Localization of light for dissipative and disordered one-dimensional systems

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The light transmission has been measured for a disordered binary multilayer system composed of cryolite and dissipative antimonite layers. The exponential decay of light transmission versus multilayer thickness is observed. The measured localization lengths l_c, coincide quite well with the calculated results. We find both experimentally and theoretically that the effects of absorption and disorder on the light transmission can be expressed by $l_c^{-1}=l_d^{-1}+l_a^{-1}$ where l_d is the localization length for a corresponding disordered multilayer in the absence of absorption and l_a the characteristic absorption length of layers. [S0163-1829(96)09534-3]

The light localization effect in a one-dimensional disordered system has received increasing attention in recent years. Sheng *et al.* have demonstrated theoretically that disordered systems in one dimension always exhibit Anderson localization.[1] This means that the exponential variation of light transmission T with the length of the disordered system L holds for all frequencies. The localization length l_c is defined as $l_c=-L/\langle \ln T\rangle$, where $\langle\rangle$ represents an average over the statistical distribution of the disordered system. The localization lengths have also been calculated for different disordered multilayers and the effects of the system's absorption and disorder on the light transmission discussed theoretically.[2–8] Experimentally, Nitta *et al.* observed an anomalous reflectance effect for a disordered binary (a-Si:H and a-Si_xN_{1-x}:H) multilayer and suggested that the effect could be related to disorder-induced light localization.[9] We have observed directly the localization effect of a binary disordered multilayer composed of cryolite ($CaCO_3$) and antimonite (Sb_2S_3) in the wavelength region of 720–900 nm.[10] Based on light localization in a one-dimensional disorder system we developed a broadband optical reflector, which possesses a much wider high reflection range than that of an ordinary $\lambda/4$ stack.[11] For this kind of reflector more layers of coating materials are required, so the absorption of the coating materials, even if small, can play an important role. Therefore, the phenomenon of light propagation through a disordered multilayer with absorption must be fully understood. However, there has not been any experimental study reported on this subject, though many interesting theoretical results have been presented. In addition, because of its potential application in photonics the effect induced by light in a disordered layered gain medium has recently attracted more attention.[12,13] The information obtained from the disordered dissipative multilayer might be useful for understanding the mechanism of the effect appearing in disordered layered gain media, which is very hard to realize practically. In this paper we describe the experiment and simulation of light localization for binary multilayers with both disorder and dissipation. Our results show that the light is still localized in such a dissipative and disordered medium and the relationship between localization length and absorption length is rather simple.

The main features of our multilayers are briefly described as follows. The multilayer consists of both cryolite and antimonite layers and is made by a conventional vacuum coating procedure. In the spectrum range below 650 nm the antimonite layers are dissipative. The disorder of the binary multilayer is produced by choosing the thickness of the antimonite layers randomly around an average value of $\langle b\rangle=d/4n_b=57$ nm, while the cryolite layers all have an identical thickness of $\langle a\rangle=d/4n_a=122$ nm. Here $n_b=2.83$ and $n_a=1.33$ are the corresponding real parts of the refractive indices and $d=650$ nm. The degree of the system's disorder is defined as

$$D=\sqrt{\sum_i (b_i-\langle b\rangle)^2/M}\Big/ l_p,$$

where b_i, M, and $l_p=\langle a\rangle+\langle b\rangle$ are the real thicknesses, total number of antimonite layers and the average multilayer period, respectively. Three types of disordered multilayers with the same $D=0.16$ and $l_p=179$ nm, but different periods $M=10$, 15, and 21 are prepared. The substrates used are the optical glasses. The designed thicknesses of the multilayers and their distribution will be given elsewhere.

The light transmission is measured at fourteen wavelengths by using two laser sources. Eight lines between 457 and 515 nm are produced from an Ar^+ laser and six wavelengths between 577 and 617 nm from an Ar^+ laser pumped Rhodamine 6G dye laser. Their linearly polarized outputs are adjusted to be 50–100 mW in the TEM_{00} mode. Due to both the absorption and disorder existing in the multilayer system the ratio of transmitted light intensity to that of input may be exceedingly small, such as 10^{-11}, so special care must be taken in the experimental setup. The laser beam goes first through a lock-in amplifier chopper with a chopping rate of 1 kHz and an aperture of 2 mm diameter, which blocks any stray radiation from the laser, and is incident normal to the multilayer, then after transmission passes through another three small apertures of 2 mm diameter which further block all scattered light caused by optical elements and holders. Finally, the beam enters a model R943-02 low noise photomultiplier. The signals are received and processed by a model SR510 lock-in amplifier. In this arrangement the detectable ratio of transmitted intensity to input can be better than 10^{-12}. The measurements are carried out for three types of multilayer at the fourteen wavelengths given above. To make experimental error as small as possible and to elimi-

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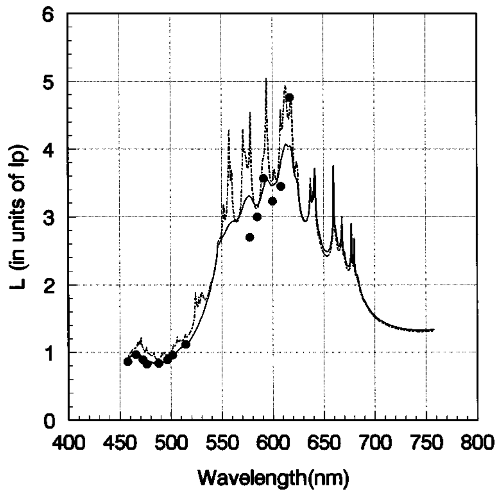

FIG. 1. The localization lengths calculated two ways. (a) solid line l_c, derived using the complex refractive index and statistical averaging over five different sequences; (b) dashed line: l_t, derived from $l_t^{-1}=l_d^{-1}+l_a^{-1}$ where l_a is the characteristic absorption length of the layers. •: measured values.

nate the influence coming from the inhomogeneity of layer thickness the measurement is repeated several times by changing the position of incidence on the multilayer. The experimental results show that the exponential variation of T^{-1} with L holds for most of the wavelengths. In the region of 457–515 nm the exponential relation is very well satisfied, but the marked deviations from the exponential relation are observed at two wavelength around 600 nm. The reason for this will be discussed later. Furthermore, based on this relationship the localization length l_c is determined by linear regression.

To compare the measurement with calculation one must know both real and imaginary parts of the refractive index of the antimonite layer, while those of cryolite are well known. A previous experiment has pointed out that the real part of the refractive index for antimonite film is almost unchanged in the visible spectrum, while the imaginary parts have been measured only at four discrete wavelengths, namely 436, 546, 578, and 640 nm.[14] In addition, the values given in Ref. 14 appear to be too small for our multilayer system, specifically in the shorter wavelength region. This situation is reasonable, because for a thin film the imaginary part of the refractive index may be sensitive to the purity of the material, the coating procedure and even to the film thickness.[15] Thus, by fitting our measured transmitted intensities for the multilayer with 21 periods to the calculated results of the transfer matrix method we obtain the imaginary parts of the index for the wavelengths between 457 and 545 nm. However, at wavelengths longer than 545 nm, i.e., in the weaker absorption region, the values in Ref. 14 are adopted to determine the imaginary part between 546 and 640 nm by a linear fitting.

In Fig. 1 the dots and solid line show, respectively, the measured localization lengths at 14 wavelengths and the corresponding calculated curve, which is computed by the transfer matrix approach with a complex refractive index and averaging over five sequences. It is found that in the blue-green spectrum the coincidence of calculation with measurement is very good, but between 570–620 nm there are some small differences. This disagreement may be attributed to the longer real localization length, $l_c=3$ to $4l_p$, in this region, compared with $l_c<1$ at the shorter wavelengths (see Fig. 1). In principle, a reliable value of l_c can be reached only if the statistical average over different sequences of the disorder multilayer is considered.[2] In practice, however, l_c is measured just in a particular layer sequence. As noted by Frigerio *et al.* in Ref. 4, the measured l_c for a thick multilayer (with many periods) is equivalent to the averaged l_c for a thin multilayer. This implies that the measured l_c will be very close to the real value, if the total length $L \gg l_c$. On the contrary, if L is not much longer than l_c the error might appear. In our case the longest L is only 21 l_p, which is not long enough compared with $l_c=3l_p$ in the region of 570–620 nm, so that the measured values may be slightly different from the real values. Secondly, due to the disorder the oscillation of l_c is rather serious around 600 nm, so a small deviation of the layer's actual thickness from the designed value may cause a considerable variation of l_c.

We now consider the case when both the absorption and disorder are present in the multilayer. As stated in the corresponding theoretical works,[2–8] the total exponential decay coefficient of light l_t^{-1} might be expressed by a simple superposition of two separate exponential decay coefficients, i.e., $l_t^{-1}=l_1^{-1}+l_2^{-1}$ Here l_1 origins only from the absorption and l_2 from the disorder. We first assume that l_1 represents the localization length of the corresponding regular dissipative multilayer, l_r, while l_2 is that for the multilayer with the same disorder, but without dissipation, i.e., $l_2=l_d$. The dependencies of transmittance on the wavelength for regular dissipative multilayers (T_r) and disordered lossless multilayer (T_d) with different periods of $M=10$, 15, and 21 are respectively calculated in terms of the transfer matrix method. The l_r and l_d are determined by linear regression in the plots of $T_{r(d)}^{-1}$–L. For a regular multilayer, rigorously speaking, the exponential relationship between T_r^{-1} and L holds only within the band gap while it fails outside the gap. However, in our case the absorption of antimonite is rather big in the wavelength shorter than 530 nm, so that T_r^{-1} is dominated by the absorption. Thus, a roughly exponential relationship may still be obtained. The comparison of experimental values of the calculated curves l_t, l_r, and l_d is given in Fig. 2. Here l_d is computed for an average over five different sequences, so that a more reliable theoretical value may be obtained. One finds from Fig. 2 that in the shorter wavelength region the curve of l_t almost overlaps with the measured one. However, in the region of 570 to 620 nm the difference between them is relatively great. This may be understood by considering the existence of a band gap for a regular binary multilayer. The dispersion relation of light for our corresponding regular multilayer is derived to be

$$2\pi l_p/\lambda = 4 \text{ arc cos}[0.435 \cos(0.276 l_p K)+0.565]^{1/2}.$$

In the first Brillouin zone, where the Bloch vector K varies from 0 to π/l_p; the photonic gap should appear at a wavelength between 526 and 849 nm. Within the gap perfect phase coherent Bragg reflection from a regular multilayer may result in a very short attenuation length. As long as the absorption of the multilayer is not very strong, according to $l_t^{-1}=l_r^{-1}+l_d^{-1}$ the total localization length is dominated by

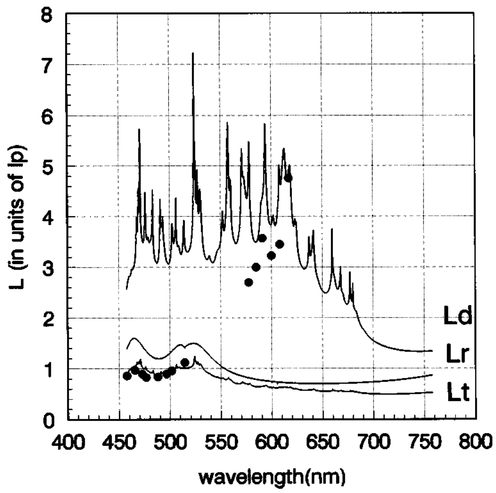

FIG. 2. The comparison of experimental localization lengths (•) with calculated l_t, $l_t^{-1}=l_d^{-1}+l_r^{-1}$ Here l_d is the localization length for a corresponding disordered multilayer in the absence of absorption and l_r that for a regular quarter-wave stack in the presence of absorption.

the Bragg reflection. Therefore, once the disorder of the multilayer destroys the perfect Bragg coherence, i.e., inside the gap the localization length becomes longer, the expression $l_t^{-1}=l_r^{-1}+l_d^{-1}$ appears to be no longer valid. To avoid the effect of perfect Bragg reflection, which does not play a big role for a disordered multilayer, and to retain the effect of absorption, l_r should be replaced by l_a, the characteristic absorption length of antimonite layers. The absorption length is directly connected to n', the imaginary part of the refractive index, by $l_a^{-1}=4\pi n'(\lambda)/\lambda$. Thus we have $l_t^{-1}=l_a^{-1}+l_d^{-1}$ The calculated results are also shown in Fig. 1 as the dashed curve. The agreement among l_c, l_t and experimental values is well established both inside and outside the gap. If more sequences of the multilayer are averaged in the calculation of l_d, the peaks appeared between 550 and 600 nm for the dashed curve (l_t) will be smoothed. Based on Figs. 1 and 2 and the above discussion it is confirmed that the total localization length can be decided by the localization length of the disordered multilayer in the absence of absorption and the characteristic absorption length of the dissipative layer. Our conclusions are coincident with the theoretical prediction given recently by Zhang in Ref. 7.

In addition, from Fig. 2 one may see that both measured and calculated localization length for a disordered multilayer with $D=0.16$ are longer than that of a regular dissipative one in the gap, i.e., $l_d>l_r$. However, the absorption is very small in $\lambda>550$ nm and it is null when $\lambda>640$ nm. So, this means that the transmittance increases if disorder exists. That behavior may be an experimental evidence of the theoretical results presented by Freilikher *et al.*[16]

In summary, we have measured the transmission of light for the dissipative disordered multilayers in the visible spectrum and found an exponential relationship between transmission and the multilayer thickness. The calculated localization lengths fit the experimental values very well. From both the experiment and computation it is found that the effects caused due to both the absorption and disorder can be simply expressed by $l_c^{-1}=l_d^{-1}+l_a^{-1}$

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