General properties of a polaron in motion

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A concise approach for static polarons is extended to study the dynamic polaron system having nonspherical symmetry. The general properties of moving polarons, such as the ground-state energy, the phonon number, the effective mass, and the density distribution of phonon momentum—studied as functions of the total polaron momentum and electron-phonon coupling strength—are calculated numerically in a wide range of intermediate coupling strengths. In particular, a modified Fröhlich Hamiltonian is derived for the problem of polaron momentum $Q \geqslant 1$. [S0163-1829(96)05542-7]

I. INTRODUCTION

Polaron problems have attracted much attention during the last 40 years, and there has been considerable progress in both theoretical and experimental studies of its properties. Each improvement in polaron theory has doubtlessly shed light on elucidating the mechanisms of the electron-phonon interaction in polar crystals, semiconductors, and noncrystalline materials. In general, three approaches were applied to static polarons, which are variational calculations including the path-integral method, perturbation theory, and the Green's function method. The path-integral method was first applied to the polaron problem by Feynman,[1] and his results was universally accepted and proved to be reliable. Recently, a Monte Carlo calculation has been performed on the polaron ground-state energy by Alexandrou *et al.*[2] Meanwhile, some fundamental theorems, deductions, and physical properties about the static polaron system have been discussed by many physicists.[3–8]

The problem of the energy-momentum dispersion relation of a Fröhlich polaron confined to two dimensions for small electron-phonon coupling strength has been studied by Peeters *et al.*[9] P. Warmenbol *et al.*[10] have investigated the effect of the polaron-induced nonparabolicity of the energy-momentum relation on the dynamics of transport electrons for AgCl using different models. It is known that the static polaron has spherical symmetry so that an original three-dimensional (3D) problem can be reduced into a 1D one. If the polaron is in motion, the symmetry of the system will be broken, and the properties of moving polarons in the horizontal and vertical directions one are intrinsically different. For this reason, the symmetry should be an axial symmetry. Thus a 3D problem then became an essentially 2D question. By physical considerations, the moving polarons may have more significance than static ones, including how their transport character becomes affected by the surrounding, defects and impurities, for example. These properties must have a close relation with the moving behavior of polarons. This is why the problem of polarons in motion is interesting.

In this paper, a concise approach is extended to a dynamic polaron system. With this method, a systematic study of the general properties of moving polarons including, the ground-state energy, the total phonon number, the effective mass and their dependent relations with the distribution of phonons, and the electron-phonon coupling strength, are made in a wide range of intermediate-coupling strengths. In Sec. II, we first perform a two-phonon correlation to phonon coherent state of the Schrödinger equation with a Fröhlich model. As a result, a self-consistent integral equation for the distribution function of the energy in terms of the wave vector $\mathbf{q}$ is derived under the consideration of axial symmetry. Finally, we solve the equation numerically. The calculated results are presented and discussed in Sec. III. In particular, we give a modified Fröhlich Hamiltonian to discuss the energy-momentum relation of polarons to the problem of polaron momentum $Q \geqslant 1$ in Sec. IV. Conclusions are given in Sec. V.

II. THEORETICAL DERIVATION

The work starts with the well-known Fröhlich Hamiltonian.[11] In this paper, the units of $2m=\omega_0=V=\hbar=1$ are adopted, and so the Hamiltonian reads

$$H=\mathbf{P}^2+\sum_{\mathbf{q}} a_{\mathbf{q}}^{\dagger}a_{\mathbf{q}}+\sum_{\mathbf{q}} V_{\mathbf{q}}(a_{\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{r}}+a_{\mathbf{q}}^{\dagger}e^{-i\mathbf{q}\cdot\mathbf{r}}), \quad (1)$$

where

$$V_{\mathbf{q}}=\frac{2\sqrt{\pi\alpha}}{|\mathbf{q}|}, \quad (2)$$

and $a_{\mathbf{q}}^{\dagger}$ and $a_{\mathbf{q}}$ are the creation and annihilation operators of the longitudinal optical (LO) phonon with wave vector $\mathbf{q}$, respectively. α is a dimensionless coupling constant.

The operator of total momentum,

$$\mathbf{Q}=\mathbf{P}+\sum_{\mathbf{q}} \mathbf{q}a_{\mathbf{q}}^{\dagger}a_{\mathbf{q}}, \quad (3)$$

commutes with the Hamiltonian H; therefore, $\mathbf{Q}$ is a conserved quantity. Following Lee, Low, and Pines,[12] we can use this fact to eliminate the electron coordinates from H; we define the unitary transformation

$$U=\exp\left[i\left(\mathbf{Q}-\sum_{\mathbf{q}} a_{\mathbf{q}}^{\dagger}a_{\mathbf{q}}\right)\cdot\mathbf{r}\right], \quad (4)$$

and calculate $H'=U^{-1}HU$, which results in

$$H'=\mathbf{Q}^2+\sum_{\mathbf{q}}(1-2\mathbf{Q}\cdot\mathbf{q}+\mathbf{q}^2)a_{\mathbf{q}}^{\dagger}a_{\mathbf{q}}+\sum_{\mathbf{q}_1,\mathbf{q}_2}\mathbf{q}_1\cdot\mathbf{q}_2 a_{\mathbf{q}_1}^{\dagger}a_{\mathbf{q}_2}^{\dagger}a_{\mathbf{q}_1}a_{\mathbf{q}_2}+\sum_{\mathbf{q}}V_{\mathbf{q}}(a_{\mathbf{q}}+a_{\mathbf{q}}^{\dagger}). \quad (5)$$

In the next important step, following Ref. 13, we introduce our phonon coherent state as follows:

$$|\rangle=|\rangle_0+\sum_{\mathbf{q}_1,\mathbf{q}_2}b(\mathbf{q}_1,\mathbf{q}_2)a_{\mathbf{q}_1}^{\dagger}a_{\mathbf{q}_2}^{\dagger}|\rangle_0, \quad (6)$$

where

$$|\rangle_0=\prod_{\mathbf{q}}\exp[F(\mathbf{q})a_{\mathbf{q}}^{\dagger}]|0\rangle \quad (7)$$

and $b(\mathbf{q}_1,\mathbf{q}_2)$ is the interchanging symmetrical function of $\mathbf{q}_1$ and $\mathbf{q}_2$. It implies the correlation between two emitted phonons. With the intermediate-coupling strength, the expansion (6) is enough to describe the polaron behavior in ionic crystals or semiconductor materials. Even if stronger coupling should be considered, one can add more correlation terms of phonons Eq. (6), which only adds to the difficulties in calculation.

Inserting Eqs. (5) and (6) into the Schrödinger equation

$$H'|\rangle=E|\rangle, \quad (8)$$

we obtain

$$E\left\{|\rangle_0+\sum_{\mathbf{q}_1,\mathbf{q}_2}b(\mathbf{q}_1,\mathbf{q}_2)a_{\mathbf{q}_1}^{\dagger}a_{\mathbf{q}_2}^{\dagger}|\rangle_0\right\}=\sum_{\mathbf{q}}V_{\mathbf{q}}F(\mathbf{q})|\rangle_0+\left\{\sum_{\mathbf{q}}(1-2\mathbf{Q}\cdot\mathbf{q}+\mathbf{q}^2)F(\mathbf{q})+V_{\mathbf{q}}+2\sum_{\mathbf{q}'}b(\mathbf{q},\mathbf{q}')V_{\mathbf{q}'}\right\}a_{\mathbf{q}}^{\dagger}|\rangle_0$$
$$+\sum_{\mathbf{q}_1,\mathbf{q}_2}\left\{\sum_{\mathbf{q}}V_{\mathbf{q}}F(\mathbf{q})b(\mathbf{q}_1,\mathbf{q}_2)+\mathbf{q}_1\cdot\mathbf{q}_2F(\mathbf{q}_1)F(\mathbf{q}_2)+[2-2\mathbf{Q}\cdot(\mathbf{q}_1+\mathbf{q}_2)\right.$$
$$\left.+(\mathbf{q}_1+\mathbf{q}_2)^2]b(\mathbf{q}_1,\mathbf{q}_2)+\mathbf{Q}^2b(\mathbf{q}_1,\mathbf{q}_2)+\mathbf{q}_1\cdot\mathbf{q}_2b(\mathbf{q}_1,\mathbf{q}_2)\right\}a_{\mathbf{q}_1}^{\dagger}a_{\mathbf{q}_2}^{\dagger}|\rangle_0, \quad (9)$$

where the $(a^{\dagger})^3$ terms are neglected. Comparing the coefficients of $|\rangle_0$, $a_{\mathbf{q}}^{\dagger}|\rangle_0$, and $a_{\mathbf{q}_1}^{\dagger}a_{\mathbf{q}_2}^{\dagger}|\rangle_0$ on two sides of Eq. (9), we have

$$E=\mathbf{Q}^2+\sum_{\mathbf{q}}V_{\mathbf{q}}F(\mathbf{q}), \quad (10)$$

$$V_{\mathbf{q}}+(1-2\mathbf{Q}\cdot\mathbf{q}+\mathbf{q}^2)F(\mathbf{q})+2\sum_{\mathbf{q}'}V_{\mathbf{q}'}b(\mathbf{q},\mathbf{q}')=0, \quad (11)$$

$$\left\{\sum_{\mathbf{q}}V_{\mathbf{q}}F(\mathbf{q})+[2-2\mathbf{Q}\cdot(\mathbf{q}_1+\mathbf{q}_2)+(\mathbf{q}_1+\mathbf{q}_2)^2]-E+\mathbf{Q}^2\right\}b(\mathbf{q}_1,\mathbf{q}_2)=-\mathbf{q}_1\cdot\mathbf{q}_2F(\mathbf{q}_1)F(\mathbf{q}_2). \quad (12)$$

From Eqs. (11) and (12), one has

$$F(\mathbf{q})=-\frac{V_{\mathbf{q}}}{1-2\mathbf{Q}\cdot\mathbf{q}+\mathbf{q}^2}+\frac{2}{1-2\mathbf{Q}\cdot\mathbf{q}+\mathbf{q}^2}\sum_{\mathbf{q}'}V_{\mathbf{q}'}\frac{\mathbf{q}\cdot\mathbf{q}'F(\mathbf{q})F(\mathbf{q}')}{2-2\mathbf{Q}\cdot(\mathbf{q}+\mathbf{q}')+(\mathbf{q}+\mathbf{q}')^2}. \quad (13)$$

Since the polaron is in motion, the phonons should be distributed over the direction of total polaron momentum. So $F(\mathbf{q})$ can be expressed in the form $F(Q,q,\theta)$, where θ is the angle between $\mathbf{Q}$ and $\mathbf{q}$. Turn the summation $\Sigma_{\mathbf{q}}$ into an integral $[1/(2\pi)^3]\int d\mathbf{q}$, the Eq. (13) can be reduced to the form as

$$F(Q,q,\theta)=-\frac{V_{\mathbf{q}}}{1-2Qq\cos\theta+q^2}+\frac{2}{(1-2Qq\cos\theta+q^2)(2\pi)^3}\int V_{\mathbf{q}'}F(Q,q,\theta)F(Q,q',\theta')qq'^3(\sin\theta\sin\theta'\cos\varphi'$$
$$+\cos\theta\cos\theta')\sin\theta'dq'd\theta'd\varphi'[2-2q(q\cos\theta+q'\cos\theta'+2qq'(\sin\theta\sin\theta'\cos\varphi'+\cos\theta\cos\theta')]^{-1}. \quad (14)$$

From Eq. (14) one can prove that if Q satisfies the condition of $Q<1$, the denominator of formula (14) will not be equal to zero. That is to say, the solution of Eq. (14) exists. For Q approaching 1.0 and the case of $Q\geqslant 1.0$, a modified calculation will be discussed in Sec. IV.

III. NUMERICAL RESULTS AND DISCUSSIONS

By Eqs. (14) and (6), an exact form for the density of phonon number can be derived as

$$n(Q,q,\theta)=|F(Q,q,\theta)|^2[1+G^2(Q,q,\theta)], \tag{15}$$

where

$$G(Q,q,\theta)=\frac{1}{2\pi^2}\int_0^{+\infty}dq'\int_0^{\pi}d\theta' F(Q,q,\theta')q'^2\sin\theta'\mathcal{R}, \tag{16}$$

$$\mathcal{R}=1-[2-2Q(q\cos\theta+q'\cos\theta')+q^2+q'^2][2-2Q(q\cos\theta+q'\cos\theta')+q^2+q'^2 +2qq'\cos(\theta+\theta')]^{-1/2}[2-2Q(q\cos\theta+q'\cos\theta'+q^2+q'^2+2qq'\cos(\theta-\theta')]^{-1/2}. \tag{17}$$

The numerical calculations are arranged as follows. First, by the iteration method, we calculate the solution of $F(Q,q,\theta)$; we then substitute it into Eq. (15) to calculate the distribution of the density of phonon numbers (DPN) as the function of phonon momentum q and its direction θ, respectively. After that, we calculate the phonon momentum distribution, total averaged number of virtual phonons, total ground-state energy, and the effective mass as functions of coupling strength α and the total momentum $\mathbf{Q}$ separately. Now we demonstrate our numerical results and give the explanation.

A. Distribution of density of phonon numbers

According to formulas (14) and (15), we first calculate the density of phonon numbers as the functions of phonon momentum $\mathbf{q}$ in different direction θ and coupling strength α by the iteration method. The results are shown in Fig. 1. The ordinate takes the logarithm unit. Each group curve means the same α and Q with a different angle of phonons. The lower group curve represents the smaller α value. It can be seen that (i) with the increasing of $\mathbf{q}$, $n(Q,q,\theta)$ descends rapidly; (ii) if the coupling strength α increases, the DPN goes up in value, and the small included angle corresponds to the large value of the DPN.

The densities of phonon number as a function of the distribution angle of phonons with different phonon momentum $\mathbf{q}$ and coupling constant α are presented in Figs. 2(a) and 2(b) separately. Figures 2(a) and 2(b) stipulate $\alpha=2$ and

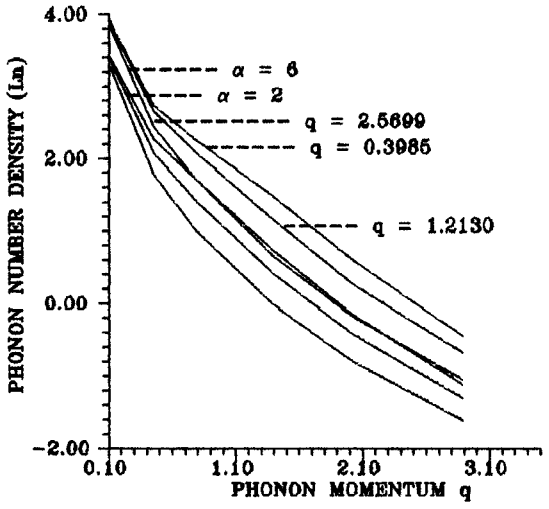

FIG. 1. Density of phonon numbers as a function of phonon momentum with different directions of phonons θ and coupling strength α.

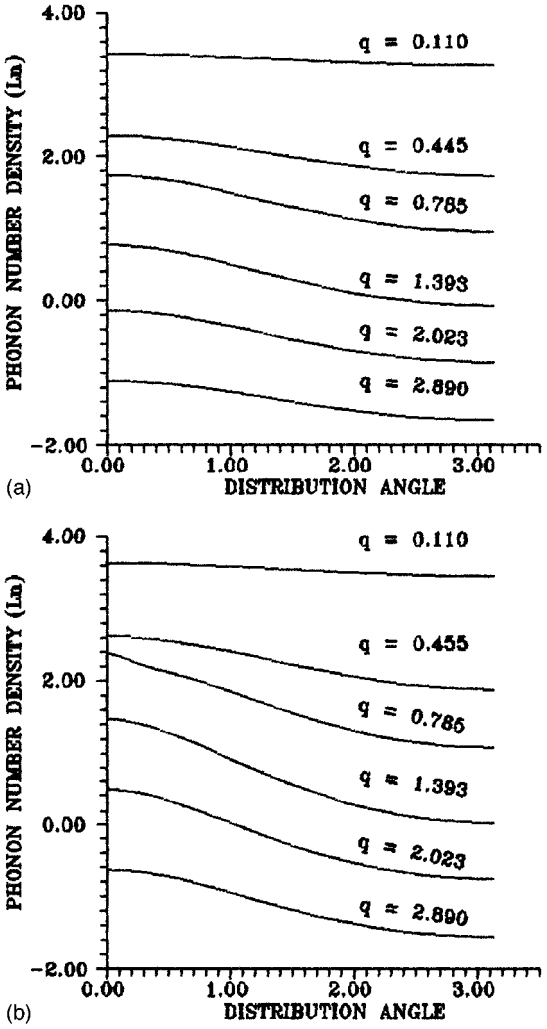

FIG. 2. Density of phonon numbers vs the phonon distribution angle θ in different q. (a) $\alpha=2$ and (b) $\alpha=6$.

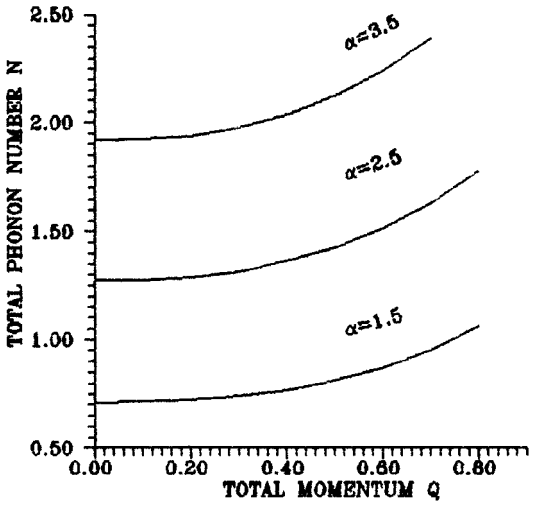

FIG. 3. Total average phonon number N vs the total polaron momentum Q with $\alpha=1$, 2, 3, and 4, respectively.

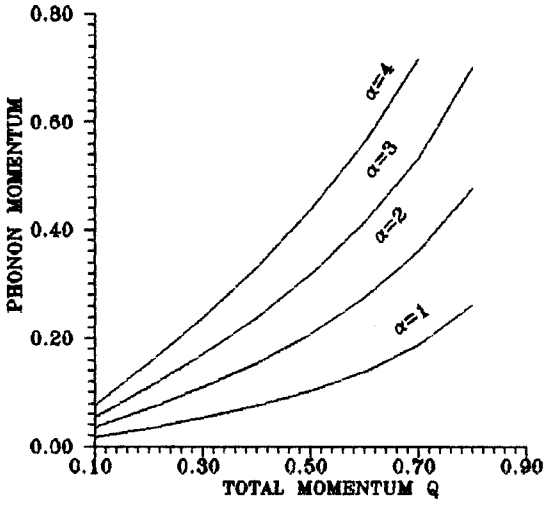

FIG. 4. Phonon momentum as a function of total momentum with different coupling constants α.

$\alpha=6$, respectively. The abscissa of two figures express the included angle between **Q** and **q**, with its unit in radians. The ordinate mark the density of phonon number n logarithmically. From two figures we noticed that (i) $n(Q,q,\theta)$ takes the maximum value in the **Q** direction and the minimum value in the opposite **Q** direction—this result is consistent with our suggestion of axial symmetry; (ii) with an increase in coupling strength, as a result of the increase of the phonon number, the change in the distribution of phonons becomes more clear with respect to the angle θ; (iii) the density of phonon numbers do not change obviously only in the case of **q** being taken both as a smaller and larger value, and in the case of a weak coupling constant.

B. Total average phonon number N

The total phonon number can be given by

$$N=\sum_{\mathbf{q}} n(Q,\mathbf{q}). \tag{18}$$

Using the results for the density of phonon number and formula (18), we calculate the total average phonon number as a function of total momentum Q. The relation between them in different coupling strengths α is demonstrated in Fig. 3. We find (i) the total average phonon number N increases with the increasing of momentum Q—there is a nonlinear relation between them—(ii) the stronger the coupling constant is, the larger the phonon number N is, a reasonable physical result. We also notice that when $\alpha=3.5$ and $Q=0.7$, N is near 2.4. This means that the two-phonon correlation is insufficient to describe the facts in such an electron-phonon system. So the reliability of the calculation in terms of the expansion (6) will be questionable. More phonon's correlation should be considered.

C. Phonon momentum

The phonon momentum $\mathbf{P}_p$ is given by

$$\mathbf{P}_p=\sum_{\mathbf{q}} n(Q,\mathbf{q})\mathbf{q}, \tag{19}$$

and the electron momentum $\mathbf{P}_e$ is determined by

$$\mathbf{P}_e=\mathbf{Q}-\mathbf{P}_p. \tag{20}$$

According to Eq. (19), we calculate the dependent relation of phonon momentum $\mathbf{P}_p$ as a function of total polaron momentum Q with different coupling strengths. Some interesting results are shown in Fig. 4. It can be seen that the phonon momentum is changing in a nonlinear relation with the polaron momentum. From the lowest solid line, we know that the proportion which phonon momentum occupies is only 1/3 of the total momentum, but when $\alpha=4$ (the highest line) the proportion will be almost 1. That is to say, the momentum of a charged carrier increases more slowly with Q when the electron-phonon coupling is weak. When the coupling strength becomes stronger, the momentum of charged carrier has a obvious change with Q. So we can conclude that the speed of a charged carrier cannot be increased by increasing the total momentum Q and coupling strength α.

D. Energy and the effective mass

The dependent relation between total energy $E(Q)$ and polaron momentum Q with different coupling strengths is presented in Fig. 5. One finds that the polaron energy has a slight increase with Q from 0 to 0.9. The possible reason can be explained as follows: with an increase of Q, the total phonon number will increase, as does the binding energy of the system; thus, it will offset most of the increase of kinetic energy, and keep the total energy almost constant with Q.

There are two deductions that can be made. (i) The inequality $E(Q=0)>E(Q\neq 0)-Q^2$ is always tenable. This conclusion had been given in theorem by Gerlach and Löwen.[6] Our quantitative results show no difference with his theorem. (ii) When the polaron is regarded as a point particle, the effective mass of this particle should be the function of total momentum **Q**.

Since the energy can be written as

$$\Delta E(Q)=E(Q+\Delta Q)-E(Q), \tag{21}$$

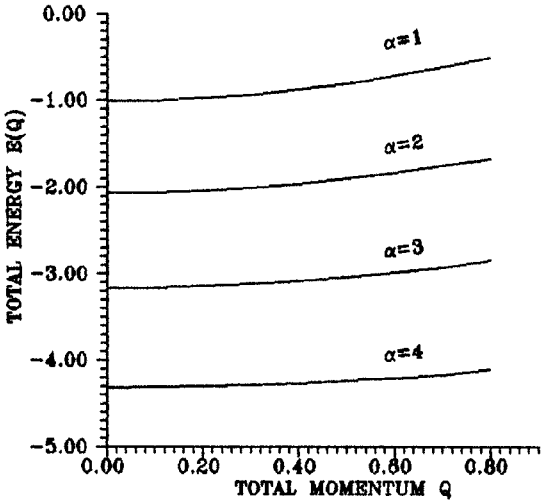

FIG. 5. Total energy $E(Q)$ vs the total momentum Q with different coupling strength for $Q<1.0$.

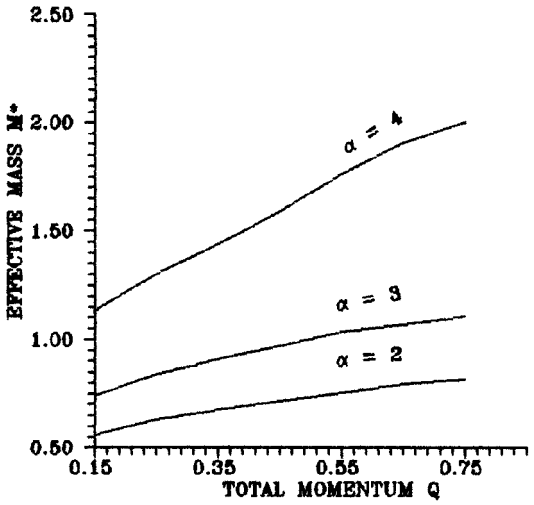

FIG. 6. The effective mass as a function of total momentum with coupling constant $\alpha=2$, 3, and 4, respectively.

the effective mass can be given by

$$m^*(Q)=\frac{Q}{\Delta E/\Delta Q}. \tag{22}$$

According to Eq. (22), we calculated the effective mass of moving polarons versus the momentum $\mathbf{Q}$. The results are displayed in Fig. 6. One may find that $m^*(Q)$ increases with an increase in Q, and the trend of this phenomenon becomes more pronounced as the coupling constant increases.

IV. ENERGY-MOMENTUM RELATION FOR $Q\geqslant 1$

It should be pointed out that the above method is not valid for $\alpha>6$ and $Q>0.9$. The enlargement of a valid range of coupling constant can be realized by introducing more phonon correlation to the ansatz. As for why $\mathbf{Q}$ cannot be equal to 1.0, the reason is that the Fröhlich Hamiltonian was derived under a one-order perturbation approximation for the potential energy. If $\mathbf{Q}=1.0$, it leads to a divergence of the energy. In order to overcome this difficulty, we expand the potential energy around the equilibrium position to the second-order term as follows:

$$U(\mathbf{r}-\mathbf{X}_\mu)=U(\mathbf{r}-\mathbf{R}_\mu)-\mathbf{u}_\mu\cdot\nabla U(\mathbf{r}-\mathbf{R}_\mu)+\frac{1}{2}\,\mathbf{u}_\mu\cdot\mathbf{u}'_\mu\,\frac{\partial^2 U(\mathbf{r}-\mathbf{R}_\mu)}{\partial\mathbf{r}\partial\mathbf{r}'}, \tag{23}$$

where $U(\mathbf{r}-\mathbf{X}_\mu)$ is the potential energy of an electron at $\mathbf{r}$ in the field of a ion at $\mathbf{X}_\mu$, and $\mathbf{X}_\mu=\mathbf{R}_\mu+\mathbf{u}_\mu$ where $\mathbf{R}_\mu$ is the equilibrium position and $\mathbf{u}_\mu$ is the atomic displacement.

After a derivation, we can write a modified Fröhlich Hamiltonian as

$$H=\mathbf{P}^2+\sum_{\mathbf{q}}\,a_{\mathbf{q}}^\dagger a_{\mathbf{q}}+\sum_{\mathbf{q}}\,V_{\mathbf{q}}(a_{\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{r}}+a_{\mathbf{q}}^\dagger e^{-i\mathbf{q}\cdot\mathbf{r}})+\sum_{\mathbf{q}}\,W(a_{\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{r}}+a_{\mathbf{q}}^\dagger e^{-i\mathbf{q}\cdot\mathbf{r}})^2, \tag{24}$$

where the forth term of the right side of the equation indicates the second-order electron-phonon interaction. W is a parameter coefficient, which is found from the calculation that relates it to the particular materials, the coupling strength, and the nonlinearity of the system.

Following a procedure similar to the above treatment, we obtain the following equations:

$$b(\mathbf{q},\mathbf{q}')=-\frac{W+\mathbf{q}\cdot\mathbf{q}'F(\mathbf{q})\mathbf{F}(\mathbf{q}')}{2-2\mathbf{Q}\cdot(\mathbf{q}+\mathbf{q}')+(\mathbf{q}+\mathbf{q}')^2}, \tag{25}$$

$$F(\mathbf{q})=-\frac{V_{\mathbf{q}}}{1-2\mathbf{Q}\cdot\mathbf{q}+\mathbf{q}^2+2W+4W\sum_{\mathbf{q}'}\,b(\mathbf{q},\mathbf{q}')}+\frac{2W}{1-2\mathbf{Q}\cdot\mathbf{q}+\mathbf{q}^2+2W+4W\sum_{\mathbf{q}'}\,b(\mathbf{q},\mathbf{q}')}\sum_{\mathbf{q}'}\,\frac{V_{\mathbf{q}'}}{2-2\mathbf{Q}\cdot(\mathbf{q}+\mathbf{q}')+(\mathbf{q}+\mathbf{q}')^2}$$
$$+\frac{2}{1-2\mathbf{Q}\cdot\mathbf{q}+\mathbf{q}^2+2W+4W\sum_{\mathbf{q}'}\,b(\mathbf{q},\mathbf{q}')}\sum_{\mathbf{q}'}\,\frac{V_{\mathbf{q}'}\mathbf{q}\cdot\mathbf{q}'F(\mathbf{q})F(\mathbf{q}')}{2-2\mathbf{Q}\cdot(\mathbf{q}+\mathbf{q}')+(\mathbf{q}+\mathbf{q}')^2}, \tag{26}$$

$$E=\mathbf{Q}^2+\sum_{\mathbf{q}}\,V_{\mathbf{q}}F(\mathbf{q})+\sum_{\mathbf{q}}\,W[F^2(\mathbf{q})+1]+2\sum_{\mathbf{q}\mathbf{q}'}\,\mathbf{W}\mathbf{b}(\mathbf{q},\mathbf{q}'). \tag{27}$$

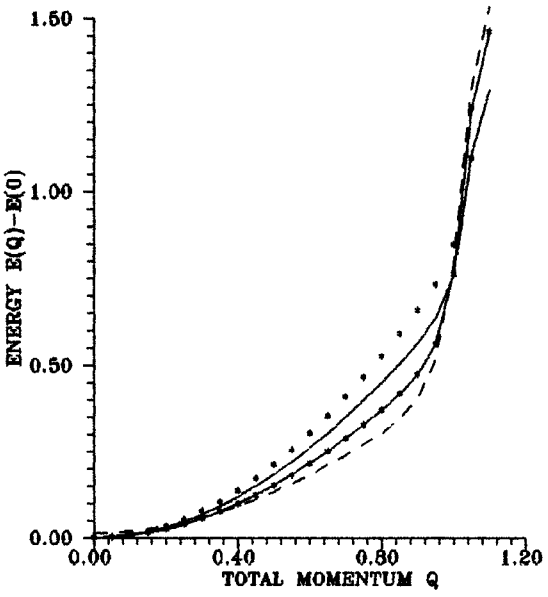

FIG. 7. Real part of the energy vs momentum from $Q=0$ to $Q=1.1$ with different coupling constants. The dotted line indicates $\alpha=1$; the solid line represents $\alpha=2$; the dot-dashed line is for $\alpha=3$; the dashed line indicates $\alpha=4$.

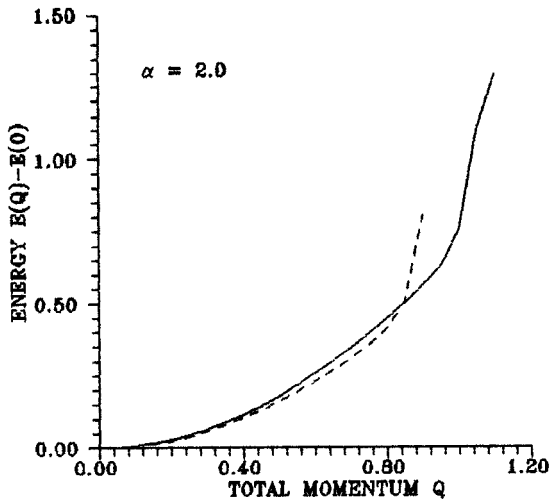

FIG. 8. Comparison for the energy-momentum relation between our original results (dashed line) and a modified calculation (solid line) with $\alpha=2$.

According to Eqs. (26) and (27), we calculate the polaron energy-momentum relation numerically from $Q=0$ to $Q=1.1$ using different coupling constants. The results are shown in Fig. 7. For comparison, we also present energy-momentum relations for the modified calculation $Q\leqslant 1.1$ and our original calculation ($Q<0.9$) in Fig. 8 as $\alpha=2.0$. It is found that, when $Q<0.5$, two results are in good agreement, though they have obvious differences when $Q>0.6$.

V. CONCLUSION

In summary, the general properties of dynamic polarons with nonspherical symmetry are investigated in a wide range of intermediate coupling constants. Some character parameters of moving polarons, such as the ground-state energy, the distribution of phonons, the effective mass, the average number of virtual phonons, and the phonon momentum, have been calculated. The calculations are completely self-consistent and the method proves to be general and effective. It also includes the results predicted by Gerlach's theorem. The reason is that, if we take $Q=0$, we are able to return to the case of the static polaron, and we can reproduce the precise results which match the previous values of sixth-order perturbation theory and are as good as the Feynman results.[13]

It is also found that phonons are distributed over the direction of moving polarons, and such a distribution decreases rapidly with the increase of the angle θ. The larger the coupling constant, the more obvious this appearance. The phonon momentum has a nonlinear relation with the total polaron momentum in a dynamic polaron system. The effective mass increases approximately linearly with the increase of total momentum, but the slope of the curve becomes large as the coupling strength increases.

It should be pointed out that, although some conclusions of this paper are similar to the pioneer works,[9,10] we use a method to deal with this problem that has enabled us to obtain more information about the moving polarons, especially about those of phonons. It can be predicted that our method is suitable for many actual polar and semiconductor materials.

ACKNOWLEDGMENTS

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