# Spontaneous interlayer coherence in double-layer quantum Hall systems: Symmetry-breaking interactions, in-plane fields, and phase solitons

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At strong magnetic fields double-layer two-dimensional–electron-gas systems can form an unusual broken symmetry state with spontaneous interlayer phase coherence. The system can be mapped to an equivalent system of pseudospin 1/2 particles with pseudospin-dependent interactions and easy-plane magnetic order. In this paper we discuss how the presence of a weak interlayer tunneling term alters the properties of double-layer systems when the broken symmetry is present. We use the energy functional and equations of motion derived earlier to evaluate the zero-temperature response functions of the double-layer system and use our results to discuss analogies between this system and Josephson-coupled superconducting films. We also present a qualitative picture of the low-energy charged excitations of this system. We show that parallel fields induce a highly collective phase transition to an incommensurate state with broken translational symmetry. [S0163-1829(96)08239-2]

## I. INTRODUCTION

The study of correlated electron systems in fewer than three dimensions continues to be an important theme in condensed matter physics. In particular, the study of strongly correlated two-dimensional (2D) electron systems on a lattice has been motivated by high-temperature superconductivity, while the study of strong correlations in continuum two-dimensional systems has been motivated by the fractional quantum Hall effect.[1] Properties of high-temperature superconductors are thought by some to be strongly influenced by the weak coupling which exists between the superconducting planes. In the fractional quantum Hall effect, early work by Halperin[2] anticipated fractional quantum Hall effects due to interlayer correlations[3] in multilayer systems. Recent technological progress has made it possible to produce double-layer two-dimensional–electron-gas systems of extremely high mobility in which these effects can be observed. The two electron layers can either be bound in separate quantum wells[4] as illustrated schematically in Fig. 1 or bound to opposite edges of a single wide quantum well.[5] In both cases the 2D electron gases are separated by a distance $d$ small enough ($d \sim 100$ Å) to be comparable to the typical spacing between electrons in the same layer.

In a large magnetic field, strong Coulomb correlations between the layers have long[2] been expected to lead to fractional quantum Hall effects. Correlations are especially important in the strong magnetic field regime because all electrons can be accommodated within the lowest Landau level and execute cyclotron orbits with a degenerate kinetic energy. The fractional quantum Hall effect occurs when the system has a gap for making charged excitations, i.e., when the system is incompressible. Theory has predicted[2,3,6] that at some Landau level filling factors, gaps occur in double-layer systems only if interlayer interactions are sufficiently strong. These theoretical predictions have been confirmed experimentally.[7] More recently, theoretical work from several different points of view[8–16] has suggested that interlayer correlations can also lead to unusual broken symmetry states with spontaneous phase coherence between layers which are isolated from each other (except for interlayer Coulomb interactions). We have argued[13] that it is spontaneous interlayer phase coherence which is responsible for the recently discovered[4] extreme sensitivity of the fractional quantum

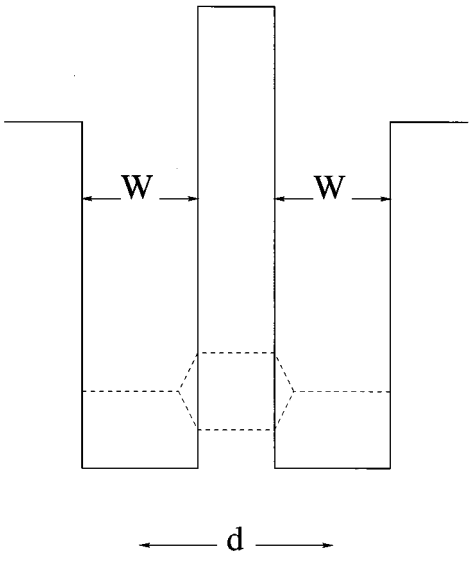


FIG. 1. Schematic conduction-band-edge profile for a double-layer two-dimensional–electron-gas system.

 

Hall effect at total Landau level filling factor $\nu=1$ to small tilts of the magnetic field away from the normal to the layers. ($\nu \equiv N/N_\phi$, where $N$ is the number of electrons and $N_\phi$ is the number of single-particle levels per Landau level.) In a previous lengthy paper[14] (hereafter referred to as I) we have developed a rather complete description of the physics associated with spontaneous interlayer phase coherence in the case where there is no tunneling between the layers. The present companion paper will analyze the case of finite tunneling between the layers and the role of magnetic field tilt. Many of the ideas presented in detail here are discussed qualitatively in Ref. 17.

## II. EXPERIMENTAL BACKGROUND

In this section we review the experimental[18] indications that the system is spontaneously ordered and exhibits excitations which are highly collective in nature. We focus here and throughout this paper on the case of Landau level filling factor $\nu=1$ (that is, 1/2 in each layer) and assume for simplicity that the electronic spin degrees of freedom are frozen out by the Zeeman energy. The schematic energy level diagram for the growth-direction degree of freedom in the double-layer system is shown in Fig. 1 for the case of noninteracting electrons. For simplicity we assume that electrons can occupy only the lowest electric subband in each quantum well. If the barrier between the wells is not too strong, tunneling from one side to the other is allowed. The lowest energy eigenstates split into symmetric and antisymmetric combinations separated by an energy gap $\Delta_{\rm SAS}$ which can, depending on the sample, vary from essentially zero to many hundreds of Kelvins. The splitting can therefore be much less than or greater than the interlayer interaction energy scale, $E_c \equiv e^2/\epsilon d$.

When the layers are widely separated, there will be no correlations between them and we expect no dissipationless quantum Hall state, since each layer has[19] $\nu=1/2$. For smaller separations, it is observed experimentally that there is an excitation gap and a quantized Hall plateau.[20,5,4] This has either a trivial or a highly nontrivial explanation, depending on the ratio $\Delta_{\rm SAS}/E_c$. For large $\Delta_{\rm SAS}$ the electrons tunnel back and forth so rapidly that it is as if there is only a single quantum well. The tunnel splitting $\Delta_{\rm SAS}$ is then analogous to the electric subband splitting in a (wide) single well. All symmetric states are occupied and all antisymmetric states are empty and we simply have the ordinary $\nu=1$ integer Hall effect. Correlations are irrelevant in this limit and the excitation gap is close to the single-particle gap $\Delta_{\rm SAS}$ (or $\hbar\omega_c$, whichever is smaller). What is highly nontrivial about this system is the fact that the $\nu=1$ quantum Hall plateau survives even when the tunnel splitting becomes arbitrarily small: $\Delta_{\rm SAS} \ll E_c$. In this limit the excitation gap has clearly changed to become highly collective in nature since the observed[20,5,4] gap can be on the scale of 20 K even when $\Delta_{\rm SAS} \sim 1$ K. As we will see below, because of a spontaneous broken symmetry,[8,9,12,13] the excitation gap actually survives the limit $\Delta_{\rm SAS} \to 0$, as illustrated in Fig. 2. This crossover from single-particle to collective gap is, as we will show, quite analogous to the result that for spin polarized single layers, the excitation gap survives the limit of zero Landé $g$ factor and hence ''$\nu=1$ is a fraction too.''[21]

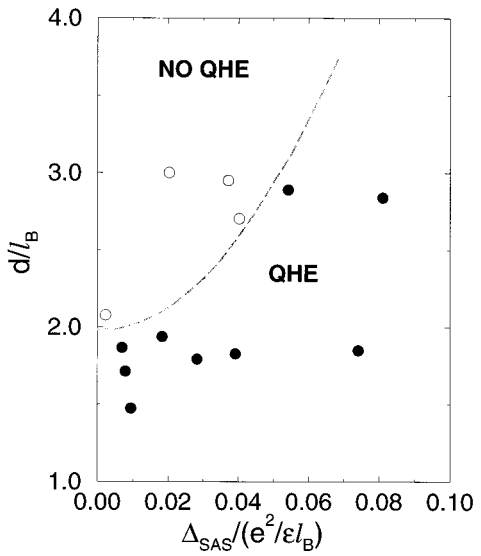


FIG. 2. Phase diagram for the double-layer QHE system [after Murphy *et al.* (Ref. 4)]. Only samples whose parameters lie below the dashed line exhibit a quantized Hall plateau and excitation gap.

A second indication of the highly collective nature of the excitations can be seen in the Arrhenius plots showing thermally activated dissipation.[4] The low temperature activation energy $\Delta$ is, as already noted, much larger than $\Delta_{\rm SAS}$. If $\Delta$ were nevertheless somehow a single-particle gap, one would expect the Arrhenius law to be valid up to temperatures of order $\Delta$. Instead one observes a rather abrupt leveling off in the dissipation as the temperature increases past values as low as $\sim 0.1\Delta$. This effect is observed both in double-well systems and wide single-well systems and is consistent with the notion of a thermally induced collapse of the order that had been producing the collective gap.

The third significant feature of the experimental data pointing to a highly ordered collective state is the strong response of the system to relatively weak magnetic fields $B_\parallel$ applied in the plane of the 2D electron gases. Within a model that neglects higher electric subbands, we can treat the electron gases as strictly two dimensional. (There is ample evidence[22] that parallel field effects due to subband mixing within a single quantum well produce only small, albeit measurable, effects.) $B_\parallel$ can then affect the system only if there are processes involving tunneling that carry electrons around closed loops containing flux. A prototypical such process is illustrated in Fig. 3. An electron tunnels from one layer to the

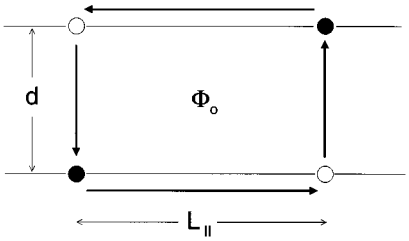


FIG. 3. A process in double-layer two-dimensional–electron-gas systems which encloses flux from the parallel component of the magnetic field. The quantum amplitude for such paths is sensitive to the parallel component of the field.

other at point $A$, and travels to point $B$. Then it (or another indistinguishable electron) tunnels back and returns to the starting point. The parallel field contributes to the quantum amplitude for this process (in the 2D gas limit) a gauge-invariant Aharonov-Bohm phase factor $\exp(2\pi i\Phi/\Phi_0)$ where $\Phi$ is the enclosed flux and $\Phi_0$ is the quantum of flux. Such loop paths evidently contribute significantly to correlations in the system since the activation energy gap is observed to decrease very rapidly with $B_\parallel$, falling by factors of order 2 – 10 (depending on the sample) until a critical field, $B_\parallel^* \sim 0.8$ T, is reached at which point the gap essentially ceases changing. To understand how remarkably small $B_\parallel^*$ is, consider the following. We can define a length $\xi_\parallel$ from the size of the loop needed to enclose one quantum of flux: $\xi_\parallel B_\parallel^* d = \Phi_0$. ($\xi_\parallel[\text{Å}] = 4.137\times 10^5/d[\text{Å}]B_\parallel^*[T]$.) For $B_\parallel^* = 0.8$ T and $d = 150$ Å, $\xi_\parallel = 0.27$ $\mu$m, which is approximately 20 times the spacing between electrons in a given layer and 30 times larger than the quantized cyclotron orbit radius $\ell \equiv (\hbar c/eB_\perp)^{1/2}$ within an individual layer. Significant drops in the excitation gap are already seen at fields of 0.1 T, implying that enormous phase coherent correlation lengths must exist. Again this shows the highly collective nature of the ordering in this system associated with spontaneous interlayer phase coherence.

## III. SPONTANEOUS PHASE COHERENCE AND SYMMETRY BREAKING BY TUNNELING

The essential physics of spontaneous interlayer phase coherence can be addressed either from a microscopic point of view[10–14] or from a macroscopic Chern-Simons field theory point of view,[8,9,14] but it is perhaps most easily explained in terms of the simple variational wave function

$$|\psi\rangle = \prod_X \{c_{X\uparrow}^\dagger + e^{i\varphi} c_{X\downarrow}^\dagger\}|0\rangle, \tag{3.1}$$

where $X$ is a state label (for instance, the Landau gauge orbital guiding center[14]) and we are using a pseudospin notation in which spin up refers to electrons in the upper layer and spin down refers to electrons in the lower layer.[14] The interpretation of this wave function is that every Landau orbital $X$ is occupied (hence $\nu = 1$), but the system is in a coherent linear combination of pseudospin up and down states determined by the phase angle $\varphi$. This means that the system has a definite total number of particles ($\nu = 1$ exactly) but an indefinite number of particles in each layer. In the absence of interlayer tunneling, the particle number in each layer is a good quantum number. Hence this state has a spontaneously broken symmetry[11,8,9,14] in the same sense that the BCS state for a superconductor has indefinite (total) particle number but a definite phase relationship between states of different particle number.

In the absence of tunneling, the energy cannot depend on the phase angle $\varphi$ and the system exhibits a global U(1) symmetry associated with conservation of particle number in each layer. One can imagine allowing $\varphi$ to vary slowly with position to produce excited states. Because of the U(1) symmetry, the effective Hartree-Fock energy functional for these states has a gradient expansion whose leading term must have the form

$$H = \frac{1}{2}\rho_s \int d^2\mathbf{r}|\boldsymbol{\nabla}\varphi|^2 + \cdots. \tag{3.2}$$

The origin of the finite ''spin stiffness'' $\rho_s$ is the loss of exchange energy which occurs when $\varphi$ varies with position. Imagine that two particles approach each other. They are in a linear superposition of states in each of the layers (even though there is no tunneling). If they are characterized by the same phase $\varphi$, then the wave function is symmetric under pseudospin exchange and so the spatial wave function is antisymmetric and must vanish as the particles approach each other. *States with spontaneous phase coherence have better interlayer electronic correlations and hence lower interlayer Coulomb interaction energy.* If a phase gradient exists then there is a larger amplitude for the particles in opposite layers to be near each other and hence the interlayer interaction energy is higher.[23] This loss of exchange energy is the source of the finite spin stiffness and is what causes the system to spontaneously ''magnetize.''

A skeptical reader might legitimately worry that the single-Slater-determinant variational wave functions in terms of which we have framed the above discussion misrepresent the physics. Indeed, the ground state of double-layer systems in the Hartree-Fock approximation has spontaneous phase coherence at all values of $\nu_T$ whereas we believe that this property actually holds only at a discrete set of total filling factors, including $\nu_T = 1$. To understand why these concerns would be misplaced, it is useful to briefly review some of the microscopic physics underlying the incompressible states whose occurrence is responsible for the fractional quantum Hall effect. For a pair of interacting electrons confined to the lowest Landau level, only one relative motion state is available for each relative angular momentum. This property means that the interaction is characterized by a discrete set of energy scales, $V_l$, first identified by Haldane[24] and known as Haldane pseudopotentials. $V_l$ is the interaction energy of a pair of electrons with relative angular momentum $l$; in the double-layer case, the pseudopotential values for interlayer and intralayer interactions will differ. Many of the largest charge gaps which occur in the fractional quantum Hall regime can be understood in terms of Haldane pseudopotentials. For example, for double-layer systems with nearby layers the largest energy scales should be the interlayer and intralayer $l = 0$ pseudopotentials. It is easy to show that it is possible to form many-body states in which pairs of particles never have $l = 0$ for $\nu_T \leqslant 1$. This is not possible for $\nu_T > 1$. Thus if $\nu_T = 1$, the energy to add a charge to the system is greater than the energy to remove one, hence the charge gap. For $\nu_T = 1$ it is easy[25] to show that the only states which completely avoid pairs with relative angular momentum $l = 0$ are, up to rotation in pseudospin space, identical to the wave functions in Eq. (3.1). This is the real reason why the variational wave functions used above are accurate. In the following we describe low-energy states of the system as spin-texture states with a position-dependent local pseudospin orientation. The property that the low-energy states are completely specified by spin textures, which we will use

frequently in following sections, depends on both the broken symmetry of the ground state *and* on the existence of the charge gap.

The U(1) symmetry leads to Eq. (3.2) which defines an effective $XY$ model which will contain vortex excitations which interact logarithmically. In a thin film of superfluid $^4$He, vortices interact logarithmically because of the energy cost of supercurrents circulating around the vortex centers. (In superconducting thin films the same logarithmic interaction appears but is cut off on length scales exceeding the penetration depth.) Here the same logarithmic interaction appears. Microscopically this interaction is due to the potential energy cost (loss of exchange) associated with the phase gradients (circulating pseudospin currents). Hartree-Fock estimates[14] indicate that $\rho_s$ and hence the Kosterlitz-Thouless critical temperature are on the scale of 1 K in typical samples. Vortices in the $\varphi$ field (''merons''[14]) are reminiscent of Laughlin's fractionally charged quasiparticles but in this case carry charges $\pm\frac{1}{2}e$ and can independently be left- or right-handed for a total of four ''flavors.''[13,14] Bound meron pairs with opposite vorticity are the lowest[26] energy charged excitations of the system. The finite pseudospin stiffness not only permits the presence of spontaneous pseudospin magnetization but leads to a finite charge excitation gap (even though the tunnel splitting is zero). Thus the QHE survives[14,26] the limit $\Delta_{\rm SAS}\rightarrow 0$.

Since the ''charge'' conjugate to the phase $\varphi$ is the $z$ component of the pseudospin $S^z$, the pseudospin ''supercurrent''

$$J=\frac{2\rho_s}{\hbar}\nabla\varphi \tag{3.3}$$

represents oppositely directed charge currents in each layer. Below the KT transition temperature, such current flow will be dissipationless (in linear response) just as in an ordinary superfluid. Likewise there will be a linearly dispersing collective Goldstone mode as in a superfluid.[11,10,8,9,13,14]

To reinforce the idea that this is not an ordinary superfluid or superconductor, it is perhaps useful to rewrite the original variational wave function as

$$|\psi\rangle=\prod_X\left[\frac{1}{\sqrt{2}}(1+e^{i\varphi}c^\dagger_{X,\downarrow}c_{X,\uparrow})\right]|\psi_\uparrow\rangle, \tag{3.4}$$

where

$$|\psi_\uparrow\rangle\equiv\prod_X c^\dagger_{X,\uparrow}|0\rangle \tag{3.5}$$

is the fully up-polarized spin state. We now see that the analogy to an excitonic insulator[27,28] is closer than the analogy to a superconductor. Only a gauge-neutral object (a particle bound to a hole) can condense and propagate freely in a strong $B$ field. A phase gradient $\nabla\varphi$ causes a flow of these neutral objects which then constitutes a spin current analogous to the charge current in a superconductor. In analogy to the excitonic insulator case, one can make a particle-hole transformation in one of the layers to produce a formal resemblance to the BCS mean field state:

$$|\psi\rangle=\prod_X\left[\frac{1}{\sqrt{2}}(1+e^{i\varphi}c^\dagger_{X,\downarrow}c^\dagger_{X,\uparrow})\right]|\psi_\uparrow\rangle, \tag{3.6}$$

but it is then clear that one has a pairing between a particle and hole; not between two particles. It is worth remarking that there is in general an exact mapping[28] between double-layer electron-hole systems and two-component electron systems in the limit of strong magnetic fields. The Hamiltonian for a double-layer electron system is mapped, up to a constant term, to that for an electron-hole system if a particle-hole transformation is made in one of the layers. The filling factor of interest here, $\nu_T=1$, corresponds to an electron-hole system with equal electron and hole densities and, as explained above, the spontaneous phase coherence state corresponds to an excitonic superfluid state. In the electron-hole case, the fact that a broken symmetry ground state can occur had been appreciated some years ago.[29] Some of the recent advances in understanding the physics associated with the spontaneous-phase-coherence ground state in the electron-electron case, have important implications for the electron-hole case which we will not explore at length here.

A finite tunneling amplitude $t$ between the layers breaks the U(1) symmetry

$$H_{\rm eff}=\int d^2r\left[\frac{1}{2}\rho_s|\nabla\varphi|^2-\frac{t}{2\pi\ell^2}\cos\varphi\right] \tag{3.7}$$

by giving a preference to symmetric tunneling states. This can be seen from the tunneling Hamiltonian

$$H_T=-t\int d^2r\{\psi^\dagger_\uparrow(\mathbf{r})\psi_\downarrow(\mathbf{r})+\psi^\dagger_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r})\}, \tag{3.8}$$

which can be written in the spin representation as

$$H_T=-2t\int d^2r S_x(\mathbf{r}). \tag{3.9}$$

(Recall that the eigenstates of $S_x$ are symmetric and antisymmetric combinations of up and down.)

We can shed further light on the spontaneous symmetry breaking by considering the tunneling Hamiltonian $H_T$ in Eq. (3.8) as a weak perturbation. Naively, since particle number is separately conserved in each layer for $t=0$, one might expect

$$\lim_{t\rightarrow 0}\frac{1}{t}\langle\psi|H_T|\psi\rangle=0. \tag{3.10}$$

That is, one might expect that the first-order term in the perturbation series for the energy due to $t$ to vanish. Instead however we find that the energy shifts *linearly* in $t$,

$$\lim_{t\rightarrow 0}\lim_{A\rightarrow\infty}\frac{1}{tA}\langle\psi|H_T|\psi\rangle$$

$$=\lim_{t\rightarrow 0}\lim_{A\rightarrow\infty}\frac{1}{A}\left\langle\psi\left|-\int d^2r 2S_x(\mathbf{r})\right|\psi\right\rangle$$

$$=-m^x, \tag{3.11}$$

where $A$ is the system area, and $m^x$ is, by definition, the magnetization which is the system's order parameter.[30] If the interlayer spacing $d$ is taken to be zero, one can readily show[14] that the variational wave function in Eq. (3.1) is exact, hence $\lim_{t\to 0} m^x = 1$, and $t = \Delta_{\rm SAS}/2$. For finite $d$, Eq. (3.1) is no longer exact and quantum fluctuations will[14] reduce the magnitude of $m^x$ and we must renormalize the hopping parameter $t$ appropriately.

As the layer separation $d$ increases, a critical point $d^*$ will be reached at which the magnetization vanishes and the ordered phase is destroyed by quantum fluctuations.[13,14] This is illustrated in Fig. 2. For *finite* tunneling $t$, we will see below that the collective mode becomes massive and quantum fluctuations will be less severe. Hence the phase boundary in Fig. 2 curves upward with increasing $\Delta_{\rm SAS}$. For $\Delta_{\rm SAS} = 0$ the destruction of long-range order and the charge excitation gap are intimately related and occur simultaneously at $d^*$ and zero temperature. For finite $\Delta_{\rm SAS}$ the system always has nonzero $m^x$ even in the phase with zero charge gap.

The effective Hamiltonian in Eq. (3.7) looks like the sine-Gordon model which is known to have a finite temperature Kosterlitz-Thouless phase transition.[31] One might be tempted to speculate then that the rapid collapse of the Arrhenius plots of the dissipation at unexpectedly low temperatures is associated with a true phase transition. We believe, however, that these are merely rapid crossovers rather than true phase transitions because the phase $\varphi$ is compact. The quantum states defined by $\varphi$ and $\varphi + 2\pi$ are microscopically identical. The interpretation of $\varphi$ in a sine-Gordon theory as an electrostatic potential for Coulomb charges or as the surface height in the solid-on-solid model requires that $\varphi$ and $\varphi + 2\pi$ be distinguishable states. Hence what we really have is an $XY$ model in a symmetry-breaking field which has no true phase transition since the vortices are linearly confined. [Note that the solid-on-solid model has no analog of open strings (see Sec. IV) terminated by vortices such as we have here.[17]]

## IV. CHARGED EXCITATIONS

At filling factor $\nu = 1$, there is an intimate connection between local distortions of the pseudospin orientation and the local charge density.[14] A simple example of this is provided by the fully spin polarized $\nu = 1$ state of Eq. (3.1). Since the Landau level is filled, the charge density is uniform and the Pauli principle forbids any (intra-Landau-level) excitations which do not flip spins. One can form a spin-flip particle-hole pair at an energy cost of[21]

$$U_0 = \Delta_{\rm SAS} + \frac{e^2}{\epsilon \ell}\left(\frac{\pi}{2}\right)^{1/2}, \tag{4.1}$$

where the first term is the tunneling energy necessary to flip the pseudospin and the second represents the loss of Coulomb exchange energy (here computed for the special case $d = 0$). The Coulomb exchange energy gives a finite cost even in the absence of tunneling and this explains the fact that the typical observed gap can be much larger than the tunnel splitting.[7,4,5] The simple spin flip is not, however, the optimal charged-pair excitation. From analogy with results of Sondhi *et al.*[21] For the case of ''real'' spins, we know that smooth local distortions of the pseudospins produce a charge density given by the remarkable formula[21,14]

$$\delta\rho(\mathbf{r}) = -\frac{\nu}{8\pi}\epsilon_{\mu\nu}\mathbf{m}(\mathbf{r})\cdot[\partial_\mu \mathbf{m}(\mathbf{r}) \times \partial_\nu \mathbf{m}(\mathbf{r})], \tag{4.2}$$

where $\mathbf{m}$ is a unit vector giving the local pseudospin orientation. $\delta\rho(\mathbf{r})$ is exactly $\nu$ times the Pontryagin index, or topological charge density.[21,31] The density in Eq. (4.2) can be viewed as the timelike component of a conserved (divergenceless) topological ''three-current''

$$j^\alpha = -\frac{\nu}{8\pi}\epsilon^{\alpha\beta\gamma}\epsilon_{abc} m^a(\mathbf{r})\partial_\beta m^b(\mathbf{r})\partial_\gamma m^c(\mathbf{r}). \tag{4.3}$$

Using the fact that $\mathbf{m}$ is a unit vector, it is straightforward to verify that $\partial_\mu j^\mu = 0$.

''Skyrmion'' (hedgehog) configurations of the order parameter carry net charge $\pm 1$ and, when $\Delta_{\rm SAS} = 0$ and $d = 0$,[21,11] a skyrmion pair has an excitation energy which is one-half of that of a simple spin-flip pair for the case of Coulomb interactions between the electrons. At finite $d$, the SU(2) symmetry is lowered to U(1) and the cheapest charged excitations are composed of merons, which are essentially vortex solutions in which the local pseudospin winds by $\pm 2\pi$ at infinity and tilts either up or down out of the $XY$ plane in the core region as shown in Fig. 9 of I. Integration of the charge density using Eq. (4.2) shows that vortices carry charge $\pm\frac{1}{2}$. They are somewhat analogous to Laughlin quasiparticles; however, they differ considerably in that, below the Kosterlitz-Thouless temperature, they are confined together in vorticity neutral pairs by their logarithmic interaction. The cheapest object with a net charge is then a vortex-antivortex pair, with each vortex carrying charge $+\frac{1}{2}$ (or $-\frac{1}{2}$) for a total charge of $+1$ (or $-1$). The charge excitation cost can be estimated by minimizing

$$E_{\rm pair} = 2E_{\rm mc} + \frac{e^2}{4\epsilon R} + 2\pi\rho_s \ln\left[\frac{R}{R_{\rm mc}}\right], \tag{4.4}$$

where $E_{\rm mc}$ is the meron core energy,[26] and $R_{\rm mc}$ is the meron core size. The optimal separation is given by[14] $R_0 = e^2/(8\pi\epsilon\rho_s)$. It is important for the discussion below that in typical double-layer systems $\rho_s$ is much smaller than the microscopic energy scale $e^2/\epsilon\ell$. For $d/\ell = 1$, $\rho_s = 6.19\times 10^{-3}(e^2/\epsilon\ell)$ in the Hartree-Fock approximation and it is further renormalized downward by quantum fluctuations.[14] Typical values of $\rho_s$ for double-layer systems are smaller than $5\times 10^{-3}(e^2/\epsilon\ell)$ so that $R_0/\ell$ will typically be larger than $\sim 8\ell$. The small values of the pseudospin stiffness allow the charged pseudospin textures to be large, as required for the validity of the long-wavelength description being employed here.

The introduction of finite tunneling amplitude destroys the U(1) symmetry and makes the simple vortex-pair configuration extremely expensive. To lower the energy the system distorts the spin deviations into a domain wall or ''string'' connecting the vortex cores as shown in Fig. (4). The spins are oriented in the $\hat{x}$ direction everywhere except in the domain line region where they tumble rapidly through $2\pi$. The domain line has a fixed energy per unit length and

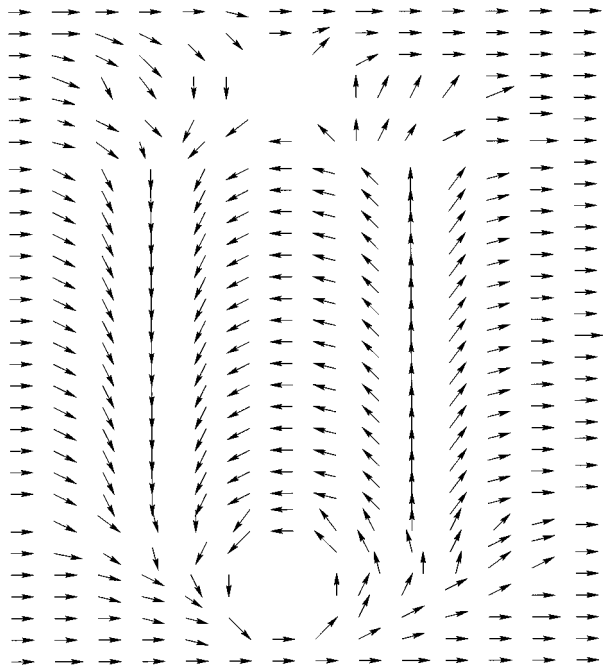

FIG. 4. Illustration of a meron pair in the presence of tunneling which confines the region of spin twist to a relatively narrow domain wall or ''string.'' Each end of the string is a vortex carrying charge $\pm 1/2$.

so the vortices are now confined by a linear potential corresponding to a fixed ''string tension'' rather than being confined only logarithmically. We can estimate the string tension by examining the energy of a domain line of infinite length. The optimal form for a domain line lying along the $y$ axis is given by

$$\varphi(\mathbf{r}) = 2\arcsin[\tanh(x/\xi)], \tag{4.5}$$

where the characteristic width of the string is

$$\xi = \left[\frac{2\pi\ell^2\rho_s}{t}\right]^{1/2}. \tag{4.6}$$

The resulting string tension is[32]

$$T_0 = 8\left[\frac{t\rho_s}{2\pi\ell^2}\right]^{1/2} = \frac{8\rho_s}{\xi}. \tag{4.7}$$

Provided the string is long enough ($R \gg \xi$), the total energy of a segment of length $R$ will be well approximated by the expression

$$E'_{\text{pair}} = 2E'_{\text{mc}} + \frac{e^2}{4\epsilon R} + T_0 R. \tag{4.8}$$

The prime on $E_{\text{mc}}$ in Eq. (4.8) indicates that the meron core energy can depend on $\Delta_{\text{SAS}}$. $E'_{\text{pair}}$ is minimized at $R = R'_0 \equiv \sqrt{e^2/4\epsilon T_0}$ where it has the value

$$E^*_{\text{pair}} = 2E'_{\text{mc}} + \sqrt{e^2 T_0/\epsilon}. \tag{4.9}$$

Note that apart from the core energies, the charge gap at fixed layer separation (and hence fixed $\rho_s$) is $\propto T_0^{1/2} \propto t^{1/4} \sim \Delta_{\text{SAS}}^{1/4}$, which contrasts with the case of free electrons, for which the charge gap is linearly proportional to $\Delta_{\text{SAS}}$. Note that because the exponent 1/4 is so small, there is an extremely rapid initial increase in the charge gap as tunneling is turned on.

The crossover between the meron-pair pseudospin texture which holds for $t \equiv 0$ and the domain line string pseudospin texture described above occurs at a finite value of $t$ which we can estimate by the following argument. For $R'_0 > R_0$ the vortices are already bound by the logarithmic attraction due to the gradient energy before the linear attraction due to the hopping becomes important at larger separations. In this regime tunneling does not play an important role in determining the nature of the lowest energy charged pseudospin texture. As $t$ increases $R'_0 \propto t^{-1/4}$ decreases and will eventually reach $R_0$ which is, of course, independent of $t$. Since

$$\frac{R'_0}{R_0} = \left(\frac{2\pi^2\rho_s}{e^2/\epsilon\xi}\right)^{1/2} = \frac{\pi\xi}{4R'_0}, \tag{4.10}$$

the characteristic width of the domain line becomes comparable to $R'_0$ in the same range of $t$ values where $R'_0$ and $R_0$ become comparable. We may conclude that the nature of the charged pseudospin texture crosses over directly from the meron pair form to the finite length domain line string form for $\rho_s/(e^2/\epsilon\xi) \sim 1/25$, or equivalently for $t \sim t_{\text{cr}}$ where

$$t_{\text{cr}} = 4\times 10^3 \left[\frac{\rho_s}{e^2/\epsilon\ell}\right]^3 \frac{e^2}{\epsilon\ell}. \tag{4.11}$$

The crossover tunneling amplitude is thus typically smaller than $5\times 10^{-4}(e^2/\epsilon\ell)$. Typical tunneling amplitudes in double-layer systems are smaller than $\sim 10^{-1}(e^2/\epsilon\ell)$ and can be made quite small by adjusting the barrier material or making the barrier wider. Nevertheless, it seems likely that $t$ will be larger than $t_{\text{cr}}$ except for samples which are carefully prepared to make $t$ as small as possible. As $t$ increases beyond $t_{\text{cr}}$, $R'_0$ will continue to decrease. When $R'_0$ becomes comparable to the microscopic length $\ell$, the description given here will become invalid and the lowest energy charged excitations will have single-particle character. However, the domain-wall string picture of the charged pseudospin texture has a very large range of validity since $R'_0 \propto t^{-1/4}$ decreases very slowly with increasing $t$. Writing $R'_0 \sim (e^2/8\epsilon\pi\rho_s)(t_{\text{cr}}/t)^{1/4}$ we find that $R'_0 \sim \ell$ only for $t \sim 10^{-2}[(e^2/\epsilon\ell)^2/\rho_s]$. Using typical values of $\rho_s$ we see that the charged excitation crosses over to single-particle character only when the hopping energy $t$ becomes comparable to the microscopic interaction energy scale. The various regimes for the charge excitations of double-layer systems are summarized in Table I. Almost all typical double-layer systems lie within the regime of the domain-wall-string pseudospin texture charge excitation.

We should emphasize that all the discussion of charged excitations above assumes that the meron core energies do not make a dominant contribution to the charge excitation energies and that meron core sizes are small compared to the overall size of the quasiparticles. Recent calculations[33] have demonstrated that these conditions are never well satisfied when the Hartree-Fock approximation is used to approximate the charged excitations. The physical pictures summarized still have some qualitative validity, however. The Hartree-Fock approximation neglects quantum fluctuation effects

TABLE I. Charged spin texture energies at $\nu_T=1$ for double-layer systems with tunneling. $\tilde{\rho}_s \equiv \rho_s/(e^2/\ell)$ and $\tilde{t} \equiv t/(e^2/\ell)$, where $\rho_s$ is the pseudospin stiffness, $t$ is the renormalized tunneling amplitude, $\ell$ is the magnetic length, $T_0 = 8\rho_s/\xi$ is the soliton string tension, and $\xi = (2\pi\ell^2\rho_s/t)^{1/2}$ is the domain wall width.

| Regime | $\tilde{t} \leq 4\times 10^3 \tilde{\rho}_s^3$ | $4\times 10^3 \tilde{\rho}_s^3 \leq \tilde{t} \leq 10^{-2}/\tilde{\rho}_s$ | $10^{-2}/\tilde{\rho}_s \leq t$ |
|---|---|---|---|
| Nature of charged excitations | Meron pairs | Finite length domain line strings | Single-particle excitation |
| Excitation size | $\sim \dfrac{e^2}{8\pi\rho_s}$ | $\sim \sqrt{\dfrac{e^2}{4T_0}} \propto t^{-1/4}$ | $\ell$ |
| Excitation energy | $\sim 2\pi\rho_s$ | $\sim \sqrt{e^2 T_0} \propto t^{1/4}$ | $t$ |

which reduce both the spin-stiffness and the order parameter, tending in both cases to increase the size of the quasiparticles and increase the appropriateness of the pictures presented here.

## V. PARALLEL MAGNETIC FIELD

Murphy *et al.*[4,18] and Santos *et al.*[5] have shown that the charge gap in double-layer systems is remarkably sensitive to the application of relatively weak magnetic fields $B_\parallel$, oriented in the plane of the 2D electron gas. Experimentally this field component is generated by slightly tilting the sample relative to the magnetic field orientation. Tilting the field (or sample) has traditionally been an effective method for identifying effects due to (real) spins because orbital motion in a single-layer 2DEG system is primarily[22] sensitive to $B_\perp$, while the (real) spin Zeeman splitting is proportional to the full magnitude of $B$. Adding a parallel field component will tend to favor more strongly spin-polarized states. For the case of the double-layer $\nu=1$ systems studied by Murphy *et al.*,[4] the ground state is known to already be an isotropic ferromagnetic state of the *true spins* and the addition of a parallel field would not, at first glance, be expected to influence the low energy states since they are already fully spin-polarized. (At a fixed Landau level filling factor $B_\perp$ is fixed and so both the total $B$ and the corresponding Zeeman energy increase with tilt.) Nevertheless experiments[4] have shown that these systems are very sensitive to $B_\parallel$. The activation energy drops rapidly (by factors varying from two up to an order-of-magnitude in different samples) with increasing $B_\parallel$. At $B_\parallel = B_\parallel^*$ there appears to be a phase transition to a new state whose activation gap is approximately independent of further increases in $B_\parallel$.

The effect of $B_\parallel$ on the *pseudospin* system can be visualized in two different pictures. In the first picture we use a gauge in which $\mathbf{B}_\parallel = \boldsymbol{\nabla}\times\mathbf{A}_\parallel$ where $\mathbf{A}_\parallel = B_\parallel(0,0,x)$. In this gauge the vector potential points in the $\hat{z}$ direction (perpendicular to the layers) and varies with position $x$ as one moves parallel to the layers. As an electron tunnels from one layer to the other it moves along the direction in which the vector potential points and so the tunneling matrix element acquires a position-dependent phase $t \to te^{iQx}$ where $Q = 2\pi/L_\parallel$ and $L_\parallel = \Phi_0/B_\parallel d$ is the length associated with one flux quantum $\Phi_0$ between the layers (defined in Fig. 3). This modifies the tunneling Hamiltonian to $H_T = -\int d^2r\, \mathbf{h}(\mathbf{r})\cdot\mathbf{S}(\mathbf{r})$ where $\mathbf{h}(\mathbf{r})$ ''tumbles'': i.e., $\mathbf{h}(\mathbf{r}) = 2t(\cos Qx, \sin Qx, 0)$. The effective $XY$ model now becomes

$$H = \int d^2\mathbf{r}\left\{ \frac{1}{2}\rho_s |\boldsymbol{\nabla}\varphi|^2 - \frac{t}{2\pi\ell^2}\cos[\varphi(\mathbf{r}) - Qx]\right\}, \tag{5.1}$$

which is precisely the Pokrovsky-Talapov (PT) model[34] and has a very rich phase diagram. For small $Q$ and/or small $\rho_s$ the phase obeys (at low temperatures) $\varphi(\mathbf{r}) \equiv Qx$; the moment rotates commensurately with the pseudospin Zeeman field. However, as $B_\parallel$ is increased, the local field tumbles too rapidly and a continuous phase transition to an incommensurate state with broken translation symmetry occurs. This is because at large $B_\parallel$ it costs too much exchange energy to remain commensurate and the system rapidly gives up the tunneling energy in order to return to a uniform state $\boldsymbol{\nabla}\varphi \approx 0$ which becomes independent of $B_\parallel$. As explained in further detail below we[13] find that the phase transition occurs at zero temperature for

$$B_\parallel^* = B_\perp (2\ell/\pi d)(2t/\pi\rho_s)^{1/2}. \tag{5.2}$$

Using the parameters of the samples of Murphy *et al.*[4] and neglecting quantum fluctuation renormalizations of both $t$ and $\rho_s$ we find that the critical field for the transition is $\approx 1.6$ T which is within a factor of 2 of the observed value.[4] Note that the observed value $B_\parallel^* = 0.8$ T corresponds in these samples to a large value for $L_\parallel$: $L_\parallel/\ell \sim 20$ indicating that the transition is highly collective in nature. We emphasize again that these very large length scales are possible in a magnetic field only because of the interlayer phase coherence in the system associated with condensation of a *neutral* object.

Having argued for the existence of the commensurate-incommensurate transition, we must now connect it to the experimentally observed transport properties. In the commensurate phase, the order parameter tumbles more and more rapidly as $B_\parallel$ increases. As we shall see below, it is this tumbling which causes the charge gap to drop rapidly. In the incommensurate phase the state of the system is approximately independent of $B_\parallel$ and this causes the charge excitation gap to saturate at a fixed value.

Recall that in the presence of tunneling, the cheapest charged excitation was found to be a pair of vortices of opposite vorticity and like charge (each having charge $\pm 1/2$) connected by a domain line with a constant string tension. In the absence of $B_\parallel$ the energy is independent of the orientation of the string. The effect of $B_\parallel$ is most easily studied by changing variables to

$$\theta(\mathbf{r}) \equiv \varphi(\mathbf{r}) - Qx. \tag{5.3}$$

This variable is a constant in the commensurate phase but not in the incommensurate phase. In terms of this new variable, the PT model energy is

$$H = \int d^2r \left\{ \frac{1}{2}\rho_s[(\partial_x\theta + Q)^2 + (\partial_y\theta)^2] - \frac{t}{2\pi\ell^2}\cos\theta \right\}. \tag{5.4}$$

We see that $B_\parallel$ defines a preferred direction in the problem. Domain walls will want to line up in the $y$ direction and contain a phase slip of a preferred sign ($-2\pi$ for $Q>0$) in terms of the field $\theta$. Since the extra term induced by $Q$ represents a total derivative, the optimal form of the soliton solution is unchanged. However, the energy per unit length of the soliton, which is the domain line string tension, decreases linearly with $Q$ and hence $B_\parallel$:

$$T = T_0\left[1 - \frac{B_\parallel}{B_\parallel^*}\right], \tag{5.5}$$

where $T_0$ is the tension in the absence of parallel $B$ field given by Eq. (4.7) and $B_\parallel^*$ is the critical parallel field at which the string tension goes to zero.[35] We thus see that by tuning $B_\parallel$ one can conveniently control the ''chemical potential'' of the domain lines. The domain lines condense and the phase transition occurs when the string tension becomes negative.

Recall that the charge excitation gap is given by the energy of a vortex pair separated by the optimal distance $R_0 = \sqrt{e^2/4\epsilon T}$. From Eq. (4.8) we have that the energy gap for the commensurate state of the phase transition is given by

$$\begin{aligned}\Delta &= 2E'_{\mathrm{mc}} + [e^2T/\epsilon]^{1/2} \\ &= \Delta_0 + \sqrt{e^2T_0/\epsilon}\left[1 - \left(\frac{B_\parallel}{B_\parallel^*}\right)\right]^{1/2}.\end{aligned} \tag{5.6}$$

As $B_\parallel$ increases, the reduced string tension allows the Coulomb repulsion of the two vortices to stretch the string and lower the energy. Far on the incommensurate side of the phase transition the possibility of interlayer tunneling becomes irrelevant. From the discussion of the previous section it follows that the ratio of the charge gap at $B_\parallel = 0$ to the charge gap at $B_\parallel \to \infty$ should be given approximately by

$$\frac{\Delta_0}{\Delta_\infty} = (t/t_{\mathrm{cr}})^{1/4} \approx (e^2/\epsilon\ell)^{1/2} t^{1/4} 8\rho_s{}^{3/4}. \tag{5.7}$$

Putting in typical values of $t$ and $\rho_s$ gives gap ratios $\sim 1.5-7$, in agreement with experiment. According to the discussion of the previous section, gap ratios as large as $\sim (t_{\max}/t_{\mathrm{cr}})^{1/4} \sim 0.07(e^2/\epsilon\ell)/\rho_s$ can be expected in the regime where the pseudospin texture picture applies. Here $t_{\max}$ is the hopping parameter at which the crossover to single-particle excitations occurs. Thus gap ratios as large as an order of magnitude are easily possible. Of course, all the discussion here neglects orbital effects (electric subband mixing) within each of the electron gas layers, and these will always become important at sufficiently strong parallel fields.

It should be emphasized that only this highly collective picture involving large length scale distortions of topological defects can possibly explain the extreme sensitivity of the charge gap to small tilts of the $B$ field. Recall that at $B_\parallel^*$ the tumbling length $L_\parallel$ is much larger than the particle spacing and the magnetic length. Simple estimates of the cost to make a local one-body-type excitation (a pseudospin-flip pair, for example) shows that the energy decrease due to $B_\parallel$ is extremely small since $\ell/L_\parallel$ is so small. As we will see in Sec. VII numerical exact-diagonalization calculations on small systems confirm the existence of this phase transition and show that the fermionic excitation gap drops to a much smaller value in the incommensurate phase.

We now discuss the commensurate-incommensurate phase transition from the microscopic point of view. At $d=0$, the $B_\parallel = 0$ Landau-gauge many-body ground state wave function is a single Slater determinant in which the single-body states are the symmetric linear combination of two single-layer states with the same guiding center. This is the state represented by Eq. (3.1) with $\varphi = 0$. Phase coherence is established (either spontaneously or in this case) by tunneling between single-layer states with the same guiding center. For many purposes this state is still a good approximation to the ground state at finite $d$ since it optimizes the tunneling energy and has good correlation energy; an electron in one layer automatically sees an exchange-correlation hole in the other layer at the same place. (It would remain an exact ground state at finite $d$ in the absence of interactions.) From a microscopic point of view it is the good interlayer correlations of states with phase coherence which leads to the broken symmetry in the absence of tunneling.

To see the effect of a parallel $B$ field, it is convenient to choose a new Landau gauge for the perpendicular field $\mathbf{A}_\perp = -B_\perp(y,0,0)$. In this gauge, a parallel field giving rise to $t \to t\exp(iQx)$ causes tunneling to couple states in the two layers whose wave vectors differ by $Q$ and whose guiding centers therefore differ by $Q\ell^2$.[36] Thus, for noninteracting electrons the exact ground state in a parallel field is one in which the exchange-correlation hole is not directly opposite its electron but rather is shifted away by $\ell^2 Q\hat{y}$ as the **B** field tilts in the $\hat{x}$ direction (i.e., the displacement is perpendicular to the direction of the in-plane field):

$$|\psi_Q\rangle = \prod_Y \{c^\dagger_{Y,\uparrow} + c^\dagger_{Y+Q\ell^2,\downarrow}\}|0\rangle. \tag{5.8}$$

This state maintains all of its tunneling energy but rapidly loses interlayer correlation energy as the field tilts. At large tilt it is better to give up on the tunneling by shifting the two layers relative to each other to put the correlation hole back next to its electron.

This shift can be seen to be the change from commensurate to incommensurate states discussed above. A straightforward computation shows that the commensurate state has the pseudospin tumbling

$$\langle\psi_Q|\psi^\dagger_\uparrow(\mathbf{r})\psi_\downarrow(\mathbf{r})|\psi_Q\rangle = e^{-(1/4)}Q^2\ell^2 e^{-iQx}, \tag{5.9}$$

while the pseudospin is constant in the incommensurate phase.

All of our discussion of the phase transition in a parallel field has been based on mean-field theory. Close to the phase

transition, thermal fluctuations will be important. At finite temperatures there is no strict phase transition at $B_\parallel^*$ in the the PT model. However, there is a finite temperature KT phase transition at a nearby $B_\parallel > B_\parallel^*$. At finite temperatures translation symmetry is restored[34] in the incommensurate phase by means of dislocations in the domain string structure. Thus there are two separate KT transitions in this system, one for $t=0$, the other for $t\neq 0$ and $B_\parallel > B_\parallel^*$. Recently Read[37] has studied this model at finite temperatures in some detail and has shown that at the critical value of $B_\parallel$ there should be a square-root singularity in the charge gap on both sides of the transition. The existing data does not have the resolution to show this, however. At zero temperature the commensurate-incommensurate[38,37] phase transition must be treated quantum mechanically. It is necessary to account for the world sheets traced out by the time evolution of the strings which fluctuate into existence due to quantum zero-point motion. Read also points out that the inevitable random variations in the tunneling amplitude with position, which we have not considered at all here, cause a relevant perturbation.

## VI. COLLECTIVE MODES AND RESPONSE FUNCTIONS

In this section we will discuss the charge neutral collective excitations of double-layer systems and some physically important response functions which have poles at the collective excitation energies. In the pseudospin language, the collective excitations are spin waves in which the pseudospin precesses around its ground state orientation. We will thus need to enlarge our description of the system by allowing the pseudospin texture to have orientations out of the $\hat{x}$-$\hat{y}$ plane. This requires that we generalize from the $XY$-model description of the system employed in previous sections, to the more complete anisotropic nonlinear $\sigma$ model description which we have discussed at length in Ref. 14 (I). (Actually this generalization is also required if want to render the physics of the meron cores.) States of the system are characterized by a pseudospin texture function $\hat{m}(\mathbf{r})$, which specifies the space dependence of the pseudospin orientation. The energy of a pseudospin texture is given by the following functional (where we retain only the leading terms in number of derivatives):

$$E[\hat{m}(\mathbf{r})]=\int d^2\mathbf{r}\left[\beta(m_z^2(\mathbf{r}))+\frac{\rho_s}{2}|\nabla m_\perp|^2-ntm_x(\mathbf{r})\right], \tag{6.1}$$

where $\hat{\mathbf{m}}(\mathbf{r})\cdot\hat{\mathbf{m}}(\mathbf{r})\equiv\mathbf{1}$ and $m_\perp$ is the projection of the unit vector onto the $\hat{x}$-$\hat{y}$ plane. The energy is with respect to the ground state in the absence of tunneling. Here $n=(2\pi\ell^2)^{-1}$ is the total electron density. This energy functional is expected to be accurate for states with spin textures which vary slowly on a microscopic scale. If the pseudospin is confined to the $\hat{x}$-$\hat{y}$ plane, its orientation is specified by its azimuthal angle and the energy functional reduces to the $XY$ functional used above. This Ginzburg-Landau energy functional differs from the one discussed in Ref. 14 only through the addition of the tunneling term. The form of the functional follows from symmetry considerations and the parameters can be considered as phenomenological constants. In Ref. 14, Eq. (6.1) was derived microscopically using a Hartree-Fock approximation, and explicit expressions for the coefficients were obtained, which become exact in the limit $d\to 0$. We now generalize the discussion of collective modes given there to the case where tunneling occurs between the layers.

### A. Collective modes with tunneling: $B_\parallel=0$

In the presence of interwell tunneling, the pseudospin-orientation in the ground state is constant and points in the $\hat{x}$ direction. To calculate response functions we use the equations of motion derived in Ref. 14. We will assume that for low-lying excitations the pseudospin orientation is always close to the $\hat{x}$ direction. Then the equation of motions of the pseudospin texture in Fourier space is

$$\frac{dm_y(\mathbf{q})}{dt}=-\frac{4\pi\ell^2}{\hbar}\frac{dE[m]}{dm_z(-\mathbf{q})}, \tag{6.2}$$

$$\frac{dm_z(\mathbf{q})}{dt}=\frac{4\pi\ell^2}{\hbar}\frac{dE[m]}{dm_y(-\mathbf{q})}. \tag{6.3}$$

In the rest of this section we will use $\ell$ as the unit of length. It is possible to linearize these equations if the pseudospin orientation is close to the $\hat{x}$ direction by letting $m_x=1-(m_y^2+m_z^2)/2+\cdots$, and dropping terms higher than first order in $m_y$ and $m_z$. For the double-layer system $m_y$ is proportional to the current flowing locally between layers and $m_z$ is proportional to the difference between the local densities in the two layers. To calculate the response functions of interest we add terms to the energy functional corresponding to pseudospin Zeeman fields in the $\hat{y}$ and $\hat{z}$ directions, $h_y$ and $h_z$. Physically $h_y$ corresponds to a perturbation in which an imaginary term is added to the tunneling amplitude and $h_z$ corresponds to a bias potential between the two layers. Linearizing and adding the Zeeman terms we find that

$$\frac{dm_y(\mathbf{q})}{dt}=-\frac{4\pi}{\hbar}(2\beta+tn)m_z(\mathbf{q})+h_z, \tag{6.4}$$

$$\frac{dm_z(\mathbf{q})}{dt}=\frac{4\pi}{\hbar}(tn+\rho_s q^2)m_y(\mathbf{q})-h_y. \tag{6.5}$$

There is a similarity between these equations of motion and those of the Josephson effect, which is connected to the similarities between interlayer phase coherence and superconductivity mentioned previously.[39] The current across a Josephson junction between two superconductors is proportional to $\sin(\phi)$ where $\phi$ is the difference in the phase of the order parameter across the junction. The current between layers in a double-layer system is similarly proportional to $m_y$, i.e., proportional to $\sin(\phi)$ where $\phi$ in the double-layer case specifies orientation of the component of the pseudospin in the $\hat{x}$-$\hat{y}$ plane. [In the linearized case we are discussing we can equate $\phi$ and $\sin(\phi)$.] More physically $\phi$ in the double-layer case specifies the difference between the phase-coherence angle and the phase angle of the interlayer tunneling amplitude. In the case of the Josephson effect

$$\frac{d\phi}{dt}=\frac{2eV}{\hbar}, \tag{6.6}$$

where $V$ is the potential drop across the junction. Thus the equations of motion for the phase angles in the Josephson effect case and in the present case differ because of the presence of the term proportional to $(2\beta+tn)m_z(\mathbf{q})$; a Josephson effect could be achieved if the tunneling current were somehow extracted from the double-layer system sufficiently quickly to prevent any difference in the charge densities of the two-layers from building up, i.e., if $m_z$ were identically zero. In the case of double-layer systems, unlike the case of a Josephson junction between two superconductors, it seems to be impossible to do this. The results we derive below are for an isolated double-layer system and the resonance frequencies we obtain are analogous to the Josephson plasma oscillations in an isolated Josephson junction.

Solving for the pseudospin magnetization induced by Zeeman fields with frequency $\omega$ we obtain the following result for the pseudospin response tensor:

$$\begin{pmatrix} m_y \\ m_z \end{pmatrix}=\frac{\hbar}{E_{\rm cm}^2-(\hbar\omega)^2}\begin{pmatrix} -i\omega & 8\pi\beta+2t \\ -(2t+4\pi q^2\rho_s) & -i\omega \end{pmatrix} \times\begin{pmatrix} -4\pi h_z \\ 4\pi h_y \end{pmatrix}, \tag{6.7}$$

where the collective mode energy is given by

$$E_{\rm cm}=[(2t+8\pi\beta)(2t+4\pi q^2\rho_s)]^{1/2}. \tag{6.8}$$

The linearly dispersing collective mode of the $t\to 0$ case acquires a gap because of the lifting of the $U(1)$ symmetry.

All components of this response tensor have poles at the collective mode energies. The response of the charge density difference to a time-dependent interlayer bias potential is given by

$$\chi_{zz}\equiv\frac{m_z}{h_z}=\frac{(4\pi\hbar)2t+4\pi q^2\rho_s}{E_{\rm cm}^2-(\hbar\omega)^2}. \tag{6.9}$$

Using the continuity equation we can evaluate the corresponding conductivity, the response in oppositely directed electric currents to oppositely directed electric fields in the two layers:

$$\sigma_{zz}(q,\omega)=e^2\omega\chi_{zz}(q,\omega)/iq^2. \tag{6.10}$$

For $t\to 0$ the real part of the conductivity has a $\delta$-function peak at zero frequency leading to spin-channel superfluidity.[14] In the presence of interlayer tunneling the $\delta$-function peak is shifted to $\omega=2t/\hbar$ and the superfluid behavior is lost.

In the static $(\omega\to 0)$ long-wavelength $(q\to 0)$ limit $\chi_{zz}$ approaches a constant:

$$\chi_{zz}(\omega=0;q=0)=\frac{1}{(8\pi\beta+2t)}. \tag{6.11}$$

A constant static interlayer bias potential will[40] simply tilt the pseudospin orientation slightly out of the $\hat{x}$-$\hat{y}$ plane. The effect of interlayer tunneling is to favor smaller tilts. On the other hand,

$$\chi_{yy}(\omega=0,q=0)=\frac{1}{2t}. \tag{6.12}$$

If the tunneling amplitude goes to zero there is no restoring force for rotations of the pseudospin in the $\hat{x}$-$\hat{y}$ plane, the addition of an infinitesimal imaginary tunneling amplitude will shift the pseudospin orientation from the $\hat{x}$ direction to the $\hat{y}$ direction, and $\chi_{yy}(\omega=0,q=0)$ diverges.

### B. Collective modes with tunneling: Commensurate state

For $B_\parallel=0$ we have been able to calculate collective mode energies and linear response functions by linearizing the nonlinear $\sigma$ model energy functional around the ground state pseudospin orientation. For $B_\parallel\neq 0$, the ground state pseudospin orientation in the commensurate state rotates with the pseudospin Zeeman field and it is necessary to linearize the nonlinear $\sigma$ model energy functional around the rotating pseudospin texture. In anticipation of our needs for the case of the incommensurate state we allow an arbitrary rate of rotation for the rotating frame pseudospin function:

$$\tilde{m}_x=m_x\cos Px-m_y\sin Px, \tag{6.13}$$

$$\tilde{m}_y=m_x\sin Px+m_y\cos Px. \tag{6.14}$$

In order to be able to treat the problem analytically, we limit ourselves to the case of a position-independent tumbling rate $P$. The Ginzburg-Landau energy functional, expressed in terms of the rotating frame pseudospins, is

$$E=\int d^2\mathbf{r}\Big[\beta m_z^2+\frac{\rho_s}{2}|\nabla\tilde{\mathbf{m}}_\perp|^2+\frac{\rho_s P^2}{2}|\tilde{\mathbf{m}}_\perp|^2 -tn[\tilde{m}_x\cos(P-Q)x+\tilde{m}_y\sin(P-Q)x] +\rho_s P\hat{z}\cdot\Big(\frac{\partial\tilde{\mathbf{m}}_\perp}{\partial x}\times\tilde{\mathbf{m}}_\perp\Big)\Big]. \tag{6.15}$$

If we allowed only translationally invariant spin textures in the rotating frame for which the lowest energy occurs, this energy functional would be minimized for small parallel fields by choosing $P=Q$ to obtain an energy per area of $\rho_s Q^2/2-tn$. This is the commensurate state. At large parallel fields the energy functional would be minimized by the state with $P=0$, which has the same energy as if no tunneling were present. These two states cross in energy when $Q^2\xi^2=2$. However, as we detail in the following section, the ground state at high parallel fields can lower its energy further by breaking translational symmetry. The commensurate state is the ground state only for $Q<Q_c$ where $Q_c^2\xi^2=16/\pi^2$. $Q=Q_c$ for $B_\parallel=B_\parallel^*$

In order for the linearization of this functional to be valid we must assume we can choose $P$ so that $\tilde{m}_x$ is close to 1 *everywhere*, both for the ground state and for the collective excitations in which we are interested. Assuming this to be the case we can apply periodic boundary conditions in the thermodynamic limit and drop the total derivative term in the energy functional. For the commensurate state the linearized energy functional simplifies to the following Fourier space version:

$$E[\tilde{m}_y,m_z]=A\left(\frac{\rho_s Q^2}{2}-tn\right)+\sum_{\mathbf{q}}\left(\beta+\frac{tn}{2}-\frac{\rho_s Q^2}{2}\right)|m_z(\mathbf{q})|^2 + \left(\frac{\rho_s}{2}q^2+\frac{tn}{2}\right)|\tilde{m}_y|^2. \tag{6.16}$$

The first term on the right-hand-side of Eq. (6.16) is the ground state energy of the commensurate state. The fact that it becomes positive at large $Q$ implies that the commensurate state eventually becomes unstable. As discussed in previous sections we expect the state to become unstable to the introduction of phase slips or solitons, which involve only the planar portion of the pseudospin texture, when $\rho_s Q^2 > 16tn/\pi^2$. We note from Eq. (6.16) that if $\beta$ is small enough the ground state will become unstable with respect to textures with $m_z \neq 0$ before the solitons are introduced. The requirement for the applicability of the scenario we introduced previously based on the $XY$ language is that $\beta \not< t(16-\pi^2)/(4\pi^3)$. From estimates of $\beta$ in Ref. 14, it is clear that for present double-layer samples this condition is secure. Nevertheless, the possibility exists that the behavior in parallel fields could be quite different from that described here, for double-layer systems with a layer spacing much smaller than what is achievable at present. We have not explored this regime in detail and will assume in what follows that $2\beta+tn-\rho_s Q^2$ is positive in the incommensurate state. The effect of $B_\parallel$ is then to reduce the collective mode energy, just as the charged excitation energies are lowered. Solving the equations of motion with the linearized energy functional of the commensurate state we find that

$$E_{\rm cm}=[(2t+8\pi\beta-4\pi\rho_s Q^2)(2t+4\pi\rho_s q^2)]^{1/2}. \tag{6.17}$$

This result agrees with collective mode energies calculated in the Hartree-Fock approximation[41] if the Hartree-Fock approximation is used for the pseudospin stiffness.

### C. Collective modes with tunneling: Incommensurate state

For $B_\parallel > B_\parallel^*$ it becomes energetically favorable to break translational symmetry and introduce phase slips, or solitons, into the pseudospin texture. In the ground state the phase slips are periodic[34,41] with a period $L_s$ which is determined by minimizing the energy. In general then we choose to work with pseudospins in a (again uniformly) rotating reference frame with

$$P=\begin{cases} Q, & Q<Q_c \\ Q-\dfrac{2\pi}{L_s}=Q-Q_s, & Q\geqslant Q_c, \end{cases} \tag{6.18}$$

where $Q=2\pi/L_\parallel \propto B_\parallel$. For $P \neq Q$ the linearized energy functional is obtained by setting $\tilde{m}_\perp^2=1-m_z^2$, $\tilde{m}_x=1-(\tilde{m}_y^2+m_z^2)/2$, and dropping terms higher than second order in $\tilde{m}_y$ and $m_z$. We obtain

$$E=\frac{A\rho_s P^2}{2}+\int d^2\mathbf{r}\Bigg[\left(\beta-\frac{\rho_s P^2}{2}\right)m_z^2+\frac{\rho_s}{2}|\nabla\tilde{m}_y|^2 + \frac{tn}{2}(\tilde{m}_y^2+m_z^2)\cos(P-Q)x-tn\tilde{m}_y\sin(P-Q)x - \rho_s P\left(\frac{\partial\tilde{m}_y}{\partial x}\right). \tag{6.19}$$

To be consistent with the linearization assumptions, $P$ must be chosen so that $\partial\tilde{m}_y/\partial x$ integrates to 0 and this term can be dropped.

In Fourier space

$$E=\frac{A\rho_s P^2}{2}+\sum_{\mathbf{p}}\left\{\left(\beta-\frac{\rho_s P^2}{2}\right)|\tilde{m}_z(\mathbf{p})|^2+\frac{\rho_s q^2}{2}|\tilde{m}_y(\mathbf{p})|^2\right\} + \frac{tn}{4}\sum_{\mathbf{p}}\,(\tilde{m}_y(\mathbf{p})\{\tilde{m}_y[-\mathbf{p}+(P-Q)\hat{x}] + \tilde{m}_y[-\mathbf{p}+(Q-P)\hat{x}]\}+m_z(\mathbf{p})\{m_z[-\mathbf{p}+(P-Q)\hat{x}] + m_z[-\mathbf{p}+(Q-P)\hat{x}]\})-\frac{tn}{2i}\sqrt{A}\sum_{\mathbf{p}}\,\{\tilde{m}_y[(P-Q)\hat{x}] - \tilde{m}_y[(Q-P)\hat{x}]\}. \tag{6.20}$$

We first need to minimize this functional to determine the ground state. Since there is no term linear in $m_z$ it follows that $m_z(q)\equiv 0$ in the ground state. For ground state calculations we could work with the $XY$ model in both commensurate and incommensurate cases. Minimizing with respect to $\tilde{m}_y$ we find that in the ground state $\tilde{m}_y$ depends only on $x$ and that its $x$ dependence is determined in Fourier space by solving

$$\rho_s q_x^2\tilde{m}_y^0(q_x)+\frac{tn}{2}[\tilde{m}_y^0(q_x+Q_s]+m_y^0(q_x-Q_s)] = \frac{t\sqrt{A}}{4\pi\not< 2i}(\delta_{q_x,Q_s}-\delta_{q_x,-Q_s}). \tag{6.21}$$

We restrict our attention here to results which are valid to leading order in $t$ so that the second term on the left-hand side of Eq. (6.20) can be dropped. This gives

$$m_y^0(q_x)=\frac{tn\sqrt{A}}{2i\rho_s Q_s^2}(\delta_{q_x,Q_s}-\delta_{q_x,-Q_s}), \tag{6.22}$$

which gives a ground state energy per unit area equal to

$$\frac{E}{A}=-\frac{1}{4}\frac{(tn)^2}{\rho_s(Q-P)^2}+\frac{\rho_s P^2}{2}. \tag{6.23}$$

Minimizing this energy with respect to $P$ we find that the overall ground state occurs for

$$\frac{P}{Q}=\frac{1}{2}\frac{(tn)^2}{\rho_s Q^4}=\frac{\pi^4}{512}\left(\frac{Q_c}{Q}\right)^4. \tag{6.24}$$

This result agrees with the analytic expression given in Ref. 34 after making several corrections of some typos in Eqs. (2.7) and (2.8). For large parallel fields $P$ approaches 0, $Q_s$ approaches $Q$, and the ground state of the system asymptoti-

cally approaches the ground state in the absence of interlayer tunneling. We emphasize that as $Q$ approaches $Q_c$ from above $\tilde{m}_y^0$ can become large and the linearization approximations will fail even in the ground state.

Having identified the ground state pseudospin functional we are able to calculate the collective mode energies and response functions. To leading order in $t$ we find that

$$\dot{\tilde{m}}_y(\mathbf{q}) = -\frac{4\pi\ell^2}{\hbar}\frac{\partial E}{\partial m_z(-\mathbf{q})} = -\frac{2}{\hbar n}\Big[-h_z(\mathbf{q}) + (2\beta + \rho_s q^2 - \rho_s P^2) m_z(\mathbf{q}) + \frac{tn}{2}[m_z(\mathbf{q}+Q_s\hat{x}) + m_z(\mathbf{q}-Q_s\hat{x})]\Big], \quad (6.25)$$

$$\dot{\tilde{m}}_z(\mathbf{q}) = +\frac{4\pi\ell^2}{\hbar}\frac{\partial E}{\partial \tilde{m}_y(-\mathbf{q})} = \frac{2}{\hbar n}\Big[\rho_s q^2 \delta\tilde{m}_y(\mathbf{q}) + \frac{tn}{2} \times [\delta\tilde{m}_y(\mathbf{q}+Q_s\hat{x}) + \delta\tilde{m}_y(\mathbf{q}-Q_s\hat{x})]\Big]. \quad (6.26)$$

The soliton lattice acts like an internal grating[41] which couples collective excitations of the ground state in the absence of interlayer tunneling whose wave vectors are separated by multiples of $Q_s \approx Q$. The collective modes of the system for small $t$ will consist of the zone-folded modes of the $t=0$ system with small corrections due to mode-mode coupling. The response functions of the system are readily evaluated numerically from the above equations. Artificial external gratings are often[42] used to allow the infrared light to couple to finite-wave-vector excitations of two-dimensional electron systems. The soliton lattice appears to offer an opportunity to couple to the Goldstone collective mode of the $t=0$ system at a wave vector which can be tuned by the application of an in-plane magnetic field. We reemphasize that as $Q$ approaches $Q_c$ from above $\tilde{m}_y^0$ can become large and the linearization approximations will fail even in the ground state.

## VII. EXACT DIAGONALIZATION STUDIES

We now turn to a discussion of exact numerical studies of the commensurate-incommensurate transition, and in particular the $B_\parallel$ dependence of the energy gap. The importance of long length scales in this transition will limit the power of the exact diagonalization approach, but we are still able to obtain some useful insights into the experimental results obtained by Murphy *et al.*[4] We simulate a bilayer 2D quantum Hall system in the presence of a tilted magnetic field with a finite number of electrons, and perform our calculations on the torus. We make use of the formalism developed by Haldane[43] for the torus to block-diagonalize the Hamiltonian in each momentum sector. As noted in Sec. V, the in-plane component of the magnetic field enters only in the tunneling matrix elements. In order to keep the momentum a good quantum number, we are constrained to use only discrete values of the magnetic field, namely, the values which correspond to an integer number of flux quanta enclosed between the layers. We therefore study the transition for a fixed value of $B_\parallel$, and continuously vary the tunneling between the layers. This is to be contrasted with the experimental situation in which the tunneling is fixed and $B_\parallel$ varies continuously.

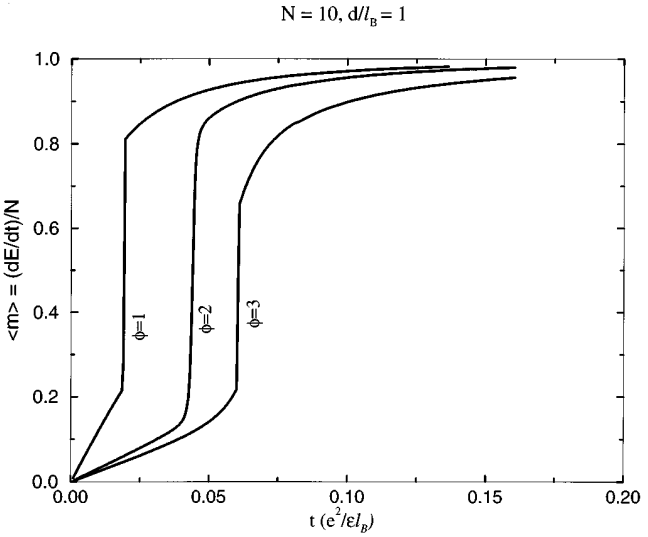


FIG. 5. Magnetization as a function of tunneling, for several integer numbers of flux quanta between the layers at $N=10$ and $d/\ell=1$. The curves are labeled by the integer number of flux quanta produced by the parallel component of the field in the finite size system.

Figure 5 shows the pseudomagnetization calculated as a function of tunneling amplitude for various fixed values of $B_\parallel$, for ten electrons and $d/\ell=1$. For fixed $B_\parallel$ the commensurate state which optimizes the tunneling energy at the cost of exchange energy occurs at large $t$. We see in Fig. 5 that the component of pseudomagnetization aligned with the effective Zeeman field increases with increasing $t$ and decreases with $B_\parallel$, even in the commensurate state. This quantum fluctuation effect is not captured in the classical field theoretic results we have presented since, strictly speaking, they apply only when the tunneling amplitude is small. The increase in the effective magnitude of the ordered moment with tunneling in the commensurate state may explain the discrepancy between the $t^{1/2}$ behavior of the ordered moment predicted here and the approximately linear behavior seen experimentally.[4]

In our finite-size studies the phase transition to the incommensurate state appears as a level crossing between states with different Haldane pseudomomenta which is accompanied by a large decrease in the component of the pseudospin aligned with the Zeeman field. In the thermodynamic limit and close to the phase transition, the pseudospin in the incommensurate state is expected to be aligned with the Zeeman field except in the domain walls of the soliton lattice. Thus the spatial average of aligned moment is expected to decrease continuously at the phase boundary.

In our finite size calculations the wave vector $Q$ corresponding to the rotating Zeeman field satisfies

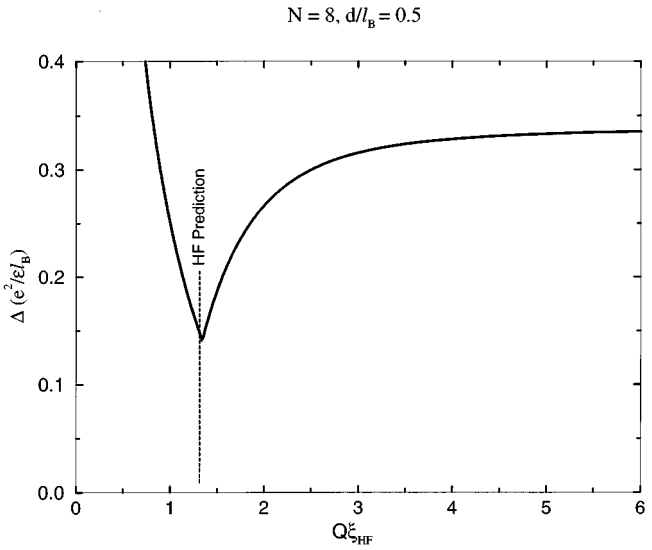


FIG. 6. The energy gap as a function of $Q\xi_{\mathrm{HF}}$, for $B_{\|}$ corresponding to one enclosed flux quantum, i.e., $N_{\phi}=1$, for $N=8$ and $d/\ell=0.5$.

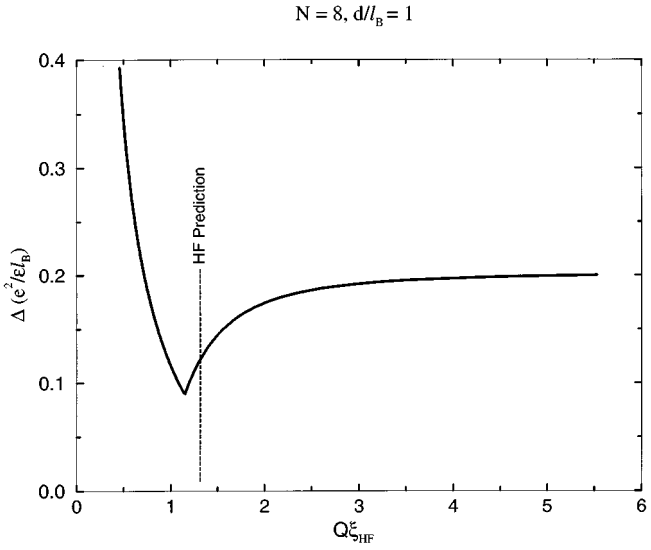


FIG. 7. The energy gap as a function of $Q\xi_{\mathrm{HF}}$, for $B_{\|}$ corresponding to one enclosed flux quantum, i.e., $N_{\phi}=1$, for $N=8$ and $d/\ell=1$.

$$Q\ell = N_{\phi}\left(\frac{2\pi}{N}\right)^{1/2}, \tag{7.1}$$

where $N$ is the number of states in a Landau level, $N_{\phi}=B_{\|}dL/\Phi_0$, and $L$ is the edge length for a square finite size square sample. For $N=10$, $Q\ell\approx 0.79$ even for $N_{\phi}=1$. Even the smallest value of $Q\ell$ that we can consider is close to the edge of the regime where we expect the long wavelength theory developed in previous sections to apply. We *do*, however, still expect the phase transition to occur outside this regime. In order to compare these microscopic calculations more directly with the long-wavelength theory we have evaluated the energy gap as a function of $t$. We present our results in terms of an estimate of the dimensionless parameter $Q\xi$ which plays the central role in the long-wavelength theory.

$$Q\xi = \frac{2\pi d\ell B_{\|}}{\phi_0}\left(\frac{2\pi\rho_s}{t}\right)^{1/2}. \tag{7.2}$$

To plot our results we have evaluated $Q\xi$ in the Hartree-Fock approximation; $t=\Delta_{\mathrm{SAS}}/2$, corresponding to full pseudospin polarization and

$$\rho_s^{\mathrm{HF}} = -\frac{1}{32\pi^2}\int_0^{\infty} dk\ V_k h(k) k^3, \tag{7.3}$$

where $V_k=(2\pi e^2/\epsilon k)\exp(-kd)$, and $h(k)=-\exp(-k^2/2)$. Both $\rho_s$ and $t$ are substantially reduced by quantum fluctuations. Previous estimates from finite-size exact diagonalization calculations[14] suggest that $t$ is reduced by a larger fraction than $\rho_s$, so that the Hartree-Fock approximation should increasingly underestimate $Q\xi$ as the layer separation increases. According to the PT model, the transition should occur when $Q\xi=4/\pi$. This will allow us to compare the HF prediction with the exact diagonalization result.

Figures 6, 7, and 8 show the $Q\xi_{\mathrm{HF}}$ dependence of the energy gap for $d/\ell=0.5$, 1, and 2, respectively. Note also that the system studied by Murphy *et al.* corresponds approximately to $d/\ell=1.85$. The cusp in the energy gap seen in these figures results from the ground state level crossing and is not obviously associated with the square root singularities in the energy gap predicted by Read[37] which would be expected to appear only in much larger systems than we are able to treat. Nevertheless, it does show the expected behavior of weakening at larger layer separations where the spin-stiffness and hence the meron-pair string energy is weakened. We also notice that Hartree-Fock theory accurately predicts the critical field for $d/\ell=0.5$, becomes worse at $d/\ell=1$ and fails badly at $d/\ell=2$. The direction of the discrepancy is in the direction anticipated by the above discussion since the value of $Q\xi_{\mathrm{HF}}$ at the transition is smaller than $4/\pi$. These results suggest that the mean source of discrepancy between the experiments of Murphy *et al.* and the

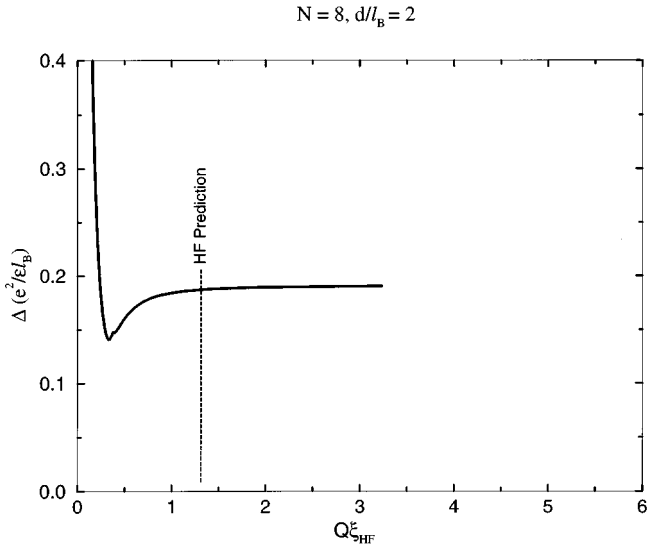


FIG. 8. The energy gap as a function of $Q\xi_{\mathrm{HF}}$, for $B_{\|}$ corresponding to one enclosed flux quantum, i.e., $N_{\phi}=1$, for $N=8$ and $d/\ell=2$.

PT theory with Hartree-Fock parameters is quantum fluctuations which reduce the ordered moment aligned by the effective magnetic field. This reduction is responsible for a proportional reduction in the energy gained by forming the commensurate state and reduces the parallel field strength at which the transition occurs. For parameters appropriate to the system studied by Murphy *et al.* our PT theory with $\rho_s$ and $t$ given by Hartree-Fock theory gives $B_\parallel = 1.3$ T whereas the experimental value is $B_\parallel^* \approx 0.8$ T. From Fig. 8, we find $B_\parallel^* \approx 0.6$ T, in substantially better agreement with experiment.

## VIII. SUMMARY

We have presented in this paper a theory of the interesting effects of a weak interlayer tunneling in double layer quantum Hall systems that spontaneously develop interlayer coherence in the absence of tunneling. We have discussed the properties of the ground state, as well as low energy collective excitations (neutral and charged) of the system, using both effective field theory and microscopic Hartree-Fock approaches. In particular, we have identified a commensurate-incommensurate phase transition driven by an in-plane component of the external magnetic field, which has been observed in recent experiments. Our theory is in good qualitative and semiquantitative agreement with experiments.

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