Thermal noise in superconducting quantum point contacts

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We present a theory for the frequency-dependent current fluctuations in superconducting quantum point contacts (SQPC) within the dc transport regime. This theory is valid for any barrier transparency between the tunnel and ballistic limits, yielding an analytical expression for the fluctuations spectrum in the subgap region. It is shown that the level of noise in a quasiballistic SQPC may have a huge increase in comparison with the case of a normal contact carrying the same average current. The effect of this high level of noise on the actual observability of the current-phase relation for a ballistic point contact is discussed in connection with recent experimental measurements.

Present technologies make it possible to fabricate superconducting point-contacts in the nanometer scale. Examples of these kind of systems are the recently developed atomic size break-junctions[1] and the split-gate superconductor–two-dimensional electron gas–superconductor junction of Takayanagi *et al.*[2] In both cases the electronic transport takes place through a reduced number of quantum channels, the contact transmission being a controllable quantity. These features make these systems very attractive for testing theoretical models of the superconducting transport beyond tunnel conditions.

Recently, there have been a number of theoretical works devoted to a detailed analysis of both the dc and ac response of a single channel point contact.[3–5] In particular, illuminating results have been obtained for the ac current in the hitherto less understood limit of small bias voltages.[3,5] However, little attention has been paid to the effect of thermal fluctuations in the transport properties of this kind of device. This is an important issue both due to its intrinsic interest as a nonequilibrium phenomena and also because they limit the observability of the measured characteristics. It is clear that thermal fluctuations have to be taken into account if a direct comparison between theory and experimental results is to be carried out. Regarding this last point, some recent experiments[6] have shown the deviation of the measured current-phase relation in a mechanically controllable break junction with respect to the theoretical predicted one.[7] Some authors have recently pointed out the importance of thermal fluctuations as a source for this deviation.[8]

The aim of this paper is to present a theory for the thermal current fluctuations of a superconducting quantum point contact (SQPC) in the dc regime valid for any contact transmission. This theory yields an analytical expression for the zero-frequency noise which, in the limit of low barrier transparencies, differs strongly from the standard tunnel theory result.[9] In the opposite limit, i.e., for a ballistic contact, we find that the current fluctuations *diverge* when the supercurrent tends to its maximum value. We claim that this fact explains the difficulties found for the experimental observation of the predicted current-phase relationship for a ballistic contact.

In recent works we have introduced a theoretical approach for the study of the transport properties of superconducting nanoscale constrictions.[3,10] In this approach the system is described by a Hamiltonian written in a site representation, from which the microscopic Bogoliubov–de Gennes equations can be derived.[10] Within this model the normal transmission coefficient through the constriction can be expressed in terms of microscopic parameters, allowing one to establish a complete correspondence with other approaches based on scattering theory.

For our present purpose of describing an atomic size contact, it will be sufficient to analyze the following Hamiltonian:[3]

$$\hat{H}=\hat{H}_L+\hat{H}_R+\sum_{\sigma}\ (te^{i\phi/2}\hat{c}^{\dagger}_{L\sigma}\hat{c}_{R\sigma}+te^{-i\phi/2}\hat{c}^{\dagger}_{R\sigma}\hat{c}_{L\sigma}), \quad (1)$$

where $\hat{H}_L$ and $\hat{H}_R$ are the BCS Hamiltonians for the uncoupled electrodes (defined as L and R), t is the hopping parameter which defines the normal transmission through the single quantum channel connecting both electrodes, and ϕ is the total superconducting phase difference between the electrodes. In the present calculations we shall neglect fluctuations in this superconducting phase difference and concentrate on the contribution to the current fluctuations arising from thermal excitation of quasiparticles.

We would like to emphasize that starting from these simple contact model results which are in complete agreement with those of scattering theory have been obtained.[3,10] A detailed discussion on the equivalence between both approaches for N-S and S-S contacts will be given elsewhere.[11]

Within this model, the operator associated with the current through the contact can be written as

$$\hat{I}(\tau)=\frac{ie}{\hbar}\sum_{\sigma}\left[te^{i\phi/2}\hat{c}^{\dagger}_{L\sigma}(\tau)\hat{c}_{R\sigma}(\tau)-te^{-i\phi/2}\hat{c}^{\dagger}_{R\sigma}(\tau)\hat{c}_{L\sigma}(\tau)\right], \tag{2}$$

where the different creation and annihilation operators appearing in Eq. (2) are the usual Heisenberg operators at a given time τ. Then, the spectral density of the current fluctuations is defined as

$$S(\omega)=\hbar\int d\tau\, e^{i\omega\tau}[\langle\delta\hat{I}(\tau)\delta\hat{I}(0)\rangle+\langle\delta\hat{I}(0)\delta\hat{I}(\tau)\rangle], \tag{3}$$

where $\delta\hat{I}(\tau)\equiv\hat{I}(\tau)-\langle\hat{I}\rangle$.

For the evaluation of the above averages, we perform a decoupling procedure which is consistent with the BCS mean-field theory. The spectrum $S(\omega)$ can then be expressed in terms of the single-particle nonequilibrium Green functions (Ref. 12) $\hat{G}^{+,-}_{\alpha\beta}(\Omega)$ and $\hat{G}^{-,+}_{\alpha\beta}(\Omega)$ (where α and β can be either L or R). In a superconducting broken symmetry representation (Ref. 13) $\hat{G}^{+-}_{\alpha\beta}(\Omega)$ is defined by

$$\hat{G}^{+-}_{\alpha\beta}(\Omega)=\int d\tau\, e^{i\Omega\tau}\hat{G}^{+-}_{\alpha\beta}(\tau,0), \tag{4}$$

with

$$\hat{G}^{+-}_{\alpha\beta}(\tau,0)=i\begin{pmatrix}\langle\hat{c}^{\dagger}_{\beta\uparrow}(0)\hat{c}_{\alpha\uparrow}(\tau)\rangle & \langle\hat{c}_{\beta\downarrow}(0)\hat{c}_{\alpha\uparrow}(\tau)\rangle\\ \langle\hat{c}^{\dagger}_{\beta\uparrow}(0)\hat{c}^{\dagger}_{\alpha\downarrow}(\tau)\rangle & \langle\hat{c}_{\beta\downarrow}(0)\hat{c}^{\dagger}_{\alpha\downarrow}(\tau)\rangle\end{pmatrix},$$

and $\hat{G}^{-+}_{\alpha\beta}(\tau,0)=[\hat{G}^{+-}_{\beta\alpha}(0,\tau)]^{\dagger}$.

In terms of the functions $\hat{G}^{+-}_{\alpha\beta}(\Omega)$ and $\hat{G}^{-+}_{\alpha\beta}(\Omega)$, $S(\omega)$ adopts the form

$$\begin{aligned}S(\omega)=\frac{e^2}{\hbar}\int d\Omega\,\mathrm{Tr}[&\hat{t}\hat{G}^{+-}_{RL}(\Omega)\hat{G}^{-+}_{RL}(\Omega+\omega)\hat{t}\\ &+\hat{t}\hat{G}^{+-}_{LR}(\Omega)\hat{G}^{-+}_{LR}(\Omega+\omega)\hat{t}-\hat{t}\hat{G}^{+-}_{LL}(\Omega)\hat{G}^{-+}_{RR}(\Omega+\omega)\\ &\times\hat{t}-\hat{t}\hat{G}^{+-}_{RR}(\Omega)\hat{G}^{-+}_{LL}(\Omega+\omega)\hat{t}+\hat{t}\hat{G}^{+-}_{RL}(\Omega+\omega)\\ &\times\hat{G}^{-+}_{RL}(\Omega)\hat{t}+\hat{t}\hat{G}^{+-}_{LR}(\Omega+\omega)\hat{G}^{-+}_{LR}(\Omega)\hat{t}\\ &-\hat{t}\hat{G}^{+-}_{LL}(\Omega+\omega)\hat{G}^{-+}_{RR}(\Omega)\hat{t}-\hat{t}\hat{G}^{+-}_{RR}(\Omega+\omega)\\ &\times\hat{G}^{-+}_{LL}(\Omega)\hat{t}],\end{aligned} \tag{5}$$

where $\hat{t}$ is the hopping interaction between the electrodes written in the (2×2) Nambu representation

$$\hat{t}=\begin{pmatrix}te^{i\phi} & 0\\ 0 & -te^{-i\phi}\end{pmatrix}. \tag{6}$$

In the present paper we concentrate in the zero-voltage case in which the average current is due to Cooper pairs. For the calculation of the Keldyhs Green functions $\hat{G}^{+,-}$ and $\hat{G}^{-,+}$ appearing in Eq. (5) we can the use the relations[10]

$$\begin{aligned}\hat{G}^{+,-}_{\alpha,\beta}(\Omega)&=[\hat{G}^{a}_{\alpha\beta}(\Omega)-\hat{G}^{r}_{\alpha\beta}(\Omega)]f(\Omega),\\ \hat{G}^{-,+}_{\alpha,\beta}(\Omega)&=-[\hat{G}^{a}_{\alpha\beta}(\Omega)-\hat{G}^{r}_{\alpha\beta}(\Omega)][1-f(\Omega)],\end{aligned} \tag{7}$$

where $f(\Omega)$ is the Fermi factor and $\hat{G}^{r,(a)}_{\alpha\beta}$ are the retarded (advanced) Green functions of the coupled contact. These last quantities can be obtained up to infinite order in the coupling parameter t by solving the following Dyson equation:

$$\hat{G}^{r,(a)}_{\alpha\beta}(\Omega)=\hat{g}^{r,(a)}_{\alpha\beta}(\Omega)\delta_{\alpha\beta}+\sum_{\gamma}\hat{g}^{r,(a)}_{\alpha\alpha}(\Omega)\hat{\Sigma}^{r,(a)}_{\alpha\gamma}\hat{G}^{r,(a)}_{\gamma\beta}(\Omega), \tag{8}$$

where $\hat{\Sigma}^{r,(a)}_{LL}=\hat{\Sigma}^{r,(a)}_{RR}=0$ and $\hat{\Sigma}^{r,(a)}_{LR}=(\hat{\Sigma}^{r,(a)}_{RL})^{*}=\hat{t}$. The indexes α, β, and γ can be either L or R, and $\hat{g}^{r,(a)}_{\alpha\alpha}$ are the retarded (advanced) Green functions corresponding to the left and right uncoupled electrodes.

For the symmetric case, both electrodes have the same modulus of the superconducting order parameter, Δ, and these Green functions can be expressed as

$$\begin{aligned}\hat{g}^{r,(a)}_{LL}(\omega)&=\hat{g}^{r,(a)}_{RR}(\omega)\\ &=\frac{1}{W\sqrt{\Delta^2-(\omega\pm i\eta)^2}}\begin{pmatrix}-\omega\pm i\eta & \Delta\\ \Delta & -\omega\pm i\eta\end{pmatrix},\end{aligned} \tag{9}$$

where W is an energy scale related to the normal density of states at the Fermi level by $\rho(\epsilon_F)=1/(\pi W)$ and η is a small energy relaxation rate that takes into account the damping of the quasiparticle states due to inelastic processes inside the electrodes. This parameter can be estimated from the electron-phonon interaction to be a small fraction of Δ.[14] It is useful to define the normal transmission coefficient of the contact, which in terms of W and t has the form $\alpha=4(2t/W)^2/[1+(2t/W)^2]^2$.[15] The spectral densities that are obtained from Eq. (8) are no longer singular at the gap edges and exhibit poles inside the superconducting gap, located at energies $\omega_S=\pm\Delta\sqrt{1-\alpha\,\sin^2(\phi/2)}$, corresponding to the interface bound states.[16] As stated in previous works, these bound states carry all the Josephson current in the limit of a short constriction.[10,17] Therefore, their contribution to the zero-voltage current fluctuations can be expected to be crucial, as is certainly found.

Once the single particle Green functions are known, the spectrum $S(\omega)$ can be calculated using Eq. (5). The typical form of this spectrum is illustrated in Fig. 1, where $S(\omega)$ is plotted for fixed temperature and three different contact transmissions. Notice that for $\omega<2\Delta$ the spectrum is formed by two resonant peaks at $\omega=0$ and $\omega=2\omega_S$, arising from the existence of the bound states at ω_S. Qualitatively, the peak at zero frequency increases with increasing transmission, while the one at $2\omega_S$ is negligible for both nearly perfect and very small transmissions, adopting its maximum value around $\alpha\sim2/3$. For $\omega>\Delta+|\omega_S|$ contributions from the continuous part of the single particle spectrum become important.

In the limit of a very weakly damped contact, i.e., $\eta\ll\alpha\Delta$, it is possible to evaluate $S(\omega)$ at $\omega=0$ and $\omega=2\omega_S$ analytically. We find

$$S(0)=\frac{2e^2}{h}\frac{\pi}{\eta}\frac{\Delta^4\alpha^2\sin^2(\phi)}{\omega_S^2}f(\omega_S)[1-f(\omega_S)] \tag{10}$$

and

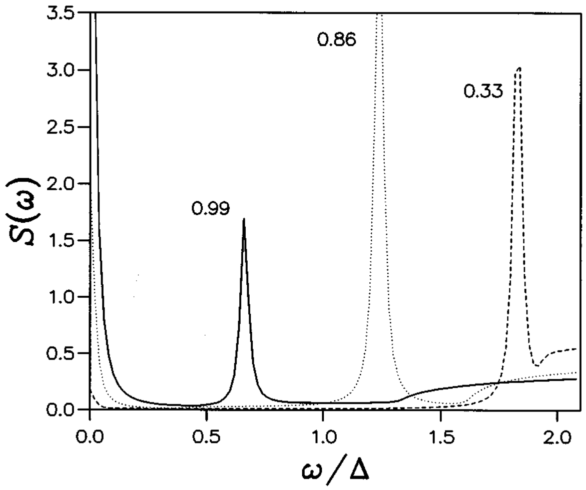

FIG. 1. Current fluctuation spectrum of a SQPC in the dc regime for three different values of the transmission. The superconducting phase difference corresponds in each case to the maximum supercurrent and the temperature is $k_B T = 0.2\Delta$.

$$S(2\omega_S) = \frac{2e^2}{h}\frac{\pi}{\eta}\frac{\Delta^4\alpha^2(1-\alpha)\sin^4(\phi/2)}{\omega_S^2} \times [f(\omega_S)^2 + f(-\omega_S)^2]. \quad (11)$$

These expressions clearly display the important role played by the interface bound states in fixing the magnitude of the current fluctuations for subgap frequencies. It should be stressed that, although the absolute size of the current fluctuations depend on the estimated value of parameter η, its precise variation with the superconducting phase difference and temperature is controlled only by the contact transmission α.

Our analytical results are strictly valid in the limit $\eta \ll \alpha\Delta$ and differ strongly from the equilibrium fluctuations obtained using standard tunnel theory,[9] which yields $S(0) \sim \alpha[1+\cos(\phi)]\ln\Delta/\eta$. This last expression becomes accurate just in the opposite limit, $\eta \gg \alpha\Delta$ which holds in the tunnel regime, i.e. $\alpha \ll 1$. In Ref. 3 we have explicitly shown that the limits $\eta \to 0$ and $\alpha \to 0$ do not commute. This behavior can be understood in the following way: when $\eta \ll \alpha\Delta$, multiple Andreev scattering processes give the dominant contribution to any dynamical quantity and should be included up to infinite order. On the other hand, when α is small enough (in such a way that $\alpha\Delta \ll \eta$) these high-order scattering events become heavily damped and the lowest term of the perturbative expansion in t gives the correct result. For a realistic SQPC in which, as commented above, η can be estimated to be a small fraction of Δ, the situation would always correspond to the weakly damped regime, except for extremely small values of α and therefore Eqs. (10) and (11) will accurately describe the low-frequency noise.

The analysis of Eq. (10) reveals some remarkable physical consequences. To begin with, and in contrast to the normal case where a reduction of noise is found (Ref. 18), $S(0)$ experiences a dramatic increase when approaching the ballistic regime. More precisely, there is a value of α, given roughly by the condition $k_B T \sim \Delta\sqrt{1-\alpha}$, above which there is an exponential increase of the thermal noise. On the other hand, in this last situation, there appears a very strong asymmetry on the phase dependence of $S(0)$, with its maximum value progressively moving from $\phi = \pi/2$ to $\phi = \pi$. This remarkable behavior should certainly have implications in the actual observability of the supercurrent-phase relation in a SQPC.

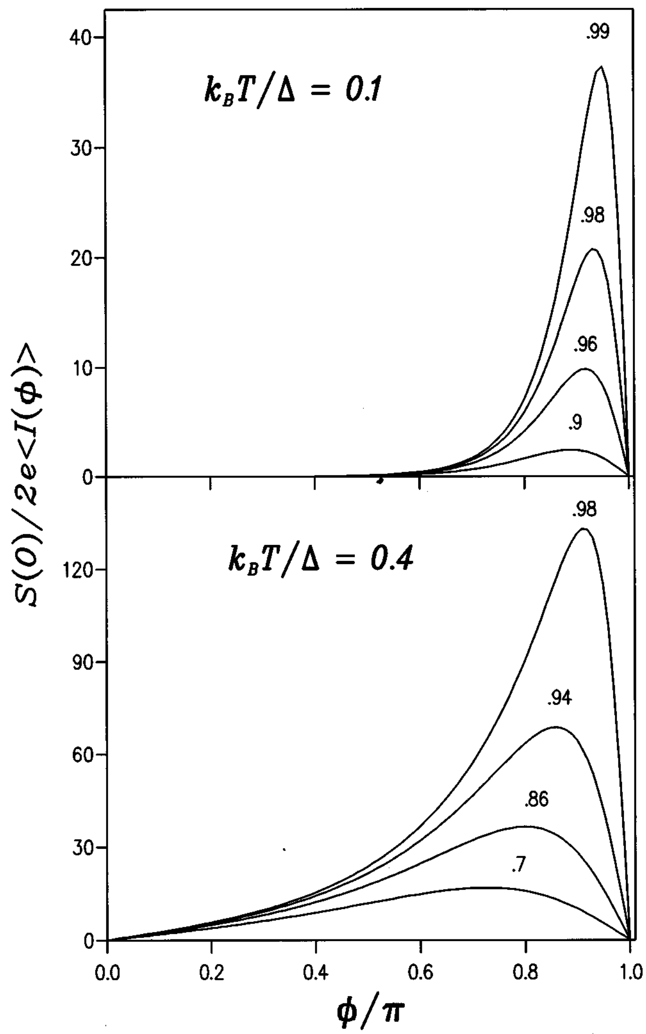

FIG. 2. The ratio between the zero frequency noise and the average supercurrent as a function of the superconducting phase difference for increasing values of the transmission coefficient.

In order to analyze the importance of these thermal fluctuations it is convenient to study the ratio $S(0)/2e\langle I(\phi)\rangle$, where $\langle I(\phi)\rangle$ is the phase-dependent average supercurrent [let us recall that for a normal contact the classical shot noise limit corresponds to $S(0) = 2e\langle I\rangle$], given by[10]

$$\langle \hat{I}(\phi)\rangle = \frac{e\pi}{h}\frac{\Delta^2\alpha\,\sin(\phi)}{|\omega_S|}\tanh\left[\frac{|\omega_S|}{2k_BT}\right]. \quad (12)$$

In Fig. 2 we plot $S(0)/2e\langle I(\phi)\rangle$ as a function of the superconducting phase difference for increasing values of the transmission α and two different temperatures. This figure illustrates the huge increase of thermal noise when the transmission becomes sufficiently large. As can be observed, for a reasonable choice of parameter η and depending on the temperature, the level of noise can reach values several orders of magnitude larger than $2e\langle I(\phi)\rangle$. When lowering the temperature this level of noise is reduced, but it will always be significant close to the ballistic case in a phase interval around $\phi=\pi$, just in the zone where the average current has its maximum at low temperatures. In fact, taking the limit $\alpha\rightarrow 1$, the ratio $S(0)/2e\langle I(\phi)\rangle$ has the form

$$\frac{S(0)}{2e\langle \hat{I}(\phi)\rangle}(\alpha\rightarrow 1)=\frac{2\Delta}{\eta}\frac{\sin[\phi/2]}{\sinh[\Delta\cos(\phi/2)/k_BT]}, \quad (13)$$

which clearly diverges when ϕ approaches π. The fact that the zero frequency noise has large values in the zone where the maximum of the average current occurs can explain the experimental difficulties found to observe the predicted $\sim\sin\phi/2$ form of the current-phase relation for junctions with direct conductivity, as reported in Ref. 6.

Finally, it is worth discussing our result for the zero frequency noise of a weakly damped contact in the light of the Callen-Welton fluctuation-dissipation theorem. In general, this theorem relates the equilibrium current fluctuations with the linear conductance G by $S(0)=4k_BTG$. This relation allows us to calculate in a straightforward way the phase-dependent linear conductance of a SQPC from Eq. (10)

$$G(\phi)=\frac{2e^2}{h}\frac{\pi}{k_BT\eta}\left[\frac{\Delta^2\alpha\sin(\phi)}{4\omega_S}\operatorname{sech}\left(\frac{\omega_S}{2k_BT}\right)\right]^2. \quad (14)$$

Equation (14) coincides exactly with the result of Ref. 3 in which a direct calculation of the linear conductance of a SQPC was performed [this expression for $G(\phi)$ has been recently rederived in Ref. 19 for the particular case of a ballistic contact].

The above relation between zero-frequency noise and linear conductance can provide a convenient way for testing the predicted unusual phase dependence of the noise by a direct measurement of $G(\phi)$. In this respect, we should mention early experiments of this kind made during the seventies for nonmesoscopic weak links (see discussion given in Ref. 3). In our opinion it should be desirable to attempt similar measurements using atomic size break junctions as the one described in Refs. 1 and 6 which are closer to the theoretical situation discussed in this work due to their reduced number of conducting channels.

In conclusion, we have developed a theory of the thermal fluctuations for a SQPC in the dc regime. The noise spectrum exhibits resonant peaks at subgap frequencies associated with the existence of bound states in the constriction region. For the case of a weakly damped contact ($\eta\ll\alpha\Delta$), we have obtained a closed analytical expression for the weight of these resonant peaks. We have shown that a striking consequence of the presence of these bound states is a huge increase of the low-frequency noise level when approaching the ballistic limit, this high level of noise being particularly important when the average supercurrent is close to its maximum value. We claim that these results may explain the reported difficulties in measuring the predicted $\sin\phi/2$ behavior for $\langle I(\phi)\rangle$ in the case of highly transmissive contacts. Finally, we have discussed the connection between the present theory and the phase-dependent conductance of a SQPC by means of the fluctuation-dissipation theorem.

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