Longitudinal magnetoresistance of superlattices caused by barrier inhomogeneity

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Classical longitudinal magnetoresistance of superlattices is calculated in the framework of a model which includes fluctuations of barrier conductivity. We found that the result depends very significantly on the fluctuations correlation length. We also found that fluctuations of the electron potential are not uniform along the superlattice, and depend on the superlattice length. A good agreement between theory and experiment is obtained. [S0163-1829(96)01639-6]

I. INTRODUCTION

In this work we consider the vertical longitudinal magnetoresistance (LMR) of a superlattice; that is, the magnetoresistance in the geometry when both electric and magnetic fields are along the growth direction. Purely classical (i.e., without any quantum effects) LMR was observed many times in experiments, but only recently has a qualitative explanation been suggested. It is obvious that in an ideal superlattice classical LMR has to be zero, because the magnetic field does not affect electron motion parallel to it. For this reason experimentally observed LMR (Refs. 1–7) has not been explained for a rather long time. The qualitative explanation suggested by Lee *et al.*.[8] attributes this to a result of nonuniform fluctuations of the superlattice barriers' width.

In this paper we present resistance calculations for a superlattice with nonuniform barriers. We consider the case of a narrow-miniband superlattice when the vertical transport can be considered as sequential tunneling. Each barrier in this case can be characterized by a conductivity fluctuating around some average value. The opposite case of wide-band superlattices where an electron tunnels across a few barriers between two successive scattering events seems to be less interesting. The effect of the barrier width fluctuations is averaged out as a result of tunneling across a few barriers.

The qualitative picture of the longitudinal magnetoresistance of superlattices with nonuniform barriers suggested by Lee *et al.*[8] is as follows. A current across each barrier is larger in places where the conductivity is larger. If high-conductivity regions of adjacent barriers are not positioned against each other, then nonuniform currents across barriers induce in-plane currents between barriers. The magnetic field perpendicular to the layers brings about a transverse magnetoresistance, reducing these in-plane currents. As a result the current across barriers cannot pass through places with maximal conductivity. In this way the magnetic field in the growth direction increases the superlattice resistance in this direction.

The effective conductivity of a spatially inhomogeneous medium has been considered many times in the literature; see, e.g., the review paper by Landauer.[9] A superlattice is just another example of such a medium with a specific geometrical structure of the inhomogeneities. We consider this problem for weak fluctuations of the barrier conductivity (the exact parameter will be shown below). We also assume that the conductivity fluctuations of different barriers are not correlated. These assumptions allow us to obtain an analytic expression for the superlattice resistance.

II. PERTURBATION THEORY FOR POTENTIAL FLUCTUATIONS

The superlattice consists of $N+1$ wells separated by N barriers, and the electric potential in the νth well is $\phi_\nu(\mathbf{r})$, where $\mathbf{r}=(x,y)$ is the in-plane coordinate. The electric current $j_{\nu,\nu+1}(\mathbf{r})$ from well ν to well $\nu+1$ is given by Ohm's law,

$$j_{\nu,\nu+1}=\sigma^{\perp}_{\nu,\nu+1}(\phi_\nu-\phi_{\nu+1}), \tag{1}$$

where $\sigma^{\perp}_{\nu,\nu+1}(\mathbf{r})$ is the conductance per unit area of the barrier following the νth well. The formulation of the problem will be completed with the charge conservation law

$$j_{\nu,\nu+1}-j_{\nu-1,\nu}=\boldsymbol{\nabla}\hat{\sigma}\boldsymbol{\nabla}\phi_\nu, \tag{2}$$

where $\hat{\sigma}\boldsymbol{\nabla}\phi_\nu$ is an in-plane electric current in the νth well, and $\hat{\sigma}$ is a two-dimensional conductivity tensor of the well. We will assume that this tensor depends on the magnetic field but does not depend on coordinates. We will also assume that the conductivity in wells is isotropic so that $\sigma_{xx}=\sigma_{yy}=\sigma^{\parallel}$, and $\sigma_{xy}=-\sigma_{yx}$. This assumption gives

$$\boldsymbol{\nabla}\hat{\sigma}\boldsymbol{\nabla}=\sigma^{\parallel}\boldsymbol{\nabla}^2, \tag{3}$$

that is the Hall conductivity does not enter into the problem. Equations (1)–(3) have to be solved with some boundary conditions. We will assume that potentials in the first and last wells are independent of $\mathbf{r}$ due to the presence of the highly doped uniform plane contacts, $\phi_0=NU=\text{const}$ and $\phi_N=0$.

Equations (1) and (2) can be solved by means of perturbation theory with respect to the fluctuations of $\sigma^{\perp}_{\nu,\nu+1}$. In the Fourier representation

$$\sigma^{\perp}_{\nu,\nu+1}(\mathbf{r})=\sigma^{\perp}+\sum_{\mathbf{q}}\delta\sigma_{\nu,\mathbf{q}}e^{-i\mathbf{q}\cdot\mathbf{r}}, \tag{4}$$

where $\delta\sigma_{\nu,\mathbf{q}}$ is considered a small quantity, and $\delta\sigma_{\nu,\mathbf{0}}\equiv 0$. The conductivity fluctuations are assumed to be uniform, with a correlation length much shorter than the superlattice plane size, so that

$$\frac{\langle \delta\sigma_{\nu,\mathbf{q}}\delta\sigma^{*}_{\nu',\mathbf{q}'}\rangle}{(\sigma^{\perp})^2}=\xi_q\delta_{\nu,\nu'}\delta_{\mathbf{q},\mathbf{q}'}, \tag{5}$$

where $\langle\rangle$ mean an ensemble average over all possible fluctuation configurations. The function ξ_q is inverse proportional to the barrier area. This conductivity fluctuations induce fluctuations of the potential

$$\phi_\nu(\mathbf{r})=(N-\nu)U+\sum_{\mathbf{q}}\delta\phi_{\nu,\mathbf{q}}e^{-i\mathbf{q}\cdot\mathbf{r}}. \tag{6}$$

The Fourier transform of Eqs. (1)–(3) can be linearized with respect to fluctuations if $\mathbf{q}\neq\mathbf{0}$

$$\begin{aligned}&[(\sigma^{\parallel}/\sigma^{\perp})q^2+2]\delta\phi_{\nu,\mathbf{q}}-\delta\phi_{\nu-1,\mathbf{q}}-\delta\phi_{\nu+1,\mathbf{q}}\\&\quad=(U/\sigma^{\perp})(\delta\sigma_{\nu-1,\mathbf{q}}-\delta\sigma_{\nu,\mathbf{q}}).\end{aligned} \tag{7a}$$

The second-order terms have to be kept in the same equations for $\mathbf{q}=\mathbf{0}$,

$$\begin{aligned}&[2\delta\phi_{\nu,\mathbf{0}}-\delta\phi_{\nu-1,\mathbf{0}}-\delta\phi_{\nu+1,\mathbf{0}}]\sigma^{\perp}\\&\quad=\sum_{\mathbf{q}}[(\delta\phi_{\nu+1,-\mathbf{q}}-\delta\phi_{\nu,-\mathbf{q}})\delta\sigma_{\nu,\mathbf{q}}\\&\qquad-(\delta\phi_{\nu,-\mathbf{q}}-\delta\phi_{\nu-1,-\mathbf{q}})\delta\sigma_{\nu-1,\mathbf{q}}].\end{aligned} \tag{7b}$$

The solution to Eq. (7a) with the boundary conditions $\delta\phi_{0,\mathbf{q}}=\delta\phi_{N,\mathbf{q}}=0$ can be expressed in terms of the Green function,

$$G_{\nu,\nu'}(q)=\frac{1}{N}\sum_{j=1}^{N-1}\frac{\sin(\pi j\nu/N)\sin(\pi j\nu'/N)}{\cosh(a_q)-\cos(\pi j/N)} \tag{8}$$

$$=\frac{1}{\sinh(a_q)\sinh(a_qN)}\times\begin{cases}\sinh(a_q\nu)\sinh[a_q(N-\nu')], & \nu\leqslant\nu',\\ \sinh(a_q\nu')\sinh[a_q(N-\nu)], & \nu\geqslant\nu',\end{cases} \tag{9}$$

where

$$\cosh(a_q)=\left(1+\frac{\sigma^{\parallel}q^2}{2\sigma^{\perp}}\right). \tag{10}$$

One has

$$\delta\phi_{\nu,\mathbf{q}}=U\sum_{\nu'=1}^{N-1}G_{\nu,\nu'}(q)\frac{\delta\sigma_{\nu'-1,\mathbf{q}}-\delta\sigma_{\nu',\mathbf{q}}}{\sigma^{\perp}}, \tag{11}$$

with $\mathbf{q}\neq\mathbf{0}$. The derivation of the above expression for the Green function is given in Appendix A.

Physical properties of the result Eq. (11) can be seen from the average value of the potential fluctuations squared,

$$\begin{aligned}\langle|\delta\phi_{\nu,\mathbf{q}}|^2\rangle&=U^2\xi_q\sum_{\nu'=1}^{N-1}G_{\nu,\nu'}(q)[2G_{\nu,\nu'}(q)-G_{\nu,\nu'-1}(q)\\&\quad-G_{\nu,\nu'+1}(q)]\\&=U^2\xi_q\left[G_{\nu,\nu}(q)+\frac{q}{2}\frac{\partial}{\partial q}G_{\nu,\nu}(q)\right],\end{aligned} \tag{12}$$

where $G_{\nu,\nu}(q)$ is given by Eq. (9).

Equation (12) describes the increase of the potential fluctuations from the contacts toward the middle of the superlattice. For a limited region of q, three situations are conceivable. The first is the case of strong in-plane conductivity, when the potential fluctuations are limited by in-plane currents, and $\langle|\delta\phi_{\nu,\mathbf{q}}|^2\rangle,\nu\neq0$, N nearly does not depend on ν. The second is the opposite case, when in-plane currents are not important, and the fluctuations of the potential are similar to the fluctuations in a series of random resistors. In the third intermediate case the fluctuations increase with the distance from the contacts, but in the internal part of the superlattice they are limited by in-plane currents.

The first case is realized under the condition $\sinh(a_q)\gg1$; then Eqs. (9) and (12) give

$$\langle|\delta\phi_{\nu,\mathbf{q}}|^2\rangle=2\xi_q\left(\frac{U\sigma^{\perp}}{\sigma^{\parallel}q^2}\right)^2,\quad \nu\neq0,N \tag{13a}$$

This result is independent of ν and inverse proportional to the in-plane conductivity. In the second case, $a_q\ll1/N$, and we have

$$\langle|\delta\phi_{\nu,\mathbf{q}}|^2\rangle=U^2\xi_q\frac{\nu(N-\nu)}{N}. \tag{13b}$$

In the third intermediate case, $1/N\lesssim a_q\ll1$, and we have to consider separately the internal region of superlattice and the regions near the contacts,

$$\langle|\delta\phi_{\nu,\mathbf{q}}|^2\rangle=U^2\xi_q\times\begin{cases}\nu, & a_q\nu\ll1\\ N-\nu, & a_q(N-\nu)\ll1\\ 1/a_q & \text{otherwise.}\end{cases} \tag{13c}$$

In order to calculate the correction to the average potential, we have to substitute Eq. (11) into Eq. (7b). This leads to a difference equation for $\delta\phi_{\nu,\mathbf{0}}$, which should be averaged with the help of Eq. (5). The solution to the obtained equation is

$$\langle\delta\phi_{\nu,\mathbf{0}}\rangle=-U\sum_{\mathbf{q}}\frac{\xi_q}{\sigma^{\parallel}q^2/\sigma^{\perp}+4}\left[\frac{\sinh[a_q(N-2\nu)]}{\sinh(a_qN)}+\frac{2\nu-N}{N}\right]. \tag{14}$$

The ν dependence of the averaged potential, $(N-\nu)U+\langle\delta\phi_{\nu,\mathbf{0}}\rangle$ becomes more smooth near the contacts; that is, near the contacts the electric field is weaker.

One can prove that the perturbation theory developed here is justified if the fluctuations of the potential are small,

$$\sum_{\mathbf{q}}\langle|\delta\phi_{\nu,\mathbf{q}}|^2\rangle\ll U^2. \tag{15}$$

The substitution of the results obtained above, Eqs. (13) and (14), to this condition leads to

$$\Xi\equiv\sum_q\xi_q\ll\max\left\{\frac{1}{N},q_0\left(\frac{\sigma^{\parallel}}{\sigma^{\perp}}\right)^{1/2},\frac{\sigma q_0^2}{\sigma^{\perp}}\right\}. \tag{16}$$

Here q_0 is the characteristic wave number of the function ξ_q, and the quantity $1/q_0$ can be considered as a conductivity fluctuation characteristic correlation length.

III. AVERAGED CURRENT AND LONGITUDINAL MAGNETORESISTANCE

The total current across a barrier,

$$j=\sigma^{\perp}U-\delta j, \tag{17}$$

is the same for all barriers and can be calculated for the first barrier. Substitution of Eqs. (4), (7), and (11) in Eq. (1) with $\nu=0$, averaging over all possible fluctuations of the barrier conductivity and summation over all q, give

$$\delta j=\sigma^{\perp}\left\{U\sum_{\mathbf{q}}\ \xi_q G_{1,1}(q)+\langle\delta\phi_{1,\mathbf{0}}\rangle\right\} \tag{18}$$

$$=\sigma^{\perp}U\sum_{\mathbf{q}}\ \frac{2\xi_q}{\sigma^{\|}q^2/\sigma^{\perp}+4}\left[\frac{N-1}{N}+\frac{\sinh[a_q(N-1)]}{\sinh(a_qN)}\right]. \tag{19}$$

This equation is the main result of our paper. It describes the change of the current due to barrier conductivity fluctuations. The sign of δj is equal to the sign of the current without fluctuation; that is, fluctuations lead to an increase of the superlattice resistance.

We are particularly interested in the application of this result to a calculation of the magnetoresistance of the superlattices. Here we consider only a weak magnetic field when $\Omega_H\tau\ll 1$, where Ω_H is the cyclotron frequency and τ is a relaxation time. In this case the magnetic-field-induced change of the in-plane conductivity is $\sigma^{\|}(0)-\sigma^{\|}(H)\approx\Omega_H^2\tau^2\sigma^{\|}(0)$, and the superlattice magnetoresistance becomes

$$\frac{R(H)-R(0)}{R(0)}=-\frac{\Omega_H^2\tau^2\sigma^{\|}}{U\sigma^{\perp}}\frac{\partial}{\partial\sigma^{\|}}\delta j. \tag{20}$$

In transport theory, surface roughness is often approximated by a Gaussian function. Such an approximation immediately gives the Gaussian form for the barrier conductivity fluctuations' correlation function, i.e.,

$$\xi_q=\frac{4\pi\Xi}{q_0^2S}e^{-q^2/q_0^2}, \tag{21}$$

where S is the area of the barrier, and Ξ is standard deviation of the normalized barrier conductivity, which is defined generally in Eq. (16). Substitution of Eq. (21) into Eq. (20) allows us to evaluate the superlattice magnetoresistance

$$\frac{R(H)-R(0)}{R(0)}=\Omega_H^2\tau^2\sum_{\mathbf{q}}\ \xi_q \times\begin{cases}\sigma^{\|}q^2N/(6\sigma_{\perp}), & \gamma N^2\ll 1\\ \dfrac{2[\sigma^{\|}q^2/\sigma^{\perp}]^{1/2}}{[\sigma^{\|}q^2/\sigma^{\perp}+4]^{3/2}}, & \gamma N^2\gg 1\end{cases}$$

$$\approx\Xi\Omega_H^2\tau^2\times\begin{cases}\gamma N, & \gamma N^2\ll 1\\ \sqrt{\gamma}, & \gamma\ll 1\\ \ln\gamma/\gamma, & \gamma\gg 1\,,\end{cases} \tag{22}$$

where $\gamma=\sigma^{\|}q_0^2/\sigma^{\perp}$. One can see from Eq. (22) that the magnetoresistance disappears for both very small and very large q_0. The reasons for this, however, are different. The former is a case of effectively ''metallic'' superlattice wells. They are almost equipotential planes, in-plane currents are small, and the magnetoresistance is also small. In addition, in this case barrier conductivity fluctuations are averaged out and δj itself goes to zero. The latter case is a case of effectively ''dielectric'' planes. The high-conductivity regions of adjacent barriers are located far from each other. In this case the conductances of the in-plane path are small, and the in-plane currents are also small.

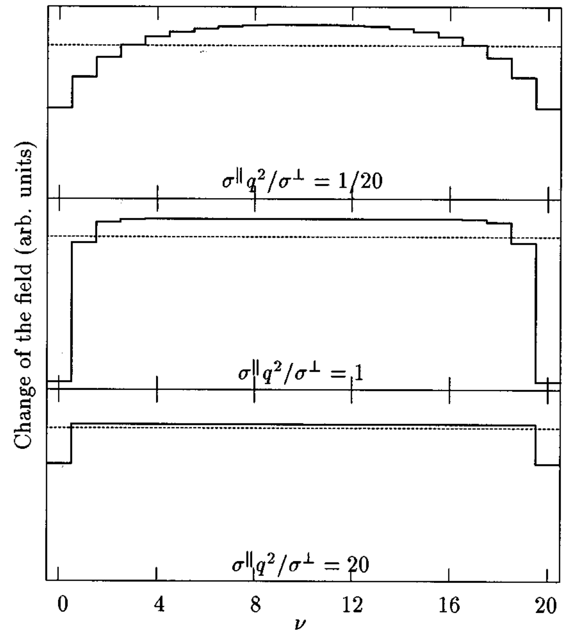

FIG. 1. Contribution of the terms with different wave vector to the change of the averaged field. One can see that the field becomes higher at the middle of superlattice and lower near the contacts. The scales on all the graphs are the same, and the dashed lines show zeros of the field change.

IV. DISCUSSION AND SUMMARY

Beside the correction to the current, the surface roughness of superlattice barriers leads to a quite unexpected result: distribution of an electric field along the superlattice appears to be nonuniform. Indeed, for the correction to the average potential drop across one barrier, Eq. (14) gives

$$\delta\phi_{\nu-1,0}-\delta\phi_{\nu,0}=U\sum_{\mathbf{q}}\ \frac{\xi_q}{\sigma^{\|}q^2/\sigma^{\perp}+4}\times\left[\frac{2}{N}-\frac{\sinh(a_q)\cosh[a_q(2\nu-1-N)]}{\sinh(a_qN)}\right]. \tag{23}$$

In the middle of the superlattice the sign of this quantity is the same as that of the potential drop without surface roughness, U, and near the contacts it is the opposite, see Fig 1. That is, because of surface roughness the field becomes stronger at the middle and weaker near the contacts. The size of the contact regions is about $1/a_q$ periods. The redistribution of the field along the superlattice is not a large effect, but it can be stronger for more pronounced surface roughness. The physical reason for the field redistribution is that the current is trying to go across the least resistive regions of the barriers. Because of the lack of correlation of the surface roughness in different barriers, this produces an in-plane current which makes the overall resistance larger. Near the contacts where the in-plane potential redistribution is not fully developed, this effect is suppressed.

The correction to the current due to surface roughness strongly depends on $\sigma^{\parallel}q_0^2/\sigma^{\perp}$. This parameter may significantly vary in experiments. Its value can be estimated interms of microscopic parameters of the superlattice. We can estimate $\sigma^{\perp}\sim(me^2\Lambda^2\tau\hbar^{-4})[1-\exp(E_F/T)]$ (Ref. 10), where e and m are the electron charge and mass, respectively, Λ is the transition amplitude between adjacent wells, τ is the relaxation time, E_F is the Fermi energy, T is the temperature, and Λ, $\hbar/\tau$ are assumed to be much less than the maximum of E_F, T. In this case we have

$$\sigma^{\parallel}q_0^2/\sigma_{\perp}\approx\frac{\hbar^2q^2}{m\Lambda^2}\max(E_F,T)\approx\frac{(lq)^2\hbar^2}{(\Lambda\tau)^2}, \qquad (24)$$

where l is the in-plane mean free path. For the conductivity in this expression we used the classical result $\sigma^{\parallel}\sim ne^2\tau/m$, where n is the two-dimensional electron concentration. This expression as well as the phenomenological Eq. (2) is correct under the condition $ql\ll1$. The other basic equation, Eq. (1), is justified only under the condition of sequential tunneling, i.e., $\hbar/(\Lambda\tau)\gg1$. That is, the right-hand side of Eq. (24) is the product of a large factor and a small factor, so that all cases in Eq. (22) are possible. In these three cases the temperature dependences of the magnetoresistance are T, $T^{1/2}$, and $\ln(T)/T$, respectively.

The condition $ql\ll1$ means that the characteristic scale of the surface roughness is much larger than the mean free path. The theory can be easily generalized for the case when this condition is not satisfied. The in-plane conductivity in Eq. (2) is a response to a uniform electric field. If the electric field is nonuniform at the scale of the mean free path, then the current conservation law, Eq. (2), holds, but the in-plane conductivity cannot be taken from the phenomenological theory and should be calculated with the help of the Boltzmann equation. The calculation is carried out in Appendix B, and the resulting conductivity depends on q. The only modification in the previous theory is that the expression $\sigma^{\parallel}q^2$ is determined now by Eq. (B5). The estimate of the magnetoresistance, Eq. (22), which was done before for the phenomenological case, is replaced in the limit $q_0l\gtrsim1$ by

$$\frac{R(H)-R(0)}{R(0)}=\Xi\Omega_H^2\tau^2\frac{\Lambda^2\tau^2}{\hbar^2q_0^2l^2}\ln\left(\frac{\hbar^2}{\Lambda^2\tau^2}\right). \qquad (25)$$

In the calculation of the conductivity we neglect quantum corrections. That is justified when the magnetic quantization is smeared by scattering, $\Omega_H\tau\ll1$, or at high enough temperature, when $\hbar\Omega_H\lesssim T$.

The comparison of the our result with available experimental data is difficult because not all parameters necessary for theoretical calculations are known. Here we compare our results with the measurements of Ref. 8, taking the relaxation time and the characteristic length of interface roughness as adjustable parameters. The widths of wells (d_w) and barriers (d_B) and measured magnetoresistance and barrier conductances in four measured samples are summarized in Table I. We assume that fluctuations of the transition amplitude between adjacent wells, $\Delta\Lambda$, result from the fluctuations of the width of the barrier by ±1 ML. The known geometry of the structures allowed us to calculate Λ, and the fluctuations of the barrier conductance were evaluated according to $\Xi\approx(2\Delta\Lambda/\Lambda)^2$. The conductivities in wells were calculated according to $\sigma^{\parallel}=ne^2\tau/m$, where the two-dimensional electron concentration $n=d_w\times10^{17}$ cm^{-3}. The dependence of the magnetoresistance on parameters of the samples can be written in the form

$$\frac{R(H)-R(0)}{R(0)H^2}\frac{m^2}{e^2\tau^2\Xi}=f\left(\frac{\sigma^{\parallel}q_0^2}{\sigma^{\perp}}\right). \qquad (26)$$

For the function on the right-hand side, Eq. (22) gives

$$f(\gamma)=\int_0^{\infty}dxe^{-x}\left(\frac{4\gamma x}{(\gamma x+4)^3}\right)^{1/2}. \qquad (27)$$

In Fig. 2 we show the theoretical curve for this function and

TABLE I. Summary of experimental data, which was used for comparison with theory.

(d_w/d_B) (Å/Å)	Ξ	$\Delta R/(RH^2)$ T^{-2}	$\sigma^{\perp}=N/(RS)$ S/cm^2
(50/50)	0.36	0.027	1200
(20/80)	0.09	0.032	5500
(80/20)	0.25	0.042	4200
(20/40)	0.19	0.047	14000

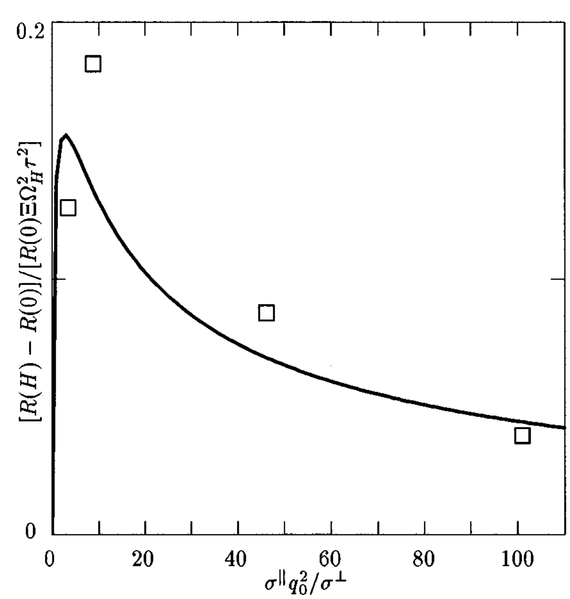

FIG. 2. Comparison of the experimental data of Ref. 8 (open squares), with the theoretical prediction of Eq. (26), solid line. Combinations $[R(H)-R(0)]/[\Xi R(0)\Omega_H^2\tau^2]$ and $\sigma^{\parallel}q_0^2/\sigma^{\perp}$ are calculated from the experimental data summarized in Table I, with two fitting parameters $\tau=5.3\times10^{-13}$ s and $q_0=3.3$ μm^{-1}.

the experimental results for four samples. The best fit is obtained for $\tau = 5.3\times10^{-13}$ s and $q_0 = 3.3\ \mu\mathrm{m}^{-1}$. We have to note an obvious qualitative and good quantitative agreement between the theory and experiment. The discrepancy (no more than 10%) can be attributed to slightly different relaxation times τ and surface roughness characteristics in different samples.

The results of the fitting give $q_0 l \approx 0.2$ that justifies the phenomenological expression for $\sigma^{\parallel}$ used in the calculations. The estimate of the surface roughness relaxation time for a quantum well with $d_w = 50$ Å and $q_0 = 3.3\ \mu\mathrm{m}^{-1}$ gives a value of 2×10^{-10} s. That is, the dominant scattering mechanism is probably impurity scattering, that explains approximately equal relaxation times in samples with different values of d_w. The surface roughness correlation length of 3000 Å seems large, but even by the order of magnitude larger values have been reported.[11]

In summary, we calculated the correction to the superlattice resistance due to nonuniform fluctuations of the conductivity of each of the barriers. Our results explain the classical longitudinal magnetoresistance of superlattices. We found that the magnetoresistance has a nontrivial dependence on characteristic length scale of fluctuations; it goes to zero for both very large-scale and very small-scale fluctuations. The fluctuations of the barrier conductivity lead also to a nonuniform distribution of electric field along superlattice. The results of the theory give a good quantitative agreement with experimental data.

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APPENDIX A: DERIVATION OF THE GREEN FUNCTION

Trigonometric sums can be calculating with the help of the equation

$$2\cosh(\lambda)u(\nu) - u(\nu+1) - u(\nu-1) = 0, \quad 0<\nu<N. \tag{A1}$$

The general solution to Eq. (A1) has the form

$$u(n) = C_1 e^{\lambda\nu} + C_2 e^{-\lambda\nu}, \tag{A2}$$

and the eigenvalues and the eigenfunctions of Eq. (28) with the boundary conditions $u_0 = u_N = 0$ are

$$\cosh(\lambda_j) = \cos(\pi j/N), \quad 1 \leqslant j \leqslant N-1,$$
$$u_j(\nu) = \left(\frac{2}{N}\right)^{1/2} \sin(\pi j\nu/N). \tag{A3}$$

Let us consider the function $G_{\nu,\nu'}$ satisfying the equation

$$2\cosh(\lambda)G(\nu,\nu') - G_{\nu+1,\nu'} - G_{\nu-1,\nu'} = \delta_{\nu,\nu'} \tag{A4}$$

with the boundary conditions $G_{0,\nu'} = G_{N,\nu'} = 0$. This can be expressed in terms of the eigenfunctions Eq. (A3),

$$G_{\nu,\nu'} = \frac{1}{N}\sum_{j=1}^{N-1} \frac{\sin(\pi j\nu/N)\sin(\pi j\nu'/N)}{\cosh(\lambda) - \cosh(\lambda_j)} \tag{A5}$$

and this is the first line in Eq. (8), with $\lambda \equiv a_q$.

The same function can be calculated in another way with the help of the general solution Eq. (29). Apparently, Eq. (A4) with the boundary conditions is satisfied by the function

$$G_{\nu,\nu'} = A\sinh(\lambda\nu), \quad \nu<\nu',$$
$$G_{\nu,\nu'} = B\sinh[\lambda(N-\nu)], \quad \nu>\nu'. \tag{A6}$$

The only problem is with the values $\nu=\nu'$. The values of A, B, and $G_{\nu',\nu'}$ can be obtained by substitution of the solutions Eq. (A6) into Eq. (A4) for $\nu=\nu', \nu'\pm1$, that leads to the system of equations

$$A\sinh(\lambda\nu') - G_{\nu',\nu'} = 0,$$
$$A\sinh[\lambda(\nu'-1)] - 2\cosh(\lambda)G_{\nu',\nu'} + B\sinh[\lambda(N-\nu'-1)] = -1, \tag{A7}$$
$$G_{\nu',\nu'} - B\sinh[\lambda(N-\nu')] = 0,$$

which has a solution

$$A = \frac{\sinh[\lambda(N-\nu')]}{\sinh(\lambda)\sinh(\lambda N)},$$
$$G_{\nu',\nu'} = \frac{\sinh(\lambda\nu')\sinh[\lambda(N-\nu')]}{\sinh(\lambda)\sinh(\lambda N)}, \tag{A8}$$
$$B = \frac{\sinh(\lambda\nu')}{\sinh(\lambda)\sinh(\lambda N)},$$

and the final expression for the Green function is

$$G_{\nu,\nu'} = \begin{cases} \dfrac{\sinh(\lambda\nu)\sinh[\lambda(N-\nu')]}{\sinh(\lambda)\sinh(\lambda N)}, & \nu\leqslant\nu' \\ \dfrac{\sinh(\lambda\nu)'\sinh[\lambda(N-\nu)]}{\sinh(\lambda)\sinh(\lambda N)}, & \nu\geqslant\nu'. \end{cases} \tag{A9}$$

This result is also an explicit expression for the sum in Eq. (A5), and this is the second expression in Eq. (9).

APPENDIX B: WAVE-VECTOR-DEPENDENT IN-PLANE MAGNETOCONDUCTIVITY

The Fourier transformation of the Boltzmann kinetic equation for the two-dimensional electron gas in the conducting layer is

$$i\mathbf{q}\cdot\mathbf{v}f_{\mathbf{q}} + \frac{e}{c}[\mathbf{v}\times\mathbf{H}]\frac{\partial f_{\mathbf{q}}}{\partial\mathbf{p}} - ie\mathbf{q}\cdot\mathbf{v}\phi_{\mathbf{q}}\frac{\partial f}{\partial E} = -\frac{f_{\mathbf{q}}}{\tau}, \tag{B1}$$

where $f(E)$ is the equilibrium distribution function, $f_{\mathbf{q}}$ is the Fourier transform of the distribution function perturbation, $\phi_{\mathbf{q}}$ is the fluctuation of the electron potential, and $\mathbf{H}$ is the magnetic field, which is considered to be perpendicular to the layers, i.e, $\mathbf{q}\perp\mathbf{H}$. By introducing $\mathbf{q}=(q,0,0)$, $\mathbf{v}=(v\cos(\theta), v\sin(\theta), 0)$, and $\mathbf{H}=(0,0,(mc/e)\Omega_H)$, we reduce the kinetic equation to

$$\Omega_H \frac{\partial f_{\mathbf{q}}}{\partial \theta} - \left[\frac{1}{\tau} + iqv\cos(\theta)\right] f_{\mathbf{q}} = -iqve\phi_{\mathbf{q}}\cos(\theta)\frac{\partial f}{\partial E}. \tag{B2}$$

It is easy to solve this first-order differential equation, and the answer is a function of v and θ:

$$f_{\mathbf{q}} = e\phi_{\mathbf{q}}\frac{\partial f}{\partial E}\left\{1 - \int_0^\infty \frac{d\theta'}{\Omega_H\tau}\exp\left(\frac{-\theta'}{\Omega_H\tau} + i\frac{qv}{\Omega_H}[\sin(\theta) - \sin(\theta'+\theta)]\right)\right\} \tag{B3}$$

For Eq. (2) only the current divergence is necessary,

$$i\mathbf{q}\cdot\mathbf{j}_{\mathbf{q}} = \int \frac{2d\mathbf{p}}{(2\pi\hbar)^2} ie\mathbf{q}\cdot\mathbf{v}f_{\mathbf{q}} = \sigma^{\parallel}q^2\phi_{\mathbf{q}}. \tag{B4}$$

The last equality is the definition of $\sigma^{\parallel}$. An integration with respect to angles in Eq. (40) can be performed, but the integration from Eq. (39) still remains

$$\sigma^{\parallel}q^2 = -\frac{e^2}{\tau}\frac{m}{\pi\hbar^2}\int_0^\infty dE\,\frac{\partial f}{\partial E} \times \int_0^\infty d\theta' e^{-\theta'}\left[1 - J_0\left(2\frac{qv}{\Omega_H}\sin\frac{\Omega_H\tau\theta'}{2}\right)\right]. \tag{B5}$$

For the case $ql \ll \max(\Omega_H\tau, 1)$, this equation gives the usual classical result,

$$\sigma^{\parallel}(H) = \frac{\sigma^{\parallel}(0)}{1+\Omega_H^2\tau^2}. \tag{B6}$$

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