

## Electron-electron scattering rate in disordered mesoscopic systems

Ya. M. Blanter\*

*Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, 76128 Karlsruhe, Germany*

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The inelastic quasiparticle lifetime due to the electron-electron interaction (out-scattering time in the kinetic equation formalism) is calculated for finite metallic diffusive systems (quantum dots) in the whole range of parameters. Both cases of “continuous” (the inelastic level broadening much exceeds the mean level spacing) and “discrete” spectrum are analyzed. In particular, the crossover between one- and zero-dimensional regimes is studied in detail. In the case of a continuous spectrum the out-scattering time is shown to be the same as the inelastic time entering expressions for universal conductance fluctuations and persistent currents. It is also found to be shorter than the phase-breaking time in two- and one-dimensional systems, while in zero-dimensional systems these two times coincide. In the case of a discrete spectrum for small enough systems a universal behavior of the scattering time is obtained. For temperatures below the mean level spacing the out-scattering rate is shown to be vanishingly small. [S0556-2821(96)05342-8]

### I. INTRODUCTION

As is well known, inelastic electron scattering plays an important role in various phenomena in disordered metallic systems (see the extensive review by Altshuler and Aronov<sup>1</sup>). It is enough to mention that it is responsible for the weak localization correction to the conductivity. It proved also to be important in mesoscopic phenomena in these systems. In particular, inelastic scattering governs the temperature dependence and crossovers between different dimensionalities for universal conductance fluctuations<sup>2,3</sup> (UCF's) and similar problems, such as persistent currents (see, e.g., Ref. 4) and correlators of persistent currents within the grand canonical ensemble (GCE) (see, e.g., Ref. 5). Recent estimations<sup>6</sup> have shown that the conductance of mesoscopic systems in the case of a discrete spectrum, at least within the canonical ensemble (CE), is also governed by the inelastic electron scattering.

The electron-electron interaction enters these problems through the inelastic scattering time. Extensive investigations carried out about 10 years ago showed that at least two relevant electron-electron scattering times exist:<sup>7</sup> (i) the out-scattering time  $\tau_{\text{out}}$  appearing in the kinetic equation formalism;<sup>8,9</sup> it has the meaning of an inverse frequency of inelastic collisions; and (ii) the phase-breaking time<sup>10</sup>  $\tau_{\phi}$  (see also Ref. 11) responsible, in particular, for the weak localization correction and for the quasi-particle decay within the Fermi liquid theory. These two times have been shown to coincide in (3D) three-dimensional systems ( $\tau_{\phi} \sim \tau_{\text{out}} \propto T^{-3/2}$ ). The phase-breaking time is well studied also in infinite 2D ( $\tau_{\phi} \propto T^{-1}$ ) and quasi-1D ( $\tau_{\phi} \propto T^{-2/3}$ ) systems.<sup>10</sup> However, the out-scattering rate in low-dimensional systems,  $\gamma(\epsilon, T) = \tau_{\text{out}}^{-1}$  has caused some controversy. It was studied for zero temperature by Altshuler and Aronov (see Ref. 1) who obtained the result  $\gamma(\epsilon, T=0) \propto \epsilon^{d/2}$ ,  $d=1$  (quasi-1D) or  $d=2$  (2D). This implies, in particular, that the Fermi liquid theory is violated in quasi-1D systems close enough to the Fermi level. An attempt to include the finite temperature makes the situation even worse: The out-scattering rate diverges for low enough energies  $\epsilon \ll T$ . In two-dimensional

systems attempts to cure this singularity have been made,<sup>12–16</sup> leading to different results.

Below this problem is revisited. The out-scattering rate  $\gamma(\epsilon, T)$  is considered for  $T \gg \epsilon$ . In principle, one should study the two-parameter problem, but in order not to make the expressions too cumbersome and not to consider a huge number of parameter ranges,  $\epsilon$  is just put to be equal to zero.<sup>17</sup> It is shown that the out-scattering time definitely does not coincide with the phase-breaking one. For the 2D case (Sec. II) just the result by Refs. 12–14,  $\gamma \propto T \ln T$ , is recovered. In the quasi-1D system (Sec. III) results are obtained: For large temperatures the out-scattering time shows the same temperature dependence as the phase-breaking one ( $\gamma \propto T^{2/3}$ ) but is parametrically shorter than the latter; moreover, for low enough temperatures (but still in the metallic regime) the out-scattering rate exceeds the temperature, and exhibits another temperature dependence  $\gamma \propto T^{3/4}$ .

Now we return to the effect of inelastic scattering on the mesoscopic phenomena in disordered systems, in particular UCF's. It is rather clear (and it is shown once more below) that the inelastic scattering time entering the UCF problem is essentially the out-scattering time. (Note that because of the controversy mentioned above it is believed also to coincide with the phase-breaking time, which is definitely not the case for 2D and Q1D systems.) So one needs to investigate the out-scattering rate in *finite* systems. The problem of electron lifetimes in finite systems has not been addressed so far, except for the only paper by Sivan, Imry, and Aronov<sup>18</sup> (to be discussed below, see Sec. IV).

The introduction of the finite size of the system leads to the appearance of two characteristic energies: the Thouless energy  $E_c = D/L^2$  (here  $D$  and  $L$  are the diffusion coefficient and the largest size of the system, respectively) and the mean level spacing  $\Delta$ . The small parameter of the standard perturbation theory (in particular, the diagram technique) is  $\gamma/\Delta$ , and, respectively, two cases should be distinguished: “continuous spectrum”  $\gamma \gg \Delta$  and “discrete spectrum”  $\gamma \ll \Delta$ .

In the three-dimensional systems (e.g. a cube of the size  $L$ ) the situation is rather simple: For  $T \gg E_c$  the inelastic level broadening  $\gamma$  much exceeds the mean level spacing  $\gamma$

(continuous spectrum), and the results actually do not depend on  $L$ . However, for  $T \ll E_c$  one obtains  $\gamma \ll \Delta$ , and the crossover to the zero-dimensional situation occurs. So in the case of 3D - 0D crossover the distinction between 3D and 0D situations is the same as the one between continuous and discrete spectra (for the exact formulation of this point see Sec. IV). However, it is not the case for 2D - 0D and Q1D - 0D crossovers. The out-scattering and the phase-breaking rates are calculated in Secs. II (2D - 0D crossover) and III (Q1D - 0D crossover) in the case of the continuous spectrum by means of the diagram technique. It is shown, in particular, that the spectrum for  $T \gg E_c$  is always continuous, and for the zero-dimensional situation (the result depends on  $L$ ) the two inelastic times coincide:  $\tau_{\text{out}} \sim \tau_\phi$ .

In the case of the discrete spectrum  $\gamma \ll \Delta$  the perturbation theory is no longer valid, and the random matrix theory (RMT) should be involved. Two cases of ‘‘small’’  $L \ll p_F l^2$  and ‘‘large’’  $L \gg p_F l^2$  dots should be distinguished. The temperature dependence of the out-scattering time is obtained,  $\gamma \sim T^2 \Delta / E_c^2$  for  $\Delta \ll T \ll E_c$ , in ‘‘large’’ dots, while ‘‘small’’ dots exhibit a universal behavior,  $\gamma \sim T^2 / \epsilon_F$ . For the temperatures below the mean level spacing  $T \ll \Delta$  in all cases the scattering rate  $\gamma$  is found to be vanishingly small (Sec. IV).

Below diffusive metallic systems are considered. ‘‘Diffusive’’ implies the following relation between the characteristic lengths of the problem:  $l \ll L_i$  with  $l$  and  $L_i$  being the elastic mean free path and linear sizes of the system, respectively. For definiteness a rectangular sample with sizes  $L_x \equiv L \gg L_y \gg L_z \equiv a$  is chosen;  $\hbar = 1$ . ‘‘Metallic’’ means that (i) the disorder is weak,  $p_F l \gg 1$ ,  $p_F$  being the Fermi momentum, and (ii) in a quasi-1D system the localization length  $\xi \sim l(p_F a)^2$  is larger than either  $L$  or the phase-breaking length  $(D\tau_\phi)^{1/2}$ . The closed systems are of primary interest. This means that the coupling to the environment is assumed to be weak, namely,  $\tau_c^{-1} \ll \Delta$ ,  $\tau_c$  being a characteristic time of the electron escape through the attached leads. For open systems the single-particle states are smeared; the magnitude of this smearing is of the order  $\gamma_c \sim \min\{E_c, \tau_c^{-1}\}$ , and, respectively, one is interested in the energy (temperature) range  $\epsilon \gg \gamma_c$ . The results obtained below are valid, however, for the open systems also provided  $\gamma \gg \gamma_c$  (see the discussion in Sec. II).

## II. CONTINUOUS SPECTRUM: GENERAL EXPRESSIONS AND 2D CASE

### A. Generalities

In the case of a continuous spectrum  $\gamma \gg \Delta$  the standard impurity diagram technique<sup>19</sup> may be involved. We use below the diffusion-Cooperon approximation.<sup>1</sup> For  $T \gg \Delta$  one cannot expect any difference between the GCE and CE; as will be shown below the spectrum is always continuous for  $T \gg E_c \gg \Delta$ , and so all calculations are performed for the more simple GCE case.

It is well known that in a 3D system the phase-breaking time

$$\tau_\phi^{-1} \sim (T/D)^{3/2} \nu_3^{-1} \quad (1)$$

coincides with the out-scattering time and is governed by a large momentum transfer  $Dq^2 \sim T$ . Here  $D = l^2 / \tau d$  is the dif-

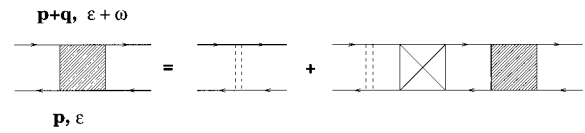


FIG. 1. The diagram representation of the diffusion propagator.

fusion coefficient ( $d$  is the dimensionality, presently  $d=3$ ) while  $\nu_3 = mp_F / \pi^2$  and  $\tau$  are the density of states and the elastic scattering time, respectively. So the 3D case is not interesting for us. Crossover to other dimensionalities occurs when one of the sizes is under  $(D/T)^{1/2}$ . Below low-dimensional systems are studied; the inequality  $T \ll D/a^2$  is assumed to be satisfied.

In the UCF theory and related problems the inelastic scattering time appears as a result of calculation of the ‘‘UCF diffusion propagator.’’<sup>3</sup> It differs from the ‘‘true’’ diffusion propagator<sup>1</sup> by the absence of electron-electron interaction lines connecting two electron propagators. A ‘‘true’’ density-density correlation function is not affected by the electron-electron interaction due to the Ward identity, which is responsible for the cancellation of interaction lines from the vertex corrections and from the corrections to electron propagators. However, if some diagrams are omitted, the Ward identity does not take place any more, and hence the electron-electron interaction renormalizes the UCF diffusion propagator. In the absence of a magnetic field the ‘‘UCF Cooperon’’ is given by exactly the same expressions as the UCF diffusion propagator.

The diagram equation in the lowest order of the electron-electron interaction for the diffusion propagator is shown in Fig. 1.<sup>8,13</sup> The shaded rectangle is the full diffusion propagator  $\Gamma(\mathbf{q}, \omega)$ , and the double-dashed line is the bare diffusion propagator

$$\Gamma^{(0)}(\mathbf{q}, \omega) = \frac{1}{\pi \nu_d \tau^2 (|\omega| + Dq^2)}, \quad (2)$$

while the crossed rectangle is the block concerned with the electron-electron interaction  $I(\mathbf{q}, \omega)$ , displayed for the UCF case in Fig. 2. The density of states in quasi- $d$ -dimensional system  $\nu_d$  (to be referred below as  $\nu$ ) is equal to  $\nu_d = \nu_3 a^{3-d}$ ; for the pure 2D system  $\nu_2 = m / \pi$ . The Matsubara diagram technique is used; it is assumed for definiteness that  $\omega + \epsilon > 0$ ,  $\epsilon < 0$ , and after the analytical continuation  $i\omega \rightarrow \omega$ ,  $i\epsilon \rightarrow \epsilon$ , the propagator corresponds to the averaged product of retarded and advanced Green’s functions  $\langle G^R(\omega + \epsilon) G^A(\epsilon) \rangle$ . The equation can be readily solved to obtain

$$\Gamma(\mathbf{q}, \omega) = \frac{1}{\pi \nu \tau^2 (|\omega| + Dq^2 + \gamma)}, \quad (3)$$

$$\gamma = -(\pi \nu \tau^2)^{-1} I(\mathbf{q}, \omega).$$

So the inelastic rate  $\gamma$  is given by the interaction block and is essentially the same as the out-scattering rate in the kinetic equation approach.<sup>8</sup>

Now we calculate the interaction block. In the arbitrary dimensionality one obtains for the diagrams of Figs. 2(a), 2(b), 2(c), and 2(d), respectively [note that for Fig. 2(a) and

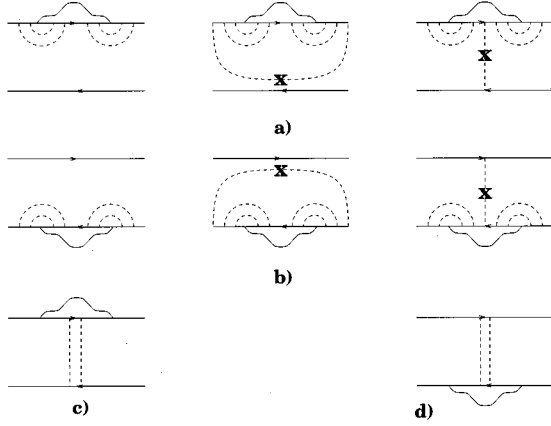


FIG. 2. The diagrams contributing to the interaction block (crossed rectangle in Fig. 1) for the UCF diffusion propagator. Wavy lines and dashed lines with a cross denote the electron-electron interaction and bare impurity scattering  $(\pi\nu\tau)^{-1}$ , respectively.

Fig. 2(b) the leading terms cancel and one has to perform an expansion over the small momenta and low frequencies],

$$\begin{aligned}
 I_a &= -\pi\nu\tau^2T \sum_{\Omega>\omega+\epsilon} \sum_{\mathbf{q}_1} \frac{U(\mathbf{q}_1, \Omega)}{(|\Omega|+Dq_1^2)^2} \\
 &\quad \times [\omega + \Omega + D(q^2 + q_1^2)], \\
 I_b &= -\pi\nu\tau^2T \sum_{\Omega<\epsilon} \sum_{\mathbf{q}_1} \frac{U(\mathbf{q}_1, \Omega)}{(|\Omega|+Dq_1^2)^2} \\
 &\quad \times [\omega - \Omega + D(q^2 + q_1^2)], \\
 I_c &= \pi\nu\tau^2T \sum_{\Omega<\epsilon+\omega} \sum_{\mathbf{q}_1} \frac{U(\mathbf{q}_1, \Omega)}{\omega - \Omega + D(\mathbf{q} - \mathbf{q}_1)^2}, \\
 I_d &= \pi\nu\tau^2T \sum_{\Omega>\epsilon} \sum_{\mathbf{q}_1} \frac{U(\mathbf{q}_1, \Omega)}{\omega + \Omega + D(\mathbf{q} + \mathbf{q}_1)^2}. \quad (4)
 \end{aligned}$$

Here  $U(\mathbf{q}, \Omega)$  is a screened Coulomb interaction.

### B. 2D case: Infinite system

As the Coulomb interaction has different forms in different dimensionalities,<sup>1</sup> the analytical continuation should be performed separately for  $d=2$  and  $d=1$ . We start from the 2D case with

$$U(\mathbf{q}, \Omega) = \frac{2\pi e^2}{q} \frac{|\Omega| + Dq^2}{|\Omega| + D\kappa q}. \quad (5)$$

Here  $\kappa = 4\pi e^2\nu_3 a$  for the quasi-2D case and  $\kappa = 2\pi e^2\nu_2$  for the 2D case.

After the analytical continuation  $i\omega \rightarrow \omega$ ,  $i\epsilon \rightarrow \epsilon$ , we obtain, for  $\omega=0$ ,  $\mathbf{q}=0$ ,

$$\begin{aligned}
 \gamma &= \frac{e^2}{\pi} \int_0^\infty D^2 \kappa q_1^3 dq_1 \int dz \left( \coth \frac{z}{2T} - \tanh \frac{z-\epsilon}{2T} \right) \\
 &\quad \times \frac{z}{(z^2 + D^2 q_1^4)(z^2 + D^2 \kappa^2 q_1^2)}. \quad (6)
 \end{aligned}$$

It is important that the terms with the hyperbolic cotangent come from the quantities  $I_c$  and  $I_d$ .

It is easy to see that for nonzero temperature the integral diverges at  $z=0$ . This singularity can be cured by the substitution of a self-consistent expression for the diffusion propagator (3) to the formulas (4) instead of  $\Gamma^{(0)}$ . The diffusion propagators entering the diagrams of Figs. 2(a) and 2(b) are ‘‘true’’ and are not renormalized by the electron-electron interaction, while those of diagrams Figs. 2(c) and 2(d) are renormalized. For  $T \gg \epsilon$  the main contribution to the integral comes from the region  $|z| < 2T$ , and one obtains the self-consistent equation for  $\gamma$  (which is supposed to be momentum independent for a while):

$$\begin{aligned}
 \gamma &= \frac{4e^2T}{\pi} \int_0^\infty D\kappa q_1 (Dq_1^2 + \gamma) dq_1 \\
 &\quad \times \int_0^{2T} \frac{dz}{(z^2 + D^2 \kappa^2 q_1^2)[z^2 + (Dq_1^2 + \gamma)^2]}. \quad (7)
 \end{aligned}$$

A straightforward calculation leads to an equation

$$\gamma = \frac{e^2T}{D\kappa} \ln(2D\kappa^2T\gamma^{-2}) = \frac{e^2T}{D\kappa} \ln\left(2D\kappa^2 \frac{(D\kappa)^2}{e^4T}\right), \quad (8)$$

where the last equality is the leading term of the solution. Substituting the explicit expressions for  $\kappa$ , one finally obtains

$$\gamma = \begin{cases} (T/2\epsilon_F\tau) \ln[8D\kappa^2(\epsilon_F\tau)^2/T] & 2D, \\ (3\pi T/4p_F^2 a l) \ln[D\kappa^2(4p_F^2 a l/3\pi)^2 T^{-1}] & \text{(quasi-2D)}. \end{cases} \quad (9)$$

Note that this rate is higher (by a logarithmic factor) than the 2D phase-breaking rate<sup>10</sup>

$$\tau_\phi^{-1} \sim (T/D\nu_2) \begin{cases} \ln(p_F l) & (2D), \\ (p_F a)^{-1} \ln(p_F^2 l a) & \text{(quasi-2D)}. \end{cases} \quad (10)$$

The result (9) is valid under the condition  $T \gg \gamma$  only. In principle, this condition (unlike the condition of the Fermi-liquid theory validity,  $T\tau_\phi \gg 1$ ) is violated in the exponentially small range of low temperatures. We consider this problem as purely academic, however, and do not write down expressions for the corresponding range.

Expression (9) was derived first in Ref. 12 as an imaginary part of the impurity-averaged self-energy. Later this result was confirmed in Refs. 13 and 14 (Fukuyama and Abrahams<sup>13</sup> have used a technique identical to that of the present paper). However, it was believed to be the expression for the phase-breaking time in a 2D system. The origin of this confusion lies in the statement<sup>13</sup> that the diagrams with the interaction between different electron lines (i.e., those not contributing to  $\gamma$ , but important for  $\tau_\phi$ , in-scattering terms in the kinetic equation approach<sup>8</sup>) are small. However, the original calculation of the phase-breaking time<sup>10</sup> allows one

to separate these two contributions (with and without interaction lines between different Green's functions) explicitly, and the result is that these contributions are of the same order. So the difference between  $\gamma$  and  $\tau_\phi^{-1}$  should be looked for in the diagrams omitted in Ref. 13. Hence the attempt<sup>15</sup> to cure this discrepancy by the introduction of a small-momenta cutoff does not seem well justified. Indeed, the momenta range proposed to be cut off is exactly the region of relevant momenta in the integral, and the introduction of finite momenta does not change the result. So we argue that the result by Fukuyama and Abrahams, Eq. (9) is true; however, it describes not the phase-breaking time, but the out-scattering time (or the UCF inelastic time). We have shown that the inelastic time entering the UCF problem is essentially the out-scattering time and differs from the phase-breaking one. It will be seen in Sec. III that the difference is even more pronounced for the quasi-one-dimensional case.

### C. 2D case: Finite system

Now we turn to the description of the 2D - 0D crossover. In the derivation of Eq. (9) it was supposed implicitly that the system is infinite. Let us now consider a 2D finite system. Under the conditions of the diffusive regime  $L \gg l$ , and in the case  $T \gg E_c$ , Eq. (7) is still valid; however, the integral should be understood as a sum over momenta with the boundary conditions taken into account. So  $q_1 = (\pi/L)(n_x, n_y)$  with integers  $n_x$  and  $n_y$  allowed or not to be equal to zero, subject to the boundary conditions. It is important that in any case the  $q_1 = 0$  mode does not contribute to the integral: For open systems (attached to the metallic junctions, i.e., in the UCF problem) this mode is forbidden by the boundary conditions, while for closed ones (e.g., in the problem of persistent currents) the conditions of the charge neutrality imply  $U(0, \omega) = 0$ . So the difference between these two cases is not quite important, and the boundary conditions will be taken into account just by the cutting off the integral for small momenta  $q < \pi/L$ .

The analysis of Eq. (7) then shows that for  $\gamma \gg (E_c D \kappa^2)^{1/2}$  this cutoff does not play any role and the result (9) is still valid. Note that this region corresponds to the 2D case from the diffusion point of view. For the opposite case, which is realized under the conditions

$$T/\epsilon_F \ll \begin{cases} (l/L)\epsilon_F \tau & (2D) \\ (l/L)(p_F a l) & (\text{quasi-2D}), \end{cases} \quad (2D)$$

one obtains

$$\gamma = \frac{e^2 T}{\kappa D} \ln \frac{2T}{\pi^2 E_c}. \quad (11)$$

It is seen that this result also differs from Eq. (8) by a logarithmic factor.

### III. QUASI-1D CASE AND Q1D-0D CROSSOVER

Let us now turn to the quasi-1D case. The screened Coulomb interaction is

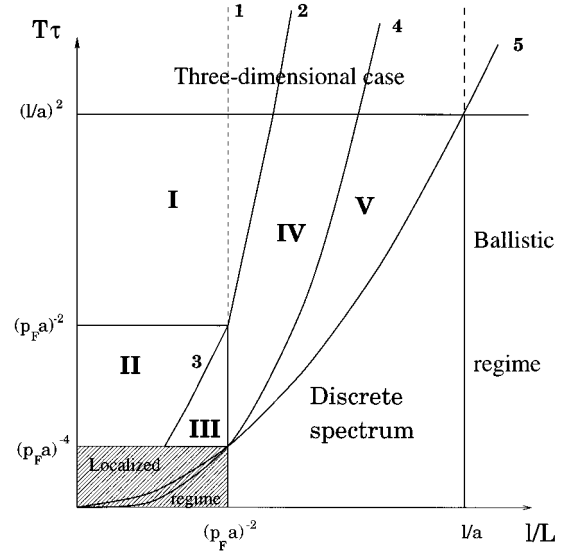


FIG. 3. Different regimes for the 1D-0D crossover. Curves 1,  $g = E_c/\Delta = 1$ ; 2,  $T\tau = (l/L)^3(p_F a)^4$  [or  $\gamma = E_c(p_F a)^2$ ]; 3,  $T\tau = (l/L)^2(p_F a)^2$  [or  $E_c = T(p_F a)^{-2}$ ]; 4,  $T\tau = (l/L)^3(p_F a)^2$  (or  $E_c \tau_\phi = 1$ ); and 5,  $T\tau = (l/L)^2$  (or  $T = E_c$ ). Regimes I-V are described in Sec. III.

$$U(\mathbf{q}, \Omega) = e^2 \ln \left( \frac{1}{q^2 a^2} \right) \frac{|\Omega| + Dq^2}{|\Omega| + Dq^2 + e^2 \nu_1 Dq^2 \ln(q^2 a^2)^{-1}}. \quad (12)$$

It is convenient to denote the denominator of this expression as  $|\Omega| + Dq^2 f(q)$  with  $f(q) = 1 + C \ln(q^2 a^2)^{-1}$ ,  $C = e^2 \nu_1$ . If, as usual,  $e^2 \sim \nu$ , the constant  $C$  occurs to be of order  $C \sim (p_F a)^2 \gg 1$ .

Prior to the calculation of  $\gamma$  for the quasi-1D system it is necessary to analyze the expression for the phase-breaking time,<sup>10</sup> which can be conveniently rewritten as

$$\tau_\phi^{-1} \sim T^{2/3} \tau^{-1/3} (p_F a)^{-4/3}. \quad (13)$$

This result was obtained under some assumptions to be analyzed here. First, the Fermi liquid theory is valid only in the case  $T\tau_\phi \gg 1$ . Second, the system is one dimensional, i.e.,  $E_c \sim D/L^2 \ll \tau_\phi^{-1} \ll T \ll D/a^2$ . Finally, it is supposed to be metallic, i.e., the correlation length  $\xi = l(p_F a)^2 \gg \min\{L, (D\tau_\phi)^{1/2}\}$ .

The last condition is violated in the localized regime (the shaded range in Fig. 3),

$$T\tau \ll (p_F a)^{-4}, \quad l/L \ll (p_F a)^{-2},$$

and the first three are consequently summarized as (Fig. 3, ranges I, II, III, and IV)

$$(l/L)^3 (p_F a)^2 \ll T\tau \ll (l/a)^2 \ll 1. \quad (14)$$

The question of much interest is what happens if the condition  $T\tau \gg (l/L)^3 (p_F a)^2$  [for  $l/L \gg (p_F a)^{-2}$ ] is violated. The expression for the phase-breaking time in an infinite system, Eq. (13) is no longer valid there. However, it is possible to derive the expressions for this regime. This is done in Appendix A. The result is

$$\frac{\gg 1}{\tau_\phi} \sim \frac{T}{(p_F a)^2} \frac{L}{l}, \quad (15)$$

$$(l/L)^2 \ll T\tau \ll (l/L)^3 (p_F a)^2. \quad (16)$$

This expression is valid until  $E_c \sim T$  (range V), as is seen from Eq. (16). As will be shown, in the whole range (16) the inequality  $\gamma \gg \Delta$  is satisfied,  $\Delta = (\nu_3 V)^{-1}$  being the mean level spacing. So the spectrum is continuous and this range is subject to the perturbative analysis. Thus, one has a ‘‘true’’ quasi-1D region (14), transition Q1D – 0D region (16), and ‘‘true’’ 0D region  $T \ll E_c$  where the spectrum is discrete. In this section both Q1D and transitional ranges are considered. It will be shown that a new splitting of the range in respect to the out-scattering time appears.

Now it is possible to start the calculation of the out-scattering time. Consideration of the diagrams of Fig. 2 leads to the divergent expression, and this divergence can be cured exactly in the same way as in 2D case. Extracting the main term for  $T \gg \epsilon$ , one obtains

$$\begin{aligned} \gamma &= \frac{4Te^2 C}{\pi^2} \int_{\pi/L}^{\infty} dq Dq^2 (Dq^2 + \gamma) \ln^2(q^2 a^2)^{-1} \\ &\times \int_0^{2T} dz \frac{1}{[z^2 + D^2 q^4 f^2(q)][z^2 + (Dq^2 + \gamma)^2]}. \quad (17) \end{aligned}$$

Here a low-momentum cutoff is introduced in the momentum integral (see the discussion for the 2D case). Equation (17) contains all information about the scattering rate and should be solved in different limiting cases (Fig. 3).

### A. Range I

Let us assume  $D(\pi/L)^2 f(\pi/L) \ll \gamma \ll T$ . In this case the cutoff is not important, and the essential momenta in the  $q$  integration are  $Dq^2 f(q) < \gamma$ . The equation simplifies to a form

$$\gamma = 4Te^2 / CDq_0 \pi, \quad Dq_0^2 f(q_0) = \gamma$$

(the inequality  $C \gg 1$  has been used). This yields the result

$$\gamma = \left( \frac{4Te^2}{\pi} \right)^{2/3} (CD)^{-1/3} \ln^{1/3} \left[ \frac{(CD)^{4/3}}{a^2} \left( \frac{\pi}{4Te^2} \right)^{2/3} \right]. \quad (18)$$

This cumbersome expression can be simplified if one assumes again  $e^2 \sim v$ . Then, as the cube root of the logarithm is always a quantity of order unity, one obtains

$$\gamma \sim T^{2/3} \tau^{-1/3} (p_F a)^{-2/3}. \quad (19)$$

It is seen that the initial assumptions  $D(\pi/L)^2 f(\pi/L) \ll \gamma$  and  $\gamma \ll T$  are satisfied only in the temperature range

$$\max\{(p_F a)^{-2}, (l/L)^3 (p_F a)^4\} \ll T\tau \ll (l/a)^2$$

(range I). In particular, this result is valid for the infinite quasi-1D system for temperatures above  $\tau^{-1} (p_F a)^{-2}$ . It should be emphasized also that in this case  $\tau_\phi \gamma = (p_F l)^{2/3} \gg 1$ , and consequently  $\tau_{\text{out}} = \gamma^{-1} \ll \tau_\phi$ . So the out-scattering time in this range of parameters shows the same temperature dependence as the phase-breaking time, but is much shorter than the latter. Nevertheless, the transi-

tion to the 0D behavior in the problem of out-scattering times occurs before the corresponding transition in the dephasing problem.

### B. Ranges II and III

If one assumes  $D(\pi/L)^2 f(\pi/L) \ll \gamma$ ,  $\gamma \gg T$ , a straightforward calculation leads to the equation

$$\gamma = \frac{4Te^2}{\pi^2} \int_{\pi/L}^{\infty} dq q \arctan \frac{2T}{Dq^2 f(q)} \frac{1}{Dq^2 + \gamma}. \quad (20)$$

A new relevant length scale appears,  $\tilde{q}$ , defined by  $D\tilde{q}f(\tilde{q}) = 2T$ . Then two limiting cases should be distinguished.

#### 1. Range II, $\tilde{q} \gg L^{-1}$

The cutoff is again unimportant, and one obtains

$$\gamma = \frac{2eT^{3/4}}{\pi^{1/2} (CD)^{1/4}} \ln^{-1/4} \left( \frac{CD}{2Ta^2} \right) \sim T^{3/4} \tau^{-1/4} (p_F a)^{-1/2}. \quad (21)$$

The second identity is the estimation, based upon assumption  $e^2 \sim v$ . All assumptions made for the derivation of Eq. (21) occur to be consistent in range II:

$$\max\{(p_F a)^{-4}, (l/L)^2 (p_F a)^2\} \ll T\tau \ll (p_F a)^{-2}.$$

This result does not contain the length of the sample again and consequently describes infinite quasi-1D systems for the temperatures just above the metal-insulator transition:  $(p_F a)^{-4} \ll T\tau \ll (p_F a)^{-2}$ . We have obtained a new temperature dependence  $\gamma \propto T^{3/4}$ , and the out-scattering time is now not only parametrically shorter than the phase-breaking one, but even  $\gamma \gg T$ . As  $\gamma$  has the meaning of a frequency of electron-electron collisions, the violation of the condition  $\gamma \ll T$  has nothing to do with the violation of Fermi liquid theory (the latter is subject to another condition,  $T\tau_\phi \gg 1$ ).

#### 2. Range III, $\tilde{q} \ll L^{-1}$

Now the scattering rate introduced to the right-hand side (RHS) of Eq. (20) for self-consistency is inessential, and one obtains

$$\gamma = \frac{4T}{\pi^{3/2}} \left( \frac{e^2 L}{CD} \right)^{1/2} \ln^{-1/2} \frac{L}{\pi a} \sim T(l/L)^{1/2} (p_F a)^{-1}. \quad (22)$$

This result is valid under the conditions (range III)

$$l/L \ll (p_F a)^{-2}, \quad (p_F a)^{-4} \ll T\tau \ll (l/L)^2 (p_F a)^{-2}.$$

Now the result contains the length of the sample, but still one has  $\gamma \gg T$  and  $\gamma\tau_\phi \gg 1$ .

### C. Ranges IV and V

In the case  $D(\pi/L)^2 f(\pi/L) \gg \gamma$  the important region in the integral over  $z$  is  $z < Dq^2 + \gamma$ , and again straightforwardly all dependence on the inelastic time cancels out on the RHS. The result is

$$\gamma = \frac{2T}{(p_F a)^2} \frac{L}{l}, \quad (23)$$

$$(l/L)^2 \ll T\tau \ll (l/L)^3 (p_F a)^4.$$

To summarize, we have described the crossover between quasi-one-dimensional and zero-dimensional behavior for the case of continuous spectrum. We have discovered five different parameter ranges (Fig. 3), which can be divided into three groups.

(1) I and II. Here both the dephasing and the out-scattering are purely quasi-one-dimensional, the results do not contain  $L$ . In the whole range the relation  $\tau_{\text{out}} \ll \tau_\phi$  holds.

(2) III and IV the transitional region. The dephasing is still quasi-one-dimensional, while the out-scattering time depends on  $L$  and is of zero-dimensional nature. Still one has  $\tau_{\text{out}} \ll \tau_\phi$ .

(3) The range V is truly zero dimensional, the expressions for the phase-breaking rate, Eq. (15), and for the out-scattering rate Eq. (23), coincide. However, the spectrum still is continuous, i.e.,  $\tau_\phi^{-1} \sim \tau_{\text{out}}^{-1} \gg \Delta$ .

The spectrum becomes discrete for the temperatures below the Thouless energy  $T \ll E_c$ , as takes place in the 3D case.

#### IV. DISCRETE SPECTRUM

The diagram technique used in Secs. II and III is valid only in the case of a continuous spectrum  $\gamma \gg \Delta$ . As was shown above, this condition in all cases is equivalent to another one  $T \gg E_c$ . So this perturbative approach is not suitable for the case of low temperatures (or, alternatively, small dots)  $T \ll E_c$ . Fortunately, this is exactly the range where the random matrix theory<sup>20</sup> (RMT) works quite well for the description of small disordered systems.<sup>21</sup> Note that usually RMT in the range of its applicability is equivalent to the zero-dimensional nonlinear  $\sigma$  model.<sup>22</sup> However, in the problem under consideration one should use the four-point correlation function which up to now was not derived by means of the supersymmetry method. So RMT (in spirit of the paper by Gor'kov and Eliashberg<sup>21</sup>) seems presently to be the only method for investigation of the electron-electron interaction in the nonperturbative regime. To avoid misunderstanding, we stress that the electron-electron interaction is taken into account *perturbatively*; however, the parameter  $\Delta/\gamma$  is no longer small and should be treated *nonperturbatively*.

In order not to overburden our expressions, we consider in this section the cubic sample with size  $L$ . The system is assumed to be charge neutral, and the results rely heavily on this fact. In principle, the methods used allow one to consider samples of arbitrary geometry, and all expressions below containing the parameters  $\Delta$  and  $E_c$  instead of  $L$  are valid for the general case (for the metallic regime). Also, we do not distinguish two types of scattering rates from each other. To understand the results better, we extend the ranges of the parameter to the clean case also. The exact result for the scattering rate in the bulk 3D case reads as<sup>8</sup>

$$\gamma(\epsilon, T=0) = \frac{3^{1/2}}{2} \frac{1}{(p_F l)^{3/2}} \frac{\epsilon^{3/2}}{\epsilon_F^{1/2}} + \frac{\pi}{8} \frac{\epsilon^2}{\epsilon_F}. \quad (24)$$

The first term is exactly Eq. (1). It exceeds the second one for small enough temperatures,  $T\tau \ll (p_F l)^{-2}$ , and in the pre-

vious sections it was implicitly supposed that this condition is satisfied. However, for  $T\tau \gg (p_F l)^2$  the second term is the most important. This is just the result in the ‘‘clean’’ case: The mean free path does not enter the expressions. So we consider in this section the case  $L \gg l$  for arbitrary temperatures: both diffusive and clean limits. The result (24) is obtained by the diagram technique and is valid consequently for  $\gamma \gg \Delta$  only; in the diffusive limit this condition is equivalent to  $T \gg E_c$ , while in the clean limit for  $T = E_c$  one has still  $\gamma \gg \Delta$ . The methods used below are valid for  $T \ll E_c$ , and so in the clean regime one has some overlap between the results to be obtained and Eq. (24). Below the out-scattering time  $\gamma^{-1}$  for both the GCE and CE cases is calculated.

Prior to the calculation it is necessary to analyze the paper by Sivan, Imry, and Aronov,<sup>18</sup> devoted to the inelastic scattering rate in finite systems. The authors of Ref. 18 calculated exactly the same quantity as we do — the scattering rate as a function of energy  $\gamma(\epsilon)$ . The methods they used are valid in the case of a ‘‘continuous’’ spectrum  $\gamma \gg \Delta$  where they have just reproduced the results (24). However, the formal application of these methods to the opposite case of a ‘‘discrete’’ spectrum ( $\gamma \ll \Delta$ ) leads to some nontrivial results that appear to be not well justified in the approach of Ref. 18. Below we present a *rigorous* calculation, valid in the case  $\gamma \ll \Delta$ , and the results in the range  $T \gg \Delta$  (where level correlations are unimportant) exactly reproduce those of Ref. 18. The reasons for this equivalence are still unclear. Also the effect of level correlations, which cannot be taken into account by the methods of Ref. 18, is automatically incorporated in the approach developed below.

A general expression for the scattering rate, based on the perturbation theory of the Coulomb interaction, is

$$\gamma_\lambda = 2\pi \sum_{\lambda_1, \lambda_2, \lambda_3} |\langle \lambda, \lambda_2 | U(\mathbf{r}_1, \mathbf{r}_2) | \lambda_1, \lambda_3 \rangle|^2 \times \delta(\epsilon_\lambda + \epsilon_{\lambda_2} - \epsilon_{\lambda_1} - \epsilon_{\lambda_3}) f_{\lambda_2} (1 - f_{\lambda_1}) (1 - f_{\lambda_3}), \quad (25)$$

$$\langle \lambda, \lambda_2 | U | \lambda_1, \lambda_3 \rangle \equiv \int d\mathbf{r}_1 d\mathbf{r}_2 U(\mathbf{r}_1, \mathbf{r}_2) \psi_\lambda^*(\mathbf{r}_1) \times \psi_{\lambda_1}(\mathbf{r}_1) \psi_{\lambda_2}^*(\mathbf{r}_2) \psi_{\lambda_3}(\mathbf{r}_2).$$

Here  $|\lambda_i\rangle \equiv \psi_{\lambda_i}(\mathbf{r})$  are exact single-particle states—single-electron states in the unique disorder realization in a quantum dot (note that all four states need to be different)— $f_{\lambda_i}$  are Fermi distribution functions (the CE case needs a more delicate treatment though; see below), and  $U$  stands for the electron-electron interaction. Expression (25) should be averaged over the disorder realizations.

##### A. Coulomb interaction in the restricted geometry

The first thing is to calculate the screened Coulomb interaction in the restricted geometry. For the temperatures (energies) below the Thouless energy one can study the static screening only. The latter is a solution to the equation as follows:

$$\nabla^2 U(\mathbf{r}_1, \mathbf{r}_2) = \kappa^2 \theta(\mathbf{r}_1) \theta(\mathbf{r}_2) U + 4\pi e^2 \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (26)$$

$\kappa = (4\pi e^2 \nu)^{1/2}$  being the inverse Debye length,  $\nu \equiv \nu_3$ . The functions  $\theta(\mathbf{r})$  are equal to unity and zero inside and outside the sample, respectively. This equation cannot be solved exactly for a rectangular sample, and one should introduce some approximations.

In order to make this approximations clear, we consider first Eq. (25) in the simplest geometry, where it can be solved. Namely, if the sample occupies a half-space  $x > 0$ , the exact solution to Eq. (25) in the region  $x > 0$ ,  $x' > 0$  has the form

$$U(\mathbf{r}_1, \mathbf{r}_2) = \int \frac{dq_x dq_y}{(2\pi)^2} \exp[iq_y(y-y')] + iq_z(z-z')] f_q(x, x'), \quad (27)$$

$$f_q(x, x') = -\frac{2\pi e^2}{p} \exp(-p|x-x'|) - \frac{2\pi e^2}{p} \frac{p-q}{p+q} \times \exp[-p(x+x')],$$

$$p = (q^2 + \kappa^2)^{1/2}. \quad (28)$$

Here the first term is essentially the screened Coulomb interaction in the continuous media; it is translationally invariant and for small screening length  $\kappa^{-1}$  is proportional to  $\delta(x-x')$ . The second term is due to the restricted geometry effects; it is important that it be essentially nonzero when both  $x$  and  $x'$  lie in a narrow layer along the boundary of the sample; the thickness of this layer is of the order of  $\kappa^{-1}$ .

Now consider another problem. Let us impose the boundary condition in Eq. (25):  $U|_{x=0} = 0$  (and, respectively,  $U$  is nonzero only when both  $x$  and  $x'$  lie inside the sample). Then both  $\theta$  factors are identically equal to unity, and we obtain

$$f_q(x, x') = -\frac{2\pi e^2}{p} \exp(-p|x-x'|) + \frac{2\pi e^2}{p} \exp[-p(x+x')]. \quad (29)$$

If another boundary condition is imposed,  $\partial U / \partial x|_{x=0} = 0$ , a solution is

$$f_q(x, x') = -\frac{2\pi e^2}{p} \exp(-p|x-x'|). \quad (30)$$

It is seen that the results the problems (28) and (29) yield *inside* the sample ( $x > 0, x' > 0$ ) differ from exact ones (27) by the contribution that is nonzero only near the boundary of the sample. Now one should recollect that the initial problem requires only the matrix elements of the screened Coulomb potential, and so one has to consider the region inside the sample only, and the contribution of the boundary term is small in comparison with the main one by a factor  $(\kappa L)^{-1} \ll 1$ . So Eq. (25) may be replaced by a more simple one, with  $\theta$  factors set equal to unity and the boundary condition imposed. Moreover, since the range of the interaction is  $\kappa^{-1}$ , i.e., is extremely small, this result is not sensitive to

either boundary conditions or the sample's geometry, and one may perform this replacement for our rectangular sample as well.

So, turning to the case of the rectangular sample, one easily obtains

$$U(\mathbf{r}, \mathbf{r}') = 4\pi e^2 \sum_q \frac{\varphi_q(\mathbf{r}) \varphi_q(\mathbf{r}')}{q^2 + \kappa^2}. \quad (31)$$

Here  $\varphi_q(\mathbf{r})$  are the eigenfunctions of the Laplace operator with appropriate boundary conditions; the eigenvalues are equal to  $-q^2$ . In the case of the specified cubic geometry one gets

$$\mathbf{q} = (\pi/L)(n_x, n_y, n_z), \quad n_i = 0, 1, 2, \dots$$

If the system is charge neutral, the  $q=0$  mode should be dropped in the summation.

## B. Calculation of the matrix element

Now we return to Eq. (25). Actually the squared absolute value of the Coulomb interaction matrix element contains the product of eight single-particle eigenfunctions, and our statement is that this combination only weakly depends on the energies of these states, and therefore can be impurity averaged separately from other, energy-dependent, factors, yielding the constant  $U^2$ . The results obtained below follow in fact from the quasiclassical approximation.<sup>21,23</sup>

One needs to average

$$J(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = \langle \psi_{\lambda_1}^*(\mathbf{r}_1) \psi_{\lambda_1}(\mathbf{r}_1) \psi_{\lambda_3}^*(\mathbf{r}_3) \times \psi_{\lambda_2}^*(\mathbf{r}_2) \psi_{\lambda_3}(\mathbf{r}_2) \psi_{\lambda_2}(\mathbf{r}_4) \psi_{\lambda_3}^*(\mathbf{r}_4) \rangle, \quad (32)$$

so that

$$U^2 = \int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4 U(\mathbf{r}_1, \mathbf{r}_2) U(\mathbf{r}_3, \mathbf{r}_4) J(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4). \quad (33)$$

In the particular case (30),  $U(\mathbf{r}, \mathbf{r}') = \nu^{-1} \delta(\mathbf{r} - \mathbf{r}')$ , one obtains

$$U^2 = \nu^{-2} \int d\mathbf{r}_1 d\mathbf{r}_3 J(\mathbf{r}_1, \mathbf{r}_1, \mathbf{r}_3, \mathbf{r}_3). \quad (34)$$

The quantity  $J$  contains the constant part, which corresponds to the calculation in the Gaussian ensemble, and coordinate-dependent contributions of higher modes.

### 1. Gaussian ensemble

The constant part can be easily calculated. It is convenient to introduce discrete notation. If one considers  $N$  electrons ( $N \sim \epsilon_F / \Delta \gg 1$ ), and consequently splits the system over  $N$  elementary volumes with positions  $\mathbf{r}_i$ , then the values of each two eigenfunctions in each two elementary volumes are in the Gaussian ensemble independent except for the constraints due to the orthogonality and normalization conditions for these eigenfunctions. In leading order in  $N^{-1}$  one has

$$J_G(\mathbf{r}_1, \mathbf{r}_1, \mathbf{r}_3, \mathbf{r}_3) = \begin{cases} V^{-4}, & \mathbf{r}_1 = \mathbf{r}_3, \\ 2N^{-2}V^{-4}, & \mathbf{r}_1 \neq \mathbf{r}_3, \end{cases} \quad (35)$$

$V$  being the volume of the system.

Hence the contribution to  $U^2$  from the Gaussian ensemble is

$$U_G^2 = \nu^{-2}V^{-2}N^{-1} = \Delta^2/N = \alpha\Delta^3/\epsilon_F. \quad (36)$$

Here  $\alpha$  is a numerical coefficient. It can be adjusted from the comparison with the clean limit in the overlap parameter range (see below); this gives  $\alpha=1/8$ . Note that the main contribution to the integral (33) comes from the range where all four coordinates coincide; in continuous notation, this means that the distance between these points is of the order of the screening length.

## 2. Higher modes

The contribution of the higher modes is concerned with the diffusion processes. In particular, the coordinate-dependent part of Eq. (32) describes the diffusion of an electron from point  $\mathbf{r}_1$  to point  $\mathbf{r}_3$ , and the diffusion of another electron from point  $\mathbf{r}_2$  to point  $\mathbf{r}_4$ . It is reasonable to assume that if points  $\mathbf{r}_1$  is far enough from point  $\mathbf{r}_2$  (in the discrete terms, these two points lie in different elementary volumes, or, alternatively, the distance between these points exceeds several interatomic distances), and point  $\mathbf{r}_3$  is far enough from point  $\mathbf{r}_4$ , these diffusion processes are independent:

$$J_{\text{HM}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = \langle \psi_{\lambda_1}^*(\mathbf{r}_1) \psi_{\lambda_1}(\mathbf{r}_1) \psi_{\lambda_3}(\mathbf{r}_3) \psi_{\lambda_1}^*(\mathbf{r}_3) \rangle \\ \times \langle \psi_{\lambda_2}^*(\mathbf{r}_2) \psi_{\lambda_3}(\mathbf{r}_2) \psi_{\lambda_2}(\mathbf{r}_4) \psi_{\lambda_3}^*(\mathbf{r}_4) \rangle. \quad (37)$$

If, however, these pairs are close to each other (in particular, this is the case for the short-ranged interaction), additional contributions from the diffusion process  $\mathbf{r}_1 \rightarrow \mathbf{r}_4$ ,  $\mathbf{r}_2 \rightarrow \mathbf{r}_3$  appear, and expression (37) acquires a coefficient 2.

The average of four eigenfunctions of the type (37) can be calculated up to the terms of order  $g^{-2}$ , with  $g = E_c/\Delta$  being the conductance. The result reads<sup>21,24</sup>

$$\langle \psi_{\lambda}^*(\mathbf{r}) \psi_{\lambda'}(\mathbf{r}') \psi_{\lambda}(\mathbf{r}') \psi_{\lambda'}^*(\mathbf{r}) \rangle \\ = (\Delta/\pi V) \int_0^\infty (W_{\mathbf{r}}(\mathbf{r}', t) - V^{-1}) dt. \quad (38)$$

Here  $W_{\mathbf{r}}(\mathbf{r}', t)$  is the probability to find an electron at the time moment  $t$  in the point  $\mathbf{r}'$  if it was in the point  $\mathbf{r}$  in the time moment  $t=0$ . This probability obeys the diffusion equation

$$\partial W/\partial t = D\nabla_{\mathbf{r}}^2 W, W(\mathbf{r}', t=0) = \delta(\mathbf{r}' - \mathbf{r}). \quad (39)$$

Integrating Eq. (39) over the time variable and taking into account that for  $t \rightarrow \infty$  the distribution tends to be uniform one,  $W = V^{-1}$ , one obtains

$$\int_0^\infty W dt = V^{-1} + \sum_{q \neq 0} (Dq^2)^{-1} \varphi_q(\mathbf{r}) \varphi_q(\mathbf{r}'). \quad (40)$$

Finally, combining Eqs. (30), (33), (37), (38), and (40), one obtains for the contribution of the higher modes to the interaction matrix element

$$U_{\text{HM}}^2 = \frac{2\Xi\Delta^4}{\pi^4 E_c^2}. \quad (41)$$

The constant  $\Xi$  is given by

$$\Xi = (\pi/L)^4 \sum_{q \neq 0} q^{-4} = \sum_{n_x+n_y+n_z > 0} (n_x^2+n_y^2+n_z^2)^{-2} \approx 5.$$

If one rewrites the contribution of the higher modes as  $U^2 \sim \Delta^2 g^{-2}$ ,  $g = E_c/\Delta$  being the conductance, it is easily seen that it has the same contribution as that of the Gaussian ensemble, except for the factor  $N^{-1}$  being replaced with  $g^{-2}$ . Consequently one obtains  $U_G^2/U_{\text{HM}}^2 \sim g^2/N$ .<sup>25</sup> In large sample,  $L \gg p_F l^2$ , we obtain  $g^2 \ll N$ , and consequently the contribution of higher modes is the leading one. In the opposite case  $L \ll p_F l^2$ , the Gaussian ensemble produces the leading term. The latter is universal, i.e., does not contain any information about the disorder. It is shown below that this contribution leads to the clean-limit result  $\gamma \gg T^2/\epsilon_F$ .

To summarize, we have obtained the following expression for the averaged matrix element of the Coulomb interaction:

$$U^2 \equiv |\langle \lambda, \lambda_2 | U | \lambda_1, \lambda_3 \rangle|^2 \\ = \begin{cases} (2\Xi/\pi^2) \Delta^4/E_c^2, & L \gg p_F l^2 \quad (g^2 \ll N), \\ \Delta^3/8\epsilon_F, & L \ll p_F l^2 \quad (g^2 \gg N). \end{cases} \quad (42)$$

## C. Uncorrelated case

Further calculations are different for the GCE and CE. We start from the most simple GCE case. In this situation the averaged sum over the three different states  $\lambda_1, \lambda_2$ , and  $\lambda_3$  in Eq. (25) is essentially the integral over three energies, corresponding to these states, multiplied by the normalizing factor  $\Delta^{-3}$ , and the four-point correlation function  $R_4$ , which is responsible for the level repulsion for small energy differences:

$$\gamma(\epsilon) = 2\pi U^2 \Delta^{-3} \int d\epsilon_1 d\epsilon_2 d\epsilon_3 f_2(1-f_1)(1-f_3) \\ \times \delta(\epsilon + \epsilon_2 - \epsilon_1 - \epsilon_3) R_4(\epsilon, \epsilon_1, \epsilon_2, \epsilon_3). \quad (43)$$

Now we set  $\epsilon=0$ . As was already mentioned, an analysis gives the same energy dependence  $\gamma(\epsilon, T=0)$  as the temperature one  $\gamma(\epsilon=0, T)$  obtained below, apart from the numerical coefficients.

The characteristic scales of the variation for the Fermi functions and the correlation function  $R_4$  are  $T$  and  $\Delta$ , respectively. So for  $T \gg \Delta$  the correlation function can be replaced with its asymptotic expressions for large values of arguments, i.e.,  $R_4=1$ .<sup>20</sup> In physical terms this means that the level correlation does not play any role for the electron-electron scattering rate provided  $T \gg \Delta$ . Note also that in this limiting case the GCE and CE situations coincide essentially, for the number of excited quasiparticles is large.

Direct calculation of the integral gives for  $T \gg \Delta$



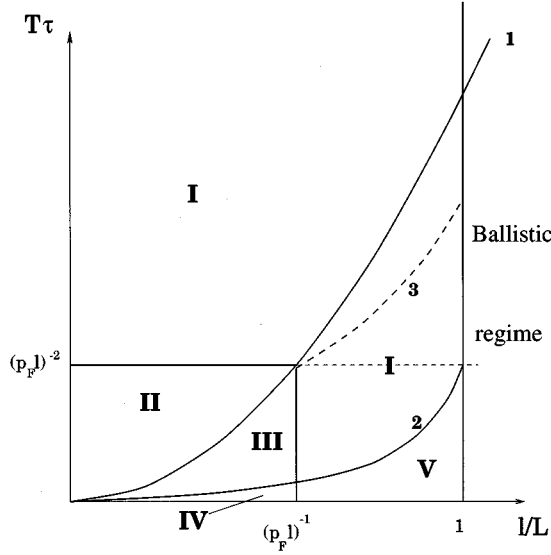


FIG. 4. 3D-0D crossover. Curves 1,  $T = E_c$ ; 2,  $T = \Delta$ . Line 3 ( $\gamma = \Delta$ ) separates the cases of continuous and discrete spectrum for  $L \gg p_F l^2$ . Regimes: I, clean limit;  $\gamma \sim T^2/\epsilon_F$ ; II, bulk diffusive limit,  $\gamma \propto T^{3/2}$ ; III, zero-dimensional diffusive limit; in ranges IV and V the level correlation is important.

$$\gamma_{uc} = \frac{\pi^3}{2} \frac{U^2 T^2}{\Delta^3} = \begin{cases} (\Xi/\pi)(T^2 \Delta/E_c^2), & L \gg p_F l^2 \quad (g^2 \ll N), \\ (\pi^3/16)(T^2/\epsilon_F), & L \ll p_F l^2 \quad (g^2 \gg N) \end{cases} \quad (44)$$

(Fig. 4). The appearance of the mean level spacing  $\Delta$  in the expression for the case where the level correlation is absent should not be misleading: It just stands for a combination  $(\nu L)^{-1}$ , and is introduced for convenience.

The upper line of Eq. (44) (range III in Fig. 4) corresponds to the case of ‘‘large’’ dots. The dependence  $\gamma \propto T^2$  is universal and is not sensitive to the geometry of the sample. However, the coefficient depends both on the sample’s size and the mean free path. It is valid for  $T \ll E_c$ , and gives in this parameter range  $\gamma \ll \Delta$ . Hence the spectrum is discrete and this result cannot be obtained in the perturbation theory. Note, however, that if one formally applies the perturbative, e.g., those developed in Ref. 8 or 18 methods to the range  $T \ll E_c$ , one obtains the same result  $\gamma \propto T^2 \Delta/E_c^2$ ,  $\gamma \ll \Delta$ . We have given above a *rigorous, self-consistent* derivation of this result.

The lower line of Eq. (44) is essentially the same result that appears in the clean limit for bulk 3D system. It does not contain either the size of the system or the diffusion properties (such as the mean free path). However, the range of validity for this result ( $L \ll p_F l^2, \Delta \ll T \ll E_c$ ) is rather different. This range includes both cases of discrete and continuous spectra, and consequently we have an overlap between the perturbation theory and the RMT calculation (region between curves 1 and 3, Fig. 4). Note that this ‘‘clean’’ behavior is observed in small enough dots even for small temperatures  $T\tau \ll (p_F l)^{-2}$  (cf. Ref. 18)

#### D. Effect of level correlation

For  $T \ll \Delta$  the correlation function can be replaced for its expansion for small arguments; in the particular case of the Gaussian unitary ensemble (GUE) one has (see Appendix B)

$$R_4(0, \omega, \omega + \Omega, \Omega)$$

$$= (\pi/\Delta)^{12} (212\,625)^{-1} (\omega^8 \Omega^4 - 2\omega^6 \Omega^6 + \omega^4 \Omega^8).$$

Straightforward calculation gives

$$\gamma_{\text{corr}} = \pi^4 \beta \frac{U^2 T^2}{\Delta^3} \left( \frac{\pi T}{\Delta} \right)^{12} = 2\pi \beta \left( \frac{\pi T}{\Delta} \right)^{12} \gamma_{uc},$$

$$\beta = \frac{59\pi^{11}}{94\,500} \sim 1000. \quad (45)$$

In particular, one obtains

$$\gamma_{\text{corr}} = \begin{cases} \beta \Xi (\Delta T^2/E_c^2) (\pi T/\Delta)^{12}, & L \gg p_F l^2 \quad (g^2 \ll N), \\ (\pi^4/8) (T^2/\epsilon_F) (\pi T/\Delta)^{12}, & L \ll p_F l^2 \quad (g^2 \gg N). \end{cases} \quad (46)$$

The 12th power in the result can be easily explained. One needs to find four energies close to each other; these four energies form six pairs, and the contribution of each pair is proportional to  $(\omega/\Delta)^2$  in the GUE,  $\omega$  being the difference between the energies in a pair. Consequently one obtains the extra factor  $(T/\Delta)^{12}$  in comparison with the uncorrelated case. In a similar way, the contribution from each pair is proportional to  $|\omega/\Delta|$  in the Gaussian orthogonal ensemble (GOE) and to  $(\omega/\Delta)^4$  in the Gaussian symplectic ensemble (GSE), and so the results are  $\gamma \propto \gamma_{uc} (T/\Delta)^6$  (GOE) and  $\gamma \propto \gamma_{uc} (T/\Delta)^{24}$  (GSE).

The CE case is more complicated. As we argue above, for  $T \gg \Delta$  it yields the same results as the GCE, and so the only interesting situation is  $T \ll \Delta$ . In expression (25) the functions  $f$  are still Fermi distribution functions; however, the chemical potential needs now to be pinned to one of the energy levels:  $\mu = \epsilon_l + 0$ . This fact changes all results considerably, as the probability to find  $\mu$  in some gap between levels depends no longer on the width of this gap.<sup>26,27</sup> The arbitrariness of the selection of the pinned level is removed by averaging with a weight function that is centered around the ‘‘mean value’’ of the chemical potential  $\bar{\mu}$  and has the support  $\delta\mu: \Delta \ll \delta\mu \ll \bar{\mu}$ . If this weight function is chosen to be a step function, after taking the limit  $\delta\mu \rightarrow 0$  one obtains

$$\gamma_\lambda = 2\pi U^2 \Delta \sum_{\lambda_1, \lambda_2, \lambda_3, l} \delta(\epsilon_\lambda + \epsilon_{\lambda_2} - \epsilon_{\lambda_1} - \epsilon_{\lambda_3}) \times \delta(\epsilon_l - \mu + 0) f_{\lambda_2} (1 - f_{\lambda_1}) (1 - f_{\lambda_3}). \quad (47)$$

In principle, the summation over  $l$  contains terms with  $l = \lambda_i$ . These particular terms are reduced after disorder averaging to the integrals over three energies with the four-point correlation function  $R_4$ . But upon setting  $\epsilon = 0$  these terms vanish just because of the correlation function, containing two equal energies ( $\epsilon_l = 0$ ). However, for nonzero  $\epsilon$  these terms yield the results:

$$\gamma(\epsilon, T) \sim \begin{cases} \gamma_{uc}(\epsilon/T)^4 (T/\Delta)^{12} & \text{(GUE),} \\ \gamma_{uc}(\epsilon/T)^2 (T/\Delta)^6 & \text{(GOE).} \end{cases} \quad (48)$$

All other terms contain the five-point correlation function and hence can be omitted. Formally for  $\epsilon = 0$  one obtains, e.g., in the GUE  $\gamma_{CE} \sim \gamma_{uc} (T/\Delta)^{20}$ .

## V. CONCLUSIONS

In conclusion, we have investigated the out-scattering time  $\gamma^{-1}$  appearing due to the electron-electron interaction. Two different cases should be distinguished: a continuous spectrum ( $\gamma \gg \Delta$ ), where perturbation theory can be applied, and the nonperturbative case of a discrete spectrum ( $\gamma \ll \Delta$ ).

In the case of a continuous spectrum the out-scattering time is essentially the same as the inelastic scattering time entering the problems of universal conductance fluctuations and persistent currents. In 3D and 0D systems it coincides also with the phase breaking time, while in the 2D and quasi-1D cases these times differ considerably. In the 2D case in some range of parameters we have recovered the earlier results,<sup>12-14</sup> but we interpret it as the out-scattering time rather than the phase-breaking time. Also an intermediate parameter range between 2D and 0D systems is investigated. For quasi-1D systems we have obtained principally the results (18)–(23). In the purely one-dimensional case for large enough temperatures  $T\tau \gg (p_F a)^2$  the out-scattering time is proportional to  $T^{-2/3}$  as well as  $\tau_\phi$ ; however, the former is considerably shorter. For lower temperatures  $(p_F a)^{-4} \ll T\tau \ll (p_F a)^{-2}$  the out-scattering rate is proportional to  $T^{3/4}$ , and becomes larger than the temperature. Also a transitional region between 1D and 0D systems exists, and we have investigated different regimes of the diffusion. In particular, for  $T \gg E_c$  the spectrum is always continuous, and close enough to  $E_c$  we obtain zero-dimensional behavior:  $\gamma \sim \tau_\phi \propto T$ . Ranges of parameters corresponding to different regimes are displayed in Fig. 3.

For temperatures below the Thouless energy the spectrum is discrete. In this case the out-scattering rate coincides with the phase-breaking one, and it is reasonable to speak just of the inelastic scattering rate. We have shown that for large enough systems  $L \gg p_F^2 l$  ( $N \sim \epsilon_F / \Delta \gg g^2$ ), the latter behaves itself as  $\gamma \sim T^2 \Delta / E_c^2$  ( $\Delta \ll T \ll E_c$ ), while in small systems  $N \ll g^2$  the universal dependence  $\gamma \sim T^2 / \epsilon_F$  is found. For  $T \ll \Delta$  we have obtained an abundance of results for small-large / CE-GCE / GUE-GOE cases. In spite of this abundance one should clearly understand that the inelastic scattering rate due to the electron-electron collisions is in this parameter range vanishingly small. So for real systems it is necessary to look for another mechanism of inelastic scattering. Electron-phonon scattering seems to be a good candidate; it has been recently discussed for mesoscopic systems.<sup>28</sup> Also a coupling to the environment produces an inelastic rate in the nearly closed system.

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*Note added.* As this paper was being prepared, a number of related works came to my attention. First, models similar to the “discrete spectrum” situation in quantum dots have

been recently studied with the help of the random band matrices, and the results are rather similar.<sup>29</sup> Then, Kamenev and Gefen<sup>30</sup> studied the role of the external environment on the inelastic broadening, and found this effect to be very strong. They relate this fact to the phenomena of the Coulomb blockade. Finally, the recent unpublished results by Altshuler, Gefen, Kamenev, and Levitov<sup>31</sup> (AGKL) and the comment on their work by Imry<sup>32</sup> are so close to the results obtained above that some discussion is required. AGKL study the same problem, and reproduce, in fact, the matrix element (41). They interpret it as an overlap between one-particle  $|\lambda\rangle$  and three-particle  $|\lambda_1, \lambda_2, \lambda_3\rangle$  states. Consequently this overlap should be compared with the three-particle level spacing  $\Delta_3 \sim \Delta^3 / \epsilon^2$ , and this comparison creates a new energy scale  $E^* = (E_c \Delta)^{1/2}$ . For  $E > E^*$  the three-particle states are well mixed by the Coulomb interaction, and the broadened peaks (which are resolved for  $E < E_c$ ) are essentially a mixture of many-particle states. On the other hand, for  $E < E^*$  the single-particle state is mixed with one three-particle state, one five-particle state, and so on. In this sense the inelastic scattering rate is zero: The state does not decay at all. AGKL describe this situation as an analog of the localization transition on the Bethe lattice. I do not want to address this problem here; however, the results obtained above for  $E < E^*$  can be interpreted rather as the width of the “envelope” formed by the many-particle states around the single-particle one. I am indebted to the authors of all these papers for the possibility to become acquainted with their results prior to publication.

## APPENDIX A: DEPHASING AND Q1D-0D CROSSOVER

Below results for the phase-breaking time in the range of parameters intermediate between the quasi-one- and zero-dimensional cases are obtained. In principle, one has to perform calculations similar to those of Ref. 10 in a restricted geometry, and this does not look quite hopeful. However, as we are interested in the result up to the numerical factor only, it is reasonable to use the method developed by Stern, Aharonov, and Imry<sup>33,34</sup> that was later applied to the calculation of the quasiparticle lifetime in a quantum dot.<sup>18</sup> In this approach the phase uncertainty  $P(t_0)$  accumulated by the electron due to the interaction with the environment (in our particular case it interacts with the electromagnetic fluctuations) is calculated; the time  $t_0$  when this phase uncertainty becomes of order unity (we will set it exactly unity) is associated with the electron lifetime (phase-breaking time). This phase uncertainty is given by<sup>18</sup>

$$P(t_0) = \frac{2}{L a^2} \int_0^{t_0} dt dt' \int_{-\infty}^{\infty} d\omega \coth \frac{\omega}{2T} \times \sum_{\mathbf{q} \neq 0} \frac{4e^2}{q^2} \text{Im} \left( \frac{1}{\epsilon(\mathbf{q}, \omega)} \right) e^{i\omega(t-t')} (S_1 + S_2 + S_3 + S_4), \quad (\text{A1})$$

with

$$S_1 = \exp\{i\mathbf{q}[x_1(t) - x_1(t')]\},$$

$$S_2 = \exp\{i\mathbf{q}[x_2(t) - x_2(t')]\},$$

$$S_3 = \exp\{i\mathbf{q}[x_1(t) - x_2(t')]\},$$

$$S_4 = \exp\{i\mathbf{q}[x_2(t) - x_1(t')]\}. \quad (\text{A2})$$

In these expressions (to be the direct generalization of those of Ref. 18 for the case of finite temperatures)  $x_1(t)$  and  $x_2(t)$  are arbitrary (quasiclassical) electron paths, and the averaging over these paths is supposed;  $\varepsilon(\mathbf{q}, \omega)$  is the dielectric susceptibility. Expression (A1) is derived for an infinite system; however, as it varies on scales of the order of the elastic mean free path  $l \ll a$ , this expression is valid in the diffusive regime, and the bulk results may be substituted for the dielectric susceptibility. It is also important that the  $\mathbf{q} = 0$  mode does not contribute to the sum (A1): It is absent in the open systems while in the closed ones the corresponding contribution is forbidden by charge neutrality. So the difference between open and closed systems is not quite important, and the system is assumed to be closed:  $\mathbf{q} = \pi(n_x/L, n_y/a, n_z/a)$ ,  $n_x + n_y + n_z > 0$ . The results for the open systems depend on boundary conditions, but in all cases differ by a numerical factor of order unity only.

After the averaging over paths is carried out in exactly the same way as in Ref. 18, one obtains

$$P(t_0) = \frac{48T}{La^2 \pi p_F^2 l} \frac{D}{E_c} \sum_{n_x + n_y + n_z > 0} \frac{1}{n_x^2 + (L/a)^2(n_y^2 + n_z^2)}$$

$$\times \int_0^{t_0} dt^+ \int_0^{t^+} \frac{dt}{t} \sin(2Tt)$$

$$\times \exp\{-\pi^2 E_c t [n_x^2 + (L/a)^2(n_y^2 + n_z^2)]\}. \quad (\text{A3})$$

Here  $D$  is the three-dimensional diffusion coefficient and we have taken into account that the main contribution to the integral over frequencies comes from the region  $|\omega| < 2T$ . The summation is restricted by the condition  $q \ll l$ . For  $D/a^2 \gg T$  (this condition excludes the 3D situation) only the values  $n_y = n_z = 0$  are important in the summation (A3).

In the limiting case  $E_c \gg t_0^{-1}$  [the inverse situation corresponds to the ‘‘true’’ quasi-1D case and the result is given by Eq. (13)] the integral can be easily calculated. One gets

$$P(t_0) = \frac{48T}{\pi p_F^2 l La^2} \frac{D}{E_c} \sum_{n=1}^{\sim L/l} \frac{1}{n^2} \left[ t_0 \arctan \frac{2T}{\pi^2 E_c n^2} - \frac{2T}{4T^2 + \pi^2 E_c^2 n^2} \right]. \quad (\text{A4})$$

The phase-breaking time  $\tau_\phi$  is defined as the time  $t_0$  when the phase uncertainty  $P(t_0)$  is equal to unity.

The case  $E_c \gg T$  corresponds to the ‘‘true’’ OD situation. One has

$$\frac{1}{\tau_\phi} = \frac{96T}{\pi^3 p_F^2 l La^2} \frac{D}{E_c^2} \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{5\pi T^2 \tau}{16p_F^2 a^2} \left(\frac{L}{l}\right)^3. \quad (\text{A5})$$

This is the result by Sivan, Imry, and Aronov<sup>18</sup> up to the numerical factor. It is seen, however, that in this region the spectrum is discrete:  $\tau_\phi^{-1} \ll \Delta$ . Thus, the OD region is not subject to our analysis and should be treated by another methods. [See, however, the discussion after Eq. (44)].

The case  $\tau_\phi^{-1} \ll E_c \ll T$  is intermediate between quasi-1D and OD regimes. Only terms with  $E_c n^2 \ll T$  are important in the summation (A4); however, due to the condition  $T \gg E_c$ , the summation can be extended to the infinity. One obtains

$$\frac{1}{\tau_\phi} = \frac{4\pi^2 TL}{(p_F a)^2 l}, \quad (\text{A6})$$

which is a new result. In the whole region  $\tau_\phi \ll E_c \ll T$  the spectrum turns out to be continuous and the result (A6) is valid.

## APPENDIX B: FOUR-POINT CORRELATION FUNCTION IN THE GUE

Below we follow the generalities given in Ref. 20. As an explicit expression for the four-point correlation is not given anywhere in the literature to the best of our knowledge, we derive it for the most simple GUE case. It is convenient to use the dimensionless energies  $x = \pi \epsilon / \Delta$  throughout this appendix.

The first step is to define the functions

$$Y_i(x_1, x_2, \dots, x_i) = \sum_P s_{12} s_{23} \dots s_{i1}. \quad (\text{B1})$$

Here

$$s_{ij} \equiv s(|x_i - x_j|), \quad s(x) \equiv \frac{\sin x}{x}, \quad (\text{B2})$$

and the summation is carried out over  $(i-1)!$  different permutations of indices. Hence,

$$Y_1(x) = 1, \quad Y_2(x_1, x_2) = s_{12}^2, \quad Y_3(x_1, x_2, x_3) = 2s_{12}s_{23}s_{31},$$

$$Y_4(x_1, x_2, x_3, x_4) = 2s_{12}s_{23}s_{34}s_{41} + 2s_{13}s_{34}s_{42}s_{21}$$

$$+ 2s_{14}s_{42}s_{23}s_{31}.$$

Now the correlation functions  $R_i(x_1, \dots, x_i)$  can be expressed as

$$R_1 = 1, \quad R_2(x_1, x_2) = -Y_2(x_1, x_2) + R_1(x_1)R_2(x_2),$$

$$R_3(x_1, x_2, x_3) = Y_3(x_1, x_2, x_3) + R_1(x_1)R_2(x_2, x_3) + R_1(x_2)R_2(x_1, x_3)$$

$$+ R_1(x_3)R_2(x_1, x_2) - 2R_1(x_1)R_1(x_2)R_1(x_3),$$

$$\begin{aligned}
R_4(x_1, x_2, x_3, x_4) = & -Y_4(x_1, x_2, x_3, x_4) + \{R_1(x_1)R_3(x_2, x_3, x_4) + R_1(x_2)R_3(x_1, x_3, x_4) \\
& + R_1(x_3)R_3(x_1, x_2, x_4) + R_1(x_4)R_3(x_1, x_2, x_3)\} + \{R_2(x_1, x_2)R_2(x_3, x_4) + R_2(x_1, x_3)R_2(x_2, x_4) \\
& + R_2(x_1, x_4)R_2(x_2, x_3)\} - 2\{R_2(x_1, x_2)R_1(x_3)R_1(x_4) + R_2(x_1, x_3)R_1(x_2)R_1(x_4) \\
& + R_2(x_1, x_4)R_1(x_2)R_1(x_3) + R_2(x_2, x_3)R_1(x_1)R_1(x_4) + R_2(x_2, x_4)R_1(x_1)R_1(x_3) \\
& + R_2(x_3, x_4)R_1(x_1)R_1(x_2)\} + 6R_1(x_1)R_1(x_2)R_1(x_3)R_1(x_4).
\end{aligned}$$

After some algebra one obtains an explicit expression for the correlation function  $R_4$ :

$$\begin{aligned}
R_4(x_1, x_2, x_3, x_4) = & 1 - 2\{s_{12}s_{23}s_{34}s_{41} + s_{13}s_{34}s_{42}s_{21} + s_{14}s_{42}s_{23}s_{31}\} \\
& + \{s_{12}^2s_{34}^2 + s_{13}^2s_{24}^2 + s_{14}^2s_{23}^2\} + 2\{s_{12}s_{23}s_{31} + s_{12}s_{24}s_{41} + s_{13}s_{34}s_{41} + s_{23}s_{34}s_{42}\} \\
& - \{s_{12}^2 + s_{13}^2 + s_{14}^2 + s_{23}^2 + s_{24}^2 + s_{34}^2\}.
\end{aligned} \tag{B3}$$

In our particular case  $x_1 + x_3 = x_2 + x_4$ , and, taking into account that the correlation function depends on three differences of arguments only, one arrives to an expression  $R_4(0, x, x + y, y)$ . From Eq. (B3) after cumbersome calculations one obtains an expansion of  $R_4$  for  $x, y \ll 1$ :

$$R_4(0, x, x + y, y) = \frac{1}{212\,625}(x^4y^8 - 2x^6y^6 + x^8y^4). \tag{B4}$$

The 12th power can be easily explained. The pair correlation function for small arguments is proportional in the GUE,  $R_2(x) \propto x^2$ . As one has in  $R_4$  six pairs of arguments close to each other and each pair produces the second power, the total expansion starts from the 12th power. In the GOE,  $R_2(x) \propto x$ , and we may easily conclude that an analogous expansion starts from the sixth power.

As could be expected, for a large value of the arguments  $x, y \gg 1$  all functions  $S_{ij}$  are small, and  $R_4 = 1$ . This fact means just that the levels are uncorrelated on distances exceeding the mean level spacing  $\Delta$ .

\*On leave from: Department of Theoretical Physics, Moscow Institute for Steel and Alloys, Leninskii Pr. 4, 117936 Moscow, Russia.

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