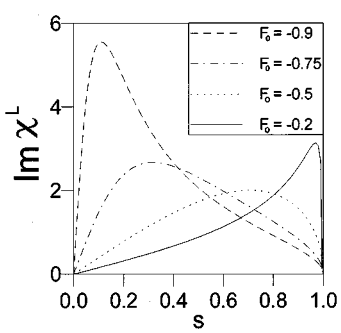

FIG. 5. Same as Fig. 4 for attractive interactions.

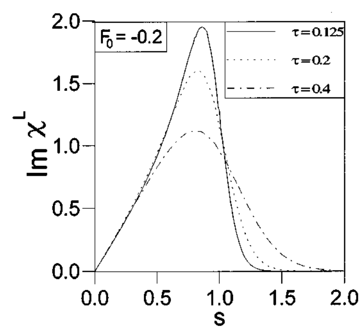

FIG. 7. Same as Fig. 6 for an attractive interaction.

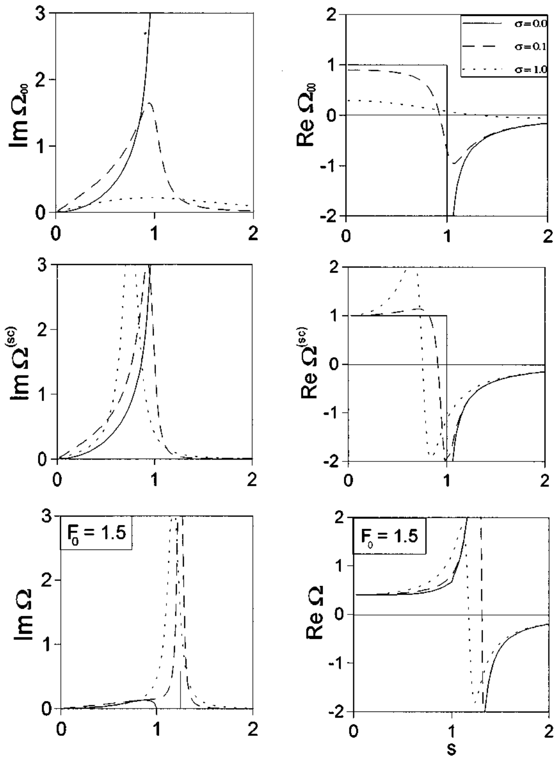

FIG. 8. Imaginary and real parts (left and right columns, respectively) of the collisional response in the Landau limit. From top to bottom, we show the free, screened, and liquid susceptibility at zero temperature for different values of the collision rate. All quantities are dimensionless.

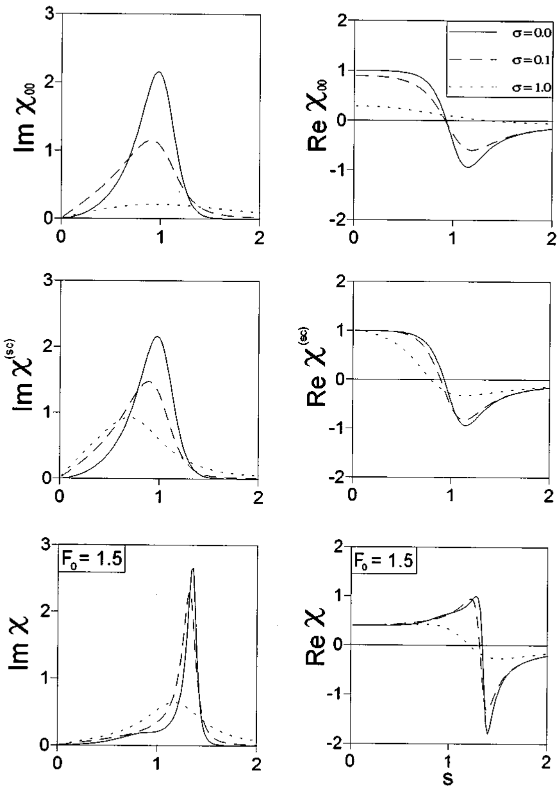

FIG. 9. Same as Fig. 8 for a temperature $T=0.2\varepsilon_F$.

$$A=i\frac{\sigma}{s'}[\chi_{00}^{(02)}\chi_{00}^{(00)}-(\chi_{00}^{(01)})^2]+\frac{s}{s'}\,\frac{N_{00}N_{02}-(N_{01})^2}{N_{00}}, \tag{5.3}$$

$$B=i\frac{\sigma}{s'}\,\frac{\chi_{00}^{(02)}N_{00}-2\chi_{00}^{(01)}N_{01}N_{00}+\chi_{00}^{(00}(N_{01})^2}{N_{00}}, \tag{5.4}$$

and

$$\alpha=\frac{1}{\omega' N_{00}}+\frac{m^*\omega}{q^2\rho}. \tag{5.5}$$

Here τ_0 is the relaxation time (given as an external parameter) and $\omega'=\omega+i/\tau_0$ is the complex frequency. The momentum-energy correlations N_{00} and the general thermal response matrix $\chi_{ll'}^{(nm)}$ are defined in Eqs. (A4) and (A5).

Some indicative results are shown in Figs. 8 and 9. In each of these figures, we plot the imaginary and real parts (left and right columns, respectively) of the p-h response χ_0^L, the screened response $\chi^{\rm sc}$, and the density-density response in the liquid, χ, as functions of the dimensionless phase velocity s for three values of the dimensionless relaxation rate $\sigma=m^*/\tau_0 q p_F$. While in Fig. 8 the temperature is $T=0$, in Fig. 9 we take $T=0.2\varepsilon_F$. Concerning the meaning of the free susceptibility χ_0^L as a function of a complex variable $s'=m^*\omega'/qp_F$, one can imagine that it represents a limiting situation where the liquid effects, mostly due to the long-range part of the two-body interaction, are sensitively weaker than the short-range effects that become apparent as two particles experience a close encounter. Surface sound propagating as a hydrodynamic mode in layers of ^{3}He, with a velocity that is consistent with vanishing or very small values of the monopole interaction,[9] represents a possible realization of these considerations. Anyway the upper part of these figures is useful as a reference to observe the evolution from the quantity χ_0^L into χ, through the coupling to $\delta\rho$ as measured by $\chi^{\rm sc}$.

The overall behavior of the dynamical susceptibility in these curves is completely equivalent to the 3D case;[28] in Fig. 8 we realize that for vanishing temperature, increasing the collision rate considerably smoothes the p-h response. Instead, the coupling to density fluctuations causes the screened response to exhibit a concentrated low-energy resonance, which drifts towards smaller phase velocities as σ becomes larger. When we regard the cool liquid, we can see that the effect of nonvanishing two-particle collisions is to broaden the collective state with moderate sensitivity of the centroid to the value of σ. On the other hand, observing Fig. 9 we learn that the combination of temperature and collisional effects gives rise to a more important dependence of the collective state with the size of the relaxation rate; indeed, the larger the value of σ, the more the height of the collective peak is lowered, and the more substantial is the increase of the width.

VI. CONCLUSIONS

In this work, we have undertaken a detailed study of the dynamical susceptibility of two-dimensional Fermi systems. Starting from the free gas exposed to an external probe that may either deposit or extract momentum and energy, we have analyzed the properties of the response of the p-h continuum in the full range of momentum, energy, and temperatures below the degeneracy threshold. The Landau limit of static and homogeneous systems has received special attention, in view of its relevance to the 3D gases. A liquid has been built introducing the simplest effective interaction, a constant one characterized by a strength F_0. We have examined the appearance of collective states and their trend in terms of all involved magnitudes, namely, transferred momentum and energy or the phase velocity in the Landau limit, as well as the temperature.

In the context of static and thermodynamic properties of 2D Fermi gases and liquids, it has been pointed out that the reduced dimensionality introduces no new effects.[19] This is not the case insofar as transport coefficients are concerned;[10] a $(\ln T)^{-1}$ factor in the thermal conductivity and spin difussion coefficients is a substantial difference with respect to the 3D system. In the present work we observe another fundamental difference, namely the fact that the dynamical structure factor of the 2D gas in the Landau limit exhibits a power singularity when the phase velocity equals the Fermi one. This is the only signature of dimensionality upon the dynamical response, since the overall tendency of all quantities here investigated, as one modifies the temperature or the transferred momentum, reproduces the behavior of the corresponding 3D magnitudes.

An important manifestation of thermal effects is the existence of finite quasiparticle lifetimes even in the vecinity of the Fermi surface of the noninteracting system. Its relevance to the broadening of collective modes in liquid ^{3}He has been remarked by many authors[33] and cannot be disregarded in any analysis of Fermi liquids at nonvanishing temperatures. The simplest approach to the problem, which makes room for the most important physical features, consists of examining the response in the collisional regime described by Landau's kinetic equation. This viewpoint allowed us to learn about the competition between thermal and collisional effects as agents for collective mode broadening in films, which are quite similar to those in 3D liquids. Generally speaking, we see that finite quasiparticle lifetimes are responsible for collective widths to a larger extent than the smearing of ther Fermi surface at nonvanishing temperatures, within the scales under consideration. In view of the results obtained in the 3D case,[28,34] which confirm the experimental observations concerning zero sound in liquid ^{3}He, it is apparent that this is a very general property of Fermi systems.

Finally, we wish to remark that experimental data on the excitation spectrum and transport properties of 2D Fermi liquids will be substantial to ascertain the validity of linear response and Landau theory restricted to the reduced dimensionality. In particular, we believe that, in spite of the simplicity of the proposed interaction, the monopolar model could describe in a first aproximation the excitation spectrum of submonolayers of liquid ^{3}He formed upon graphite or bulk ^{4}He.

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APPENDIX

The fundamental equations for the response in the presence of non-negligible two-quasiparticle collisions consist of an infinite coupled system for the multipolar amplitudes of the population fluctuations δn_l, together with the density and temperature ones, $\delta\rho$ and δT. While the relationship among the variations in the occupation numbers is derived from the LKE, following a well-known textbook procedure,[27] the coupling of these variations to those in density and temperature can be expressed by means of the mass and energy conservation laws. These ideas have been already applied to a 3D Fermi liquid in which a simplified momentum-dependent SP interaction acts between Landau quasiparticles.[25] The generalization to arbitrary momentum dependence of the interaction has been performed in Ref. 28, where it has been shown that starting from the LKE in the relaxation time approximation plus the mass-energy conservation laws, after some lengthy algebraic steps,[28] one reaches the set of equations

$$\delta n_l+\sum_{l'}\frac{s}{s'}\left[\chi_{ll'}^{(00)}+i\frac{\sigma}{s'}N_{00}\delta_{ll'}\right]\delta n_{l'}f_{l'}$$
$$=-i\frac{\sigma}{s'}\left\{\delta\rho\left[\mu_\rho(\chi_{l0}^{(00)}-\delta_{l0}N_{00})+\frac{\omega\omega' m_\mu^*}{q^2\rho}\chi_{l0}^{(00)}\right]\right.$$
$$\left.+\delta T[\mu_T\chi_{l0}^{(00)}+\chi_{l0}^{(01)}]\right\}-\chi_{l0}(s')\,\delta U \tag{A1}$$

for the variations in the occupation numbers,

$$0=-\frac{s}{s'}\sum_l\,[\chi_{l0}^{(00)}-N_{00}\delta_{l0}]f_l\delta n_l$$
$$-\delta\rho\left\{i\frac{\sigma}{s'}\left(\mu_\rho+\frac{\omega\omega' m_\mu^*}{q^2\rho}\right)\chi_{00}^{(00)}+\frac{s}{s'}\mu_\rho N_{00}\right\}$$
$$-\delta T\left\{i\frac{\sigma}{s'}[\mu_\rho\chi_{00}^{(00)}+\chi_{00}^{(01)}]+\frac{s}{s'}(\mu_T N_{00}+N_{01})\right\}$$
$$-\chi_{00}^{(00)}\delta U \tag{A2}$$

for the mass conservation law, and

$$0=-\frac{s}{s'}\sum_l\,[\chi_{l0}^{(01)}-N_{01}\delta_{l0}]f_l\delta n_l$$
$$-\delta\rho\left\{i\frac{\sigma}{s'}\left(\mu_\rho+\frac{\omega\omega' m_\mu^*}{q^2\rho}\right)\chi_{00}^{(01)}+\frac{s}{s'}\mu_\rho N_{01}\right\}$$
$$-\delta T\left\{i\frac{\sigma}{s'}[\mu_\rho\chi_{00}^{(01)}+\chi_{00}^{(02)}]+\frac{s}{s'}(\mu_T N_{01}+N_{02})\right\}$$
$$-\chi_{00}^{(01)}\delta U \tag{A3}$$

for energy conservation.

In these expressions we have introduced the momentum-energy correlation

$$N_{nm}=\frac{1}{A}\sum_{\mathbf{p}}\left(-\frac{\partial n_{\mathbf{p}}}{\partial \varepsilon_{\mathbf{p}}}\right)p^n\left(\frac{\varepsilon_{\mathbf{p}}-\mu}{T}\right)^m \tag{A4}$$

and the general thermal response matrix

$$\chi_{ll'}^{(nm)}(s')=\frac{1}{A}\sum_{\mathbf{p}}\left(-\frac{\partial n_{\mathbf{p}}}{\partial \varepsilon_{\mathbf{p}}}\right)\Omega_{ll'}[s'(p)]p^n\left(\frac{\varepsilon_{\mathbf{p}}-\mu}{T}\right)^m. \tag{A5}$$

In particular, $N_{00}\equiv N_0(T)$ is the averaged density of states whose zero-temperature value is $N(0)$, while $\chi_{00}^{(00)}$ is the density-density response of the free 2D Fermi gas in Eq. (3.5). Furthermore, $s'(p)$ is the complex dimensionless phase velocity of the perturbation, in units of the velocity of a quasiparticle having momentum **p** and effective mass $m^*(p)$,

$$\begin{aligned} s'(p)&=s(p)+i\sigma(p)\\ &=\frac{\omega' m^*(p)}{qp}\\ &=\frac{\omega m^*(p)}{qp}+i\frac{m^*(p)}{\tau_0 qp}, \end{aligned} \tag{A6}$$

while $s'=m^*\omega/qp_F+im^*/\tau_0 qp_F$ is taken at the Fermi level.

Here τ_0 is the assumed relaxation time of the liquid; thus σ is the dimensionless collision rate. In addition, we have written the parameters[28]

$$\mu_\rho=\frac{1+N_{00}f_0}{N_{00}} \tag{A7}$$

and

$$\mu_T=-\frac{N_{01}}{N_{00}}, \tag{A8}$$

and the effective mass $m_\mu^*=m^*(p_\mu)$, p_μ being the momentum at which the quasiparticle energy $\varepsilon_{\mathbf{p}}$ equals the chemical potential μ.

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