Continuum electrodynamics of type-II superconductors in the mixed state: The dc and ac response

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The dc and ac response of the ideal type-II superconductor in the mixed state is analyzed in the frame of a continuum electrodynamics, in which all fields are averaged on a scale exceeding the intervortex distance. The results of previous calculations are brought together and compared, while paying special attention to the role of the vortex line tension and the normal current. The electromagnetic response is studied in the whole range of magnetic fields and frequencies. The possible effect of the normal current on vortex motion is discussed. We argue in this respect that existing theories, where the Lorentz force involves the normal current, are not consistent with Onsager relations. Due to vortex line tension the external fields penetrate into a superconductor as a superposition of two modes with different complex wave numbers (the two-mode electrodynamics). Obtained expressions for the surface impedance should permit one to determine the parameters of the theory from the experiment and to discriminate different models of vortex motion. [S0163-1829(96)02134-0]

I. INTRODUCTION

Observation of the dc and ac responses of superconductors to the external electromagnetic fields is a powerful method of their experimental investigation. Therefore, a lot of work has been done to analyze and calculate this response.[1–9] This analysis can be done within the frame of the continuum approach which deals with the macroscopic fields averaged on a large scale compared with the vortex line (VL) spacing a. However, there is a problem to write equations of continuum electrodynamics properly, i.e., not to forget some relevant terms or forces.

In principle, one can derive these equations on averaging from a more basic theory for smaller scales, like the London or Ginzburg-Landau (GL) theories. This yields not only the structure of equations, but also the magnitudes of all parameters which enter the continuum theory. However, in this case, one is restricted to some simple situations or vortex configurations, which are not always adequate to the variety of experimental objects. The second way is to derive the continuum electrodynamics from the general conservation laws and symmetry arguments. This method has been successfully exploited for derivation of continuum hydrodynamics of rotating He II (a counterpart of the continuum electrodynamics for neutral superfluids).[10] For superconductors such a theory has been initiated by Abrikosov, Kemoklidze, and Khalatnikov[11] and later developed in an essentially more general form by Mathieu and Simon (MS).[12,13]

First works on the a.c. response[3,4] usually neglected normal currents and the VL tension. Neglecting line tension amounts to only retaining that part, $B^2/2\mu_0$, of the elastic energy which is associated with the macroscopic average field $\vec{B}$. The theory was reduced to that for the normal nonmagnetic conductor, but with the flux-flow resistivity ρ_f. For the first time, the VL tension has been taken into account properly in the ac response calculation in Ref. 9. The most important outcome of this work was that the external electromagnetic field penetrates the superconductor in the form of the superposition of two modes (two-mode electrodynamics): one has a long penetration depth equal to the skin depth determined by ρ_f and is common for any conductor. The second mode is related to the vortex line degree of freedom and penetrates to a much shorter distance slightly more than the intervortex spacing. Earlier this distance, which we shall call the vortex length λ_V, appeared in the vortex dynamics for rotating superfluids[14,15] and for superconductors in Ref. 12. It was shown that the second mode is crucial for incorporating the surface pinning into the theory. However, the normal currents have been neglected in Ref. 9, that restricts validity of the theory to low magnetic fields.

The normal currents have been taken into account by Coffey and Clem (CC).[7,8] They included also vortex bulk pinning and creep into their theory, but neglected the VL tension assuming that the latter would yield only small correction to magnetization. In order to introduce the normal currents into the theory, one should decide if there is any force of the normal current $\vec{J}_n$ on a vortex. CC assumed that this force was like that of the supercurrent $\vec{J}_s$, so that they wrote the Lorentz force with the total current $\vec{J}=\vec{J}_s+\vec{J}_n$. However, as will be shown in the present paper, the introduction of any force from the normal current on a vortex implies that there is also a reciprocal force driving the normal current. The latter is required by Onsager symmetry. This force was ignored by CC. Thus the effect of the normal currents also needs to be revised.

This paper has two objects: (i) to extend the continuum-electrodynamics theory of the linear response in the mixed state over the whole field range (from 0 to B_{c2}), by introducing a normal component $\vec{J}_n$ and retaining line tension effects; (ii) to bring out and discuss the common features and discrepancies in the basic equations for vortex motion given by different authors, which lead to different dispersion equations.

Moreover it will be instructive to discuss the role and the place of the vortex elasticity in continuum electrodynamics. A widely used approach is to present the electrodynamical equations in terms of the only vortex displacements $\vec{u}(\vec{r},t)$; following Brandt[16] this compels to introduce nonlocal k-dependent elastic moduli. In a sense this procedure is against the spirit of the elasticity theory which is supposed to be a *local* theory presented in terms of differential equations only. Furthermore, as will be shown below, when normal currents are involved, the nonlocal moduli should depend not only on wave number, but also on frequency, i.e., they are nonlocal in space and time. But it is possible to connect elastic moduli only with the vortex line energy, while treating the energy of the macroscopic magnetic field and the transport currents separately. This leads to k-independent elastic moduli as introduced in Ref. 9. This procedure is analogous to what is usually done with the long-range Coulomb interaction in the elasticity theory of atomic crystals. One introduces the electrical mean field and the corresponding electrostatic energy which depends on the atomic displacements over the whole crystal. After that, the additional deformation energy may be given in terms of local elastic moduli. In fact, the local elastic moduli were already involved in the general phenomenological theory of MS, but in different terms and notations.

In the MS theory, the local vortex structure is described by the vector $\vec{\omega}=n\varphi_0\vec{\nu}$ gathering the flux quantum φ_0, the vortex density n (the vortex number per unit area in a plane normal to vortices or, in other terms, the length of VL per unit volume), and the direction $\vec{\nu}$ of the vortex lines; similarly, in the Bekarevich-Khalatnikov (BK) theory of the He II, $\vec{\omega}=n\kappa\vec{\nu}$ where κ is the quantum of circulation.[10] By using the only parameter $\vec{\omega}$ to describe the vortex lattice, differences in free energy between different lattices of same density (triangular or square for example) are deliberately ignored in the BK and MS theories. Thus the shear rigidity given by the minute shear modulus C_{66} is ignored.[17] Otherwise, the MS equations account for all elastic effects associated with line-tension and compression of vortices.

In most of this paper we shall restrict our discussion to isotropic materials and perfect homogeneous samples, free from all surface or volume defects. In particular, this means that the sample surface is assumed to be ideally smooth. Indeed, the basic problems of vortex dynamics discussed in the present paper are independent of pinning problems, and it is worth first considering the behavior of an ideal sample. However, there is no doubt that defects play a dominant role in determining the ac response of an actual sample, and the main purpose of the CC theory, in this respect, was precisely to include pinning effects in the vortex equations of motion. Therefore these effects are discussed in the present paper also, with the emphasis on the surface pinning which can be incorporated into the theory only in the two-mode electrodynamics.

To be more explicit in our predictions, we shall refer to the following *standard conditions*: in the usual geometry of a slab (or an half-space) in normal applied field $\vec{B}(0,0,B)$, we shall consider (steady or oscillatory) one-dimensionnal vortex motions ($\partial/\partial x=\partial/\partial y=0$). Furthermore, we assume low to moderate currents (and/or exciting fields), so that the sample is quasi-isothermal and the vortex array only undergoes slight deformations with respect to the uniform array of straight vortices parallel to z, with the equilibrium density $n=1/\pi a^2=B/\phi_0$.

The plan of the paper is the following. In Sec. II, the MS formalism is presented and dependences of different parameters on the magnetic field are discussed. Section III is devoted to the dc response in the flux-flow regime. In Sec. IV, the dc response is reconsidered so as to account for a possible J_n component of the Lorentz force; this requires one to introduce a crossterm into the linear dynamical laws (γ term in the text). The effect of such a cross-term in the ac response is discussed in the Appendix in the frame of the one-mode electrodynamics ignoring the line tension effect. It is shown that the analysis of Coffey and Clem[7] does not satisfy the Onsager relations and their results concerning the ac response are revised.

In Sec. V, we analyze the ac response including the vortex line tension, which results in the two-mode electrodynamics. The two-mode effects are especially important at low magnetic fields and low frequencies. In the end of this section the relation of our theory to different concepts of the elastic moduli is discussed. Section VI extends the presented theory to include vortex pinning. Other effects relevant for real superconductors are also discussed. Finally, Sec. VII contains the resume of theory and conclusions.

II. THE MS FORMALISM

Neglecting space charge and electrostatic effects on a macroscopic scale, and assuming that the superfluid density, $n_s=\rho^2(\vec{r},t)$, satisfies rigidly its equilibrium conditions, as if it relaxed instantaneously, the free-energy density $\overline{F}$ can be expressed in terms of a reduced number of local macroscopic variables, namely the magnetic field $\vec{B}$, the supercurrent density $\vec{J}_s$ (or the superfluid velocity field $\vec{V}_s$), the vortex field $\vec{\omega}$ and the temperature T. Here $\vec{B},\vec{J}_s,\vec{V}_s$ stand for the macroscopic averages of the corresponding "microscopic" magnetic field $\vec{b}$, supercurrent $\vec{j}_s$ and superfluid velocity $\vec{v}_s$; $\vec{v}_s,\vec{j}_s$ are defined from the order parameter $\psi=\rho e^{i\theta}$ by

$$\vec{p}_s=\hbar\vec{\nabla}\theta=2m\vec{v}_s-2e\vec{a}, \tag{1}$$

$$\vec{j}_s=-2e\rho^2\vec{v}_s, \tag{2}$$

where $\vec{p}_s$ is the momentum field of the supercurrent, m and $-e$ are the electronic mass and charge, and $\vec{a}$ is the vector potential. Equation (2) holds in isotropic materials.

In the mixed state (in rotating He II) curl$\vec{p}_s\equiv0$ (curl $\vec{v}_s\equiv0$) everywhere except at the vortex cores. Averaging and taking account of the core singularities gives[13]

$$\vec{B}-\frac{m}{e}\mathrm{curl}\vec{V}_s=\vec{\omega}, \tag{3}$$

[curl$\vec{V}_s=\vec{\omega}$ in rotating He II (Ref. 10)]. The macroscopic London equation (3) states the crucial distinction to be made in general between vortex lines and field lines.[12,13] Thus the vortex field $\vec{\omega}$ and the magnetic field $\vec{B}$ must be regarded as two independent variables, when writing a local thermodynamic identity, as in Eq. (4) below. We note that the vortex field $\vec{\omega}$ was called ''vortex induction'' in Ref. 18 and ''local vortex magnetic induction'' in Refs. 7 and 8. Here we consider the simple term ''vortex field'' as more suitable. Equation (3) together with Maxwell equations intervene as constraints limiting the possible spatial variations of currents and fields.

In the presence of a vortex lattice, the macroscopic thermodynamics identity for the free-energy density $\overline{F}$ reads[13]

$$d\overline{F}=-\overline{\sigma}dT+\frac{1}{\mu_0}\vec{B}\cdot d\vec{B}-\frac{m}{e}\vec{J}_s\cdot d\vec{V}_s+\vec{\varepsilon}\cdot d\vec{\omega}. \qquad (4)$$

Two simple results have come out: (i) $\partial\overline{F}/\partial\vec{B}=\vec{B}/\mu_0$, and (ii) $\vec{J}_s$ is the conjugate variable of $\vec{V}_s$. Otherwise, explicit calculations of $\overline{F}$ and/or approximations are required to obtain both equations of state $\vec{J}_s(T,\vec{\omega},\vec{V}_s)$ and $\vec{\varepsilon}(T,\vec{\omega},\vec{V}_s)$. Several expressions for $\overline{F}$ are available in the literature, in the special case of an uniform regular array where $\vec{V}_s\equiv0$, $\vec{J}_s\equiv0$, $\vec{\omega}\equiv\vec{B}\equiv\overrightarrow{\mathrm{const}}$: in isotropic materials, $\overline{F}(T,\vec{B},\vec{V}_s=0,\vec{\omega}=\vec{B})=\varphi(T,B)$, $\vec{\varepsilon}=\varepsilon\vec{\nu}$ is directed along vortices and $\varepsilon(T,\omega=B)=\partial\varphi/\partial B-B/\mu_0$. This quantity is usually referred to (except for the sign) as the magnetization, $\vec{\varepsilon}=-\vec{M}$, by a formal analogy with ferromagnetics. However, one should be careful with this analogy as explained in Ref. 13. Therefore it is more advisable to call $\vec{\varepsilon}$ the *vortex potential*. All the same, it will be convenient to introduce, as a short notation, an auxiliary vector $\vec{h}$, defined as $\vec{\omega}/\mu_0+\vec{\varepsilon}$, which is neither more nor less artificial than the $\vec{H}$ field in magnetism.

In a homogeneous sample the macroscopic chemical potential of the electrons $\overline{\mu}$ may be assumed to be uniform, and the equations for equilibrium (or nondissipative currents) are[13]

$$\vec{E}'=\vec{E}+\frac{\nabla\overline{\mu}}{e}\simeq\vec{E}=0, \qquad (5)$$

$$\vec{C}=\vec{J}_s+\mathrm{curl}\vec{\varepsilon}=0. \qquad (6)$$

These equations should be complemented by the boundary condition for the ideal surface:

$$\vec{\varepsilon}\times\vec{N}=0, \qquad (7)$$

where $\vec{N}$ is the normal unit vector: taking $\vec{\varepsilon}=\varepsilon\vec{\nu}$ (isotropic materials and low $\vec{V}_s$), Eq. (7) requires that vortex lines terminate perpendicular to the sample surface. Equation (6) states, on a macroscopic scale, that the local supercurrent $\vec{j}_s$ at the vortex cores, which includes the contribution induced by vortices themselves if they are curved, is zero.[13]

As we are possibly concerned with very high frequency vortex motion, we must pay some attention to the hypothesis that n_s should relax instantaneously to its equilibrium values as assumed in our theory. Strictly $\rho=\rho_0 f$ (ρ_0^2 is the zero-field equilibrium value of the superfluid density, and f is the reduced order parameter) obeys some time-dependent Ginzgurg-Landau (GL) equation, such as that considered by Schmid:[19]

$$\frac{\partial f}{\partial t}=\frac{L}{2\tau_R}=\frac{1}{2\tau_R}\left[f-f^3-f\xi^2\left(\vec{\nabla}\theta+\frac{2e}{\hbar}\vec{a}\right)^2+\xi^2\Delta f\right], \quad (8)$$

where L is the left-hand side of the first GL equation, acting as a generalized force, and τ_R a relaxation time, which does not exceed 10^{-12} sec at temperatures not too close to T_c. We may consider that f hardly departs from equilibrium, as far as $L\ll f$, or, otherwise stated, $L\simeq0$ within an accuracy L/f. If $\vec{v}_L$ is the vortex line velocity,

$$L\sim\tau_R\dot{f}\sim\tau_R\frac{v_L}{\xi}f,$$

so that $L/f\leqslant10^{-4}$ in standard conditions of dc flux flow. For small vortex displacements $ue^{-i\Omega t}$ the condition $L/f\ll1$ reads

$$\frac{L}{f}\sim\Omega\tau_R\frac{u}{\xi}\ll1, \qquad (9)$$

which turns out to be a limitation on linearity rather than frequency. Taking $\tau_R\sim10^{-12}$ sec, $\xi\sim100$ Å, and vortex displacements typically less than 1 Å, condition (9) is still fulfilled at $\Omega/2\pi\sim10$ GHz ($L/f<10^{-3}$). In contrast, but consistently, the contribution of time relaxation effects to dissipation remains significant (see Sec. III). We also assume that, at any time, ρ satisfies the equilibrium GL boundary condition $\vec{N}\cdot\vec{\nabla}\rho=0$. This is consistent with the fact that vortex lines (lines $\rho=0$) must end perpendicular to the sample surface [condition (7)].

Figure 1 shows the general shape of $\varepsilon(\omega)$ for a uniform vortex array in an isotropic material.[20] In the low field limit ($\omega\ll B_{c1}$), $\varphi\simeq\omega H_{c1}$ so that $\varepsilon=H_{c1}-\omega/\mu_0$. Here H_{c1} and H_{c2} merely stand for B_{c1}/μ_0 and B_{c2}/μ_0, where $\mu_0=4\pi\times10^{-7}$.

At high fields, ε decreases linearly when ω approaches to B_{c2} (the Abrikosov line):[21]

$$\varepsilon=\frac{H_{c2}-\omega/\mu_0}{\beta_A(2\kappa^2-1)+1}\simeq\frac{H_{c2}-\omega/\mu_0}{2\kappa^2\beta_A} \quad (\kappa\gg1), \qquad (10)$$

where $\beta_A=1.16$ for a triangular lattice. In an extended London model ($\omega\gg B_{c1}$):

$$\varepsilon=\frac{\varphi_0}{4\pi\mu_0\lambda^2}\ln\frac{a}{\xi^*}, \qquad (11)$$

where $\lambda=\lambda_0(\overline{f^2})^{-1/2}$ is the field-dependent London penetration depth ($\mu_0\lambda^2=m/2n_s e^2$);[22] $\lambda_0(T)$ is the zero-field penetration depth; and $\xi^*\geqslant\xi$ is an effective core radius. The field-dependent London penetration depth λ tends to λ_0 at $\omega=0$, and diverges at $\omega\to B_{c2}$. Following this ''mean-field'' approximation we shall employ the London-like equation of state:

$$\vec{J}_s=-2eN_s(T,\omega,V_s^2)\vec{V}_s, \qquad (12)$$

where $N_s=\overline{\rho^2}$ is the mean superfluid density; Eq. (12) follows at once from Eq. (2) if it is assumed that

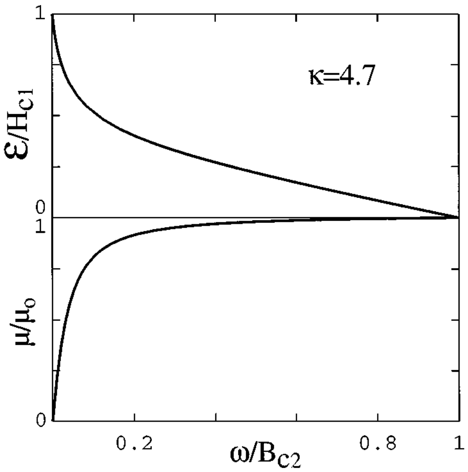

FIG. 1. The local vortex potential ε defined in Eq. (4) as the local thermodynamic variable conjugate of the vortex field, for a uniform vortex array ($\omega=B$). $\varepsilon(\omega,T)$ is the fundamental equation of state, from which reversible magnetization curves of simply shaped samples can be deduced. In particular, $\varepsilon(\omega=B=B_0)$ coincides, except for the sign, with the magnetization curve of a slab in a normal field B_0. An interpolating formula has been used to join the Abrikosov line (10) and the low-field line $\varepsilon=H_{c1}-\omega/\mu_0$ (Ref. 38). It is convenient to introduce an h field defined in the text as $h=\omega/\mu_0+\varepsilon$. The ''permeability'' μ defined in Eq. (40) as the ratio ω/h is directly calculated from the upper curve.

$(\overline{f^2\vec{v}_s})=\overline{f^2}\vec{V}_s$. Inserting Eq. (12) in the thermodynamic identity suggests a quadratic V_s dependence of the vortex potential, according to $\partial\varepsilon/\partial V_s{}^2=m\partial N_s/\partial\omega$. However, in the linearized equations describing small standard vortex motions, such as defined in the Introduction, $V_s{}^2$ terms in ε or N_s, being of second order, may be systematically ignored.

Equation (12) can be rewritten as

$$-\frac{m}{e}\vec{V}_s=\mu_0\lambda^2\vec{J}_s. \tag{13}$$

One can use Eq. (13) with the approximate linear law $\overline{f^2}=1-\omega/B_{c2}$. A better approximation in the high field range, using Abrikosov's results, is[22]

$$\lambda^2=\frac{\lambda_0{}^2}{\overline{f^2}}=\frac{\lambda_0{}^2\beta_A}{1-\omega/B_{c2}},\quad (\text{high } \omega \text{ limit, } \kappa\gg 1) \tag{14}$$

and, in the low field range

$$\lambda^2=\frac{\lambda_0{}^2}{1-(\omega/B_{c2})\ln\kappa^*},\quad (\text{low } \omega \text{ limit, } \omega\ll B_{c2}), \tag{15}$$

where $\ln\kappa^*=\ln\kappa+0.52=2\kappa^2(B_{c1}/B_{c2})$.

Under dissipative conditions, vortices are moving, $\vec{E}\neq 0$, $\vec{C}\neq 0$ (dissipative part of the supercurrent density), and $\vec{J}=\vec{J}_s+\vec{J}_n$, where $\vec{J}_n$ is the normal current density. Dissipation is governed by the constitutive equations[13]

$$\vec{J}_n=\vec{\vec{\sigma}}\cdot\vec{E}=\sigma\vec{E}_\perp+\sigma'\vec{E}_\parallel, \tag{16}$$

$$\vec{E}+\frac{m}{e}\frac{\partial\vec{V}_s}{\partial t}=\vec{\varphi}=\beta\omega\vec{C}_\perp, \tag{17}$$

and the dissipative function $R=T\dot{\sigma}$ is

$$R=\vec{J}_n\cdot\vec{E}+\vec{\varphi}\cdot\vec{C}. \tag{18}$$

Here $\vec{E}_\parallel$, $\vec{E}_\perp$, and $\vec{C}_\perp=(\vec{\nu}\times\vec{C})\times\vec{\nu}$ denote components parallel or normal to vortices; σ is the normal fluid conductivity, and β is a kinetic coefficient which is the analog of the mutual friction coefficient B in rotating He II, and $-\vec{\varphi}$ is a ''friction force'' field, which is the analog of the mutual friction force.

Electric fields $\vec{e}$ and nomal currents $\vec{j}_n$ are induced by vortex motion inside and around the vortex core, which contribute to dissipation. However, as all fields in our continuum approach, $\vec{J}_n$ is a current averaged over the vortex-array cell. Thus $\vec{J}_n\cdot\vec{E}$ in Eq. (18) does not involve all the dissipation associated with normal currents. So it is worth noting that a significant part of this dissipation, i.e., $\langle\vec{j}_n\cdot\vec{e}\rangle-\vec{J}_n\cdot\vec{E}$, comes within the second term, through β [except near H_{c2}, see Eq. (26) below], as has been already demonstrated in the Bardeen-Stephen model.[2]

In Eq. (17) the absence of any significant Hall effect has been assumed. Also, cross terms in the above linear dynamical laws between fluxes $(\vec{J}_n,\vec{\varphi})$ and associated affinities $(\vec{E},\vec{C})$ have been left out deliberately. The possible occurrence of such cross terms is discussed in Sec. IV, in connection with the expression of the Lorentz force.

Collecting Eqs. (3), (13), (16), (17), and Maxwell equations, we obtain a complete set of equations for the *four unknown fields* $\vec{E}$, $\vec{B}$, $\vec{\omega}$, $\vec{J}_s$:

$$(\text{I})\quad \text{curl}\vec{E}=-\frac{\partial\vec{B}}{\partial t},$$

$$(\text{II})\quad \text{curl}\frac{\vec{B}}{\mu_0}=\vec{J}_s+\vec{\vec{\sigma}}\cdot\vec{E},$$

$$(\text{III})\quad \vec{\omega}=\vec{B}+\text{curl}\mu_0\lambda^2\vec{J}_s,$$

$$(\text{IV})\quad \vec{E}=\frac{\partial\mu_0\lambda^2\vec{J}_s}{\partial t}+\beta\omega\vec{C}_\perp. \tag{19}$$

From Eqs. (19) (I), (III), and (IV), we find

$$\frac{\partial\vec{\omega}}{\partial t}=-\text{curl}(\beta\omega\vec{C}_\perp)$$

which has to be identified with the transport equation for vortices $\partial\vec{\omega}/\partial t=\text{curl}(\vec{v}_L\times\vec{\omega})$, with the line velocity $\vec{v}_L$. Whence

$$(\text{V})\quad \vec{v}_L=-\beta\vec{\nu}\times\vec{C}\quad (\vec{\varphi}=\beta\omega\vec{C}_\perp=\vec{\omega}\times\vec{v}_L). \tag{20}$$

This equation can be rewritten as a force equation in the form

$$\text{(V)} \quad \eta \vec{v}_L = \frac{\varphi_0}{\beta} \vec{v}_L = \vec{C} \times \varphi_0 \vec{\nu} = \vec{J}_s \times \varphi_0 \vec{\nu} + \text{curl} \vec{\varepsilon} \times \varphi_0 \vec{\nu}. \tag{21}$$

It may be interpreted as the balance of forces per unit length: a Lorentz force $\vec{J}_s \times \varphi_0 \vec{\nu}$, a restoring force $\text{curl}\vec{\varepsilon} \times \varphi_0 \vec{\nu}$ resulting from a line tension $\varepsilon \varphi_0$, and a viscous drag force $-\eta \vec{v}_L$.

Let us now consider the standard conditions such as defined in the Introduction. Quantities relating to the equilibrium reference state are written without indices: $\vec{E}=0$, $\vec{B}(0,0,B)$, $\vec{\omega}=\omega \vec{\nu}(0,0,\omega=B)$, $\vec{J}_s=0$, $\vec{J}_n=0$, $\vec{C}=0$, $\vec{\varepsilon}=\varepsilon \vec{\nu}[0,0,\varepsilon(\omega)]$. Small changes in the fields are labeled 1. Keeping terms of first order in one-indexed fields in Eqs. (19)(I)–(IV) and (21)(V), gives the following set of linearized equations:

$$\text{(I)} \quad \text{curl} \vec{E}_1 = -\frac{\partial \vec{B}_1}{\partial t},$$

$$\text{(II)} \quad \text{curl} \frac{\vec{B}_1}{\mu_0} = \vec{J}_{s1} + \vec{\sigma} \cdot \vec{E}_1,$$

$$\text{(III)} \quad \vec{\omega}_1 = \vec{B}_1 + \mu_0 \lambda^2 \text{curl} \vec{J}_{s1},$$

$$\text{(IV)} \quad \vec{E}_1 = \mu_0 \lambda^2 \frac{\partial \vec{J}_{s1}}{\partial t} + \beta \omega \vec{C}_{1\perp},$$

$$\text{(V)} \quad \vec{v}_{L1} = -\beta \vec{\nu} \times \vec{C}_{1\perp}, \tag{22}$$

where $\vec{C}_{1\perp} = (\vec{\nu} \times \vec{C}_1) \times \vec{\nu}$ is normal to the z axis.

Moreover we restrict our attention to one-dimensional (1D) vortex motions $(\partial/\partial x = \partial/\partial y \equiv 0)$. The varying fields $\vec{E}_1$, $\vec{B}_1$, $\vec{\omega}_1 = \omega \vec{\nu}_1$, $\vec{J}_{s1}(z,t)$ have no z component, neither have $\vec{\varepsilon}_1 = \varepsilon \vec{\nu}_1$ and $\vec{C}_1 = \vec{J}_{s1} + \varepsilon \text{curl} \vec{\nu}_1$. As easily seen, two linearly polarized motions $(E_y, B_x, \nu_{1x}, J_{sy}, v_{Lx})$ and $(E_x, B_y, \nu_{1y}, J_{sx}, v_{Ly})$ can be considered separately and equivalently. For definiteness, we shall use the former, and drop indices x, y, 1, if there is no ambiguity.

Thus we obtain :

$$\text{(I)} \quad \frac{\partial E}{\partial z} = \frac{\partial B}{\partial t},$$

$$\text{(II)} \quad \frac{1}{\mu_0} \frac{\partial B}{\partial z} = J_s + \sigma E,$$

$$\text{(III)} \quad \omega \nu_x = -\mu_0 \lambda^2 \frac{\partial J_s}{\partial z} + B_x,$$

$$\text{(IV)} \quad E = \mu_0 \lambda^2 \frac{\partial J_s}{\partial t} + \beta \omega \left(J_s + \varepsilon \frac{\partial \nu_x}{\partial z} \right),$$

$$\text{(V)} \quad v_L = \beta \left(J_s + \varepsilon \frac{\partial \nu_x}{\partial z} \right). \tag{23}$$

Later on we shall consider small vortex displacements $\vec{u}[u(z,t),0,0]$ from the reference equilibrium state. In this case $\nu_x = \partial u/\partial z$, and $\varphi_0[J_s + \varepsilon(\partial^2 u/\partial z^2)]$ in Eq. (23)(V) stands for the thermodynamic force $\vec{C} \times \varphi_0 \vec{\nu}$ per unit length of vortex line. If $\vec{\omega}$ and $\vec{B}$ are not distinguished, and if the normal current in Eq. (23)(II) is ignored, $J_s = (\omega/\mu_0)(\partial^2 u/\partial z^2)$, so that the force $n(\vec{C} \times \varphi_0 \vec{\nu}) = \vec{C} \times \vec{\omega}$ per unit volume becomes identical with the classical elastic force $C_{44}(\partial^2 u/\partial z^2)$, where $C_{44} = h\omega$ (or HB) is the Labusch tilt modulus. As a matter of fact, the introduction of renormalized[9] or wavelength-dependent[16] elastic moduli amounts to making allowance for the macroscopic London equation (23)(III) (see also the discussion at the end of Sec. V).

Concluding this section we discuss what conditions restrict application of our theory. First of all, this is a condition for using the phenomenological approach, both in the Meissner state and in the mixed state: the frequency should be less than the microscopic frequencies like the inverse of the quasiparticle relaxation time, or the superconducting gap. This condition restricts the validity of the theory at very high frequencies. Another limitation is that the spatial scales derived from the theory should not be small compared with the intervortex distance since the theory deals with variables averaged over the vortex-array cell. We shall return back to this restriction in Sec. V after these relevant spatial scales were obtained. Our approach does not require the London theory to be valid: close to the upper critical field where the latter does not hold one can use the Ginzburg-Landau theory to derive the parameters of the theory, or take them from the experiment. Since we do not consider effects of shear rigidity which discriminate the vortex crystal and the vortex liquid, we can apply our theory only if shear rigidity is not essential, like in the perpendicular geometry considered throughout the present paper (the standard conditions defined in Sec. I). But in this case our theory addresses both the solid and the liquid state of the vortex array.

III. THE dc RESPONSE

Let us consider the dc flux flow in a perfect slab normal to the applied field $\vec{B}$ $(0,0,B)$. When a low to moderate dc current is applied in the y direction (Fig. 2), Eqs. (23) apply (with $\partial/\partial t = 0$ and $\omega = B$). The electric field and $\vec{J}_n$ are uniform. On integrating Eq. (23)(IV) over the thickness d of the slab, the supercurrent i_s per unit length along Ox is found to be $i_s = Ed/\beta\omega$. Note that this simple result is obtained regardless of the detailed current distribution $J_s(z)$ and deformations of the vortex array: the line tension term $\varepsilon d\nu_x/dz$ in Eq.(23)(IV) vanishes by integration, thanks to the boundary condition $\vec{\nu} \times \vec{N} = 0$ $(\nu_x = 0)$. Then, as the normal current $i_n = \sigma E d$, the measured flux-flow conductivity $\sigma_f = i/(Ed)$, where $i = i_s + i_n$, is given by

$$\sigma_f = \sigma + \frac{1}{\beta\omega} = \sigma + \frac{1}{\beta B}. \tag{24}$$

Figure 3 shows the behavior of the flux-flow resistivity $\rho_f(\omega = B)$ such as commonly observed in an alloy.[1,23] Let r be the reduced slope of the resistivity curve,

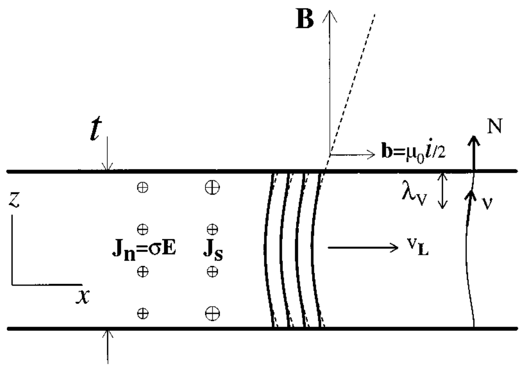

FIG. 2. A sketch of the moving vortex lattice in an ideal slab perpendicular to the applied field. When driven by a low dc current, the configuration of the vortex and field lines, $\nu_x(z)$ and $B_x(z)$, together with the supercurrent distribution $J_{sy}(z)$, are governed by the linearized set of equations (23)(II)–(IV), where $J_n=\sigma E=\text{const}$. Beyond a small healing length λ_V defined in the text [see Eq. (39) and Fig. 5] the vortex curvature and J_s are uniform, and vortex lines coincide with field lines. The total current $i=i_n+i_s$ is obtained on integrating J_s and J_n over the thickness d, and the experimental flux-flow conductivity, defined as the ratio $i/(Ed)$ is given by Eq. (24).

$$r=\frac{\partial\rho_f/\rho_n}{\partial B/B_{c2}},$$

where ρ_n is the resistivity of the normal state. At low fields, $\rho_f(B)$ approximately follows a linear law with $r=r_0\lesssim 1$. The limiting slope r_1 at B_{c2} is observed to be larger than unity,[23] with $r_1(T=0)=1.7$ and $r_1(T_c)=2.5$ in good agreement with theory.[24]

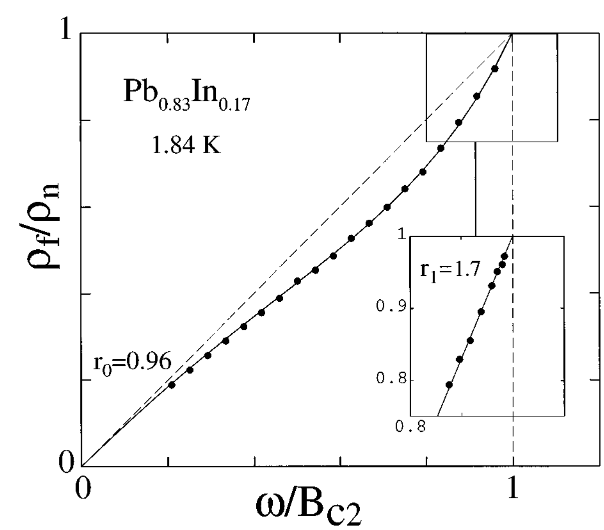

FIG. 3. The field dependence of the flux-flow resistivity. Full circles are experimental data taken with a lead-indium alloy $Pb_{0.83}In_{0.17}$ at $t=T/T_c=0.265$: $\kappa=3.5$, $\kappa_1(t)=4.7$, $B_{c2}=4800$G, $\rho_n=1.04\times10^{-7}\ \Omega$ m. The inset shows data near H_{c2} in magnified scales. On fitting these data, we find the reduced slopes $r_0=0.96$ and $r_1=1.7$. As explained in the text, from the measured slope r_1, and using Eq. (28), we obtain an estimation of the relaxation time of the order parameter $\tau_R=5.5\times10^{-13}$ sec.

Since $\sigma\lesssim\sigma_n$, Eq. (24) reduces to $\sigma_f\simeq 1/\beta\omega$, or $\rho_f\simeq\beta\omega$, at the zero-field limit. Experimental results (Fig. 3) suggest that

$$\beta=\frac{r_0}{\sigma_n B_{c2}}\lesssim\frac{1}{\sigma_n B_{c2}}\quad\text{(zero-field limit)}. \tag{25}$$

The corresponding viscous-drag coefficient $\eta=\varphi_0/\beta$ is nothing but that introduced in the Bardeen-Stephen model at $T=0$.[2] A time-dependent GL model near T_c, such as that developed by Schmid,[19] yields $r_0=0.64$ in the dirty limit. As pointed out above, dissipation from both relaxation effects and eddy normal currents around the vortex core contribute to β.

As $\sigma\to\sigma_n$ at B_{c2}, Eq. (24) implies that β diverges in the high field limit. As a matter of fact, Schmid's expression for σ_f near H_{c2} (and $T\to T_c$) is of the general form (24) by taking

$$\sigma=\sigma_n=\text{const},\quad \beta=\frac{\xi^2}{2\tau_R\varepsilon}\quad(H_{c2}\text{limit}), \tag{26}$$

where τ_R and ε have been introduced in Eqs. (8) and (10), so that $\beta^{-1}\to 0$ as $B_{c2}-B$.

At $t=T/T_c<1$, let us adopt this relationship between the kinetic coefficient β and the relaxation time τ_R, while accounting for the field and temperature dependence of the normal fluid conductivity σ through the two-fluid expression[7]

$$\sigma=\sigma_n[1-\overline{f^2}(1-t^4)]. \tag{27}$$

Whence we find

$$r_1(t)=-\left[\frac{\partial\sigma_f/\sigma_n}{\partial B/B_{c2}}\right]_{B_{c2}}$$

$$=\frac{1}{\beta_A}\left[\frac{\tau_R}{\tau_j}-(1-t^4)\right],\quad(\kappa\gg 1), \tag{28}$$

where $\tau_j=\mu_0\sigma_n\lambda_0^2$. From the measured slope r_1 (see inset of Fig. 3) we thus obtain an experimental estimation of the relaxation time $\tau_R\sim 10^{-13}-10^{-12}$ sec (for $t^4\ll 1$), consistent with Schmid's expression for $\tau_R=(\pi/16)(\hbar/kTc)[1/(1-t)]$.[19]

The above estimated values of σ and β (see Fig. 4) will be useful when discussing the importance of various terms in the dispersion equation (Sec. VI). Now, one may hope to obtain, from accurate measurements of surface impedance, more precise information about the relative weight of the normal and superconducting channels in σ_f, as well as the need for introducing (or not) a third transport coefficient γ as discused in Sec. IV and the Appendix.

IV. ON A NORMAL CURRENT CONTRIBUTION TO THE LORENTZ FORCE

It is possible to regain some normal-current vortex interaction in the MS formalism, on the condition that cross terms in the linear dynamical equations (16) and (17) are taken into account. Equations (16) and (17) can be rewritten in the following generalized form, in accordance with the Onsager symmetry:[25]

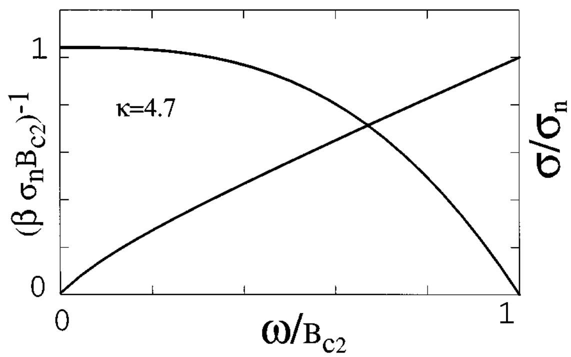

FIG. 4. The field-dependence of the normal fluid conductivity σ and of the kinetic coefficient β, here displayed as dimensionless quantities. According to Eq. (24), σ and $1/\beta\omega$ represent the normal and superfluid components of the flux-flow conductivity. Referring to data of Fig. 3 ($t=0.265, t^4 \ll 1$), $\sigma(\omega)$ is a plot of the two-fluid expresion (27). Near T_c, σ would be of the order of σ_n over the whole field range. Then β is calculated from $\sigma_f-\sigma$, by using the fitting line of Fig. 3

$$\vec{J}_n=\sigma\vec{E}_\perp+\sigma'\vec{E}_\parallel-\gamma\vec{C}_\perp, \tag{29}$$

$$\vec{E}+\frac{m}{e}\frac{\partial\vec{V}_s}{\partial t}=\vec{\varphi}=\vec{\omega}\times\vec{v}_L=\gamma\vec{E}_\perp+\beta\omega\vec{C}_\perp, \tag{30}$$

where γ is a dimensionless coefficient. Note that γ does not enter the dissipative function (18) directly, but it influences, nevertheless, the rate of dissipation via the currents and affinities in the dissipative function.

From Eqs. (29) and (30) we then obtain the generalized force equation

$$\frac{\varphi_0\sigma\omega}{\beta\sigma\omega+\gamma^2}\vec{v}_L=\vec{C}\times\varphi_0\vec{\nu}+\frac{\gamma}{\beta\sigma\omega+\gamma^2}\vec{J}_n\times\varphi_0\vec{\nu}$$
$$=(\vec{C}+\alpha\vec{J}_n)\times\varphi_0\vec{\nu}. \tag{31}$$

Equation (21) corresponds to $\gamma=0$. If $\gamma\neq 0$, the linearized equations (23) become

$$\text{(I)}\quad \frac{\partial E}{\partial z}=\frac{\partial B}{\partial t},$$

$$\text{(II)}\quad \frac{1}{\mu_0}\frac{\partial B}{\partial z}=(1-\gamma)J_s+\sigma E-\gamma\varepsilon\frac{\partial\nu_x}{\partial z},$$

$$\text{(III)}\quad \omega\nu_x=-\mu_0\lambda^2\frac{\partial J_s}{\partial z}+B_x, \tag{32}$$

$$\text{(IV)}\quad (1-\gamma)E=\mu_0\lambda^2\frac{\partial J_s}{\partial t}+\beta\omega\left(J_s+\varepsilon\frac{\partial\nu_x}{\partial z}\right),$$

$$\text{(V)}\quad v_L=\beta\left(J_s+\varepsilon\frac{\partial\nu_x}{\partial z}\right)+\frac{\gamma}{\omega}E.$$

On applying the dc response, and following the same procedure as in Sec. III, we find

$$J_n=\sigma\left(1-\frac{\gamma(1-\gamma)}{\beta\sigma\omega}\right)E, \tag{33}$$

$$J_s+\varepsilon\frac{\partial\nu_x}{\partial z}=\frac{1-\gamma}{\beta\omega}E, \tag{34}$$

$$\sigma_f=\sigma+\frac{(1-\gamma)^2}{\beta\omega}. \tag{35}$$

Equation (35) is a general expression for the flux-flow conductivity assuming arbitrary normal-current contribution to the Lorentz force. The latter could be specified either from the experiment, or from the microscopical theory.

Coffey and Clem write down the two-fluid equation $\vec{J}=\vec{J}_s+\vec{J}_n$, as also the macroscopic London equation, in the form (19)(III). But the CC equation of vortex motion is different from Eq. (21); in the absence of pinning, this reads

$$\eta_{CC}\vec{v}_L=\vec{J}\times\varphi_0\vec{\nu}, \tag{36}$$

where $\vec{J}$ is the *total current* density and the viscous-drag coefficient η_{CC} is directly related to the experimental flux-flow conductivity $\eta_{CC}=\varphi_0\sigma_f\omega$. In contrast, the force equation (21) only involves the supercurrent through $\vec{C}=\vec{J}_s+\mathrm{curl}\vec{\varepsilon}$. In order to restore the Lorentz force $\vec{J}\times\varphi_0\vec{\nu}$ in the equation of vortex motion (the CC model), one should assume that $\alpha=\gamma/(\beta\sigma\omega+\gamma^2)=1$ in Eq. (31). But strong discrepancy between CC results and ours still remains: they neglected the term $\propto\gamma$ in the right-hand side of Eq. (29). Therefore the CC equations violate the Onsager symmetry. This affects both the dc response and the ac response. In the Appendix the latter is analyzed in the frame of a one-mode electrodynamics (ε terms neglected) to compare with the CC results.

Now if the condition $\alpha=1$, or $\gamma=\gamma^2+\beta\sigma\omega$, is prescribed, so as to restore the Lorentz force $\vec{J}\times\varphi_0\vec{\nu}$ in the equation of vortex motion (the CC model is revised to satisfy the Onsager symmetry), Eq. (33) turns back to $J_n\equiv 0$, and Eq. (35) reduces to $\sigma_f=\sigma/\gamma$. The latter result and the discontinuity implied at H_{c2} look rather difficult to believe. Thus we are led to the conclusion that the naive concept of a Lorentz force $\vec{J}\times\varphi_0\vec{\nu}$ driving the vortices involving the total current hardly holds. In the following section γ will be assumed to be zero.

V. THE ac RESPONSE AND THE PENETRATION OF EXTERNAL FIELDS: THE TWO-MODE ELECTRODYNAMICS

Here we consider small vortex displacements $\vec{u}[u(z,t),0,0]$ from the reference equilibrium state. We are looking for solutions of Eqs. (23) in the form $e^{ikz}e^{-i\Omega t}$; e, b, j_s, ν_x, u denoting complex amplitudes. Equations (I), (II), (III), and (V) from Eq. (23) become a set of homogeneous and linearly independent equations for e, b, j_s, $\nu_x=iku$ $[v_L=-i\Omega u=-(\Omega/k)\nu_x]$:

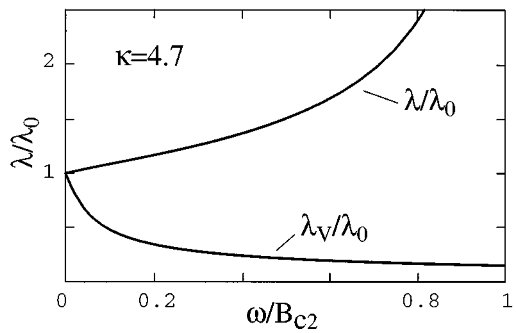

FIG. 5. The field-dependent London "penetration depth" $\lambda=\lambda_0(\overline{f^2})^{-1/2}$ according to the Clem "mean-field" approximation. An interpolating formula has been used to join the low and high-field expressions of $\overline{f^2}$ [see Eqs. (14) and (15)]. While being useful in various extensions of the London model, λ should not be regarded as any screening length. According to the MS model, the actual penetration depth for nondissipative supercurrents is that λ_V given by Eq. (39). Note that λ diverges at H_{c2} whereas λ_V decreases to $\xi/\sqrt{2}$.

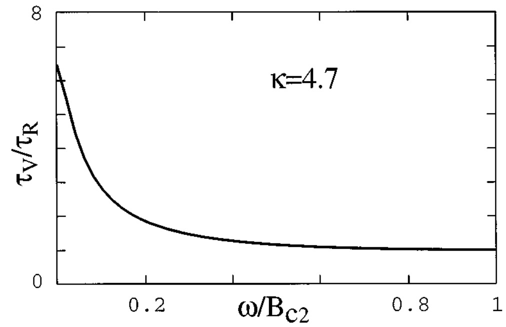

FIG. 6. The small field-dependent characteristic time τ_V, defined in Eq. (41), which governs the frequency dependence of the ac response. It is to be compared with the relaxation time of the order parameter, $\tau_R\sim10^{-12}$ sec.

$$\text{(I)}\quad ke=-\Omega b,$$

$$\text{(II)}\quad \sigma e+j_s=ik\frac{b}{\mu_0},$$

$$\text{(III)}\quad ik\mu_0\lambda^2 j_s+\omega\nu_x=b,$$

$$\text{(V)}\quad j_s+ik\varepsilon\nu_x+\frac{\Omega}{\beta k}\nu_x=0. \tag{37}$$

Note that one would obtain an equivalent system from equations (I), (II), (III), and (IV). Indeed, the Euler equation (IV) can be obtained here as a linear combination of equations (I), (III), and (V).

On stating that the set (37) of homogeneous equations has nonzero solutions, we obtain the following biquadratic dispersion equation which connects the frequency Ω and the wave number k. It is clear that the term $ik\varepsilon$ is responsible for the existence of a second mode (k^4 term) :

$$\lambda_V^2k^4+(1-i\Omega\mu_0\sigma\lambda_V^2-i\Omega\tau_V)k^2 -i\Omega\mu\sigma_f-\Omega^2\mu_0\sigma\tau_V=0, \tag{38}$$

where σ_f is the flux-flow conductivity [Eq. (24)], and

$$\lambda_V^2=\frac{\lambda^2\varepsilon}{h},\quad \left(h=\frac{\omega}{\mu_0}+\varepsilon\right), \tag{39}$$

$$\mu=\frac{\omega}{h}, \tag{40}$$

$$\tau_V=\frac{\lambda^2}{\beta h}=\frac{\lambda_V^2}{\beta\varepsilon}. \tag{41}$$

The length λ_V, labeled d in Ref. 13 and l_E in Ref. 26, is nothing but the mixed state penetration depth for diamagnetic and nondissipative dc currents ($\vec{J}_s-\vec{C}=-\text{curl}\vec{\varepsilon}$) such as first introduced in Ref. 12. The length λ_V is a monotonic decreasing function of ω from λ_0 to $\xi/\sqrt{2}$ (Fig. 5). It should be noted that λ_V, in contrast to λ (see Eq. (14)), does not diverge at H_{c2} (Fig. 5). The expression for λ_V in an extended London model ($\mu_0\varepsilon\ll\omega$ or $a\ll\lambda_0$),

$$\lambda_V=\sqrt{\frac{1}{4\pi n}\ln\frac{a}{\xi^\star}} \tag{42}$$

coincides with that of the characteristic healing length for distortions of the vortex array in rotating He II. Such distortions occur in collective Kelvin waves,[14] or, at equilibrium, near a wall inclined to the axis of rotation.[15] Whereas λ_V in superconductors is always smaller than $\lambda_0\sim100$ nm, λ_V in He II is only limited by the size of the rotating vessel. Accurate second sound measurements at angular velocities $\Omega\sim1\,\text{sec}^{-1}$ ($\lambda_V\sim0.1$ mm) have proved the correctness of Eq. (42).[15] The ratio $\mu=\omega/h$ is an equilibrium property directly deduced from the equation of state $\varepsilon(\omega)$ (Fig. 1). It was first introduced by Sonin *et al.*[9] as a permeability accounting for the diamagnetism of the mixed state.

According to Eqs. (25) and (26) for β, the high and low field limits of the short time τ_V are, respectively, $\tau_V=\tau_R$ (at $\omega=B_{c2}$), and $\tau_V\simeq\tau_j(B_{c2}/B_{c1})=\mu_0\sigma_n\lambda_0^2(B_{c2}/B_{c1})$ ($\omega\rightarrow0$). Figure 6 shows the ω dependence of τ_R in an alloy, such as deduced from those of ε,β and λ_V (Figs. 1, 4, and 5); this relates to the dirty limit where $\tau_R=2.89\tau_j$ (Ref. 19) and $\tau_R\gg\tau$, the electronic relaxation time ($\sigma_n=ne^2\tau/m$). Excepting a small temperature interval close to T_c, in the pure limit $\tau_R\ll\tau$ and $\tau_j\sim\tau$, so that $\tau_V\ll\tau$. Then the condition $\Omega\tau_V\ll1$ holds in the whole frequency range where Ohm's law itself is valid ($\Omega\tau\ll1$). In contrast, in the dirty limit the condition $\Omega\tau\ll1$ holds at rather high frequencies where $1\lesssim\Omega\tau_V$, and one can apply our dispersion relation up to $\Omega/2\pi\sim10-100$ GHz.

The dispersion equation (38) can be rewritten as

$$(k^2\lambda_V^2+1-i\Omega\tau_V)(k^2-i\mu_0\sigma\Omega)-i\Omega(\mu\sigma_f-\mu_0\sigma)=0. \tag{43}$$

When comparing different terms of Eqs. (38) or (43) it is worth noting the inequalities

$$\mu_0\sigma\leqslant\mu_0\sigma_n\leqslant\mu\sigma_f\leqslant\frac{\tau_V}{\lambda_V^2},\quad \mu_0\sigma\ll\frac{\tau_V}{\lambda_V^2}, \tag{44}$$

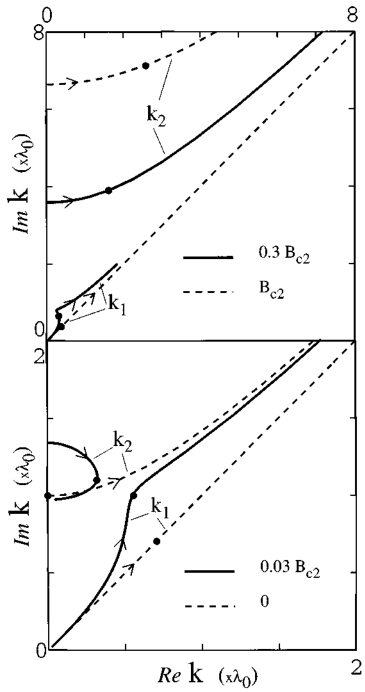

FIG. 7. Full lines represent, in the complex plane, typical variations of the wave vector k for each of the two modes, as function of the frequency $\Omega/2\pi$ at a given field ($\omega \gg B_{c1}$ upper curve; $\omega \lesssim B_{c1}$ lower curve). In this figure, the frequency is increased, in the direction of the arrow, from $\Omega\tau_V = 10^{-2}$ to $\Omega\tau_V = 10^2$. Full dots indicate the values at $\Omega\tau_V = 1$. Dashed lines represent the limiting dispersion curves at $\omega = B_{c2}$ and $\omega = 0$. Concerning the low frequency range ($\Omega\tau_V \ll 1$), it is more convenient to refer to simplified and explicit expressions for k_1 and k_2, Eq. (47).

as confirmed by a numerical calculation. From Eq. (43) it is clear that, at H_{c2} where $\sigma_f = \sigma = \sigma_n$ and $\mu = \mu_0$, a second evanescent mode k_2 appears besides the damped mode k_1 commonly observed in any conductor :

$$k_1^2 = i\mu_0\sigma_n\Omega = \frac{2i}{\delta_n^2},$$

$$k_2^2 = -\frac{1}{\lambda_V^2}(1 - i\Omega\tau_V) = -\frac{2}{\xi^2}(1 - i\Omega\tau_R)(\text{at } H_{c2}) \quad (45)$$

These two modes can be followed continuously down to $\omega = 0$ (Fig. 7).

In the zero-field limit we have

$$k_1^2 = i\Omega\frac{\tau_V}{\lambda_0^2} = \lim_{\omega\to 0}(i\Omega\mu\sigma_f),$$

$$k_2^2 = -\frac{1}{\lambda_0^2} + i\mu_0\sigma\Omega, \quad (\omega \to 0) \quad (46)$$

The k_2−mode at $\omega = 0$ is nothing but the one mode emerging from the classical electrodynamics of the Meissner state,[27] whereas now, the damped, normal-like, k_1-mode results from the onset of the mixed state (in the absence of pinning).

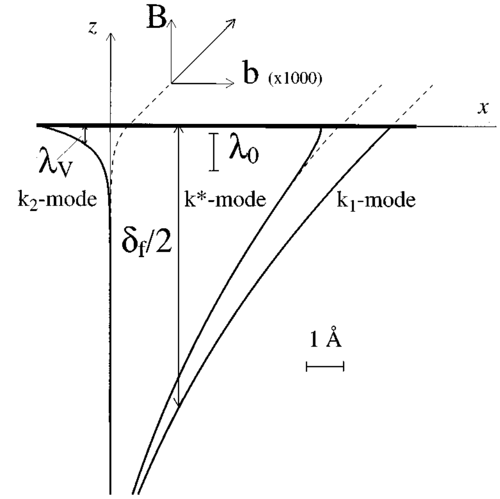

FIG. 8. Small vortex motions in xz planes near a surface $z = 0$ perpendicular to the applied dc field (superconducting half-space $z < 0$). The exciting field $b(0)\cos(\Omega t)$ is along the x direction. The full line, labeled k^*, represents the vortex profile $u_x(z, t=0)$ such as calculated for an ideal surface from Eqs. (50) and (54), taking $\omega \simeq 2B_{c1}$, $\Omega\tau_V = 10^2$. The full line labeled k_1 (k_2) represents the profile, which would be obtained for a pure k_1 mode (k_2 mode) only satisfying the continuity of B_x [but $\nu_x(0) \neq 0$]. For clarity, x components of displacements and fields have been magnified by a factor of 1000. The low vortex density has been chosen so as to illustrate the weighting effect of the factor $\mu < \mu_0$ in Eq. (54).

Let us consider *low to moderate frequencies* such that $\Omega\tau_V \ll 1$. It should be noted that the condition $\Omega\tau_V \ll 1$ is not so restrictive as it is satisfied in most practical conditions: $\Omega\tau_V \lesssim 10^{-2}$ for $\Omega/2\pi \lesssim 1$GHz in alloys. Taking into account inequalities (44), simplified expressions for k_1 and k_2 then follow immediately

$$k_1 = \pm\frac{1+i}{\delta_f},$$

$$k_2 = \pm\frac{i}{\lambda_V}(\Omega\tau_V \ll 1) \quad (47)$$

where

$$\delta_f = \sqrt{\frac{2}{\mu\sigma_f\Omega}} < \delta_n \quad (48)$$

is the skin depth related to the flux-flow conductivity that incorporates the permeability μ. In so far as $\sqrt{\Omega\tau_V} \ll 1$ Eq. (46) implies that $\delta_f \to \lambda_0\sqrt{2/\Omega\tau_V} \simeq \delta_n\sqrt{B_{c1}/B_{c2}} \gg \lambda_0$, so that $\lambda_V < \lambda_0 \ll \delta_f$ (or $k_1 \ll k_2$). We retain that, under practical conditions, the $k_1$mode can penetrate into a sample a relatively large depth δ_f whereas the k_2 mode dies off over the small depth λ_V.

Let us consider a half space $z < 0$ subject to a small exciting field $B_x = b(0)e^{-i\Omega t}$. From the first three equations (37) the vortex profile $u(z)$ can be calculated for each mode [including the k_{CC} mode; see Eq. (57) below] as function of the field amplitude (Fig. 8):

$$iku = v_x = \frac{b}{\omega}(1 + k^2\lambda^2 - i\mu_0\sigma\Omega\lambda^2). \tag{49}$$

Using simplified expressions (47) for k_1 and k_2 ($\Omega\tau_V \ll 1$), we obtain

$$v_{1x} = \frac{b_1}{\omega}, \quad v_{2x} = \frac{-b_2}{\omega}\frac{\omega}{\mu_0\varepsilon},$$

$$u_1 = \frac{b_1}{\omega}\frac{\delta_f}{1-i}, \quad u_2 = \frac{-b_2}{\omega}\frac{\omega\lambda_V}{\mu_0\varepsilon}(\Omega\tau_V \ll 1). \tag{50}$$

The new length scale $\lambda_S = \omega\lambda_V/\mu_0\varepsilon$ in the k_2 mode is relevant for surface pinning (see Sec. VI). In determining the amplitude of the displacement in the k_2 mode, this scale plays the same role as δ_f for the k_1 mode.

In the k_2 mode, field lines and vortex lines incurve in opposite directions (Fig. 8). In the wide frequency range concerned ($\Omega\tau_V \ll 1$), the k_2 mode has a quasistatic behavior, while the k_1 mode is spatially uniform over the depth λ_V ($u_1 \simeq$ const, if $\lambda_V \ll \delta_f$). None of them fulfils the boundary condition (7) for an ideal surface $z=0$, i.e., $v_x = 0$. Therefore, we have to combine both modes, $B_x = (b_1 e^{ik_1 z} + b_2 e^{ik_2 z})e^{-i\Omega t}$, while requiring $b_1 + b_2 = b(0)$ and $v_x = v_{1x} + v_{2x} = 0$. We thus obtain

$$\frac{b_2}{b_1} = -\frac{1 + k_1^2\lambda^2 - i\mu_0\sigma\Omega\lambda^2}{1 + k_2^2\lambda^2 - i\mu_0\sigma\Omega\lambda^2} \quad (v_x = 0). \tag{51}$$

Note in this respect that a one-mode theory cannot satisfy the boundary condition $v_x = 0$.

A quantity of interest in the ac response is the surface impedance, usually expressed in terms of a complex effective penetration depth $\lambda^* = 1/ik^* = \lambda' + i\lambda''$:

$$\begin{aligned} Z &= \frac{\mu_0 e(0)}{b(0)} = \frac{\mu_0(e_1 + e_2)}{b_1 + b_2} \\ &= \frac{b_1}{b(0)}\frac{\mu_0 e_1}{b_1} + \frac{b_2}{b(0)}\frac{\mu_0 e_2}{b_2} \\ &= -\frac{\mu_0\Omega}{k^*} = -i\mu_0\Omega\lambda^* \\ &= -i\mu_0\Omega(\lambda' + i\lambda''). \end{aligned} \tag{52}$$

It appears, from the second line, that Z is the mean of the surface impedances for pure k_1 and k_2 modes weighted with their respective contribution $b_1/b(0)$ and $b_2/b(0)$ to the screening.

Using Eqs. (37)(I) and (51), we have

$$\begin{aligned} Z &= -\frac{\mu_0\Omega}{k_1}\left(1 + \frac{b_2}{b_1}\frac{k_1}{k_2}\right)\left(1 + \frac{b_2}{b_1}\right)^{-1} \\ &= -\frac{\mu_0\Omega}{k_1}\frac{1 - i\mu_0\sigma\Omega\lambda^2 + \lambda^2(k_2^2 + k_2 k_1 + k_1^2)}{\lambda^2 k_2(k_2 + k_1)}. \end{aligned} \tag{53}$$

In the low frequency limit the above formulas become simplified. For a pure k_2 mode, Z would be purely inductive ($\lambda' = \lambda_V, \lambda'' = 0$). For a pure k_1 mode, $\lambda' = \lambda'' = \delta_f/2$. From the right combination of modes,

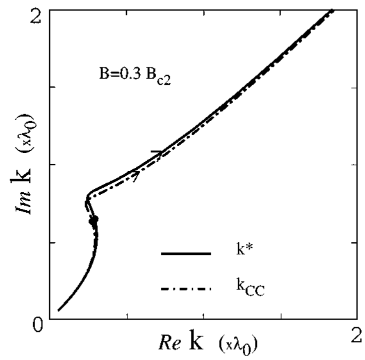

FIG. 9. Typical high frequency graph of the effective wave vector k^* for an ideal surface, such as resulting from the right combination of the two modes. The frequency has been increased, in the direction of the arrow, from $\Omega\tau_V = 10^{-2}$ to $\Omega\tau_V = 10^2$; the full dot corresponds to $\Omega\tau_V = 1$. Numerical calculations show that the Coffey-Clem wave vector k_{CC}, in the absence of pinning (dashed line), is close to k^*, at fields such that $\mu \simeq \mu_0$ ($\omega \gg B_{c1}$).

$$b_1 = b(0)\frac{\mu}{\mu_0}, \quad b_2 = b(0)\left(1 - \frac{\mu}{\mu_0}\right) \quad (\Omega\tau_V \ll 1), \tag{54}$$

we infer the effective complex penetration depth of an ideal surface

$$\begin{aligned} \lambda^* = \frac{1}{ik^*} &= \left(1 - \frac{\mu}{\mu_0}\right)\lambda_V + \frac{\mu}{\mu_0}(1+i)\frac{\delta_f}{2} \\ &\simeq \lambda_V + \frac{\mu}{\mu_0}(1+i)\frac{\delta_f}{2} \quad (\Omega\tau_V \ll 1) \end{aligned} \tag{55}$$

or equivalently, neglecting the small λ_V term, the surface impedance:

$$Z \simeq \sqrt{\frac{-i\Omega\mu}{\sigma_f}}. \tag{56}$$

This is an expression for the surface impedance of a conductor with the conductivity σ_f and the magnetic permeability μ well known from electrodynamics of continuous media.[28]

At higher frequencies ($\Omega\tau_V > 10^{-2}$), we have to use rather cumbersome expressions for k^*, which are not worth writing. Figure 9 shows an example of the high-frequency behavior of $k^* = 1/i\lambda^*$ at intermediate to large fields ($\omega \gg B_{c1}$), such that $\mu \simeq \mu_0$ (say to better than 5%; see Fig. 1). Numerical calculations confirm that $k^* \simeq k_1$ (and $\lambda' \simeq \lambda''$ in this range), within the same precision, as it is evident on the ''low frequency'' expression (54).

Let us compare our results with the traditional one-mode theory.[6–8] In the one-mode theory the effective penetration depth $\lambda^* = i/k^*$ is directly connected with the complex wave number k^* of the only mode penetrating into the sample from the surface. In contrast, the two-mode effective penetration depth given by Eq. (56) does not relate directly to the actual field penetration, this being described in the two-modes electrodynamics by *two* penetration depths $1/|k_1|$ and $1/|k_2|$. In the one-mode theory of Clem and Coffey

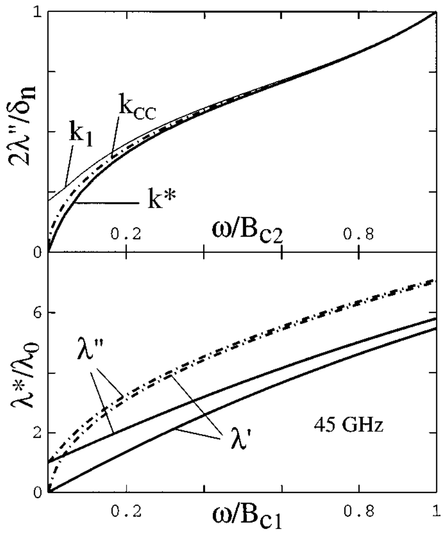

FIG. 10. The field dependence of the complex penetration depth $\lambda^* = \lambda' + i\lambda''$. λ' (λ'') corresponds to the inductive (resistive) part of the surface impedance. Upper curves show the low frequency ($\Omega \tau_V < 10^2$) behavior of λ'' in three cases: (i) k^*, combination of two modes for an ideal surface; (ii) pure k_1 mode; (iii) k_{CC}, one-mode Coffey-Clem theory, in the absence of pinning. At H_{c2}, $\lambda'' \to \delta_n/2$ where δ_n is the normal skin depth. It is worth noting that the finite limit at $\omega = 0$ of λ'' for the k_1 mode, corresponds, in our model to a large flux-flow skin depth ($\delta_f \lesssim \delta_n$) in contrast with the vanishing CC skin depth. The lower figure magnifies, at 45 GHz, the low field dependence of λ^*, for comparison with the CC theory.

$$k^{*2} = k_{\mathrm{CC}}^2 = i\mu_0 \sigma_f \Omega \frac{1 - i\mu_0 \sigma \Omega \lambda^2}{1 - i\mu_0 \sigma_f \Omega \lambda^2}. \tag{57}$$

At $\omega = 0$, as $\sigma_f \to \infty$, k_{CC}^2 tends to the same limit as k_2^2 [Eq. (46)]. At $\omega = B_{c2}$, as $\lambda^2 \to \infty$, k_{CC}^2 tends to the same limit as k_1^2 [Eq. (45)]. We emphasize that the flux-flow skin depth defined in the CC paper as $\delta_{0f} = \sqrt{2/\mu_0 \sigma_f \Omega}$ differs from that given by Eq. (48) by the factor $\sqrt{\mu/\mu_0}$. Whereas the former vanishes at $\omega \to 0$, the latter remains generally large compared with λ_0.

Effects of low ''permeability'' μ are most important at low vortex densities ($\omega \ll B_{c1}$),[9] when k^* deviates from both k_1 and k_{CC} significantly (Fig. 10). Measurements of $\lambda''(\omega)$ have been performed by Berezin *et al.*,[29] where a low pinning Pb-In sample exhibits, at $\Omega \simeq 10^9$ rd/sec, a linear dependence $\lambda'' \propto \omega$ (to be compared with $\lambda'' \propto \sqrt{\omega}$ in the one-mode electrodynamics). This result may corroborate the two-mode response, in so far as the sample may be regarded as an ideal one.

It is worth discussing now the conditions restricting application of our theory. It is accurate enough until the spatial scales (the penetration depths of the two modes) exceed the intervortex distance, since the latter plays a role of a ''microscopical'' scale for our continuum approach. From Eq. (42) it appears that λ_V may be of the same order as the vortex spacing a. Then our theory may not describe the field variation in the k_2 mode accurately. But, in many experimental situations, the amplitude of such a mode is more important than the details of its spacial variation. The same situation arises with the Debye screening theory for metals in which the amplitude of the jump in the electrical field is enough to obtain a reliable electrostatic solution, despite that the Debye radius may be of the same order or even less than the interatomic distance.

Concluding this section, we discuss the relation of the presented analysis with the traditional approach in terms of the nonlocal elastic moduli.[16] Following this approach, we reduce the set of equations (37) to one equation for the vortex displacement u_x ($\nu_x = iku_x$, $v_L = -i\Omega u_x$):

$$-\eta \frac{\omega}{\varphi_0} v_L = C_{44} k^2 u_x. \tag{58}$$

This equation expresses the balance between the viscous friction force on ω/φ_0 vortices per unit volume and the elastic force determined by the tilt elastic modulus

$$C_{44} = \frac{\omega^2}{\mu_0} \frac{1 - i\Omega \mu_0 \sigma / k^2}{1 - i\Omega \mu_0 \sigma \lambda^2 + \lambda^2 k^2} + \omega \varepsilon. \tag{59}$$

One can see that the elastic modulus is k and Ω dependent, i.e., nonlocal not only in space, but also in time. However, the nonlocal contribution is due to the energy of the average magnetic field $B = \omega$ and the transport currents which has nothing to do with the true vortex line tension proportional to the vortex potential ε. We prefer to relate the tilt modulus with the vortex line tension only, rewriting Eq. (58) as

$$-\eta \frac{\omega}{\varphi_0} v_L = -\omega j_s + C_{44}^* k^2 u_x. \tag{60}$$

Here $C_{44}^* = \omega \varepsilon$ is the renormalized tilt modulus[9] which does not depend on either k or Ω, as one might expect for any elastic modulus. The long-range intervortex interaction responsible for nonlocality is incorporated by the Lorentz force [the first term on the right-hand side of Eq. (60)] proportional to the supercurrent j_s averaged over the vortex cell.

VI. TOWARDS REAL SUPERCONDUCTORS: BULK AND SURFACE PINNING

Now our theoretical predictions for ideal samples may appear as a somewhat academic discussion. Nevertheless, it is an essential step before tackling the difficult and actual problem of the ac response in the presence of pinning. As is well known, the smallest critical currents alter the linear ac response altogether: for instance, in a large low-frequency domain below the so-called depinning frequency ($\sim 10-100$ MHz), the surface impedance is nearly inductive, $\lambda' \simeq$ const, $\lambda'' \simeq 0$, where typically $\lambda' \sim 10$ μm. Remember that, in this range, the response of an ideal surface corresponds to a classical skin effect with a much larger skin depth.

In order to account for such a radical change in the surface impedance, the classical way, initiated by Campbell,[4] consists in introducing a bulk pinning force $-Ku$ in the force equation (36) for small vortex motions. This means that one should replace the friction coefficient η_{CC} in this equation, as well as in all following equations, by $\eta_{\mathrm{CC}} - K/i\Omega$, or, in other terms, to replace the flux-flow conductivity σ_f by $\sigma_f - 1/i\Omega \mu_0 \lambda_C^2$, where $\lambda_C = \sqrt{\omega \varphi_0 / \mu_0 K}$ is the Campbell length directly related to K. As a result, there is still one mode, but its penetration depth is strongly reduced down to λ_C.

The same substitution $\eta\rightarrow\eta-K/i\Omega$, or $\sigma_f\rightarrow\sigma_f-1/i\Omega\mu_0\lambda_C^2$ in our final expressions incorporates the Campbell approach into the two-mode theory. In particular, the skin depth δ_f entering Eq. (47) has to be replaced by δ_v using the expression

$$k_1^2=\frac{2i}{\delta_v^2}=\frac{2i}{\delta_f^2}-\frac{\mu}{\mu_0\lambda_C^2}. \tag{61}$$

Note that the incorporation of the permeability μ again prevents for the divergence of the second term at vanishing vortex density ($\mu_0\lambda_C^2/\mu\rightarrow\varphi_0 H_{c1}/K$ as $\omega\rightarrow 0$), so that $\delta_v \gg \lambda_V$ and Eq. (54) for ideal boundary conditions holds. As a result, according to Eq. (55) (with δ_v instead of δ_f), the effective penetration depth is linear in the vortex density at low field $\lambda^*\simeq\lambda_V+\omega\sqrt{\varphi_0/B_{c1}K}$.

Another way is offered by a new interpretation of vortex pinning, that follows rather naturally from the MS theory as explained in Refs. 12 and 13. According to this MS model of the critical state, which relies on a number of experiments,[13,30–33] critical currents of soft samples (in fact, of most standard samples) are well accounted for by only considering the surface roughness. In spite of unavoidable surface irregularities on a scale comparable to or smaller than the vortex spacing, MS have suggested that the continuum description can be maintained, provided that the boundary condition (7) be released and replaced by a new empirical surface condition. The new boundary condition should lead to another combination of the two modes, enhancing the k_2 mode. This must change the frequency dependence of the surface impedance Z.[9,34]

An explicit form of this boundary condition must depend on actual properties of the surface, and, as illustration of the effect on the ac response, we restrict ourselves here to the case of extreme surface pinning when the ends of vortices are literally pinned to defects so that they cannot move along the surface at all, i.e., $u_1+u_2=0$. Then, according to Eq. (49), the ratio of the two-mode amplitude is [c.f. Eq. (51)]:

$$\frac{b_2}{b_1}=-\frac{k_2}{k_1}\frac{1+k_1^2\lambda^2-i\mu_0\sigma\Omega\lambda^2}{1+k_2^2\lambda^2-i\mu_0\sigma\Omega\lambda^2}. \tag{62}$$

Surface impedance is given by, instead of Eq. (53),

$$Z=\mu_0\Omega\frac{\lambda^2(k_2+k_1)}{1-i\mu_0\sigma\Omega\lambda^2-\lambda^2 k_1 k_2}. \tag{63}$$

In the low-frequency limit, $(\mu_0\sigma\Omega\lambda^2,\Omega\tau_V)\ll 1$, the effective complex penetration depth reads

$$\lambda^*=(\lambda_S+\lambda_V)\left(1+\sqrt{-2i}\frac{\lambda_S}{\delta_f}\right)^{-1}. \tag{64}$$

Here λ_S, introduced after Eq. (50), plays the role of the Campbell length. The low frequency expansion of Eq. (64), $\lambda^*\approx(\lambda_S+\lambda_V)[1+i(\lambda_S/\delta_f)]$, yields the surface resistance $\mathrm{Re}Z\propto\Omega^{3/2}$ different from that for an ideal sample ($\propto\sqrt{\Omega}$) or for the pinning case in the frame of the Campbell approach ($\propto\Omega^2$). This might be used to discern between the bulk and surface pinning.[9] Also there may be some noticeable differences concerning the size effects. Experiments are now in progress, in the Laboratoire de Physique de la Matière Condenśee, which ought to decide between the two approaches for some superconducting materials.

There are also other effects crucial for description of real superconductors, especially high-T_c materials.

(i) *Flux creep*. This may be incorporated into the theory simply by using a proper expression for the conductivity σ_f as done by Clem and Coffey.[7,8] Then σ_f is not the flux-flow conductivity anymore, but the *thermal activated flux-flow* (TAFF) conductivity. This does not change structure and qualitive conclusions of our theory, but can modify quantitative results which should be discussed for any material separately.

(ii) *Hall effect*. In the classical superconductors the Hall effect is usually very weak, but it becomes strong in superclean high-T_c superconductors as recent experiments have revealed.[35] The Hall effect has been incorporated into the two-mode theory in Ref. 36 in order to explain magnetoresistance resonances observed in the superclean Bi compounds.[37]

(iii) *Anisotropy, thin-film applications*. It is another important extension of our theory. The two-mode theory for the anisotropic thin films has been developed in Ref. 36 mentioned a few lines above.

VII. CONCLUSION

The continuum electrodynamics of the mixed state of type-II superconductors has been presented on the basis of the MS equations of vortex motion.[12] This has yielded the two-mode electrodynamics advanced in Ref. 9, while extending it in the whole field range and including normal currents. In this generalized form, our theory of the ac linear response can be compared with the previous one-mode theories (Campbell,[4] Brandt,[6] Coffey and Clem,[7,8]) by first considering the simpler case of ideal samples. The main difference is that, in the two-mode electrodynamics, the external fields penetrate into the superconductor as a superposition of two modes which satisfies the boundary condition (7) for the ideal surface, whereas it is ignored in the one-mode theories. Thereby the latters are not able to discriminate the ideal and the rough surface, which is crucial for the analysis of the effects of the surface pinning.

Concerning the effects of the normal current on the dc and ac responses, the main differences with Clem and Coffey,[7,8] when writing the equations vortex motion, are the following: (i) while assuming the existence of a the Lorentz force from the normal current on the vortex, Clem and Coffey ignored the force from the vortex on the normal fluid, as required by the Onsager relations; (ii) the assumption of a Lorentz force involving the total current seems to be questionable, as argued in Sec. IV. However, the final decision on this question should be given by the microscopical theory or by the careful observation of the field dependence of the ac response at different frequencies, as discussed in the Appendix.

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APPENDIX: THE ac RESPONSE AND THE PENETRATION OF EXTERNAL FIELDS: THE ONE-MODE ELECTRODYNAMICS

In this appendix, in order to simplify the analysis of the effect of the normal current on the ac response, and then to compare with the analysis of CC,[7,8] we shall neglect the vortex-line tension systematically. The effects of the vortex-line tension are represented in the equations by the terms involving the vortex potential ε. We thus study the ac response in the frame of a one-mode electrodynamics.

We refer to the set of equations (32), while taking $\varepsilon=0$ and $\gamma\neq 0$, so that Eqs. (37) are replaced by

(I) $ke=-\Omega b$,

(II) $\sigma e+(1-\gamma)j_s=ik\dfrac{b}{\mu_0}$,

(III) $ik\mu_0\lambda^2 j_s+\omega \nu_x=b$,

$$\text{(V)}\quad \frac{\gamma}{\beta\omega}e+j_s+\frac{\Omega}{\beta k}\nu_x=0. \qquad \text{(A1)}$$

Then, using the same notations as in Sec. V, the dispersion equation now reads

$$(1-i\Omega\tau_V)k^2-i\Omega\mu_0\sigma_f-\Omega^2\mu_0\sigma\tau_V=0, \qquad \text{(A2)}$$

where σ_f is given by Eq. (35). Solving Eq. (66) with respect to k, we can find the complex effective penetration depth $\lambda^*=1/ik$,

$$\lambda^{*2}=\frac{\Lambda^2+\dfrac{i}{2}\delta_{0f}^2}{1-2i\Lambda^2/\delta^2}, \qquad \text{(A3)}$$

in terms of the three lengths: (i) the CC flux-flow penetration depth $\delta_{0f}=(2/\mu_0\Omega\sigma_f)^{1/2}$, (ii) the normal-fluid penetration depth $\delta=(2/\mu_0\Omega\sigma)^{1/2}$, and (iii) a new real length Λ:

$$\Lambda^2=\frac{\tau_V}{\mu_0\sigma_f}=\frac{\lambda^2}{(1-\gamma)^2}\frac{\sigma_f-\sigma}{\sigma_f}. \qquad \text{(A4)}$$

Taking $\Lambda=\lambda$, expression (A3) for λ^* coincides with that given by Clem and Coffey in the absence of pinning: see Eq. (4) in Ref. 7 at the infinite Campbell length λ_C. However, Λ differs from the London penetration depth λ. So, as far as one-mode electrodynamics is concerned, it is seen that various models of vortex motion lead to the same general form of the surface impedance. Discrepancies between them will only result from different relations between Λ and λ. Let us discuss them.

In the Clem-Coffey theory,[7,8] the Lorentz force is assumed to be proportional to the total current. As discussed in Sec. IV, this assumption implies the relationship $\gamma=\gamma^2+\beta\sigma\omega$. Using Eq. (35), this condition reads $\gamma=\sigma/\sigma_f$, and Eq. (68) reduces to

$$\Lambda^2=\lambda^2\frac{\sigma_f}{\sigma_f-\sigma}. \qquad \text{(A5)}$$

Expression (A5) for Λ, at variance with the CC result $\Lambda=\lambda$, should properly reflect the assumption that the normal current and the supercurrent enter the Lorentz force symmetrically. This disagreement is due to the fact that the CC analysis, assuming a Lorentz force from the normal current on a vortex, did not take into account the force from the vortex on the normal component, such as required by the Onsager symmetry.

It is to be noted that, irrespective of Λ, Eq. (A3) leads to the same limiting expressions for λ^* at both low and high frequencies: in the low-frequency limit, $\lambda^{*2}=i\delta_{0f}^2/2$; in the high-frequency limit, $\lambda^{*2}=i\delta^2/2$; that is an usual skin effect described by the flux-flow conductivity σ_f and by the normal-fluid conductivity σ, respectively. Therefore, in order to discriminate between different models by experiment, one should investigate the intermediate range of frequencies (typically $\Omega\tau_V\sim 10^{-2}-1$). Furthermore, the difference between the CC result ($\Lambda=\lambda$) and expression (A5) becomes especially important at high magnetic fields approaching B_{c2}. In this limit, $\lambda^2\propto 1/(B_{c2}-\omega)$ [see Eq. (14)], whereas $\sigma_f-\sigma\propto(B_{c2}-\omega)$. Thus the divergence of the length $\Lambda\propto 1/(B_{c2}-\omega)$ at B_{c2} is stronger than that for λ itself, $\lambda\propto 1/\sqrt{B_{c2}-\omega}$. On the other hand, the simple assumption $\gamma=0$, adopted in Secs. II, III and V, which neglects the interaction between the normal current and the vortex, yields no divergence of Λ at $\omega\to B_{c2}$ at all. Thus the observation of the field dependence of the surface impedance at different frequencies should define the role of the normal current in the vortex dynamics.

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